

CMS

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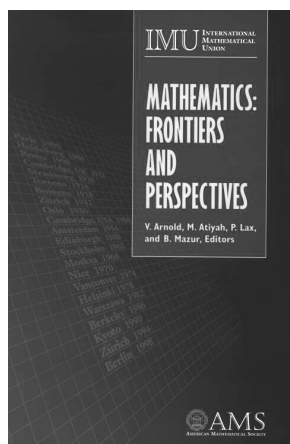
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A Smorgasbord of Delicacies

Book Review by A. J. Coleman,
Queen's University

Mathematics: Frontiers and Perspectives

V. Arnold, M. Atiyah, P. Lax and
B. Mazur, Editors
American Mathematical Society,
Providence, 1999
xi + 459 pages.



This book, commissioned by the International Mathematical Union, was published by the AMS as a contribution to the celebration of 2000 as the Year of Mathematics. The mere names of the editors ensures that it is fascinating and exciting.

“The 20th century has transformed mathematics from a cottage industry run by a few semi-amateurs into a world-wide industry run by an army

of professionals” Having lived through 82% of the century, I would qualify this observation of Atiyah in the Preface only by suggesting that the transformation took place since 1945. I base this opinion chiefly on the changes in the quantity and quality of book exhibitions at the math conferences I attended since 1940. These attest not only to the burgeoning magnitude of the mathematical enterprise but also to the not totally healthy effect of the almighty dollar about which, unfortunately little is said in this book.

Arnold does refer to the social context of mathematics. My philosophical mentor, Alfred North Whitehead (young colleagues often chide me by noting that everything I write contains at least one allusion to Whitehead!), stated, “It is more important that a proposition be interesting than that it be true.” Here is Arnold’s opening paragraph:

“All mathematics is divided into three parts: *cryptography* (paid for by CIA, KGB and the like), *hydrodynamics* (supported by manufacturers of atomic submarines) and *celestial mechanics* (financed by military and other institutions dealing with missiles, such as NASA).”

In a veritable *tour de force*, Arnold shows that this proposition is at least 2/3 true and then proceeds to demonstrate that nearly all interesting mathematics may be classified by Coxeter-Dynkin graphs! Do read this, pp.403 - 416.

(see REVIEW—page 15)

CMS NOTES
NOTES DE LA SMC

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EDITORIAL



Peter Fillmore

The record of accomplishment of the CMS, relative to its size, is something that Canadian mathematicians can justly be proud of. There is however an area which needs more attention, namely what might be called "the profession". As the leading national organization, the Society has the obligation, in matters affecting mathematics and mathematicians, to collect data, monitor developing situations and, when appropriate, formulate and publicize positions.

A few years ago we looked at the Australian scheme of accreditation for professional mathematicians, but no action was taken. At the recent meeting in Vancouver, the question of whether the CMS ought to take a public position on accreditation standards for school mathematics teachers, raised by one of our task forces, was briefly discussed—and we hope for more discussion of this in our pages in the future. Also in Vancouver it was decided to formally abandon the CMS Annual Survey (last completed, it appears, for the year 1996), with the hope that the AMS might take it over. This may be seen as an instance of the understandable but sometimes regrettable inclination to leave things to Uncle Sam, as indeed might the fact that the only national organization representing Canadian school mathemat-

ics teachers is the US National Council of Teachers of Mathematics.

Clearly the situation is far from satisfactory.

Pour sa taille, la SMC a une feuille de route dont les mathématiciens canadiens peuvent être fiers, et avec raison. Un aspect du travail de notre association, par contre, mériterait qu'on s'y attarde d'avantage : le côté «professionnel» proprement dit. À titre d'association nationale, la Société a l'obligation de recueillir des données, de surveiller les tendances du domaine et, lorsque le besoin s'en fait sentir, de formuler et de faire connaître sa position sur les questions qui touchent les mathématiques et les mathématiciens.

Nous avons étudié, il y quelques années, le processus d'agrément des mathématiciens professionnels en Australie, mais le dossier n'a pas progressé davantage. Lors de notre dernière Réunion, tenue à Vancouver, nous avons brièvement abordé la question de savoir si la SMC devait prendre position sur les normes d'agrément des enseignants de mathématiques, soulevée par l'un de nos groupes de travail (nous espérons d'ailleurs aborder le sujet plus souvent dans les numéros à venir des Notes). Toujours à Vancouver, nous avons officiellement décidé d'abandonner le sondage annuel de la SMC (dont le dernier remonterait à 1996) en espérant que l'AMS le reprenne. Certains y verront sans doute une illustration de notre tendance, compréhensible, mais parfois regrettable, à laisser l'initiative à notre voisin du Sud, tendance qui pourrait expliquer aussi pourquoi le US National Council of Teachers of Mathematics est la seule association nationale qui représente les enseignants de mathématiques du Canada.

De toute évidence, la situation est loin d'être idéale.

Some Trends in Modern Mathematics and the Fields Medal

by Michael Monastyrsky

This talk was presented at the symposium "The Legacy of John Charles Fields," which was held in Toronto, June 7–9, 2000.

Introduction

The Fields Medal is now indisputably the best known and most influential award in mathematics. Sometimes it is compared with the Nobel prize, since there is no Nobel prize for mathematics. Publishers and journalists especially like this comparison. It seems to me that such a comparison is not adequate. The Fields medal was established on different principles. Unlike the Nobel prize, which is mostly awarded to mature scientists to crown their careers, the Fields medal is awarded to young scientists, less than 40 years old. The prize is intended not only to recognize results already obtained, but also to stimulate further research. Besides this it is awarded only every four years, at the International Mathematical Congress.

The first Fields Medal was awarded in 1936 in Oslo and the second one 14 years later, in 1950, in Cambridge, Massachusetts. So mathematicians born during 1900–1910 were automatically excluded from the list of candidates, for example brilliant mathematicians like A. Kolmogorov, H. Cartan, A. Weil, J. Leray, L. Pontryagin, S. S. Chern, and H. Whitney. Nevertheless, if we look at the achievements of Fields laureates from the point of view of the development of mathematics in the 20th century, we see an impressive picture.

The founder of the prize, John Charles Fields, considered two fundamental principles for the award: (a) the solution of a difficult problem and (b) the creation of a new theory enlarging the fields of applications of mathematics. Both these principles are important for the development of mathematics. It is quite clear that they are not independent. Very often the solution of a concrete difficult problem is based on the creation of a new mathematical theory and, conversely, the creation of a new theory may lead to the solution of an old classical problem.

It is absolutely impossible to cover in a one-hour talk the results of Fields laureates even in a condensed form. In this talk I shall take a stroll through modern mathematics, giving a kaleidoscopic view of some exciting pictures. I shall try to explain the characteristic features of the mathematics of the 20th century, what kind of mathematics is considered important in this or that period, and how the results of the Fields medallists look from this point of view.

The role of prizes, like the role of international recognition in general, is important for individual scholars. Despite Franz Neumann's beautiful quote, "The discovery of new truth is the greatest joy; recognition can add almost nothing to it," this wise idea is only partially true. According to

Niels Bohr, the opposite conclusion is also valid. Recognition is especially important to young researchers. Selecting young mathematicians supports the continuing development of mathematics. The Fields Committees consist of outstanding mathematicians of the older generation, which makes their assessment of the creativity of the young all the more interesting.

As I already mentioned, the first Fields Medal was awarded in 1936, and the next one in 1950, so with one exception the medals are connected with the second half of the 20th century. The second world war greatly affected the development of society and science in general, mathematics especially. The development of mathematics is a good illustration of the more general thesis about the continuous but "non-differentiable" nature of the development of science. If we consider the graph of the development of mathematics, we evidently see the changes of interest in the periods of the world wars. It is natural for science to develop continuously, a fact based both on internal factors and the succession of generations. Also, science is characterized by some conservatism, which I consider in general as a robust phenomenon. Great ideas appear in the world by noiseless steps, as Nietzsche said. The acceptance of new ideas proceeds against great obstacles and requires long testing. As Max Planck joked, "a new scientific truth does not triumph by convincing its opponents and making them see the light, but rather because its opponents eventually die, and a new generation grows up with it." That each tragic world war destroyed a whole generation of scientists accelerated in addition an apparently objective process to accept new points of view in mathematics.

If we look at the prizes of 1936 and 1950 from this point of view we can see that new waves such as the explosion of interest in algebraic topology and geometry in the first years after the Second World War are not yet reflected in the first postwar award. The 1950 prize was awarded to L. Schwartz (for the theory of distributions) and to A. Selberg for his remarkable achievements in number theory, namely, the distribution of zeros of Riemann ζ -function and an "elementary" proof of the asymptotic distribution of primes. But in 1954 the prize was awarded to K. Kodaira and J. P. Serre for postwar achievements. Hermann Weyl, who chaired the Fields committee in 1954, delivered a speech on the papers of Kodaira and Serre. Curiously, Weyl had difficulty distinguishing the areas of research of the two mathematicians. He said, "The uninitiated may get the impression that our committee erred in awarding the Fields Medals to two men whose research runs on such closely neighboring lines. It is the task of the Committee to show that, despite some overlap in methods, they give the solutions of completely different, extremely difficult problems."

In the subsequent awards, we see a definite balance between the two leading principles established by the founder of the prize. For example, in 1958 Klaus Roth was honoured for the proof of a delicate estimate that refines the Thue-Siegel theorem on the approximation of algebraic numbers by rational numbers. **Roth's theorem:** If α is any algebraic number, not itself rational, then for any $\nu > 2$ the inequality

$$\left| \frac{p}{q} - \alpha \right| < \frac{1}{q^\nu}$$

has only a finite number of solutions in rational p/q .

The second medalist was René Thom, who constructed a powerful method in topology known as the cobordism theory.

In 1962 the prize winners were Lars Hörmander and John Milnor. Hörmander developed the general theory of linear partial differential equations, including hypoelliptic operators. The work of the other laureate was absolutely astonishing and has had great influence on the future development of topology. It is very difficult to find an analogous invention in the past to his beautiful construction of the different differential structures on the seven-dimensional sphere. Later, the result became the cornerstone of a new branch of topology — differential topology. The original proof of Milnor was not very constructive but later E. Briscorn showed that these differential structures can be described in an extremely explicit and beautiful form.

Four medals were awarded in 1966. Among those honoured was Paul Cohen, who showed that if the Zermelo-Fraenkel axioms are consistent, then the negation of the axiom of choice or even the negation of the continuum hypothesis can be adjoined and the theory will remain consistent. It was the first and the last time that the award was given to a specialist in mathematical logic. Alexander Grothendieck, one of the most original and puzzling mathematicians of our time, revolutionized algebraic geometry. The concept of schemes that he introduced raised algebraic geometry to a new level of abstraction, beyond the reach of mathematicians with a traditional education. The theory of sheaves, spectral sequences, and other innovations in the late 1940's and earlier 1950's are subsumed by this complicated technique. But if certain mathematicians could console themselves for a time with the hope that all these complicated structures were “abstract nonsense” (in algebra, the term “abstract nonsense” has a definite meaning without any pejorative connotation), the later papers of Grothendieck and others showed that classical problems of algebraic geometry and the theory of numbers, the solutions of which had resisted efforts of several generations of talented mathematicians, could be solved in terms of the Grothendieck K -functor, motives, l -adic cohomology, and other equally complicated concepts.

Two remarkable mathematicians are present at this conference. The traditions of a scientific community are rather different from those of writers, movie stars, and fashion models.

It is not an accepted practice to compliment a renowned scientist in his presence. So I really will not touch on the results of the mathematicians present here, but make some exception and say some words about the results of Steven Smale and Michael Atiyah, because they beautifully characterised the level of the prize and the realisation of its principles.

The results of Smale are especially near to me, since I started my own career in mathematics as a student of the well-known Russian mathematician Dmitry Anosov, and his first advice was to study the papers of Smale about dynamical systems.

