CMS NOTES de la SMC

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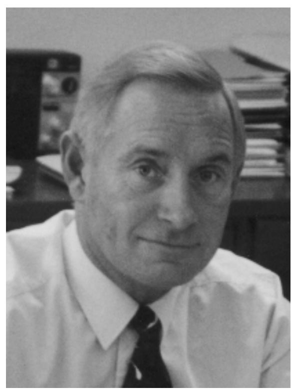
FROM THE EXECUTIVE DIRECTOR'S DESK



Graham Wright

2000 Annual Report

In March 1998, under the direction of the then president Katherine Heinrich, the Society began an extensive review of all of its activities and operations. Richard Kane, CMS President 1998 - 2000, and Jonathan Borwein, the current President, have overseen this important strategic planning exercise. The final reports of seven Task Forces (Budget and Policy, Board Representation, CMS Endowment Fund, Publications, Finances and Fundraising, Support of the Mathematics Community and Office Strategies) and one ad-hoc committee (Electronic Services and Camel) are available on Camel (www.cms.math.ca/Projects/).

The Executive Committee has been charged to review all of the reports and recommendations and to develop an overall structure and strategy for the CMS for the next several years. During 2000, each standing committee, as well as members of the Society, were given an opportunity to comment upon the reports and recommendations. The Executive has already began the process of reviewing all the submissions and will present its conclusions to the Board of Directors in June 2001. A great deal of effort has gone into the present strategic planning approach and although the final stages of the work are not yet completed, numerous changes have already been implemented. When the process is completed, the CMS will have taken the necessary steps to ensure our activities and services are delivered in the most effective, efficient and cost-effective manner.

The 2000 Annual Reports from each standing committee chair indicate the extensive range of research, publications and educational activities supported by the Society. All of these activities are possible because of the invaluable assistance received from both members and others.

Over the past few years the number of sessions at our semi-annual meetings have increased markedly, as has the number of delegates attending these meetings. Although the Executive

(see EXEC–page 20)

CMS NOTES
NOTES DE LA SMC

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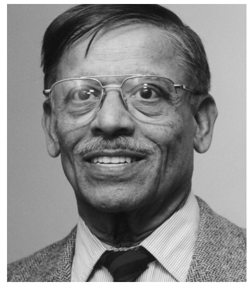
EDITORIAL



S. Swaminathan

One of my students approached me recently for advice on whether he should pursue graduate studies in mathematics. Here is a part of our conversation:

Student: What are the prospects for a job after I graduate with a degree in mathematics?

S. S.: The prospects are expected to be very good when you graduate with a Ph. D. five or six years hence. The rate of retirement is increasing in many universities opening up positions for new graduates.

S: Which is better - mathematics or computer science?

S. S.: Such comparisons are difficult. It depends on your aptitude and attitude. I note that you have a better aptitude for mathematics although you are doing very well in both subjects. Moreover more students are opting for computer science. Hence competition with them for jobs will be tougher.

S: I see that computer science is very useful to our technologically oriented society. We cannot say the same about mathematics.

S. S.: In general the usefulness of mathematics to society is not well understood. Actually mathematics has a significant part to play in almost every aspect of society. Let me give you just one example. Earlier this month (February), almost exactly two hundred years after the first asteroid was discovered, a spacecraft bearing the acronym NEAR (Near Earth Asteroid Rendezvous), launched from Cape Canaveral, Florida, in 1996, made a smooth crash-landing on the asteroid Eros which is hurtling in orbit at a tremendous speed some 196 million miles away. This made-on-Earth contraption jumped on the rugged rock at commands from Earth by scientists who based their calculations using advanced mathematical techniques. Further, since Andrew Wiles' proof of Fermat's Last Theorem, there is greater awareness among the public about mathematics. A successful play called *Proof* was staged in New York in Fall 2000, in which three of the four characters were mathematicians. It was followed by a symposium in New York University in which there were three panel discussions by mathematicians and nonmathematicians on some of the themes and ideas raised in the play. More can be said about the usefulness of mathematics but I suppose this much will suffice.

S.: I hear that computer scientists can make a lot of money. Can mathematicians become rich?

S. S.: It is true that computer scientists are paid more both in universities and in industry. As a mathematician you can become a millionaire if you solve one or more of the Clay problems. See page 18 for these problems.

*****

Il y a quelque temps, un étudiant est venu me demander s'il devait poursuivre des études supérieures en mathématiques. Voici une partie de notre conversation :

Étudiant : Quelles seraient mes perspectives d'emploi à la fin de mes études?

S. S. : On s'attend à ce qu'il y ait beaucoup de travail pour les titulaires d'un doctorat qui arriveront sur le marché du travail dans cinq ou six ans. De plus en plus de professeurs d'université prennent leur retraite, et il faut les remplacer.

Étudiant : Qu'est-ce qui est mieux : les mathématiques ou l'informatique?

(voir EDITORIAL–page 21)

CRM / Fields Prize Lecture

The Renormalization Group Approach in Spectral Analysis and the Problem of Radiation

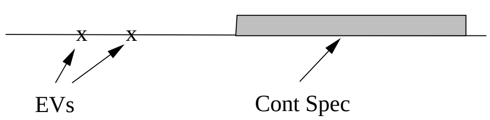
I.M. Sigal, University of Toronto

This article includes the slides of the talk I gave at the CRM on November 10, 2000. I gave a related talk at the Fields Institute on November 4, 2000. I completed sentences indicated on the slides, added a few explanations of the notation and concepts presented which I gave orally during the talks and inserted brief literature comments and a list of references. Apart from this I changed nothing. As a result the paper retains the informal style of the talk.

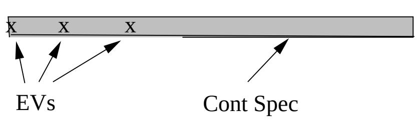
SPECTRAL ANALYSIS

I want to address the problem of perturbation of spectra of operators. For example, consider the problem of perturbation of a *single eigenvalue*. There are two possible cases:

1 Isolated eigenvalues



2 Embedded eigenvalues



In physical applications the second situation is generic, while the first one arises as a crude idealization when one considers a small part of a system in question.

Let us consider several examples of the second case.

HOPF BIFURCATION FROM SOLITONS

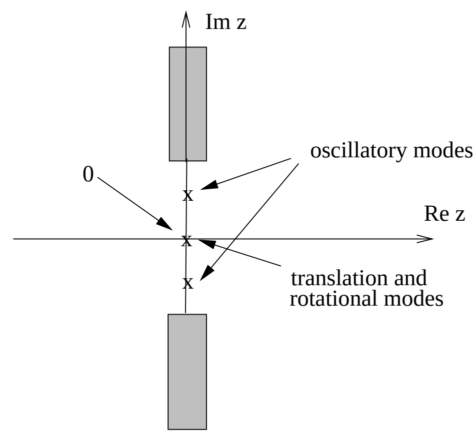
Consider the nonlinear Schrödinger equation

$$i\frac{\partial\psi}{\partial t} = -\Delta\psi + g\big(|\psi|^2\big)\psi,$$

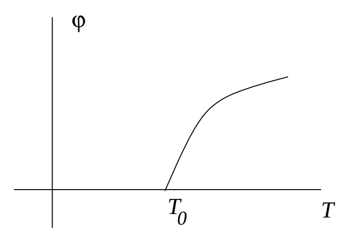
where ψ: $\mathbb{R}^n \times \mathbb{R} \to \mathbb{C}$. This equation has soliton solutions

$$\psi_{sol}(x,t) = e^{i\Phi(x,t)}\ f(x-vt)$$

where $\Phi(x,t)$ is some real phase depending on the velocity v. The spectrum of fluctuations around ψ_{sol}, i.e. of the linearization, $L_{\psi_{\mathrm{sol}}}$, of the r.h.s. around ψ_{sol}, is



Do oscillatory modes lead to the bifurcation of time-periodic solutions?