S. Smale was honoured mostly for two of his achievements. The first one is the solution of the Poincaré conjecture in higher dimensions. The Poincaré conjecture is among the most difficult problems in topology. It can be stated as follows in modern terms:

Poincaré conjecture closed smooth simply connected manifold M^n with the homology groups of the sphere S^n is homeomorphic to S^n .

Poincaré stated his conjecture in three dimensions. He believed that a stronger assertion was true, namely that M^n is diffeomorphic to S^n . But as follows from the existence of Milnor's exotic spheres, the conjecture is not true in this form. Smale proved a more general theorem on h -cobordism, from which it follows that Poincaré conjecture holds for dimensions $n \geq 5$. In dimensions 5 and 6, a stronger conjecture is true: M^n is diffeomorphic to S^n .

At first sight it seems paradoxical that the proof of the Poincaré conjecture for higher-dimensional spaces is more accessible than for three- and four-dimensional manifolds. The reason is that a map of a surface into a manifold of fewer than five dimensions cannot be approximated by an embedding. The situation is similar to the classification of manifolds. This indisputably classical result corresponds to the first principle of the Fields award.

The second achievement of Smale is connected with the theory of dynamical systems. This field has its origin in classical mechanics and the theory of ordinary differential equations. It was developed at the beginning of the twentieth century by H. Poincaré, G. D. Birkhoff, J. Hadamard, and I. Bendixson. In the middle 30's, remarkable results were obtained by E. Hopf, G. Hedlund, M. Morse, A. Andronov, L. Pontryagin, and some others. But almost all of them were of a two-dimensional nature. Smale substantially developed a multidimensional case. He showed that so-called structurally stable dynamical systems in higher dimensions have radically different properties. Unlike two-dimensional systems, studied by Andronov and Pontryagin, in a multidimensional situation structurally stable systems may have infinite number of singular points, limit cycles, etc. His first construction was the famous horseshoe, generated by discrete automorphisms of the torus. He proposed a very interesting hypothesis about the structural stability of geodesic flows on compact mani-

folds of negative curvature, later proved by Anosov. These results led to the creation of the theory of multidimensional dynamical systems, a new field of mathematics still actively being developed. These results of Smale are an excellent illustration of the second Fields principle.

The other laureate of this year, M. Atiyah, was recognised for his work in algebraic topology, especially for the proof of the index theorem which is known as the Atiyah-Singer Theorem. This theorem is remarkable from several points of view. Firstly, it generalized the long sequence of famous theorems beginning with the Euler theorem on polyhedra and including the Riemann-Roch Theorem and the Poincaré-Hopf Theorem about the singularities of vector fields.

The original proof of Atiyah and I. M. Singer was extremely complicated and used a wide spectrum of mathematical concepts developed in algebraic topology, geometry, and partial differential equations in previous years. Later, essential simplifications were obtained and, especially remarkable, in recent years the relation between this theory and important problems in quantum field theory, for example the problem of quantum anomalies, became clear.

The work of Atiyah and Singer, Grothendieck, F. Hirzebruch, and many other mathematicians established a new field of mathematics, where the ideas of algebraic topology and geometry and complex analysis are so intertwined that traditional division is absolutely impossible now. Using a nice phrase Atiyah said, "topologists used to study simple operators on complicated manifolds while analysts studied complicated operators on simple spaces." The time has arrived to study complicated operators on complicated spaces.

These results not only raised mathematics to a very high level of abstraction, but proved the fruitfulness of these methods in the solution of long standing unsolved classical problems. One of the best examples is the solution by J. Adams of the famous problem of the existence of division algebras. From the time of Cayley, the following division algebras were known: real numbers, complex numbers, quaternions, and Cayley numbers. As the dimension grows we lose some properties, e.g. quaternions are non-commutative. A natural question is: Are there other division algebras? The negative answer was obtained only in the 1960s and proved to be closely related to the following topological problem: find all spheres on which the number of independent, continuous vector fields is equal to its dimension. There are only three such: S^1 , S^3 , S^7 .

I hope that this gives at least a hint of how the two principles of Fields are linked in the work of M. Atiyah. Mathematics is a single subject, a fact that is not always obvious when you study the daily reality of research. It becomes clear, however, when you become acquainted with results of great mathematicians. This realization is one by-product resulting from an analysis of the works of the Fields medallists. Although honours went to authors of the greatest achievements obtained in the years immediately preceding each congress and sometimes in areas of mathematics widely separated from one another, truly wonderful connections between them were discovered with the passage of time. For that reason an ϵ -grid over the works of the Field medalists covers a significant portion of the achievements of modern mathematics.

Editors' note: This article will be continued in the next issue.

Endowment Grants Committee

Reports on Projects supported in 1999
Chair, James Timourian, University of Alberta

In its first competition, the Endowment Grants Committee supported five projects. Here are reports on four of them.

Mathématiques An 2000

Le document Mathématiques An 2000 a été produit par l'Institut des sciences mathématiques en collaboration avec l'Association mathématique du Québec à l'occasion de l'Année mathématique mondiale. Il est composé de six articles rédigés par huit mathématiciens québécois qui présentent un modeste survol de quelques-uns des principaux domaines des mathématiques à l'intention du grand public. Nous avons choisi de restreindre le nombre de sujets présentés en tentant dans chaque cas d'approfondir les questions abordées. Par ailleurs, chaque article est accompagné d'une bibliographie pour ceux ou celles qui désirent approfondir davantage le sujet.

Le document a été distribué dans l'envoi d'avril de la revue Interface (ACFAS - Association Canadienne-Française pour l'Avancement des Sciences), dans toutes les universités membres de l'ISM (Concordia, Laval, McGill, Université de Montréal, Université de Sherbrooke et UQAM), au Méga-congrès tenu à l'Université Laval au mois de mai, congrès réunissant toutes les associations mathématiques du Québec et dans tous les cégeps. Selon la demande, il est bien possible qu'on procède à une réimpression l'automne prochain de quelques milliers d'exemplaires.

Enrichment in Number Theory and Encryption

by Keith F. Taylor

On January 21, 2000, I was informed, by the Chair of the CMS EGC, that a grant of \$5000 was made for my proposal to

develop a web based course entitled Enrichment in Number Theory and Encryption. The proposal was for a three year development period and the project began on May 1, 2000. So this report comes before one quarter of the time period has passed.

I should note right away that I have applied to the EGC in this fall's competition for the second and third years of funding and I have submitted related applications to other organizations for major funding to support the development of a much larger enrichment curriculum using the EGC grant as a seed. To this point in time, about \$4500 from the Endowment Fund grant and about \$2000 from my Discretionary account have been spent to employ students to set up an architecture of web pages, enter content and write appropriate Java Applets. We have completed Lesson 1 (Divisibility and the Greatest Common Divisor) and have written a collection of applets that will be used in later lessons. The structure of the later lessons has been established (web pages and content outline) and the Christmas break will find me writing several additional lessons.

Lesson 1 has been tested with several young people from the target group and has received a generally positive response. However, they found the writing too formal and "stuff". I will be modifying the language to make it more appropriate for the 13-16 age range.

To view Lesson 1, go to <http://math.usask.ca/encryption> and to play with preliminary versions of some of the applets, go to <http://math.usask.ca/encryption/applet.html>

If I receive funding for the next two years of this project, I feel that a useful enrichment resource will be completed and it will serve as a sample of what could be accomplished on a much broader scale.

(Added to this report: The Endowment Grants Committee has awarded this project a further \$5,000 of support in each of the next two years.)

Congrès mathématique de l'an 2000 Université Laval – 5-7 mai 2000

Le Congrès mathématique de l'an 2000, tenu les 5, 6 et 7 mai derniers à l'Université Laval sous la présidence d'honneur du Ministre de l'Éducation du Québec a été une réussite. En effet, huit cents congressistes ont pris part au congrès alors que plus de deux cents ateliers et conférences étaient au programme. Déjà, on étudie la possibilité que les associations organisent à nouveau un événement conjoint dans quelques années.

Rappelons que ce congrès était une première. En effet, dans le cadre de l'Année mondiale des mathématiques, toutes les associations québécoises qui en mathématiques et en enseignement des mathématiques avaient convenu de la tenue exceptionnelle d'un congrès conjoint à l'Université Laval, les 5, 6 et 7 mai 2000. Il aura fallu harmoniser des manières de

procéder différentes pour arriver à organiser cette rencontre importante. Ces associations sont, par ordre alphabétique :

- Association mathématique du Québec (AMQ)
- Association des promoteurs de l'avancement de la mathématique à l'élémentaire (APAME)
- Groupe des chercheurs en sciences mathématiques (GCSM)
- Groupe des didacticiens de la mathématique (GDM)
- Groupe des responsables de la mathématique au secondaire (GRMS)
- Mouvement international pour les femmes et l'enseignement des mathématiques (MOIFEM)
- Quebec Association of Mathematics Teachers (QAMT)

La conférence d'ouverture du Congrès a été prononcée par Bernard Hodgson alors que Denis Guedj donnait la conférence grand public à la salle Albert-Rousseau. Plus de cent soixante-cinq personnes ont donné un ou des ateliers ou conférences, notamment des chercheurs tels Jean-Marie De Koninck, François Lalonde, Gilbert Labelle, Jacques Labelle, Christiane Rousseau et Yvan Saint-Aubin.

L'appui de la SMC nous aura permis d'offrir des frais de participation réduits aux étudiants et ce alors que certains des autres commanditaires n'avaient pas encore confirmé leur appui. Les autres commanditaires du congrès ont été le Ministère de l'Éducation du Québec, le programme national de conférences des instituts de recherche mathématique (CRM, Fields et PIMMS), Hydro-Québec, Waterloo-Maple et 3-Soft, et Hydro-Québec. Le congrès a été une réussite sur le plan financier et nous aura permis de contribuer (plus que prévu) à la bonne santé financière des trois grandes associations du Québec, soit l'AMQ, l'APAME et le GRMS.

Le congrès aura fortement contribué à un rapprochement entre les associations du Québec, et entre les enseignants et les chercheurs.

Website for the Newfoundland and Labrador Senior Mathematics League by Bruce Shawyer

We requested an Endowment Fund Grant for the development of a WEB site for the Newfoundland and Labrador Senior Mathematics League. We did not spend the entire amount of the grant in the first year, because of other funding we were able to obtain, using the leverage provided by the CMS. We will be spending the money in 2000 and maybe in 2001 to make further progress on the site, so this report should be viewed as an interim one. We employed a Memorial University student, Alasdair Graham, for summer 2000 to do the necessary work. He surpassed his own expectations, and with some assistance from our department's Systems Manager, has produced a preliminary version of the site, that makes the "games" available to any school in Newfoundland and Labrador that registers. The address

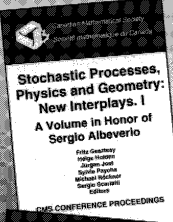
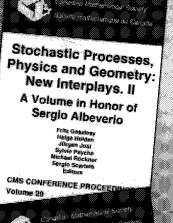
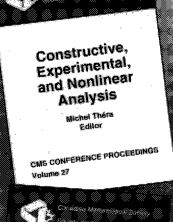
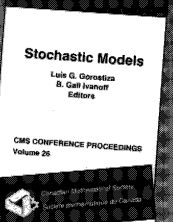
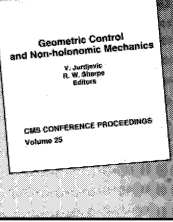
is <http://www.math.mun.ca/~mleague/game1.html> We invite members of the CMS to try this out (we welcome comments and criticism). In order to do so, they should e-mail me at bshawyer@math.mun.ca and I will send them a username and password for temporary access.

Should this method of making a cooperative mathematics competition available to more schools prove successful, and should the computer facilities at Memorial University prove to be sufficient, then we will be able to make this available across Canada (and even beyond).

AMERICAN MATHEMATICAL SOCIETY

Conference Proceedings, Canadian Mathematical Society

This series is published for the Canadian Mathematical Society by the AMS. It consists of the proceedings of internationally attended conferences on pure and applied mathematics sponsored by the CMS. CMS members may order at the AMS member prices. (ISSN 0731-1036) Softcover.

NEW! Volume 29
Stochastic Processes, Physics and Geometry: New Interplays. II
 A Volume in Honor of Sergio Albeverio

NEW! Volume 28
Stochastic Processes, Physics and Geometry: New Interplays. I
 A Volume in Honor of Sergio Albeverio

Fritz Gesztesy, *University of Missouri, Columbia*,
Helge Holden, *Norwegian University of Science and Technology, Trondheim*, **Jürgen Jost**, *Max Planck Institut für Mathematik, Leipzig, Germany*, **Sylvie Paycha**, *Université Blaise Pascal, Aubiere, France*, **Michael Röckner**, *Universität Bielefeld, Germany*, and **Sergio Scarlatti**, *Università G. D'Annunzio, Pescara, Italy*, Editors

These volumes present state-of-the-art research currently unfolding at the interface between mathematics and physics. Included are select articles from the international conference held in Leipzig (Germany) in honor of Sergio Albeverio's sixtieth birthday. The theme of the conference, "Infinite Dimensional (Stochastic) Analysis and Quantum Physics", was chosen to reflect Albeverio's wide-ranging scientific interests. The articles in these books reflect that broad range of interests and provide a detailed overview highlighting the deep interplay between stochastic processes, mathematical physics, and geometry.

The contributions are written by internationally recognized experts in the fields of stochastic analysis, linear and nonlinear (deterministic and stochastic) PDEs, infinite dimensional analysis, functional analysis, commutative and noncommutative probability theory, integrable systems, quantum and statistical mechanics, geometric quantization, and neural networks. Also included are applications in biology and other areas.

Most of the contributions are high-level research papers. However, there are also some overviews on topics of general interest. The articles selected for publication in these volumes were specifically chosen to introduce readers to advanced topics, to emphasize interdisciplinary connections, and to stress future research directions. Volume I contains contributions from invited speakers; Volume II contains additional contributed papers.

Volume 29; 2000; 647 pages; Softcover; ISBN 0-8218-1960-7; List \$125; Individual member \$75; Order code CMSAMS/29CMS01
 Volume 28; 2000; 333 pages; Softcover; ISBN 0-8218-1959-3; List \$75; Individual member \$45; Order code CMSAMS/28CMS01

Constructive, Experimental, and Nonlinear Analysis
Michel Théra, *University of Limoges, France*, Editor

This volume presents twenty original refereed papers on different aspects of modern analysis, including analytic and computational

number theory, symbolic and numerical computation, theoretical and computational optimization, and recent development in nonsmooth and functional analysis with applications to control theory. These papers originated largely from a conference held in conjunction with a 1999 Doctorate Honoris Causa awarded to Jonathan Borwein at Limoges. As such they reflect the areas in which Dr. Borwein has worked. In addition to providing a snapshot of research in the field of modern analysis, the papers suggest some of the directions this research is following at the beginning of the millennium.

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Leopold Vietoris – At 109 the oldest Austrian alive

by Gilbert Helmborg

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Leopold Vietoris was born on June 4, 1891 in Radkersburg, the son of a civil engineer. He began his studies as a student of civil engineering, but from his second semester on he was captivated by projective geometry at the Vienna University of Technology and by mathematics at Vienna University. Before completing his studies, he was conscripted and wounded in the East. In 1916, he was sent to South Tyrol and trained to be an army mountain guide. He earned his doctorate in 1919 at Vienna University, supervised by Escherich and Wirtinger with a dissertation that he completed while a prisoner of war in Italy.

In his thesis, he introduced the concepts of directed set and filter basis (with different names). These concepts later became well-known through publications of E. H. Moore, H. L. Smith and Garrett Birkhoff. Also, the characterization of compactness of a topological space through filter bases appeared later in French publications. But the topological separation axiom introduced by Vietoris was – at least abroad – known as the Vietoris Axiom.

After an interlude as a grammar school teacher, he started in 1922 to work as an assistant at Graz Technical University and later at Vienna University, where he habilitated in 1923 with a continuation of his work in set theoretical topology.

In a perhaps somewhat daring social scientific analogy, Vietoris pointed out how one can gain understanding of relationships between social groups in a state by starting from the relationships amongst individuals. A Rockefeller stipend allowed him in 1925 to take advantage of three research semesters at the University of Amsterdam and to develop his groundbreaking contribution to algebraic topology, the theory of homology groups. Today, this is firmly connected to the name Vietoris. But even in 1986, the well known American mathematician Saunders MacLane had to point out, in an article in the *Journal of Pure and Applied Algebra*, that Leopold Vietoris gave at least as important an impulse to the development of this theory as did Emmy Noether and other contemporary mathematicians.

In 1928 Vietoris accepted a chair at the Vienna University of Technology, and, in 1930 a chair at Innsbruck University, which (or was it the mountains?) never let him go. He and his family were hit hard by fate in 1935 when his wife Klara died in childbirth, their sixth daughter. In Klara's sister Maria, he found his second wife and a caring mother of his children. In an article in the *Tiroler Tageszeitung*, he is quoted as follows: "One must be lucky to stay healthy – and must be lucky to get

the right woman – I had this luck twice."

Besides teaching and research, Vietoris devoted himself to the natural scientific foundations of mountain sports, in particular orienteering in the mountains and the surveying of glaciers. In the second world war, he was wounded again, but he assumed the office of a dean of natural science for the second time in the reconstruction years 1945/46. In 1935, Vietoris was elected a corresponding member of the Austrian academy of science. In 1960, he became a regular member. In 1970, he celebrated his golden doctoral jubilee at Vienna University. The Austrian emblem of honour for science and arts – the highest in Austria – was awarded to him in 1973. Later he was elected an honorary member of the Austrian and German mathematical societies, and received honorary degrees from the Vienna Institute of Technology and the University of Innsbruck. His last – for the time being – paper was published in 1990 in the notices of the Austrian Alpenverein under the title "Die Hangstellung als Orientierungsmittel". It sets forth why a mountain hiker who's equipped with an altimeter, compass and a map doesn't have to become desperate even in fog.

During his life, Professor Vietoris had to cope with situations many of us have never been in, situations that we would possibly face helplessly. In times of automation of leisure and professional activities, of canalization of thoughts and emotional expressions, who else but he could give us answers to questions like the following:

- How does one write a dissertation at the front and as a prisoner of war?
- How does one ski with only one pole?
- How does one find the inspiration for a new mathematical theory?
- What emotions does one have when one's own scientific insights are becoming more and more important, whereas the priority of discoveries and the creation of terms is disregarded?
- How does one raise six daughters while being a teacher, researcher and "Glacier menial" (to use an expression coined by Professor Vietoris)?
- How does one avoid adverse health effects when camping at over 80?
- How does one remember birthdays for a family of more than 50 descendants, an average of one per week?

2000 COXETER-JAMES PRIZE LECTURE

Results and methods of transcendental number theory

Damien Roy, Université d'Ottawa

1. Liouville inequality.

A basic problem in Diophantine approximation and transcendental number theory is, given a class of numbers $\mathcal{C} \subset \mathbf{C}$ and a complex number $\theta \in \mathbf{C}$, to decide whether or not θ belongs to \mathcal{C} . If $\theta \notin \mathcal{C}$, we also want to know how far is θ from elements of \mathcal{C} , and how closely we can approximate it by elements of \mathcal{C} .

For example, when $\mathcal{C} = \mathbf{Q}$, we ask if θ is a rational number, and if not, we want to estimate the distance $|\theta - p/q|$. When $\mathcal{C} = \overline{\mathbf{Q}}$ is the algebraic closure of \mathbf{Q} in \mathbf{C} , we ask if θ is algebraic, and if not, we want to estimate $|\theta - \alpha|$ for $\alpha \in \overline{\mathbf{Q}}$ or $|P(\theta)|$ for a non-zero polynomial $P \in \mathbf{Z}[X]$.

It is known since the Pythagorean school, that $\sqrt{2} \notin \mathbf{Q}$. This fact can be quantified in the following way:

Let p/q be a rational number with denominator $q \geq 1$. We have $p^2 \neq 2q^2$ since 2 divides the left hand side with an even exponent and the right hand side with an odd exponent. So, $p/q \neq \sqrt{2}$ and also

$$\left| \sqrt{2} - \frac{p}{q} \right| \left| \sqrt{2} + \frac{p}{q} \right| = \frac{1}{q^2} |p^2 - 2q^2| \geq \frac{1}{q^2},$$

since a non-zero integer has absolute value ≥ 1 . Assuming $|\sqrt{2} - p/q| \leq 1$, we find $|\sqrt{2} + p/q| \leq 4$ which, combined with the above, implies

$$\left| \sqrt{2} - \frac{p}{q} \right| \geq \frac{1}{4q^2}.$$

J. Liouville observed that similar considerations apply to any algebraic number:

Theorem (Liouville, 1844). If $\alpha \in \overline{\mathbf{Q}}$ is algebraic of degree $d \geq 2$ over \mathbf{Q} , then for any rational number p/q with $q \geq 1$, we have

$$\left| \alpha - \frac{p}{q} \right| \geq \frac{c(\alpha)}{q^d}$$

with a constant $c(\alpha) > 0$ depending only on α .

He inferred from that the existence of transcendental numbers. More precisely, a number like

$$\theta = \sum_{k=1}^{\infty} 10^{-k!}$$

cannot be algebraic because, contrary to all algebraic numbers, it admits rational approximations

$$\sum_{k=1}^n 10^{-k!} = \frac{p_n}{q_n} \quad \text{where} \quad q_n = 10^{n!},$$

that satisfy

$$\frac{\log \left| \theta - \frac{p_n}{q_n} \right|}{\log q_n} \leq \frac{\log (2q_n^{-(n+1)})}{\log q_n} \longrightarrow -\infty$$

as $n \rightarrow \infty$.

2. Height and size

If $P \in \mathbf{Z}[X]$ is a polynomial with integer coefficients, we define its *height*, denoted $H(P)$, as the maximum of the absolute values of its coefficients, and its *size* by

$$t(P) = \deg(P) + \log H(P).$$

For an algebraic number $\alpha \in \overline{\mathbf{Q}}$, we define the degree $\deg(\alpha)$, height $H(\alpha)$ and size $t(\alpha)$ of α as the degree, height and size of its irreducible polynomial over \mathbf{Z} , respectively.

For example, if α is a rational number p/q , with $\gcd(p, q) = 1$, its irreducible polynomial over \mathbf{Z} is $qX - p$ and so $H(\alpha) = \max\{|p|, |q|\}$.

These definitions are such that, for a given real number $T > 0$, there are only finitely many polynomials $P \in \mathbf{Z}[X]$ of size $\leq T$ and therefore only finitely many algebraic numbers of size $\leq T$.

3. Transcendence of e and π

The transcendental numbers constructed by Liouville were in a sense artificial but it was suspected that numbers like e and π were also transcendental. The transcendence of e was proven by Hermite in 1873 and that of π by Lindemann in 1882, by an adaptation of Hermite's method. Since $e^{2\pi i} = 1$, both results follow from the so-called

Hermite-Lindemann Theorem. If $\beta \in \overline{\mathbf{Q}} - \{0\}$, then $e^\beta \notin \overline{\mathbf{Q}}$.

Quantitatively, we have the following irrationality measures

$$\left| e - \frac{p}{q} \right| \gg \frac{1}{q^2 \log q} \quad \text{and} \quad \left| \pi - \frac{p}{q} \right| \gg q^{-8.017}.$$

The first is based on the knowledge of the continued fraction expansion $[2; 1, 2, 1, 1, 4, 1, \dots]$ of e and goes back to Euler, while the second due to Hata in 1993 is the sharpest actual measure for π . We also have transcendence measures

$$\log |e - \alpha| \gg -t(\alpha)^2 (\log t(\alpha))^3,$$

$$\log |\pi - \alpha| \gg -t(\alpha)^2 (\log t(\alpha))^2$$

due to G.V. Chudnovsky and N.I. Feld'man respectively.