Following the Hopf bifurcation analysis we have to consider the Floquet operator

$$-T^{-1}\frac{\partial}{\partial t} + L_{\psi_{\mathrm{sol}}} \quad \text{on} \quad L^2(\mathbb{R}^n \times S^1)\ ,$$

where S^1 is the unit circle and T is an unknown period of the bifurcating periodic solution we are looking for. The spectrum of this operator is

$$\mathrm{spec}(L_{\psi_{\mathrm{sol}}}) + iT^{-1}\mathbb{Z}, \tag{$*$}$$

where $\mathrm{spec}(L_{\psi_{\mathrm{sol}}})$ is shown on the figure preceding the one above. Spectrum $(*)$ consists of a continuum filling in the entire imaginary axis and translation/rotation and oscillatory eigenvalues repeated periodically and embedded into this continuum.

Thus the answer to the question of what kind of solution bifurcates from oscillatory modes depends on an understanding of what happens to embedded oscillatory modes under a nonlinear perturbation.

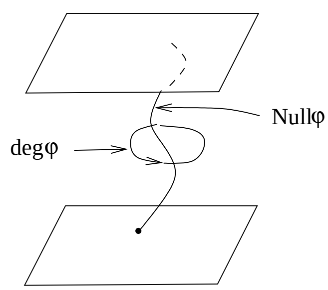
VORTEX SPECTRUM

Consider the Ginzburg-Landau equation

$$\Delta\varphi + (1 - |\varphi|^2)\varphi = 0$$

$$\varphi : \mathbb{R}^3 \to \mathbb{C}$$

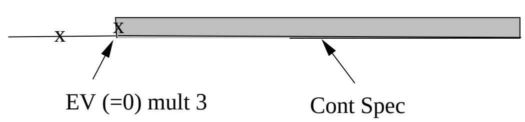
with the boundary condition that $|\varphi| \to 1$ as $|x^\perp| \to \infty$, where $x^\perp = (x_1, x_2)$ for $x = (x_1, x_2, x_3)$. Solutions of this equation can be specified by smooth curves of zeros of φ and a topological degree of φ with respect to these curves.



This equation has special–equivariant–solutions called *vortices*

$$\varphi_n = f_n(r)e^{in\theta} .$$

where (r, θ) are cylindrical coordinates. The spectrum of the linearized equation (i.e. of vortex fluctuations) is



(The negative eigenvalues are present for $|n| > 1$ and absent for $|n| = 1$.)

A detailed analysis of perturbation of the zero embedded eigenvalue is a key to understanding the dynamics of many (interacting) vortices.

QUANTUM SPECTRUM OF GEODESICS

Consider a space of curves given by their parameterizations, φ. Let $V(\varphi)$ be an energy of a curve φ. A quantization of $V(\varphi)$ yields the Schrödinger operator

$$-\Delta_\varphi + V(\varphi) \quad \text{on} \quad L^2(S', d\mu_C)\,, \qquad (*)$$

where $d\mu_C$ is a Gaussian measure on the Schwartz space $S' = S(\mathbb{R}^n)$ and the meaning of the "Laplacian", Δ_φ, acting on functionals of the field $\varphi \in S'(\mathbb{R}^n)$ will be alluded at later.

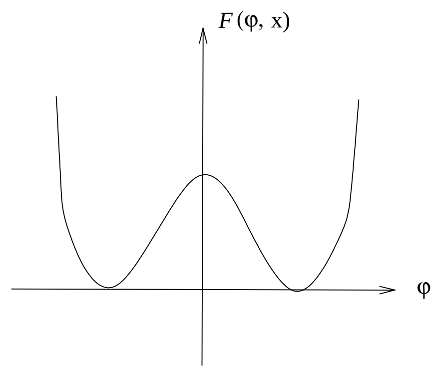
Now, let φ_{CP} be a critical point of $V(\varphi)$. The question we want to ask is: What are the *quantum corrections to the energy of* φ_{CP}*?*

Answering this question involves understanding the low energy spectrum of $(*)$ near the classical energy $V(\varphi_{CP})$ which in turn leads to a perturbation of embedded eigenvalues and the nearby spectrum.

In a special situation φ_{CP} could be a geodesic or, more generally, a minimal submanifold.

An important example of the situation above is that of quantum vortices. In this case φ: $\mathbb{R}^3 \to \mathbb{C}$ and $V(\varphi)$ is of the form

$$V(\varphi) = \int \frac{1}{2}|\nabla\varphi|^2 + F(\varphi, x) \qquad (**)$$



double-well potential

The (line) vortices arise as critical points of $V(\varphi), \varphi : \mathbb{R}^3 \to \mathbb{C}$, satisfying certain topological conditions (see above). The latter conditions imply that the null sets of these critical points are curves which are geodesics in a certain Riemannian metric (see a figure above).

One can think of the dynamics of vortices as motion of their centers – Null φ – with relatively rigid vortex rigging around them.

Another interesting case is that of functional $(**)$ with x-independent $F \geq 0$ and for $\varphi : [0,1] \to \mathbb{R}^m$. In this case, critical points of $V(\varphi)$ are (modulo parametrization) geodescics in the Riemannian metric $ds^2 = F(y)\,dy^2$ (Jacobi metric). The latter fact is related to Maupertuis principle in Classical Mechanics.

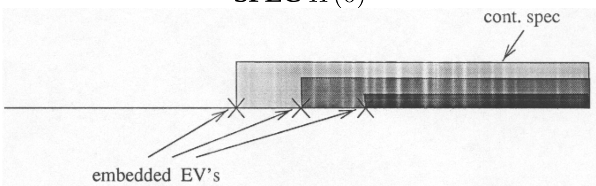
PROBLEM OF RADIATION

I want to present an example of a common physical situation when a small system (with finite number of degrees of freedom) is coupled to a large system (of infinite number of degrees of freedom) – the problem of radiation. This problem is reduced to finding the low energy spectrum of the quantum *Hamiltonian for the system of matter and radiation*

$$H(e) = \sum \frac{1}{2m_j} p_{j,eA}^2 + V(x) + H_{\mathrm{rad}}$$

on $\mathcal{H}_{\mathrm{matter}} \otimes \mathcal{H}_{\mathrm{rad}}$ (Schrödinger equation coupled to quantized Maxwell equations).

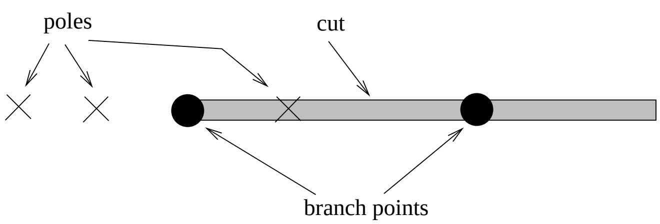
SPEC $H(0)$



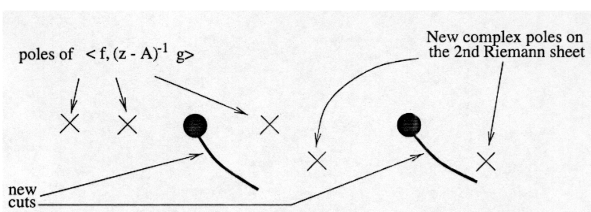
The spectrum of the unperturbed (=uncoupled) Hamiltonian $H(0)$ contains eigenvalues sitting on the top of the thresholds of continuous spectrum. They correspond to bound states of an atom in a vacuum. Are these bound states stable or unstable (when $e \neq 0$)?

REFINEMENT OF NOTION OF SPECTRUM

Standard notions of spectral analysis are insufficient for treating perturbation of embedded eigenvalues. We extend the notion of spectrum as follows. Consider a self-adjoint operator H on a Hilbert space $\mathcal{H}$. Then *point and continuous spectra are poles and cuts of* $\langle f, (z-H)^{-1}g\rangle \; \forall f, g \in \mathcal{H}$



Consider the *Riemann surface* of $\langle f, (z-H)^{-1}g\rangle$ for f and g in some dense set $\mathcal{D} \subset \mathcal{H}$. In other words we want to continue this analytic function from, say, $\mathbb{C}^+$ across the cut (continuous spectrum of H) into the second Riemann sheet:



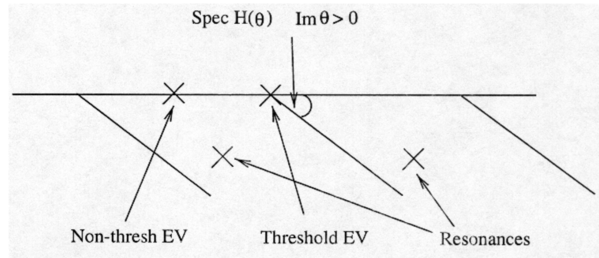
We see that non-threshold eigenvalues of H become isolated poles of this analytic continuation while new complex poles, not seen before, are revealed. Clearly, real poles coming from embedded eigenvalues and complex poles must be treated on the same footing.