4. Dirichlet box principle

The proof of the transcendence of e by Hermite used explicit formulas and this was an obstacle to further progress. The solution to avoid them came from the box principle of Dirichlet.

Among other applications, Dirichlet used his principle in 1842 to show the existence of good rational approximations to a given real number α . He proved that, for any integer $Q > 1$, there exist $p, q \in \mathbf{Z}$ with $1 \leq q \leq Q$ such that $|q\alpha - p| \leq 1/Q$. In particular, if $\alpha \notin \mathbf{Q}$, this provides infinitely many rational numbers p/q with

$$\left| \alpha - \frac{p}{q} \right| < \frac{1}{q^2}.$$

When α is algebraic of degree $d > 2$, this upper bound is much bigger than the lower bound $c(\alpha)q^{-d}$ given by Liouville inequality. This gap was successively reduced by the work of Thue, Siegel, Dyson and Gel'fond, until Roth showed in 1955 that, for such an algebraic number and for any given $\epsilon > 0$, the inequality

$$\left| \alpha - \frac{p}{q} \right| < \frac{1}{q^{2+\epsilon}} \quad (1)$$

has only finitely many solutions $\frac{p}{q} \in \mathbf{Q}$. This illustrates the efficiency of the box principle. Note that, while the constant $c(\alpha)$ in Liouville inequality can be made entirely explicit in terms of the degree and height of α , a major open problem related to Roth theorem is to give an explicit upper bound for the largest height of a solution p/q of (1).

For applications to transcendence, the box principle is used in the following form:

Thue-Siegel Lemma. Let m, n be integers with $n > m > 0$, let $A = (a_{ij}) \in \text{Mat}_{m \times n}(\mathbf{R})$, and let $\epsilon > 0$. Then, the system

$$\begin{cases} |a_{11}x_1 + \cdots + a_{1n}x_n| < \epsilon \\ \vdots \\ |a_{m1}x_1 + \cdots + a_{mn}x_n| < \epsilon \end{cases}$$

admits a non-zero solution $x = (x_1, \dots, x_n) \in \mathbf{Z}^n$ with maximum norm

$$\|x\| = \max_i |x_i| \leq U = \left(1 + \frac{\|A\|}{\epsilon}\right)^{\frac{m}{n-m}}$$

where $\|A\| = \max_i \sum_j |a_{ij}|$.

Note that the result does not provide an explicit solution to the system but only a bound on the maximum norm $\|x\|$ of such a solution. In particular, when A has integral coefficients and $\epsilon = 1$, it provides an upper bound on the maximum norm of a non-zero integral solution to the system $Ax = 0$.

The general idea behind the proof of this result is quite simple and interesting. Put $T = [U]$ and $\ell = [U^{n/m}]$. The matrix A maps the $(T+1)^n$ integral points in $[0, T]^n$ into an m -dimensional cube of side $\|A\|T$. Subdivide this cube

into ℓ^m cubes of side $\|A\|T/\ell < \epsilon$. Since $(T+1)^n > \ell^m$, there are at least two integral points $x' \neq x''$ with images Ax' and Ax'' in the same sub-cube. Their difference $x = x' - x''$ is a non-zero integral point with $\|x\| \leq T \leq U$ and $\|Ax\| = \|Ax' - Ax''\| < \epsilon$ as required.

5. Algebraic approximations

Fix a number $\theta \in \mathbf{C}$. To construct a polynomial $P \in \mathbf{Z}[X]$ of degree $< D$ having small absolute value at the point θ , one has to solve in integers a linear inhomogeneous equation in the D unknown coefficients of P . Applying a version of Thue-Siegel lemma (over \mathbf{C}) to this situation, one finds that, for any sufficiently large positive integer T , there exists a non-zero polynomial $P \in \mathbf{Z}[X]$ with integral coefficients such that

$$t(P) \leq T \quad \text{and} \quad |P(\theta)| \leq \exp\left(-\frac{T^2}{3}\right).$$

We may expect that such a polynomial P has a root close to θ . In general, we can only hope for a root with distance of the order of e^{-T} . But for infinitely many T , we have approximations which compare with the above:

Theorem (Durand, 1978). For infinitely many integers $T \geq 1$, there exists $\alpha \in \overline{\mathbf{Q}}$ with

$$t(\alpha) \leq T \quad \text{and} \quad |\theta - \alpha| \leq c(\theta)e^{-T/80}.$$

In view of this result, the reader will note that the transcendence measures for e and π indicated in Section 3 are essentially best possible. Extensions of this result where degree and height are separated have shown to be useful in algebraic independence to show that certain numbers generate fields of transcendence degree ≥ 2 (see [1], [7] and [8]).

6. Algebraic independence

In this presentation, we concentrate mainly on the part of the theory related to the usual exponential function e^z . Here are three important results in this context.

Theorem (Lindemann-Weierstrass, 1885).

If $\alpha_1, \dots, \alpha_n \in \overline{\mathbf{Q}}$ are linearly independent over \mathbf{Q} , then $e^{\alpha_1}, \dots, e^{\alpha_n}$ are algebraically independent over \mathbf{Q} .

Theorem (Baker, 1966). If $\log \alpha_1, \dots, \log \alpha_n$ are \mathbf{Q} -linearly independent logarithms of algebraic numbers, then $1, \log \alpha_1, \dots, \log \alpha_n$ are linearly independent over \mathbf{Q} .

A. Baker gave as well a measure a linear independence for these numbers, that is a lower bound for $|\beta_0 + \beta_1 \log \alpha_1 + \cdots + \beta_n \log \alpha_n|$ when $\beta_0, \dots, \beta_n \in \overline{\mathbf{Q}}$ are not all zero. This lower bound has since been improved in several directions and shown to be useful in several branches of number theory, especially in solving Diophantine equations. Baker's theorem was generalized to commutative algebraic groups by G. Wüstholz and this generalization was, in turn, incorporated by M. Waldschmidt in his vast theorem of the algebraic subgroup.

The third result (refining earlier work of many authors including Yu. Nesterenko and P. Philippon) represents our actual knowledge concerning a famous conjecture of Gel'fond.

Theorem (Diaz, 1987). Let $\alpha, \beta \in \overline{\mathbf{Q}}$ with $\alpha \neq 0$ and $d = \deg(\beta) \geq 2$. Choose a non-zero determination of $\log \alpha$ and, for $z \in \mathbf{C}$, define $\alpha^z := e^{\log(\alpha)z}$. Then

$$\text{tr.deg}_{\mathbf{Q}}(\alpha^\beta, \alpha^{\beta^2}, \dots, \alpha^{\beta^{d-1}}) > \frac{d-1}{2}.$$

The transcendence of α^β was the question asked by D. Hilbert in his seventh problem. It was solved in 1934 independently by A. O. Gel'fond and Th. Schneider. In the above notations, Gel'fond conjectured that the $d-1$ numbers $\alpha^\beta, \dots, \alpha^{\beta^{d-1}}$ are algebraically independent over \mathbf{Q} and proved this in the case $d=3$.

This conjecture as well as all the other results are contained in the following vast conjecture:

Conjecture (Schanuel, 1966). Let $x_1, \dots, x_n \in \mathbf{C}$ be linearly independent over \mathbf{Q} . Then

$$\text{tr.deg}_{\mathbf{Q}}(x_1, \dots, x_n, e^{x_1}, \dots, e^{x_n}) \geq n.$$

In particular, this statement implies the so-called

Main conjecture for logarithms. Any set of \mathbf{Q} -linearly independent logarithms of algebraic numbers are algebraically independent over \mathbf{Q} .

7. Rank of matrices with entries in L

Not much is known towards the main conjecture for logarithms of algebraic numbers. The set of these numbers

$$L = \{ \lambda \in \mathbf{C}; e^\lambda \in \overline{\mathbf{Q}} \}$$

is a vector space over \mathbf{Q} , and the main conjecture asserts that any basis of L over \mathbf{Q} is a transcendence basis of the field $\mathbf{Q}(L)$. Equivalently, if $A = \text{Sym}_{\mathbf{Q}}(L)$ denotes the symmetric algebra of L over \mathbf{Q} , it says that the inclusion map of L into \mathbf{C} extends to a \mathbf{Q} -algebra isomorphism from A to $\mathbf{Q}[L] \subset \mathbf{C}$. In this context, we have the following result [9]:

Theorem (Waldschmidt, 1981). For any matrix M with entries in L , one has

$$\text{rank}_A(M) \geq \text{rank}(M) \geq \frac{1}{2} \text{rank}_A(M)$$

where $\text{rank}_A(M)$ denotes the rank of M viewed as a matrix with entries in A .

So, the rank of any matrix with entries in L is at least half of the value predicted by the main conjecture. In connexion with Leopoldt's conjecture, a p -adic version of this result shows that the p -adic rank of the units of a number field is at least half of its rank over \mathbf{Z} . This result can also be used to study points with coordinates in L on affine algebraic varieties, by expressing such a variety as a determinantal variety [5]. It is

also a strong evidence for the main conjecture because the latter is equivalent to the statement that $\text{rank}(M) = \text{rank}_A(M)$ for any matrix M with entries in $L + \mathbf{Q}$ (see [5]).

However, in the present state of knowledge, we still do not know if the field $\mathbf{Q}(L)$ has transcendence degree ≥ 2 over \mathbf{Q} . Assuming $\text{tr.deg}_{\mathbf{Q}} \mathbf{Q}(L) = 1$, one can replace the inequality $\text{rank}(M) \geq (1/2)\text{rank}_A(M)$ of the above theorem by a strict inequality when $M \neq 0$, with the following consequence [7]:

Theorem (Roy-Waldschmidt, 1995).

Suppose $\text{tr.deg}_{\mathbf{Q}} \mathbf{Q}(L) = 1$. If $\lambda_1, \dots, \lambda_n \in L$ are \mathbf{Q} -linearly independent, then $P(\lambda_1, \dots, \lambda_n) \neq 0$ for any non-zero homogeneous polynomial $P \in \mathbf{Z}[X_1, \dots, X_n]$ of degree 2.

It is also possible to give lower bounds for the rank of matrices whose entries are linear combinations of 1 and logarithms of algebraic numbers with coefficients in $\overline{\mathbf{Q}}$. Such matrices appear for example in studying Leopoldt's conjecture for Galois extensions of \mathbf{Q} or its generalization by F. Jaulent.

Theorem (Roy, 1993). Let $\mathcal{L} = \overline{\mathbf{Q}} + \overline{\mathbf{Q}} \cdot L \subset \mathbf{C}$ and let $A = \overline{\mathbf{Q}} \otimes_{\mathbf{Q}} A = \text{Sym}_{\overline{\mathbf{Q}}}(\overline{\mathbf{Q}} \cdot L)$. For any matrix M with entries in \mathcal{L} , one has

$$\text{rank}_A(M) \geq \text{rank}(M) \geq \frac{1}{2} \text{rank}_A(M).$$

8. Criteria of algebraic independence

A typical transcendence argument constructs a large supply of points with small absolute value, belonging to a fixed subfield K of \mathbf{C} . When K is algebraic over \mathbf{Q} , one uses an appropriate version of Liouville's inequality to show that these numbers are in fact equal to 0. When K has a positive transcendence degree, a substitute is given by the following so-called criterion of algebraic independence [4]:

Theorem (Philippon, 1986). Let $Z \subseteq \mathbf{C}^m$ be an algebraic set defined over \mathbf{Q} of dimension $k \geq 0$, and let $\theta \in Z$. Assume that, for each integer $T \gg 1$, there exists a subset $\mathcal{F}_T \neq \emptyset$ of $\mathbf{Z}[X_1, \dots, X_m]$ consisting of polynomials P with

$$t(P) \leq T \quad \text{and} \quad |P(\theta)| \leq \exp(-cT^{k+1})$$

where $c = c(\theta, Z) \geq 1$. Then, for infinitely many T , there exists a zero α of \mathcal{F}_T on Z such that

$$\|\theta - \alpha\| \leq \exp(-T^{k+1}).$$

To understand how this result is used in practice, one has to say that a typical transcendence argument is based on the choice of a large integer parameter T and, for a given T , the values that are produced are not only elements of K , but in fact elements of a finitely generated subring $\mathbf{Z}[\theta_1, \dots, \theta_m]$ of K (with generators θ_j independent of T). We will see an example of this later. Expressing these values as polynomials in $\theta = (\theta_1, \dots, \theta_m)$, one gets a family \mathcal{F}_T as in the above theorem. For the variety Z , one takes the smallest algebraic set defined over \mathbf{Q} containing θ . The conclusion is not that

these numbers are zero – a counter-example due to Cassels shows that this may not happen. It is that, for infinitely many T , this family of polynomials have a common zero in Z that is close to θ . This is a satisfactory explanation for those T , but unfortunately, the result says nothing for the other values of T .