DEFORMATION OF SPECTRA

Now I outline a constructive tool used in the study of the Riemann surface for a given operator H–the spectral deformation method. It goes as follows. Consider the orbit

$$H \to H(\theta) = U(\theta)HU(\theta)^{-1}$$

of H under a one-parameter group, $U(\theta)$, of unitary operators, s.t. $H(\theta)$ has an *analytic continuation in* θ *into a neighbourhood of* $\theta = 0$. The spectrum of such a continuation looks typically as on the figure below.

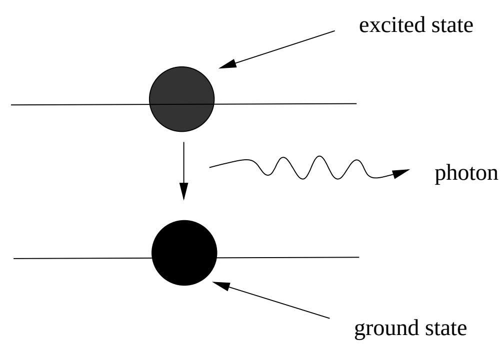


The resolvent $\big(H(\theta)-z\big)^{-1}$ provides the desired information about the Riemannian surface of the operator H. In particular, the real eigenvalues of $H(\theta)$ coincide with the eigenvalues of H, i.e. with the real poles mentioned above, while the complex eigenvalues of $H(\theta)$ are related to the complex poles on the second Riemann sheet. These complex eigenvalues are called the *resonances* of H.

Thus the problem of understanding the behaviour of embedded eigenvalues and the continuous spectrum of H under a perturbation is reduced to the problem of understanding the complex spectrum of the operator $H(\theta)$ for complex θ's.

MATHEMATICAL PROBLEM OF RADIATION

The goal here is to construct a *mathematical theory of emission and absorption of electro-magnetic radiation by systems of non-relativistic matter s.a. atoms and molecules:*



Mathematically, this translates into the problem of understanding the bound state–resonance structure of the quantum Hamiltonian of a system of quantum matter coupled to quantum radiation.

QUANTIZED MAXWELL EQUATIONS

I review quickly a mathematical framework of quantum theory of radiation. First, I describe the quantized Maxwell equations and then their coupling to quantum matter.

The quantized Maxwell equations can be presented as the Schrödinger equation $\dfrac{\partial\phi}{\partial t} = H_{\mathrm{rad}}\phi$ with the quantum Hamiltonian operator

$$H_{\text{rad}} := \frac{1}{2}\int : E^{\text{op}}(x)^2 + \big(\text{curl}A^{\text{op}}(x)\big)^2 : d^3x$$

acting on $\mathcal{H}_{\text{rad}} = L^2(S', d\mu_C)$. Here S' is the Schwartz space of the transverse vector fields, $A(x)$, $\text{div}A(x) = 0$, on $\mathbb{R}^3$, $d\mu_C$ is the Gaussian measure on S' with the mean 0 and covariance $C = (-\Delta)^{-\frac{1}{2}}$, $A^{\text{op}}(x)$ and $E^{\text{op}}(x)$ are the quantum operators of the vector potential and electric field in the Couloub gauge,

$$A^{\text{op}}(x) = \text{operator of multiplication by } A(x)$$

$$\text{E}^{\text{op}}(x) = -i\frac{\delta}{\delta\, A(x)} + iC^{-\frac{1}{2}}A(x)\ ,$$

and the double colons signify the Wick, or normal, ordering, i.e. some sort of deformation of the quantization procedure.

Now we explain briefly an origin of this Hamiltonian.

The Maxwell equations in a vacuum is an infinitely dimensional Hamiltonian system with the *Hamiltonian functional*

$$H(A, E) = \frac{1}{2}\int E^2 + (\text{curl}\, A)^2$$

defined on the phase-space $H_1^{\text{transv}} \times L_2^{\text{transv}}$ equipped with a standard symplectic structure. Here $A(x)$ is the vector potential in the Coulomb gauge and $E(x)$ is the electric field (the field conjugate to $A(x)$), which are transverse vector fields on $\mathbb{R}^3$ (i.e. $\text{div}\, A(x) = 0$ and $\text{div}\, E(x) = 0$), and H_1^{transv} and L_2^{transv} are the Sobolev space of order 1 and L_2-space of transverse vector fields.

A naive quantization of this dynamical system patterned on the quantization of the Newton equations goes as follows.

Concept	CFT	QFT
Phase/state space	$H_1^{\text{transv}} \otimes L_2^{\text{transv}}$	"$L^2(H_1^{\text{transv}}, DA)$"
Symplectic structure	Poisson brackets	commutators
Canonic. variables (w.r. to the symplectic structure)	$A(x)$	$A^{\text{op}}(x) =$ operator of multiplication by $A(x)$
	$E(x)$	$E^{\text{op}}(x) = -i\frac{\delta}{\delta\, A(x)}$
Observables	Real functionals $f(A, E)$ on $H_1^{\text{transv}} \otimes L_2^{\text{transv}}$	Self-adjoint operators $A = f(A^{\text{op}}, E^{\text{op}})$ on "$L^2(H_1^{\text{transv}}, DA)$"
Dynamics	Ham.fnctnl $H(A, E)$	Ham.opr $H(A, E)$

The third column does not make sense mathematically, but its natural modification leads to the formulation presented at the beginning of this section.

MATTER

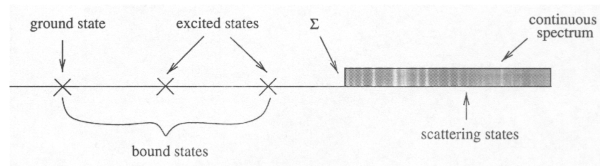
Quantum non-relativistic matter is described by a Schrödinger operator of the form (in units: $\hbar = 1$, $c = 1$, and $m_e = 1$)

$$H_{\text{matter}} = \sum_1^N \frac{1}{2m_j}p_j^2 + V(x)$$

on $\mathcal{H}_{\text{matter}}$(e.g. $L^2(\mathbb{R}^{3N})$). Here $p_j = -i\nabla_{x_j}$, $m_j > 0\ \forall j$ and $x = (x_1, \ldots, x_N)$.

SPECTRUM OF H_{matter}

Typically, the spectrum of H_{matter} is of the form (Hunziker-van Winter-Zhislin Theorem)



MATTER + RADIATION

To introduce the coupling between matter and radiation, we think about the vector potential $A(x)$ as a *quantum connection* on $\mathbb{R}^3$ and pass to the covariant derivatives

$$p \to p_A = p - eA(x)\ .$$

This leads to the *Hamiltonian for the system of matter and radiation*

$$H(e) = \sum \frac{1}{2m_j}p_{j,A}^2 + V(x) + H_{\text{rad}}$$

on $\mathcal{H}_{\text{matter}} \otimes \mathcal{H}_{\text{rad}}$. This operator is not well defined. To remedy this we introduce

Ultraviolet cut-off: $\quad A(x) \to \chi{*}A(x), \int |\chi|^2 d^3k < \infty$

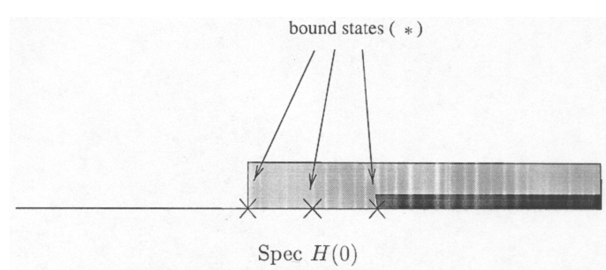
in the interaction terms $p_{j,A}^2$. The resulting operator (which we still denote by the same symbol $H(e)$) is *self-adjoint* and is bounded from below.