The proof of this result is based on properties of Chow forms which, in this context, were first studied by Yu. V. Nesterenko.

9. Approximation by algebraic sets

Fix a point $\theta \in \mathbf{C}^m$. The Thue-Siegel lemma produces for each T a polynomial $P_T \in \mathbf{Z}[X_1, \dots, X_m]$ of size $\leq T$ taking small absolute value at the point θ . However, these values are not small enough in order to apply the above criterion. We cannot conclude directly, that, for infinitely many T , these polynomials have a zero which is close to θ . However, if we assume that they do not have such a zero, then their partial derivatives as well are small at the point θ . In joint work with M. Laurent, we developed a criterion which takes these derivatives into account [2]. Applying this criterion to the above situation, we obtained:

Theorem (Laurent–Roy). For infinitely many integers $T \geq 1$, there exists a non-zero polynomial $P \in \mathbf{Z}[X_1, \dots, X_m]$ of size $t(P) \leq T$, and a zero α of P with

$$\|\theta - \alpha\| \leq \exp(-cT^{m+1})$$

where c is a positive constant depending only on m and $\|\theta\|$.

This result can be viewed as the case $k = m - 1$ of the following conjecture, where, for an algebraic set $Z \subseteq \mathbf{C}^m$ defined over \mathbf{Q} , $t(Z)$ denotes the size of a Chow form of Z .

Conjecture. Let k be an integer with $0 \leq k < m$. For infinitely many integers $T \geq 1$, there exists an algebraic set $Z \subset \mathbf{C}^m$ defined over \mathbf{Q} , of dimension k , and a point $\alpha \in Z$ with

$$t(Z) \leq T^{m-k} \quad \text{and} \quad \|\theta - \alpha\| \leq \exp(-cT^{m+1})$$

where $c = c(m, \|\theta\|) > 0$.

10. Zero estimates

After application of Liouville's inequality or of a criterion of algebraic independence as above, one gains the additional information that many numbers which were known to be small are in fact equal to 0. The purpose of a zero estimate is to analyze this new information. In important applications, like for example the theorem of the algebraic subgroup of M. Waldschmidt, these numbers are values of a polynomial at points of a commutative algebraic group G . To give a concrete example, let us consider the case of the additive group $G_{\mathbf{a}}^n(\mathbf{C}) = \mathbf{C}^n$.

Let $\Gamma = \mathbf{Z}\gamma_1 + \dots + \mathbf{Z}\gamma_\ell$ be a finitely generated subgroup of \mathbf{C}^n . Define the *Dirichlet exponent* $\mu(\Gamma)$ of Γ as the minimum of all ratios

$$\frac{\text{rank}((\Gamma + V)/V)}{\dim(\mathbf{C}^n/V)}$$

where V is a proper subspace of \mathbf{C}^n . Then, we have the following result [3].

Theorem (Masser, 1981). Let N be a positive integer and let $P \in \mathbf{C}[X_1, \dots, X_n]$ be a non-zero polynomial such that

$$P(m_1\gamma_1 + \dots + m_\ell\gamma_\ell) = 0$$

for any choice of integers $0 \leq m_1, \dots, m_\ell \leq N$. Then we have

$$\deg(P) \geq \left(\frac{N}{n}\right)^{\mu(\Gamma)}.$$

The generalizations of this statement to arbitrary commutative algebraic groups, mainly due to Masser–Wüstholz and Philippon, play an important role in the theory.

11. A proof of Hermite-Lindemann theorem

To illustrate how all the preceding tools enter into a transcendence argument, let us look at a proof of Hermite-Lindemann theorem. For this purpose, we require the following special case of a general construction of auxiliary function due to M. Waldschmidt [9]:

Lemma. Let $s, u \in \mathbf{R}$ with $1 < s < u < 5/4$. For any integer $N \gg 1$, there exists a polynomial $P_N \in \mathbf{Z}[X, Y]$ with

$$\deg_X(P_N) \leq N, \quad \deg_Y(P_N) \leq \sqrt{N}, \quad H(P_N) \leq e^N$$

such that the function $f_N(z) = P_N(z, e^z)$ satisfies

$$\sup \left\{ |f_N^{(k)}(z)|; 0 \leq k \leq N^s, |z| \leq N^{s/2} \right\} \leq e^{-N^u},$$

where $f_N^{(k)}$ denotes $\frac{d^k f}{dz^k}$.

The proof of this goes in three steps. First, one chooses P_N using Thue-Siegel lemma so that f_N has small derivatives at 0. Then, one uses an interpolation formula to bound the supremum norm of f_N on the disk of radius $N^{s/2} + 1$ centered at 0, and afterwards Cauchy's formulas to bound the norm of its derivatives on the disk of radius $N^{s/2}$.

Sketch of proof of Hermite-Lindemann theorem.

Let $y \in \mathbf{Q}$ be $\neq 0$. Suppose $\alpha = e^y \in \overline{\mathbf{Q}}$. We look for a contradiction. The identity

$$f_N^{(k)}(z) = (\mathcal{D}^k P_N)(z, e^z) \quad \text{where} \quad \mathcal{D} = \frac{\partial}{\partial X} + Y \frac{\partial}{\partial Y}$$

implies

$$f_N^{(k)}(my) = (\mathcal{D}^k P_N)(my, \alpha^m) \in \mathbf{Z}[y, \alpha] \subseteq \overline{\mathbf{Q}}$$

for any integers $k, m \geq 0$. Moreover, the choice of f_N implies

$$|f_N^{(k)}(my)| \leq e^{-N^u}$$

whenever $0 \leq k \leq N^s$ and $0 \leq m|y| \leq N^{s/2}$. Applying Liouville's inequality, one concludes

$$(\mathcal{D}^k P_N)(my, \alpha^m) = 0$$

for the same values of k and m provided $N \gg 1$. A zero estimate then gives $\deg_Y(P_N) \gg N^s$ or

$$\deg_X(P_N)(1 + \deg_Y(P_N)) \gg N^{3s/2}.$$

This is the required contradiction, since $s > 1$.

12. A final digression

Let $y_1, \dots, y_\ell \in \mathbf{C}$ be linearly independent over \mathbf{Q} . Put $\alpha_1 = e^{y_1}, \dots, \alpha_\ell = e^{y_\ell}$. With the above notations we have:

$$\begin{aligned} & f_N^{(k)}(m_1 y_1 + \dots + m_\ell y_\ell) \\ &= (\mathcal{D}^k P_N)(m_1 y_1 + \dots + m_\ell y_\ell, \alpha_1^{m_1} \dots \alpha_\ell^{m_\ell}) \\ & \in \mathbf{Z}[y_1, \dots, y_\ell, \alpha_1, \dots, \alpha_\ell] \end{aligned}$$

and

$$\left| (\mathcal{D}^k P_N)(m_1 y_1 + \dots + m_\ell y_\ell, \alpha_1^{m_1} \dots \alpha_\ell^{m_\ell}) \right| \leq e^{-N^u},$$

whenever $0 \leq k \leq N^s$ and $0 \leq m_1, \dots, m_\ell \ll N^{s/2}$.

So, the polynomial P_N takes small values as well as its derivatives with respect to \mathcal{D} on many points of the subgroup Γ of $(G_a \times G_m)(\mathbf{C}) = \mathbf{C} \times \mathbf{C}^\times$ generated by $(y_1, \alpha_1), \dots, (y_\ell, \alpha_\ell)$, and these values lie in the ring $\mathbf{Z}[y_1, \dots, y_\ell, \alpha_1, \dots, \alpha_\ell]$.

However, if $\ell > 1$, these values are not small enough in order to apply the present criteria of algebraic independence. Still, one may wonder if the above data have a non-trivial content. Can we hope to conclude from them that the field $\mathbf{Q}(y_1, \dots, y_\ell, \alpha_1, \dots, \alpha_\ell)$ has transcendence degree $\geq \ell$? In fact, this happens to be equivalent to Schanuel's conjecture. To be more precise, consider the following statement.

Conjecture 2. Let $y_1, \dots, y_\ell \in \mathbf{C}$ be linearly independent over \mathbf{Q} and let $\alpha_1, \dots, \alpha_\ell \in \mathbf{C}^\times$. Assume that, for any integer $N \gg 1$, we have

$$\max_{\substack{0 \leq k \leq N^s \\ 0 \leq m_j \leq N^{s/2}}} \left| (\mathcal{D}^k P_N) \left(\sum_{j=1}^{\ell} m_j y_j, \prod_{j=1}^{\ell} \alpha_j^{m_j} \right) \right| \leq e^{-N^u},$$

Then, $\text{tr.deg}_{\mathbf{Q}} \mathbf{Q}(y_1, \dots, y_\ell, \alpha_1, \dots, \alpha_\ell) \geq \ell$.

The preceding observations show that the hypotheses of Conjecture 2 are satisfied if $\alpha_j = e^{y_j}$ for $j = 1, \dots, \ell$, and then the conclusion of Conjecture 2 is exactly what is predicted by Schanuel's conjecture. So Conjecture 2 implies the

latter. The converse is also true and uses a new interpolation formula. This interpolation formula shows that, under the hypotheses of Conjecture 2, there exists an integer $d \geq 1$ such that $\alpha_j^d = e^{dy_j}$ for $j = 1, \dots, \ell$. Assuming that Schanuel's conjecture is true, this implies

$$\text{tr.deg}_{\mathbf{Q}} \mathbf{Q}(dy_1, \dots, dy_n, \alpha_1^d, \dots, \alpha_n^d) \geq \ell,$$

and the conclusion of Conjecture 2 follows (see [6] for more details).

Note that Conjecture 2 is a purely arithmetic statement in the sense that it does not involve the exponential function (unlike Schanuel's conjecture). One may hope to prove it by refining the existing criteria of algebraic independence. Note also that, on the basis of Lindemann-Weierstrass theorem, one can show, as above, that Conjecture 2 is true in the case where y_1, \dots, y_ℓ are algebraic numbers.

Work for this paper was partially supported by NSERC and CICMA.

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For more details and references on the whole subject, the reader is referred to the surveys [10–13] by M. Waldschmidt. The paper [10] on the origins of the theory is very pleasant reading.

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RESEARCH NOTES

Ian Putnam, Column Editor

2000 CRM-Fields Institute Prize



Michael Sigal

Michael Sigal, the Norman Stuart Robertson Chair Professor of Applied Mathematics of University of Toronto, is the most recent winner of the CRM - Fields Institute Prize. He has been teaching at Toronto since 1985. His work has been rewarded by many honours including several invited lectures to the International Congress on Mathematical Physics and the International Congress of Mathematicians. He was a Killam Research Fellow in 1989-91 as well as the Jeffery-Williams lecturer of the Canadian Mathematical Society in 1992. He received the John Synge Award of the Royal Society of Canada in 1993.

Most of Sigal's contributions are aimed at constructing the mathematical framework and developing analytical tools for the physical theory of quantum dynamical processes. This development is at the frontier of modern analysis.

In large measure due to these contributions, the areas of scattering theory, theory of radiation and quantum dynamics at positive temperatures have become some of the most active areas of mathematical physics. In the last two years alone, 7 conferences and summer schools were organized which either were exclusively devoted to those subjects or featured them prominently.

The following is a detailed description of Sigal's work in 9 areas.

Scattering Theory: Proof (jointly with A. Soffer) of the main mathematical problem of quantum scattering theory – the conjecture of asymptotic completeness for N-body systems which states that any evolution of a system of particles results in a break up of the system into stable freely moving fragments.

Non-linear Partial Differential Equations: Proof of structural instability of time-periodic solutions of wave equations and related equations under arbitrarily weak nonlinear perturbations.

Theory of Radiation: Construction (jointly with V. Bach and J. Fröhlich) of the mathematical theory of quantum radiation processes. The latter addresses physical phenomena standing at the origin of quantum theory – emission and absorption of radiation by systems of non-relativistic matter such as atoms and molecules. The mathematical theory mentioned above gives the first consistent and effective method for computation of radiative corrections (Lamb shift) and life-times.

Spectral Renormalization Group Method: Development (jointly with V. Bach and J. Fröhlich) of the Renormalization-Group approach to spectral problems including the introduction of a new concept of spectral renormalization-group flow (acting directly on a space of equations).

Return to Equilibrium: Proof (jointly with V. Bach and J. Fröhlich) of property of return to equilibrium for quantum systems for positive temperatures. The property of return to equilibrium states that a system disturbed from its equilibrium state converges back to this state with the progress of time.

Non-linear wave equation: Establishing (jointly with R. Pyke) a general limitation on periods of time-periodic solutions of a wide class of nonlinear wave equations and related equations. This result generalized a previous one-dimensional result due to J.-M. Coron and A. Weinstein.

Theory of Vortices: Proof (jointly with S. Gustafson) of

the long-standing Jaffe-Taubes conjecture which states that in type I superconductors the magnetic vortices are stable for any vorticity n ; while in type II superconductors they are stable for $|n| = 3D1$ and unstable for $|n| > 1$. Implicitly the corresponding properties were assumed by physicists since the foundational paper by A. Abrikosov of 1957. Jointly with Yu. Ovchinnikov, a general framework for the description of vortex dynamics for nonlinear Schroedinger equation (or, time-dependent Ginzburg-Landau equation) and related equations.

Theory of Quantum Resonances: Development, jointly

with P. Hislop and simultaneously with B. Helffer & J. Sjöstrand and J.-M. Combes, P. Duclos, M. Klein & R. Seiler, of the mathematical theory of tunneling resonances, one of the major constructs of quantum physics, underpinning such phenomena as nuclear instability.

Theory of Large Coulomb Systems Proof of instability of large negative ions and, jointly with V. Ivrii, proof of the Scott Conjecture regarding the behavior of ground states of large molecules.

by Man-Duen Choi, University of Toronto

(from REVIEW—page 1)

Several of the 29 authors touch on the nature and/or future of mathematics. Ruelle treats this matter with whimsy having had the privilege of entertaining a being from extra-galactic space who (which?) is working on a Ph.D. thesis on “human mathematics” which is obviously inferior to that of extra-galactic mathematics. Steven Smale proposes 18 principal and 3 additional “Problems for the next century” which are clearly stated and seem sufficiently difficult to keep us all fully occupied for many decades.

Of course, the most interesting essays are those whose authors get out on a limb and dogmatically announce, as saving truth, propositions radically different from common opinion. For me the most startling was David Mumford whom hitherto I have regarded as the ultimate algebraist of the algebraists. But no! Away with algebra! Statistics must now be given pride of place. Stochasticity is the lead topic and slogan for the new century - for any mathematician who truly reads the tea-leaves in our afternoon cups! RANDOM VARIABLE will supplant such antedeluvian concepts as *function*, *space*, *group*! Mumford being Mumford argues his case VERY persuasively. My whole world reels. I am in a state of shock fearing for his life at the hands of the algebra community even as the statisticians put wreaths of flowers around his neck!

Mathematical physics, my personal [1] area of competence, is discussed by Edward Witten, Cumrun Vafa, and Roger Penrose in this book and was the topic of a key-note lecture by Atiyah at the International Conference on Mathematical Physics at Imperial College in August 2000. From Witten we learn that since 1925 the fundamental physical theory changes roughly every generation of physicists. In pp. 348 - 352, we are told that *String Theory* was the theory in the previous generation and that *M-theory* is now the hot topic. String theory which was the first of the theories about everything has the virtue of combining both quantum mechanics and general relativity theory. The great outstanding problem of contemporary physics is how to relate QM and GRT in a unified theory. The solution of this problem is the Holy Grail the discovery of which has preoccupied several of the ablest minds of the past 50 years.

There are five rather rigid string theories and no definitive experimental evidence for any one of them. Enter *M-theory*. This is a super-combination of all five string theories! These theories have engaged the attention of many mathematicians chiefly because ideas promulgated by Witten have led to remarkable discoveries in enumerative geometry and stimulated Donaldson’s discussion of four-dimensional topology. Atiyah, in the above-noted speech, gave vivid expression of his belief that pure mathematics has profited greatly by adopting ideas recently developed by physicists in quantum field theory.

So all serious mathematicians must now master string theory if we take to heart the views of Atiyah, Vafa and Witten! However, Roger Penrose is not a true believer. He observes that while an ideal theory has no arbitrary constants, the widely accepted Standard Model has 17 adjustable parameters. This reminds me of my experience in first year physics labs. My lab partner and I became quite proficient, when we knew the expected results, at demonstrating that our data produced the ‘correct’ result within reasonable experimental error!

Penrose even embraces the heretical idea that supersymmetry, which is essential to M-theory, has no relevance to the real world.

However I do not reject string theory totally since, as a Whiteheadian, I know that the world consists not of *particles* but rather of quantized *events* with rich internal character. Strings, as the ultimate minimal constituents of being have a distinct advantage over the point-particles of classical mechanics. They do have internal structure.

I left the August mathematical physics meeting at Imperial College with the depressed feeling that the problem of combining GRT and QM was proving impossible and that, as a make-work project, physicists, captured by the beauty and fascination of pure mathematics, have essentially abandoned physics as we have known it attempting to become pseudo-mathematicians. Even some mathematicians have been enticed into this morass by the illusion that they can become famous by finding the Holy Grail.

But I do not want to leave the impression that the book un-

der review is principally about quantum field theory. Among the 29 authors there are many famous pure mathematicians whose contributions constitute a smorgasbord of delicacies sufficient to satisfy every taste. Do sample them!

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EDUCATION NOTES

Ed Barbeau and Harry White, Column Editors

A word from the new co-editor

I am happy to join the editorial team of the CMS Notes, and collaborate with Ed Barbeau on the Education Notes. Some colleagues and myself will try to give you information on and/or share our preoccupations about topics related to mathematics education. I will write in French and provide short English abstracts.

In my first contribution, I will discuss changes that the Quebec Ministry of Education (MEQ) wants Quebec universities to implement in programs for future secondary school teachers. Universities in Quebec have to follow rules for preparing teachers that are set by the government. Students planning to teach mathematics will now need only one subject (a major in mathematics), rather than the two (major and minor) that have been required since 1994. Also, the vocabulary is changing; we talk about “professional competencies” instead of “objectives”. I remind you that the baccalaureate for teaching at the secondary level consists of a four-year program (120 15-hour credits) that includes practice teaching.

La formation à l'enseignement

Une importante consultation que l'on a appelée les « États généraux » s'amorçait au printemps 1995 et a permis de diagnostiquer l'état de la situation de l'éducation au Québec, et d'enclencher un important chantier de réformes en tenant compte des nouvelles réalités politiques, sociales, démographiques, économiques et culturelles.

Même si la formation des maîtres a connu des changements majeurs au cours de la dernière décennie, il devenait essentiel d'harmoniser les programmes de formation des enseignants et enseignantes en fonction des transformations en cours dans tout le système d'éducation québécois, afin de mieux adapter les programmes aux nouvelles réalités qui caractériseront le monde scolaire pour les années à venir.

C'est pourquoi le MÊQ (ministère de l'éducation du Québec) a défini les orientations en regard de la formation à l'enseignement, le référentiel de compétences professionnelles attendues au terme de la formation initiale ainsi que les profils de sortie. L'établissement de ces balises constitue la première étape d'un processus qui comprend ensuite l'élaboration des programmes par les universités, l'agrément de ces programmes, et la reconnaissance d'aptitude à l'enseignement.

Cette réforme permettra d'offrir une formation plus

avancée en mathématiques car on reconnaît que les mathématiques constituent une matière de base au secondaire, et de ce fait, les futurs maîtres en mathématiques seront formés dans une seule discipline (les mathématiques) au lieu de deux disciplines comme c'est le cas depuis 1994. De plus, ils devront avoir une meilleure formation en didactique des mathématiques et en adaptation scolaire (difficultés d'apprentissage).

Somme toute, cette réforme répond en partie à une demande du milieu universitaire québécois pour une meilleure formation de base en mathématiques.

The formation of teachers: your opinion, please

It has been recommended by one of the Task Forces that the Canadian Mathematical Society produce a statement on the mathematical preparation of teachers. The Education Committee has been struggling with the advisability of doing this, and wonders whether it might be a good idea first to invite members of the Society comment on this issue in order to see what consensus exists before anyone presumes to speak on behalf of the membership. We encourage you to write letters to the editors of the Notes and submit items on this issue to this column. If you would like to write a longer and more considered essay, please get in touch with Ed Barbeau (barbeau@math.utoronto.ca) about what you have in mind. One idea is that the Society might publish a collection of essays that touch on this issue.

Un groupe de travail de la SMC a recommandé de produire un document relatif à la préparation des futurs maîtres. Le Comité d'Éducation se demande s'il ne serait pas plus approprié, dans un premier temps, d'inviter les membres de la Société à se prononcer sur la question afin de connaître les tendances actuelles avant que le comité de travail présente un avis officiel.

Nous encourageons fortement les membres à donner leurs opinions sur la question par une lettre ouverte dans cette rubrique en écrivant aux éditeurs des Notes. Pour un écrit plus élaboré, s'il vous plaît, veuillez rejoindre Ed Barbeau (barbeau@math.utoronto.ca). Éventuellement, la SMC pourrait publier un document qui contiendrait les écrits présentés.

Symposium About Mathematical Understanding Friday June 1 at Saskatoon

On the day prior to the beginning of the CMS Summer Meeting in Saskatoon a Symposium about Mathematical Understanding will occur. The Symposium will be the first day of

a three day conversation about mathematical understanding, at the Mathematics Education sessions, at the CMS meeting.

This is a very exciting time for all aspects of mathematics education and we believe that this Symposium will be a wonderful opportunity for researchers from the fields of developmental and cognitive psychology, mathematics education, and mathematics to come together to further our conversations about the many different perspectives in thinking about the phenomenon of mathematical understanding.

We intend that the first day of this Symposium will investigate the different ways of thinking about the phenomenon of mathematical understanding. We've invited four Canadian researchers to lead the Symposium on June 1. Three of the four have confirmed their attendance: Anna Sierpiska from Concordia University; Jamie Campbell from the University of Saskatchewan and Peter Taylor from Queen's University.

The format for June 1 will be 4 major presentations with time for dialogue following each session. On the second and third days, there will be a number of thirty minute sessions where specific examples and research will be shared by a number of different researchers.

This Symposium is scheduled to begin two days after the closing of the annual meeting of the Canadian Mathematics Education Study Group, which will be held in Edmonton, AB.

In addition, we would very much like to publish the proceedings of this Symposium as we believe these conversations

will prove to be a great resource for faculty and students alike.