MATHEMATICAL PROBLEM

Problem of Radiation: Fate of the bounded states of matter

$$\Big(\underset{\substack{\text{bound state of matter}\\ \text{with energy } E_j}}{\varphi_j} \otimes \underset{\substack{\text{vacuum of}\\ \text{quantized EM field}}}{\Omega}\Big)e^{-iE_jt}\ ,\ \forall j \qquad (*)$$

as charges are "turned on".

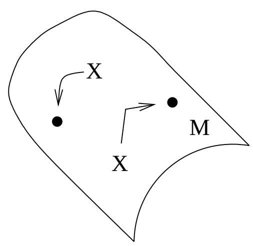


E.g. one would like to show that an atom in an excited state in a vacuum is unstable, that it emits a photon and descends into the stable ground state.

RENORMALIZATION GROUP APPROACH

Assume we want to study a part of the spectrum of the operator $H(e)$ near E_0. We proceed as follows:

- Pass to a metric space M of operators
- Construct a flow Φ_τ on M s.t. - Φ_τ eliminates "inessential" degrees of freedom - Φ_τ is isospectral in $|z - E_0| \leq e^{-\tau}$
- Find fixed points of Φ_τ and their stability.



Observe now that

- Isospectrality of Φ_τ allows us to transfer the spectral information we have about fixed points to the initial operator $H(e)$;

- Classify possible behaviour of physical systems in question according to the fixed points to which they are attracted.

Since ϕ_τ eliminates "inessential" degrees of freedom (a kind of partial dissipation) we expect that the Hamiltonians $H_\tau := \phi_\tau(H)$ at levels τ simplify as $\tau \to \infty$ and in particular that the fixed points are especially simple.

DECIMATION MAP

In order to define the RG-flow we first define a map, called the decimation map, which *eliminates inessential degrees of freedom*. First, we observe that the map

$$H \to P_\tau H P_\tau \ ,$$

where P_τ is the spectral projector associated with the inequality $|H(0) - E_0| \leq e^{-\tau}$, eliminates the part of H acting on $(\mathrm{Ran} P_\tau)^\perp$ but it *distorts Spec* H. We modify this map in order to restore the spectral fidelity in a small neighbourhood of $z = 0$:

$$D_\tau : H \to P_\tau(H - HR^\perp H)P_\tau \ ,$$

where $R^\perp = P_\tau^\perp(P_\tau^\perp H P_\tau^\perp)^{-1}P_\tau^\perp$ and $P_\tau^\perp = \mathbf{1} - P_\tau$. This new map is called the *decimation map*.

Trade-off: D_τ is isospectral at $z = 0$, but is *non-linear*.

RESCALING

Next, we introduce the rescaling map S_τ acting on operators by a unitary conjugation, $S_\tau : H \to U_\tau H U_\tau^{-1}$, which rescales the photon momenta as $k \to e^{-\tau}k$. As a result we have

$$S_\tau : |H(0) - E_0| \leq e^{-\tau} \to |H(0) - E_0| \leq 1 \ .$$

RG - FLOW

Now we are ready to define the RG-flow:

$$\Phi_\tau = E_\tau \circ S_\tau \circ D_\tau \ ,$$

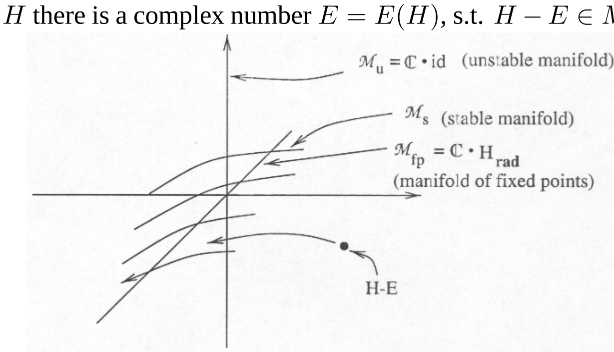
where $E_\tau(A) = e^\tau A$, a normalization map.

Observe that Φ_τ has the following properties

- Φ_τ is a semi-flow
- Φ_τ projects out $|H(0) - E_0| \geq e^{-\tau}$ and magnifies the result
- Φ_τ is isospectral in $\{z \in C||z| \leq e^{-\tau}\}$ modulo the factor e^τ.

RG FLOW DIAGRAM

The figure below shows fixed point, stable, and unstable manifolds of Φ_τ. The point here is that the fixed point manifold is very simple: $\mathcal{M}_{fp} = \mathbb{C} \cdot H_{\mathrm{rad}}$, while the stable manifold, $\mathcal{M}_s$, has the codimension 2. Hence given an operator H there is a complex number $E = E(H)$, s.t. $H - E \in M_s$.



Now proceed as follows. Apply the RG-flow to $H - E$. Then $\Phi_\tau(H - E)$ converges to the fixed point manifold so that for τ sufficiently large

$$\Phi_\tau(H - E) \approx w H_{\mathrm{rad}} \text{ for some } w \in \mathbb{C}$$

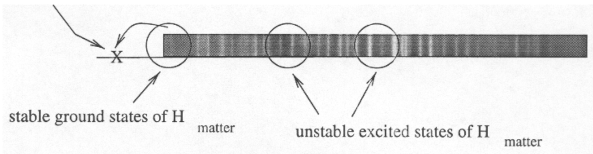
and as a result H is isospectral to $wH_{\mathrm{rad}} - E$ in $\{|z - E| \leq e^{-\tau}\}$. This way we transfer the spectral information about H_{rad}, which is available to us, to the operator H.

MATHEMATICAL RESULTS (Bach-Fröhlich-IMS)

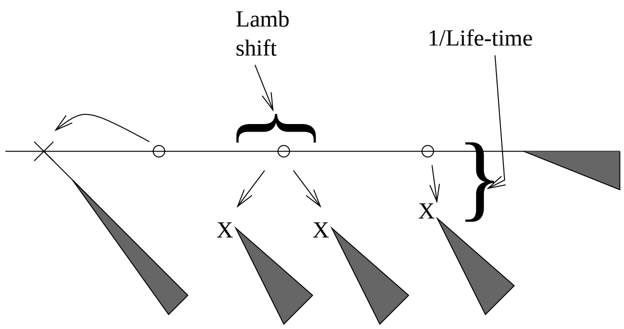
Assume that $|e|$ is sufficiently small and, in the third statement below, that the particle potential, $V(x)$, is confining, i.e. $V(x) \to \infty$ as $|x| \to \infty$. Then we have

I. **Binding.** $H(e)$ has a ground state ψ. $\|e^{\alpha|x|}\psi\| < \infty$ $\alpha > 0$.

II. **Instability.** $H(e)$ has no EVs near the excited EVs of H_{matter}.



III. **Resonances.** The excited states of H_{matter} bifurcate into resonances of $H(e)$.



Projection of Riemann surface of $H(e)$ onto $\mathbb{C}$

(EV stands for "eigenvalue".)

REMARKS

The result on the vortex spectrum mentioned is equivalent to the property of (linearized) stability/instability of vortices (see [G, GS, LL, M, OS1]). Recent results on dynamics of vortices are reviewed in [OS2].

The spectrum of critical points of the potential in Quantum Mechanics was found [Sim, BCD, Sj].

One can also try to understand spectra of critical points of the action functional

$$S(\phi) = \int \left(\int \frac{1}{2}|\dot{\phi}|^2 \, dx - V(\phi) \right) dt$$

which are periodic in time.

The method of spectral deformation and the theory of resonances based on it were proposed by Aguilar, Balslev, Combes and Simon and extended in works of Balslev, Hunziker, Jensen, Sigal, Simon and others. See [HisSig, HunSig] for recent reviews. A different approach was proposed by Helffer and Sjöstrand (see [HeSj, HeM]).

For a text on rigorous quantum field theory see [GJ] and for a physical discussion of the problem of radiation, [C-TD-RG].

A recent review of the quantum theory of many particle systems can be found in [HunSig].

The renormalization group approach and the results on the radiation problem presented here were obtained in [BFS1-3], where the reader can also find many references to the earlier or simultaneous work. Some improvements of these results are given in [BFS4]. The result on the ground state was improved in [G, GLL].

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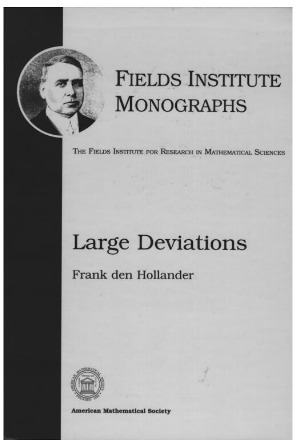
How Rare is Rare?

Book Review by C.C.A. Sastri, Dalhousie University

Large Deviations
by Frank den Hollander
Fields Institute Monograph, 14
American Mathematical Society,
Providence, 2000
x + 143 pages



Large Deviations is a branch of probability theory that deals with rare events. It is a very active field with applications to several areas including statistics, communication networks, computing, statistical physics, and mathematical finance. (For an example of a rare event, consider the problem of species sampling: suppose that one wishes to determine the species of fish native to a body of water and that, after repeated sampling, one identifies a certain number of species. Suppose that n trials have been conducted. The problem is to estimate the probability, for large n, that one would observe a new species on the next trial.) Roughly speaking, one may know, in a given situation, that the probabilities of certain events tend to zero as some parameter goes to ∞ (or zero). The question arises as to how fast the probabilities converge. Large deviation techniques enable one to find the rate of convergence. Central to the subject is the large deviation principle (LDP), enunciated by S.R.S. Varadhan in 1966. A natural context for stating the LDP is that of Borel probability measures on a Polish (i.e., a complete, separable metric) space. The reason is that the space of Borel probability measures on a Polish space can be metrized in such a way that it itself becomes a Polish space. The topology induced by the metric is the so-called topology of weak convergence of measures.

A sequence $\{P_n\}$ of Borel probability measures on a Polish space S is said to satisfy the LDP with a rate function I if there exists a function $I : S \longrightarrow [0, \infty]$ such that

(i) I has compact level sets, i.e., the set $\{s \mid I(s) \leq \ell\}$ is compact in S, for every ℓ

(ii) for every closed set $F \subset S$, $\limsup_{n\to\infty} \frac{1}{n} \log P_n(F) \leq -\inf_{s\in F} I(s)$,

and

(iii) for every open set $G \subset S$, $\liminf_{n\to\infty} \frac{1}{n} \log P_n(G) \geq -\inf_{s\in G} I(s)$.

Prior to Varadhan's work, large deviation results, including such important theorems as those due to Cramer and Sanov, lay scattered in the literature; the LDP brought them all under one rubric. Thus a typical theorem in large deviations states that, in a certain setting, the LDP holds.

An important application of large deviation techniques is to the asymptotic evaluation of certain expectations or function space integrals. This is based on a variational formula, also due to Varadhan, which plays a key role in large deviation analysis. In fact, a converse of Varadhan's result, due to W. Bryc (1990), can be used as a starting point for the application of weak convergence ideas to large deviation theory. This is the approach followed by P. Dupuis and R.S. Ellis in their book *A Weak Convergence Approach to the Theory of Large Deviations* (1997).

There exist several books on large deviations, written from different points of view and at different levels of depth. Nevertheless, the book by den Hollander is a welcome addition. It is ideally suited for nonspecialists interested in learning the subject as well as for graduate students who have had some exposure to measure-theoretic probability. One advantage it has over the other books is its brevity. The basic ideas and principles are presented in the first part of the book, which takes up only sixty pages. The topics covered include the theorems of Cramer and Sanov, which deal with independent identically-distributed sequences, and the Gärtner-Ellis theorem, which deals with weakly dependent sequences. The second part of the book, which is only slightly longer than the first, contains applications to a variety of areas such as hypothesis testing and interacting diffusions. There are exercises throughout the book. The appendix, containing solutions to the exercises, is an attractive feature, making the book suitable for self-study. The constraint of brevity forced den Hollander to leave out important topics such as Schilder's theorem, but still the resulting book provides a quick, relatively painless introduction to the subject. Indeed, the book is user-friendly, as den Hollander says in his preface. One can read it in conjunction with the books by Dembo and Zeitouni (*Large Deviations Techniques and Applications* (1998)), Ellis (*Entropy, Large Deviations, and Statistical Mechanics* (1985)) or Shwartz and Weiss (*Large Deviations for Performance Analysis* (1995)). One can then go on, if one wishes, to the book by Deuschel and Stroock (*Large Deviations* (1989)) or perhaps to the papers of Donsker and Varadhan.

Some Trends in Modern Mathematics and the Fields Medal

by Michael Monastyrsky

This is the second and concluding part of this article. The first part appeared in last month's issue.

If we very quickly review the results of the Fields medallists, keeping in mind the fundamental principles of J. Fields, we can observe several interesting developments:

1. Allocation of stable fields of interest.
2. Succession of mathematics.
3. Zigzags of mathematical fashion.

I will try to illustrate these theses with excerpts from the Fields medallists' results.

1. Allocation of Interest

Indisputably, if we divide the mathematics of the second half of the century into two parts, the first thirty years is mainly concentrated around problems of algebraic topology, algebraic geometry, and complex analysis. Here new concepts and methods appeared, and this evidently is reflected in the list of Fields medallists. A definite change in this tendency, a return to the more classical topics, but of course on a new level, can be observed in mathematics from the end of the 70's. With some delay, this has been reflected in the Fields medals awarded at the last two congresses.

It is important to note the new convergence between mathematics and physics. The traditional contacts between mathematics and physics are well known. If we consider the parallel development of mathematics and fundamental physics, we are astonished that the most revolutionary theories in 20th century physics are based on mathematics, which was especially developed for this purpose. It is enough to mention Einstein's special and general relativity based on the classical differential geometry of Riemann spaces, quantum mechanics and Hilbert spaces and the theory of linear operators, the Schrödinger equation and spectral theory, and so on. This connection was broken, somewhere in the 30's, at the time of the solution of several more concrete problems in physics, when it seemed to physicists that most of their problems could be solved without the application of sophisticated and abstract modern mathematics. The development of pure mathematics in the period between the two world wars, and especially in the post-World War II period, was also characterized by weak connections with applied science, in particular with physics. This association was especially true of the areas of mathematics in which many Fields medalists worked. It was difficult to imagine that the concepts of sheaf, étale cohomology, J-functor, and the like would ever be applied in physics. It was still more difficult to imagine that physics could assist algebraic topology and geometry.

This point of view was widespread. The French mathematician Jean Dieudonné, one of the founders of Bourbaki, expressed himself unambiguously on this subject in 1962. "I would like to stress how little recent history has been willing to conform to the pious platitudes of the prophets of doom who regularly warn us of the dire consequences that mathematics is bound to incur by cutting itself off from applications to other sciences. I do not intend to say that close contact with other fields, such as theoretical physics, is not beneficial to all parties concerned; but it is perfectly clear that of all the striking progress I have been talking about, not a single one, with the possible exception of distribution theory, had anything to do with physical applications." (Quoted from an address delivered at the University of Wisconsin in 1962, in which Dieudonné gave a survey of the achievements of the preceding decade in pure mathematics. He emphasized algebraic topology, algebraic geometry, complex analysis, and algebraic number theory.) But as often happens with globally expressed opinions, the situation underwent a vast change ten years later.