Florence Glanfield, University of Saskatchewan

National Math Trail (USA)

This is the second year for the National Math Trail in the United States. On its website, <http://www.nationalmathtrail.org>, FASE Productions, the medium arm of the Foundation for Advancements in Science and Education, has posted community-based problems submitted by teachers and students across the USA, along with photographs, illustrations, audio and video clips, and webpages. Now it is inviting material from around the world, in preparation for *National Math Trail Week*, May 14-18, 2001. Awards will be given to winning submissions.

The web site provides a complete explanation on creating a math trail activity, and includes a video on an activity in East Harlem by teacher Kay Toliver. A *Technology Tutorial* presents simple ways to utilize technology in preparing entries, including a template for the creation of web page submissions and an online self-teaching guide on digital communication. The resources on the site will be expanded this year.

This resource is free to teachers, because of support provided by the US Department of Education's Star School Program and Texas Instruments. The host of the site is The Futures Channel (www.thefutureschannel.com), a digital content service for teachers.

UPCOMING CONFERENCE

Second Annual Colloquiumfest on Real Algebraic Geometry and Model Theory

University of Saskatchewan – March, 2001

The month of March will be a period of special activity for our algebra group. Two PhD students, Matthias Aschenbrenner (Urbana, Illinois) and Markus Schweighofer (Konstanz, Germany) will be visiting the department for the whole of March. Activities will center around the week of March 19-24. Daily seminars will be held March 19-22. The week will conclude with a mini-conference March 23-24, featuring the following speakers:

- Matthias Aschenbrenner, Urbana, Illinois. Ideal membership in polynomial rings over the integers: Kronecker's Problem.
- Isabelle Bonnard, Angers, France. Nash constructible functions.
- Raf Cluckers, Leuven, Belgium. Semi-algebraic p-adic geometry.
- Max Dickmann, Paris 7, France. Title to be announced.
- Victoria Powers, Emory University, Atlanta, Georgia. Convex optimization and real algebraic geometry.
- Claus Scheiderer, Duisburg, Germany. Sums of squares and the moment problem.
- Markus Schweighofer, Konstanz, Germany. Bounded polynomials on unbounded real algebraic sets and the iterated real holomorphy ring.
- Other participants will include: Eberhard Becker, Dortmund, Germany; Alex Prestel, Konstanz, Germany; Niels Schwartz, Passau, Germany.

See the website <http://math.usask.ca/fvk/Mb2.htm> for the latest information including arrival and departure times. If you wish to attend please contact the organizers by e-mail fvk@math.usask.ca, skuhlman@math.usask.ca, marshall@math.usask.ca or by mail: Franz-Viktor Kuhlmann, Salma Kuhlmann, Murray Marshall, Department of Mathematics & Statistics, University of Saskatchewan, 106 Wiggins Road, Saskatoon, SK S7N 5E6.



Calendar 2001

PIMS Thematic Programme in Nonlinear Partial Differential Equations

Viscosity Methods in PDEs, July 2 – 10 at PIMS-UBC
Phase Transitions, July 11–18 at PIMS-UBC
Concentration Phenomena and Vortex Dynamics,
 July 19–27 at PIMS-UBC
Variational Methods, July 30 – August 7 at PIMS-UBC
Geometric PDEs, August 8–17 at PIMS-UBC
 Contact: pde@pims.math.ca

PIMS Thematic Programme in Theoretical, Numerical and Industrial Fluid Dynamics

3rd Annual PIMS Fluid Dynamics Summer School
 May 27 – June 8, University of Alberta
 Contact: fdss@math.ualberta.ca
**Wave Phenomena III: Waves in fluids from the micro-
 scopic to the planetary scale**
 June 11 – 15, University of Alberta
 Contact: waves3@math.ualberta.ca
**International Conference on Theoretical and
 Numerical Fluid Mechanics II**
 August 20–25, Coast Plaza Suite Hotel, Vancouver
 Contact: heywood@math.ubc.ca

Education Activities

Evening of Math, March 29, SFU, Burnaby
Greater Vancouver Science Fair, April 5–7, UBC
Evening of Math, April 12, SFU, Harbour Centre
A Staging of Lakatos' "Proofs and Refutations",
 May 8–12, PIMS-SFU
Changing the Culture, May 11, Harbour Centre, SFU
PIMS Elementary School Math Contest, May 26, UBC
The Alberta High School Mathematics Competition,
 Part I of the 2001–2002 season, November 20
Math Fairs, November, University of Alberta

Scientific Activities

Design Theory Workshop, May 15–19, PIMS-SFU
Black Holes III: Theory and Mathematical Aspects,
 May 19–23, Kananaskis, Alberta
**9th Canadian Conference on General Relativity and
 Relativistic Astrophysics**, May 24–26, U. Alberta
Inverse Problems Workshop, June 9–10, PIMS-UBC
Designs, Codes, Cryptography and Graph Theory,
 July 9–14, University of Lethbridge
**PIMS Summer Workshop on Particles, Fields and
 Strings**, July 16–27, SFU
**International Conference on Scientific Computa-
 tion and Differential Equations**, July 29 – August 3,
 Vancouver
**13th Canadian Conference on Computational Ge-
 ometry**, August 9–11, University of Waterloo
2001 Canada-China Math Congress, August 20–25,
 Vancouver
**Aspects of Symmetry on the 60th birthday of Robert
 Moody**, August 26–29 Banff, Alberta
PIMS PDF meeting, December 8–9

Industrial Activities

**4th PIMS Graduate Industrial Mathematics Mod-
 elling Camp**, June 11–15, University of Victoria
5th PIMS Industrial Problem Solving Workshop,
 June 18–22, University of Washington
PIMS Industrial Case Study Workshop, June, Centre
 for Operations Excellence, UBC

Deadlines for PIMS Opportunities

March 16: Applications to the National Programme
 Committee
September 14: Applications to the National Pro-
 gramme Committee, Applications to the Fall Com-
 petition of the PIMS Scientific Review Panel
September 28: Nominations for the PIMS prizes

AWARDS / PRIX

Un doctorat honoris causa pour Robert Moody



Professeur à l'Université de l'Alberta, Robert Vaughan Moody arrive au Canada de la Grande-Bretagne alors qu'il est encore très jeune. En 1966, il reçoit son doctorat de l'Université de Toronto et se distinguera par la suite comme mathématicien grâce à la découverte d'une classe d'algèbres de dimension infinie nommée les algèbres de Kac-Moody. Son travail exceptionnel lui mérite d'ailleurs des honneurs conjoints avec V. G. Kac, soit la Médaille Eugene Wigner. En 1978 et en 1995, La Société mathématique du Canada l'honorera, d'une part en lui offrant de livrer la conférence inaugurale Coxeter-James, un privilège qui revient aux mathématiciens qui se distinguent, d'autres parts, d'être conférencier lors de la remise du Prix Jeffery-Williams à l'occasion du 50ième anniversaire de La Société

mathématique du Canada. Régulièrement invité au Centre de recherches mathématiques, il y séjournait pendant un an en 1980, l'année même où il est élu à la Société Royale du Canada. En 1998, Robert Moody devient lauréat du Prix CRM/Fields Institute pour son travail exceptionnel sur l'ordre apériodique. En septembre 1999, il donne une conférence remarquée au Fields Institute sur l'ordre apériodique et les quasicristaux. Finalement, le 26 mai 2000, l'Université de Montréal lui attribue un doctorat honoris causa pour sa contribution exceptionnelle aux sciences.

Le Prix Aisenstadt 2000

C'est avec grand plaisir que le CRM annonce la remise du Prix de mathématiques André-Aisenstadt de l'année 2000 à Eckhard Meinrenken de l'Université de Toronto. Monsieur Meinrenken a obtenu son doctorat de l'Universität Freiburg en 1994. Il a donné une conférence sur ses travaux le 9 février 2001. Voici un résumé de sa conférence: **Matrices, Moment Maps, and Moduli Spaces** – Moment maps are a mathematical generalization of angular momentum in classical mechanics. The abstract notion of a moment map was introduced in the late 1960's by Souriau, and developed by Guillemin, Kirillov, Kirwan, Kostant, Marsden, Sternberg, Weinstein, and many others. This lecture will be concerned with a "non-linear" theory of moment maps, introduced in 1998 in collaboration with Alekseev and Malkin. We will explain the main properties of non-linear moment maps, and discuss their applications to eigenvalue problems for matrices and to moduli spaces of flat connections over a surface.

NEWS FROM DEPARTMENTS

Concordia University, Montreal, PQ

Appointment: Dr. Malcolm Harpe (Research Professor, Number Theory, Jan–May, 2001).

Visitor: Nadia Stehlikova (Czech Republic, Mathematics Education, Jan–May 2001).

University of Western Ontario, London, ON

Appointments: Dan Christensen (Assistant Professor, Homotopy theory, July 2000), David Riley (Associate Professor, Algebra, July 2000), Cezar Joita (Imperial Oil Postdoctoral Fellow, Complex analysis, July 1, 2000), Hugh Thomas (Imperial Oil Postdoctoral Fellow, Algebraic geometry and combinatorics, July 2000).

Retirements: Prof. Anne Bode, June 30, 2000. Her first appointment at UWO began in 1960. She was a revered teacher,

and served as Associate Dean of the Faculty of Science from 1975 to 1985. Prof. Irvine Robinson, June 30, 2000. His first appointment UWO began in 1963. He is a well respected teacher of Mathematics.

Visitor: T. Hales (Univ. of Michigan, Automorphic forms, January–March, 2000).

Other News: S. Lichtenbaum, Brown University spoke in the Distinguished Lecture Series, March 13, 14, 2000, on "An introduction to motives and motivic cohomology I,II." R. Jardine (with M. Kolster, McMaster) organized the 6th Great Lakes K-theory Conference, held at the Fields Institute, March 25–26, 2000. D. Christensen and R. Jardine organized the Fall, 2000 session of the Ontario Topology Seminar, held at UWO October 14–15, 2000.

CALL FOR NOMINATIONS / APPEL DE CANDIDATURES

Associate Editors - CJM and CMB / Rédacteurs associés - JCM et BCM

The Publications Committee of the CMS solicits nominations for three Associate Editors for the Canadian Journal of Mathematics (CJM) and the Canadian Mathematical Bulletin (CMB). The appointment will be for five years beginning January 1, 2002. The continuing members (with their end of term) are below.

CJM Editors-in-Chief / Rédacteurs-en-chef du CJM :

Henri Darmon and/et Niky Kamran, McGill (2006)

Rédacteurs-en-chef du BCM/ CMB Editors-in-Chief:

James Lewis, Arturo Pianzola; Alberta and/et Noriko Yui; Queen's (2005)

Le comité des publications de la SMC sollicite des mises en candidatures pour trois postes de rédacteur associé du Journal canadien de mathématiques (JCM) et Bulletin canadien de mathématiques (BCM). Le mandat sera de cinq ans et débutera le 1 janvier 2002. Les membres qui continuent suivent.

Associate Editors/Rédacteurs associés :

J. Bland, Toronto (2002)	M. Barlow, UBC (2004)
F. Lalonde, UQAM (2003)	P. Borwein, SFU (2004)
J. Millson, Maryland (2003)	N. Pippenger, UBC (2004)
C. Sulem, Toronto (2003)	G. Elliott, Toronto (2005)
F. Shahidi, Purdue (2005)	

The deadline for the submission of nominations is **April 15, 2001**. Nominations, containing a curriculum vitae and the candidate's agreement to serve should be sent to the address below.

L'échéance pour proposer des candidats est le **15 avril 2001**. Les mises en candidature, accompagnés d'un curriculum vitae ainsi que du consentement du candidat(e), devrait être envoyées à l'adresse ci-dessous.

James A. Mingo
Chair–CMS Publications Committee / Président–Comité des publications
Department of Mathematics and Statistics
Queen's University, Kingston
Ontario K7L 3N6

Coxeter-James / Jeffery-Williams / Krieger-Nelson Prize Lectureships Prix de conférence Coxeter-James / Jeffery-Williams / Krieger-Nelson

The CMS Research Committee is inviting nominations for three prize lectureships.