At the begining of the 70's, both in mathematics and physics, results were obtained that absolutely changed this point of view. Among the mathematicians who quickly understood the new opportunities and challenges hidden in the new physics were some Fields laureates. It is enough to mention S. Novikov, S. T. Yau, A. Connes, S. Donaldson, and E. Witten. Witten was the first physicist to be awarded a Fields medal. Among the results of Fields laureates which were inspired by physical ideas, let us mention first of all the work of Simon Donaldson. After the work of Milnor on differential structures on S^7, the paper of Donaldson appearing in 1983 had a similar striking impact. Donaldson proved the existence of different differential structures on simply-connected 4-dimensional manifolds. (Unfortunately the case of S^4 is not covered by his method and is still open.) Immediately after the work of Donaldson, in papers of R. Gompf and C. Taubes, the following remarkable result was proved: There exist an infinite number of different differential structures on R^4. This result, that the "well-known" space R^4 hides such deep structures, is absolutely astonishing. It has very deep consequences for quantum gravity, where integrating over all metrics and so over different differential structures is necessary. It is not less important than the proof, which is based on earlier discoveries in field theory, mostly in the gauge theory of strong and weak interactions. Such interactions in the world of elementary particles are described by highly nonlinear equations with deep topological properties—the so called Yang-Mills equations. These equations were invented by the physicists C. N. Yang and R. Mills in 1954, but for many years were considered as nonphysical and attracted very little attention from physicists. Only the newest development of the theory of elementary particles—the creation of the theory of weak and strong interactions based on the Yang-Mills equations—led physicists to a deeper study of the structure of these equations.

In the early 70's, the physical-mathematical union lessened the gap in the transmission of information, leading to the final score in this striking mathematical achievement. These and other more recent results led to a new and deeper connection between mathematics and physics. The value of this union for modern mathematics is indispensible and is based on a series of achievements of the first rank. It is enough to mention the results of V. Drinfeld, M. Kontsevich, and many others.

2. Mathematical Succession

The best confirmation of continuity and fruitfulness in the development of mathematics is the solution of deep classical problems left by the previous generations of mathematicians. And here the results of Fields medalists confirm this idea nicely.

The first recipient of the Fields Medal was Jesse Douglas, who solved the classical two-dimensional Plato problem. It is necessary to say that this problem was solved simultaneously by Tibor Rado, but Douglas' solution was considered as deeper and could be applied to higher dimensions.

Mathematicians of this mind-set include the famous number theorists like A. Selberg, K. Roth, and A. Baker. In the latest period, we see this tradition in the works of Gregory Margulis and Pierre Deligne.

The most important result of Margulis is his proof of Selberg's conjecture that a certain class of discrete subgroups of the group of motions of symmetric spaces of higher rank with finite volume is arithmetic. While the conjecture can be stated rather easily, its proof required a virtuoso mastery of the technique of the theory of algebraic groups, use of the multiplicative ergodic theorem, the theory of quasi-conformal mappings, and much more. In recent years Margulis has examined the properties of discrete groups in different and sometimes unexpected areas. By combining ideas from the theory of discrete groups and ergodic theory, he recently solved an old problem of the geometry of numbers: Oppenheim's conjecture on the representation of numbers by indefinite quadratic forms.

Pierre Deligne received the prize for a proof of a conjectures of A. Weil on zeta functions over finite fields. His results are included as a special case of the proof of the classical Ramanujan conjecture.

Ramanujan Conjecture: Consider the parabolic form

$$2\pi^{-12}\Delta(z) = x\Pi_{n=1}^{\infty}(1-x^n)^{24} = \sum_{n=1}^{\infty}\tau_n x^n$$

where $x = \exp(2\pi i z)$. Then $|\tau_p| \leq 2p^{11/2}$ for all primes p.

The proof of Deligne is one of the most brilliant and striking examples of the unity and continuity of mathematics. It is striking in its beauty and complexity, but required the application of the wealth of techniques accumulated in algebraic geometry over preceding years.

The last example which I give, but only to mention in passing to support this thesis, is the proof of the "Moonshine hypothesis" by Richard Borcherds. Here the statement regarding the relations between the coefficients of special modular forms, dimensions of the representations of the Monster group and some infinite-dimensional Kac-Moody algebras led to the proof by applying methods from different fields of mathematics. It was inspired by the recent development of string theory.

3. Zigzags in Mathematics

What I mean are the zigzags of mathematical fashion. I already talked about the domination of three mathematical diciplines in the list of Fields awards. Some reaction to this bias, even beside some objective background, appeared at the two last congresses. The awardees were mathematicians working in more classical fields. Let us mention here Jean Bourgain and Tim Gowers–Banach spaces, harmonic analysis, combinatorics; P-L. Lions–partial differential equations; Jean-Cristoph Yoccoz, Curtis McMullen–dynamical systems, holomorphic dynamics; and the algebraist Efim Zelmanov, who solved the classical restricted Burnside problem. This result capped off an extended period in group theory. J. Bougain and T. Gowers solved several classical problems in the theory of Banach spaces, discovered in very deep structures. J. C. Yoccoz and C. McMullen got important results in the so-called holomorphic dynamics. Here the study of sequences of mappings of complex sets led to the theory of dynamical systems. A typical problem of holomorphic dynamics is to describe the limiting sets of points of the mapping $z \rightarrow R(z)$, where $R(z)$ is a rational function and z is in C or $\overline{C}$. Even the the study of sequences of iterations of such a seemingly simple map as $f_c(z) = z^2 + c$ conceals highly nontrivial results. This theory is placed at the meeting point of many beautiful mathematical theories, such as dynamical systems, Kleinian groups, Fricke-Teichmüller spaces and many others, including computer graphics.

This theory is very remarkable and instructive if you look at it from a historical perspective. Created in the end of the 19th and the beginning of the 20th centuries in the works of the famous mathematicians P. Fatou, P. Montel, and G. Julia, it was seriously forgotten for more than forty years and was restored only in modern times. Now, besides being a very interesting theory, it has a wide field of applications in physics. Let us mention the famous universality law of Feigenbaum which has important applications in turbulance.

The unity of mathematics is shown best with these seemingly simple yet extraordinary complicated examples.

To finish this very sketchy review of some of the achievements of modern mathematics in the light of Fields medals, let me say that the results honored by Fields medals substantially determined the development of mathematics in our time and its laureates are worthly representives of the mathematical community. Whether or not the Fields medal can be compared with Nobel prize, Fields' idea of awarding it to the young has met with complete success.

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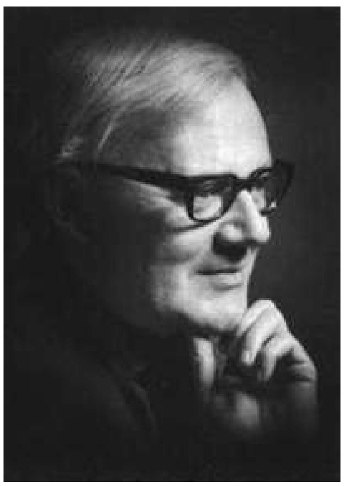
AWARDS / PRIX

2001 CRM-Fields Prize



William T. Tutte

The Centre de recherches mathématiques and the Fields Institute has announced the winner of the CRM-Fields prize for 2001: Professor William T. Tutte from the University of Waterloo.

William T. Tutte is one of the leading experts in graph theory and matroid theory worldwide. He is responsible for some of the most fundamental results in both of these fields; to mention just a couple, in graph theory, he established the fundamental theorems of matching theory, an important branch of combinatorial optimization, and in matroid theory, he characterized regular matroids in terms of excluded minors, one of the deepest results in the field.

Professor Tutte received his education at Cambridge University, graduating with his Ph.D. in 1948. He joined the faculty of the University of Toronto in 1948 and moved to the University of Waterloo in 1962. He was for a long time the editor of the Journal of Combinatorial Theory and is the Honorary Director of the Centre for Applied Cryptographic Research. He is a Fellow of the Royal Societies of London and Canada and was recently made an Officer of the Order of Canada.

The CRM-Fields prize recognizes exceptional achievement in mathematical research conducted primarily in Canada or in affiliation with a Canadian university.

As a CRM-Fields prize winner, Professor Tutte will present a lecture at both Institutes in Fall 2001. For further information, visit the Institute websites: www.fields.utoronto.ca or www.crm.umontreal.ca .

Three Honoured for Outstanding Research

The Canadian Mathematical Society (CMS) has selected Priscilla Greenwood as the winner of the 2002 Krieger-Nelson Prize, Edwin Perkins as the winner of the 2002 Jeffery-Williams Prize and Kai Behrend as the winner of the 2001 Coxeter-James Prize.

CMS 2002 Krieger-Nelson Prize - Dr. Priscilla Greenwood (Arizona State University)

The Krieger-Nelson Prize recognizes outstanding research by a female mathematician.

Dr. Priscilla Greenwood was born in Kansas, USA and obtained her B. A. in mathematics from Duke University in 1959, and her Ph. D from the University of Wisconsin (Madison) in 1963. She held positions at the University of Wisconsin and North Carolina College and, from 1966 to 2000, at the University of British Columbia. In 2000 she moved to Arizona State University. She was elected a Fellow of the Institute of Mathematical Statistics (IMS) in 1985 and served on the IMS Council from 1988-2000.

Over 35 years, Priscilla Greenwood's research career has spanned a broad range of topics in probability and statistics and she has brought together ideas from different fields in a sustained research program. During the past 15 years she has made fundamental contributions to mathematical statistics and, in particular, to the statistics of general stochastic processes. Her continued interest in the applications of probability and statistics led to her leadership of a large interdisciplinary research project at the University of British Columbia (1997 to 2000) which was funded by the Wall Institute and involved dozens of scientists in mathematics, geophysics, psychology, zoology, and physics.

Dr. Greenwood will present the 2002 Krieger-Nelson Prize Lecture at Laval University (Québec) in June 2002.

CMS 2002 Jeffery-Williams Prize - Dr. Edwin Perkins (University of British Columbia)

The Jeffery-Williams Prize recognizes mathematicians who have made outstanding contributions to mathematical research.

Dr. Edwin Perkins was born in Toronto and obtained his B.Sc. in mathematics from the University of Toronto in 1975, and his Ph.D. from the University of Illinois (Urbana) in 1979. He has been at the University of British Columbia since 1979. He was awarded the Rollo Davidson prize for young probabilists in 1983 and the CMS Coxeter-James Prize in 1986. He was elected as a Fellow of the Royal Society of Canada in 1988, and was awarded an NSERC Steacie Fellowship in 1992.

Edwin Perkins has made outstanding contributions to probability theory. Early in his career he pioneered the application of non-standard analysis to probability theory, and proved several spectacular results on the sample paths of Brownian motion. Perkins has been a world leader in the study of "superprocesses", or measure valued diffusions. Using non-standard analysis, he found a particle description of super-Brownian motion which enabled the establishment of very accurate estimates of the space-time behaviour of this process. Recently, with Richard Durrett (Cornell) and Theodore Cox (Syracuse) he has made a very exciting link between particles systems and superprocesses.

Dr. Perkins will give the 2002 Jeffery-Williams Prize Lecture at Laval University (Québec) in June 2002.

CMS 2001 Coxeter-James Prize - Dr. Kai Behrend (University of British Columbia)

The Coxeter-James Prize recognizes young mathematicians who have made outstanding contributions to mathematical research.

Dr. Kai Behrend obtained his M.A. from the University of Oregon in 1984, his Diploma from the University of Bonn in 1989 and his Ph.D. from the University of California at Berkeley in 1991. He was a Moore Instructor at MIT from 1991 to 1994 before joining the staff at the University of British Columbia. He has held visiting positions at the Max-Planck-Institut für Mathematik, Bonn, and at Research Institute for Mathematical Sciences, Kyoto, Japan.

Kai Behrend is one of the world's leading experts in the theory of algebraic stacks and the geometry of moduli spaces of stable maps, which has become one of the most important areas in algebraic geometry because of the unexpected predictions in enumerative algebraic geometry made by physicists based on string theory. Two of his publications, one with Barbara Fantechi (Trento, Italy) are among the most widely cited works in algebraic geometry over the last five years. Kai's recent work on differential graded stacks is an extremely important contribution to the construction of extended moduli spaces.

Dr. Behrend will present the 2001 Coxeter-James Prize Lecture at York University (Toronto) in December 2001.

Fifth Canadian Open Challenge

The annual Canadian Open Mathematics Challenge (COMC) is organized and administered by the Canadian Mathematical Society (CMS) in collaboration with the Centre for Education in Mathematics and Computing (University of Waterloo). Over 4,500 high school students from all across Canada participated in the Fifth Open on November 29, 2000. Students had to solve 12 questions during the two and one-half hour time limit.

The top seven winners in the 5th Open are: Daniel Brox, Sentinel Secondary School, West Vancouver BC; Paul Cheng, West Vancouver Secondary School, West Vancouver BC; Lino Demasi, St. Ignatius High School, Thunder Bay ON; Nima Kamoosi, West Vancouver Secondary School, West Vancouver BC; Roger Mong, Don Mills Collegiate Institute, Don Mills ON; Henry Pan, East York Collegiate Institute, Toronto ON; Feng Tian, Vincent Massey Secondary School, Windsor ON.

The top contestant in each region is selected as a Provincial Gold Medalist. The 2000 Open Provincial Gold Medalists are: Peter Du, Sir Winston Churchill High School, Calgary AB; Daniel Brox, Sentinel Secondary School, West Vancouver BC; Michael Hirsh, St. John's-Ravenscourt School, Winnipeg MN; Ryan Kabir, Saint John High School, Saint John NB; Timothy Hopkins, Queen Elizabeth Regional High School, Foxtrap NF; Jeremy Nicholl, Horton High School, Wolfville NS; Brian Choi, Markville Secondary School Markham ON; Nicolae Petrescu, Lisgar Collegiate Institute, Ottawa ON; Ilinca Popovici, Lisgar Collegiate Institute, Ottawa ON; Wayne Thompson, Earl of March Secondary School, Kanata ON; Henry Pan, East York Collegiate Institute, Toronto ON; Lino Demasi, St. Ignatius High School, Thunder Bay ON; Feng Tian, Vincent Massey Secondary School, Windsor ON; Rui Dong, Marianopolis College, Montréal PQ; Alexandre Gadbois, Champlain St-Lawrence, Ste-Foy PQ; Jordon Wan, Aden Bowman Collegiate Institute, Saskatoon SK.

Approximately 80 of the top students from the 2000 Canadian Open Mathematics Challenge will be invited to write the 2001 Canadian Mathematical Olympiad (CMO). The CMO, Canada's premier mathematics competition, is organized and administered by the CMS. Results in the CMO help determine the six students who will represent Canada at the 2001 International Mathematical Olympiad in Washington DC.

EDUCATION NOTES

Ed Barbeau and Harry White, Column Editors

Associations of Mathematics Teachers in Quebec

There are four associations of mathematics teachers and four "groups of interest" that are also concerned with mathematics teaching. Here is the list:

AMQ: Association mathématique du Québec

GRMS: Groupe des responsables de la mathématique au secondaire

APAME: Association des promoteurs de l'avancement de la mathématique à l'élémentaire

QAMT: Quebec Association of Mathematics Teachers

GCSM: Groupe des chercheurs en science mathématiques

GDM: Groupe de didactique des mathématiques

GMA: Groupe de mathématiques appliquées

MOIFM: Mouvement international pour les femmes et l'enseignement des mathématiques

Each has its particular objectives, but is represented in CQEM (*Conseil québécois de l'enseignement des mathématiques*), a lobbying group for promoting mathematics teaching at the Ministry of Education and School Boards.

In this issue, I will give some information about the AMQ, the oldest association of teachers of mathematics in Quebec.

AMQ: Historique

Le 25 avril 1923 est créé à Montréal un organisme appelé *Société des mathématiques et d'astronomie du Canada* dans le but de faire avancer et encourager les études de mathématiques et d'astronomie. Un mois plus tard, le 23 mai, au Cercle universitaire, a lieu une rencontre en vue de créer une Fédération des sociétés savantes canadiennes-françaises qui deviendra *l'Association canadienne-français pour l'avancement des sciences* (ACFAS). D'autres groupes verront le jour tels *la Société de physique et de mathématiques de Montréal* et *la Société de mathématiques de Montréal* dans les années 1940, de même que *la Société de mathématiques de Québec* animé par Adrien Pouliot de 1929 jusqu'à la fin des années 1950, avant d'en arriver à la fondation de l'AMQ le 5 juin 1958.