The Coxeter-James Prize Lectureship recognizes outstanding young research mathematicians in Canada. The selected candidate will deliver the prize lecture at the Winter 2001 Meeting in Toronto, Ontario. Nomination letters should include at least three names of suggested referees.

The Jeffery-Williams Prize Lectureship recognizes outstanding leaders in mathematics in a Canadian context. The prize lecture will be delivered at the Summer 2002 Meeting in Québec, Québec. Nomination letters should include three names of suggested referees.

The Krieger-Nelson Prize Lectureship recognizes outstanding female mathematicians. The prize lecture will be delivered at the Summer 2002 Meeting in Québec, Québec. Nomination letters should include three names of suggested referees.

The deadline for nominations is **September 1, 2001**. Letters of nomination should be sent to the address below:

Le Comité de recherche de la SMC invite les mises en candidatures pour les trois prix de conférence de la Société, la Conférence Coxeter-James, la Conférence Jeffery-Williams et la Conférence Krieger-Nelson.

Le prix Coxeter-James rend hommage à l'apport exceptionnel des jeunes mathématiciens au Canada. Le candidat choisi présentera sa conférence lors de la réunion d'hiver 2001 à Toronto (Ontario). Les lettres de mises en candidatures devraient inclure les noms d'au moins trois répondants possibles.

Le prix Jeffery-Williams rend hommage à l'apport exceptionnel des mathématiciens d'expérience au Canada. La Conférence sera présentée lors de la réunion d'été 2002 au Québec, (Québec). Les lettres de mises en candidature devraient inclure les noms d'au moins trois répondants possibles.

Le prix Krieger-Nelson rend hommage à l'apport exceptionnel des mathématiciennes au Canada. La Conférence sera présentée lors de la réunion d'été 2002 au Québec, (Québec).

Les lettres de mises en candidatures devraient inclure les noms d'au moins trois répondants possibles.

La date limite pour les mises en candidatures est le 1

septembre 2001. Les lettres de mises en candidatures devraient être envoyées à :

Douglas Stinson, CMS Research Committee / Comité de recherche de la SMC
Department of Pure Mathematics, University of Waterloo
200 University Ave West, Waterloo, ON Canada N2L 3G1

2001 Adrien Pouliot Award /Prix Adrien-Pouliot 2001

Nominations of individuals or teams of individuals who have made significant and sustained contributions to mathematics education in Canada are solicited. Such contributions are to be interpreted in the broadest possible sense and might include: community outreach programmes, the development of a new program in either an academic or industrial setting, publicizing mathematics so as to make mathematics accessible to the general public, developing mathematics displays, establishing and supporting mathematics conferences and competitions for students, etc.

Nominations must be submitted on the "Nomination Form" available from the CMS Office. To assure uniformity in the selection process, please follow the instructions precisely. Documentation exceeding the prescribed limits will not be considered by the Selection Committee. Individuals who made a nomination in 2000 can renew this nomination by simply indicating their wish to do so by the deadline date. Only materials updating the 2000 Nomination need be provided as the original has been retained.

Nominations must be received by the CMS Office no later **April 30, 2001.** Please send six copies of each nomination to the following address:

The Adrien Pouliot Award / Le Prix Adrien-Pouliot
Canadian Mathematical Society / Société mathématique du Canada
577 King Edward, Suite 109, P.O. Box 450, Station A / C.P. 450, Succ. A
Ottawa, Ontario K1N 6N5

Nous sollicitons la candidature de personnes ou de groupe de personnes ayant contribué de façon importante et soutenue à des activités mathématiques éducatives au Canada. Le terme "contributions" s'emploie ici au sens large; les candidats pourront être associés à une activité de sensibilisation, un nouveau programme adapté au milieu scolaire ou à l'industrie, des activités promotionnelles de vulgarisation des mathématiques, des initiatives, spéciales, des conférences ou des concours à l'intention des étudiants, etc.

Les candidatures doivent nous être transmises via le "Formulaire de mise en candidature" disponible du bureau de la direction de la SMC. Pour garantir l'uniformité du processus de sélection, veuillez suivre les instructions à la lettre. Toute documentation excédant les limites prescrites ne sera pas considérée par le comité de sélection. Il est possible de renouveler une mise en candidature présentée l'an dernier, pourvu que l'on en manifeste le désir avant la date limite. Dans ce cas, le présentateur n'a qu'à soumettre des documents de mise à jour puisque le dossier original a été conservé.

Les mises en candidature doivent parvenir au bureau de la SMC avant **le 30 avril 2001.** Veuillez faire parvenir vos mises en candidature en six exemplaires à l'adresse suivante:

Letters to the Editors/Lettres aux Rédacteurs

The Editors of the *Notes* welcome letters in English or French on any subject of mathematical interest but reserve the right to condense them. Those accepted for publication will appear in the language of submission. Readers may reach us at notes-letters@cms.math.ca or at the CMS Executive Office.

Les rédacteurs des *Notes* acceptent les lettres en français ou en anglais portant sur un sujet d'intérêt mathématique, mais ils se réservent le droit de les compresser. Les lettres acceptées paraîtront dans la langue dans laquelle elles nous sont parvenues. Les lecteurs pourront nous joindre au bureau administratif de la SMC ou à l'adresse suivante: notes-lettres@smc.math.ca.

CALENDAR OF EVENTS / CALENDRIER DES ÉVÉNEMENTS

MARCH 2001

8–11 Workshop on Population Genetics at the Molecular Level, (CRM, Montréal)
<http://www.CRM.UMontreal.CA/biomath/>

25–30 Sixth International Conference on Approximation and Optimization (Guatemala City, Guatemala)
<http://www.ing.usac.edu.gt/apopt6/>

26–April 7 Symplectic and Contact Topology, Field Theory and Higher Dimensional Gauge Theory, in the Symplectic Topology, Geometry, and Gauge Theory Program (Fields Institute, Toronto and CRM, Montréal)
<http://www.fields.utoronto.ca/symplectic.html>

APRIL 2001

4 33rd Canadian Mathematical Olympiad 2001/ 33e Olympiade Canadienne de mathématiques 2001

7 51st Algebra Day (Carleton University, Ottawa)
<http://www.math.carleton.ca/AlgebraDay.htm>

25–26 Workshop on Mathematical Formalisms for RNA Structure, (CRM, Montréal)
<http://www.CRM.UMontreal.CA/biomath/>

MAY 2001

25–29 Annual meeting of the Canadian Mathematics Education Study Group (University of Alberta, Edmonton)
<http://cmesg.math.ca>

25–27 Annual meeting and special session on French mathematics, Canadian Society for History and Philosophy of Mathematics / Société canadienne d'histoire et de philosophie des mathématiques (Université Laval, Québec)
<http://www.cshpm.org>

JUNE 2001

2–4 CMS Summer Meeting / Réunion d'été de la SMC (University of Saskatchewan, Saskatoon, Saskatchewan)
<http://www.cms.math.ca/CMS/Events/summer01>

2–5 One Hundred Years of Russell's Paradox (Munich)
<http://www.lrz-muenchen.de/godeherd.link/russell1101.html>,
 Ulrich.Albert @lrz.uni.muenchen.de

4–8 International Conference on Computational Harmonic Analysis (City University of Hong Kong)
malam@cityu.edu.hk

4–13 Hamiltonian Group Actions and Quantization, in the Symplectic Topology, Geometry, and Gauge Theory Program (Fields Institute, Toronto and CRM, Montréal)
<http://www.fields.utoronto.ca/symplectic.html>

MARS 2001

JULY 2001

1–14 42nd International Mathematical Olympiad (Washington D.C., USA)
imo2001.usa.unl.edu

9–20 Séminaire de mathématiques supérieures NATO Advanced Study Group (Université de Montréal)
<http://www.dms.umontreal.ca/sms>

16–21 COCOA VII - The Seventh International Conference on Computational Commutative Algebra (Queen's University, Kingston)
 A. Geramita (tony@mast.queensu.ca)
<http://cocoa.dima.unige.it/>

22–25 International Symposium on Symbolic and Algebraic Computation, (University of Western Ontario, London, Ontario)
<http://www.oreca.on.ca/issac2001/>

23–Aug.3 Combinatorics and Matrix Theory, (Laramie, Wyoming)
sfallat@math.wm.edu, <http://math.uwyo.edu/>

AUGUST 2001

7–9 Nordic Conference on Topology and its applications, NORDTOP 2001 (Sophus Lie Centre at Nordfjordeid, Norway)
nordtop2001@mail.mathatlas.yorku.ca

12–18 Thirty-ninth International Symposium on Functional Equations (Sandbjerg, Denmark, organized by Aarhus University) Henrik Stetkaer: stetkaer@imf.au.dk
<http://www.imf.au.dk/isfe39>

13–15 13th Canadian Conference on Computational Geometry, (University of Waterloo)
<http://compgeo.math.uwaterloo.ca/cccg01>

13–15 Second Gilles Fournier Memorial Conference / Seconde Conférence à la mémoire de Gilles Fournier (Université de Sherbrooke, Sherbrooke, Québec)
<http://www.dmi.usherb.ca/evenements>

15–18 Second Workshop on the Conley Index and related topics / Deuxième atelier sur l'indice de Conley et sujets connexes (Université de Sherbrooke, Sherbrooke, Québec)
<http://www.dmi.usherb.ca/evenements>

20–23 Second Canada-China Mathematics Congress (Vancouver)
<http://www.pims.math.ca/science/2001/canada-china/>

SEPTEMBER 2001

22–26 Applications of Discrete Mathematics, Australian Mathematical Society (Australian National University, Canberra) Ian Roberts: iroberts@darwin.ntu.edu.au
 or Lynn Batten: lbatten@deakin.edu.au

JUILLET 2001

AVRIL 2001

AOÛT 2001

JUIN 2001

SEPTEMBRE 2001

DECEMBER 2001**DÉCEMBRE 2001**

8–10 CMS Winter Meeting / Réunion d'hiver de la SMC
(Toronto Colony Hotel, Toronto, Ontario)

<http://www.cms.math.ca/CMS/Events/winter01>

MAY 2002**MAI 2002**

3–5 AMS Eastern Section Meeting (CRM, Université de Montréal)

<http://www.ams.math.org/meetings/>

JUNE 2002**JUIN 2002**

6–8 CAIMS 2002 (University of Calgary)

Samuel Shen: shen@maildrop.srv.ualberta.ca

15–17 CMS Summer Meeting / Réunion d'été de la SMC
(Université Laval, Québec, Québec)

Monique Bouchard: meetings@cms.math.ca

24–28 Special Activity in Analytic Number Theory (Max Planck Institute, Bonn) moroz@mpim-bonn.mpg.de

AUGUST 2002**AOÛT 2002**

20–28 International Congress of Mathematicians (Beijing, China) <http://icm2002.org.cn/>

DECEMBER 2002**DÉCEMBRE 2002**

8–10 CMS Winter Meeting / Réunion d'hiver de la SMC
(University of Ottawa / Université d'Ottawa, Ottawa, Ontario)

Monique Bouchard: meetings@cms.math.ca

JUNE 2003**JUIN 2003**

CMS Summer Meeting / Réunion d'été de la SMC
(University of Alberta, Edmonton, Alberta)

Monique Bouchard: meetings@cms.math.ca

DECEMBER 2003**DÉCEMBRE 2003**

CMS Winter Meeting / Réunion d'hiver de la SMC
(Simon Fraser University, Burnaby, British Columbia)

Monique Bouchard: meetings@cms.math.ca

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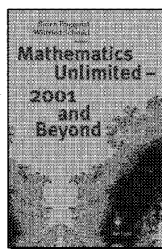
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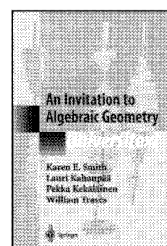
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