Les 125 premiers membres provenaient de toutes les régions du Québec. La moitié appartenait à des communautés religieuses qui ont beaucoup fait pour l'enseignement des mathématiques au Québec. Le premier *Bulletin de l'AMQ* (journal officiel de l'AMQ) est paru en avril 1959, le premier congrès a eu lieu le 16 avril 1959, et le premier concours mathématique, le 16 mai 1959. La Société mathématique du Canada avait offert deux prix de 100$ pour ce concours.

Dans les années 60, l'AMQ est incorporée comme une société à but non lucratif, et propose un plan de recyclage des enseignants et enseignantes qui sera à l'origine des programmes de perfectionnement des maîtres en mathématiques. Au début des années 70, le GDM (*groupe de didactique des mathématiques*) est fondé et devient un groupe d'intérêt de l'AMQ. Il en sera de même pour le GRMS (groupe des responsables de la mathématique au secondaire) et de l'APAME (association des promoteurs de l'avancement de la mathématique à l'élémentaire) qui se préoccupent de façon plus spécifique de la cause de l'enseignement des mathématiques à l'ordre secondaire et à l'ordre primaire. Le GCSM (*groupe des chercheurs en sciences mathématiques*), un autre groupe d'intérêt de l'AMQ, voit le jour en 1976. Ce groupe est à l'origine des Annales des sciences mathématiques du Québec, une revue internationale de recherche en mathématiques.

D'autres développements dignes de mention apparaîtront au cours des années : les Camps mathématiques pour les élèves de l'ordre collégial qui obtiennent les meilleurs résultats au concours mathématique, et depuis l'an dernier, il y a un camp mathématique pour les élèves de l'ordre secondaire ; de même, des prix seront institués pour souligner des contributions remarquables : *le prix Abel-Gauthier* pour la personnalité de l'année dans le domaine des mathématiques, *le prix Roland-Brossard* pour le meilleur article publié dans le Bulletin, *le prix Adrien-Pouliot* pour le meilleur livre édité, *le prix Frère-Robert* pour le meilleur matériel non édité, et *le prix Dieter-Lunkenbein* pour la meilleure thèse en didactique des mathématiques. Le CQEM (*conseil québécois de l'enseignement des mathématiques*) verra le jour au début des années 90. Cet organisme a produit plusieurs avis et a servi de table de concertation pour les diverses associations et les différents groupes qui en font partie. L'année mondiale de mathématiques fut l'occasion d'un méga-congrès organisé conjointement par toutes les associations et les groupes d'intérêt associés.

Références

Bulletin AMQ (octobre 1982) et conférence de Bernard Courteau lors du 40e congrès de l'AMQ.

Buts de l'AMQ

Regrouper en association les personnes intéressées aux mathématiques.

Contribuer à l'étude des mathématiques et au progrès de son enseignement.

S'engager pédagogiquement et socialement dans la défense et la promotion de mathématiques au Québec.

Organiser et tenir des conférences, réunions, assemblées, expositions pour la promotion, le développement et la vulgarisation des mathématiques.

Imprimer, éditer, distribuer toute publication pour les fins ci-dessus, et établir en bibliothéque des publications se rapportant ou connexes aux mathématiques.

Veiller aux intérêts de ses membres.

Site web: http://www.Mlink.NET/∼amq/AMQ/index.html

Harry White

A message from the Pacific

In December, at the Vancouver meeting, I had the pleasure to meet Kanwal Neel, currently president of the British Columbia Association of Mathematics Teachers (website: `www.bctf.bc.ca/bcamt`), who was one of the speakers at the education session. His organization has been very active, particularly in informing the general public about numeracy and how it can be fostered in the classroom. In the following article, he describes the initiatives taken in his province and how they are in accord with the goals enunciated by NCTM. (EJB)

Numeracy Initiatives in British Columbia

Almost everyone knows what math is but what is numeracy? Defined by the British Columbia Association of Mathematics Teachers (BCAMT) as “the combination of mathematical knowledge, problem solving and communication skills required by all persons to function successfully within our technological world." Numeracy is a significant part of literacy.

Over the past few years, numeracy has been a topical issue with parents, teachers, Ministry of Education, NCTM, media, and other organizations.

In order to put theory into practice the BCAMT has launched a number of numeracy initiatives such as producing a brochure, presenting “Numeracy Workshops" to a wide range of audiences such as students, parents, teachers, administrators, business, media, etc. BCAMT’s initiatives have also influenced the Ministry of Education’s report on the “Mathematics Task Force" and the “BC Numeracy Performance Standards."

Our numeracy initiatives are aligned with the National Council of Teachers of Mathematics (NCTM) *Principles and Standards for School Mathematics 2000*. Overall, the NCTM 2000 Standards documents advocate a broader and more meaningful mathematics curriculum that is responsive to changing societal priorities and to changes in instructional practice that meet the needs of a far greater proportion of the student population than has been true in the past. The six Principles and ten Standards constitute a vision to guide educators as they strive for the continual improvement of mathematics education in classrooms, schools, and educational systems.

The six Principles for school mathematics address overarching themes; they are:

The Equity Principle: Excellence in mathematics education requires equity, high expectations and strong support for all students.

The Curriculum Principle: A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades.

The Teaching Principle: Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.

The Learning Principle: Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.

The Assessment Principle: Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.

The Technology Principle: Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students’ learning.

The ten Standards describe what mathematics instruction should enable students to know and do. They specify the understanding, knowledge, and skills that students should acquire from pre kindergarten through grade 12.

The five Content Standards are:

Standard 1: Number and Operations
Standard 2: Algebra
Standard 3: Geometry
Standard 4: Measurement
Standard 5: Data Analysis and Probability

The five Process Standards are:
Standard 6: Problem Solving
Standard 7: Reasoning and Proof
Standard 8: Communication
Standard 9: Connections
Standard 10: Representation

Senses of Numeracy We believe, to be numerate, an individual should possess a variety of mathematical skills, knowledge, attitudes, abilities, understanding, intuition and experience which could be expressed as the following senses:
Spatial Sense

Statistical Sense
Sense of Relationship
Number Sense

Teaching Numeracy. There is a direct association between students' attitudes and their levels of success. The mathematics classroom should be an environment that engages the interest and imagination of the learners. Learning activities should help students understand that mathematics is a changing and evolving subject to which many cultural groups have contributed. Teaching strategies should also place mathematics in context allowing students to see how understanding one mathematical idea can help them understand others.

Early Numeracy Project. Dr. Heather Kelleher of the Department of Curriculum Studies, Faculty of Education, UBC has embarked on a three year *Early Numeracy Project*. This project proposes to address the best possible start for those children most at risk in mathematics. We believe that a successful start in mathematics is essential to future success and that early attention is preferable to later remediation.

Future direction and Challenges. We want our graduates to enter the work force with a STRONG sense of numeracy and the ability to effectively communicate their understanding. They should be able to use estimation skills, solve problems, and engage in effective reasoning. Our challenge as mathematics educators is to provide all students with these qualities. In order to achieve this we offer the following suggestions:

Specific numeracy standards should be listed that represent the minimum level of numeracy required for high school graduates.

Minimum acceptable learning outcomes or stages of numeracy at grades 4, 7, and 10 also need to be outlined. The intent is not to create standardized tests but instead, performance indicators at these grade levels.

Multiple forms of assessment with the integration of technology should be used to assess students' numeracy level.

School mathematics experiences should provide a sufficiently broad understanding so that graduates will be prepared to pursue a variety of careers in their lifetime.

Establish what constitutes Early Numeracy and develop support materials for the at-risk learners and provide a companion program for parents.

Kanwal Neel, President, BCAMT

Millenium Prize Problems

To celebrate mathematics in the new millenium, the Clay Mathematics Institute (www.claymath.org) has identified seven old and important questions that have resisted all past attempts to solve them. The Board of Directors of CMI have designated a $7 million prize fund for the solution to these problems, with $1 million allocated to each.

The P versus NP Problem

It is Saturday evening and you arrive at a big party. Feeling shy, you wonder whether you already know anyone in the room. Your host proposes that you must certainly know Rose, the lady in the corner next to the dessert tray. In a fraction of a second you are able to cast a glance and verify that your host is correct. However, in the absence of such a suggestion, you are obliged to make a tour of the whole room, checking out each person one by one, to see if there is anyone you recognize. This is an example of the general phenomenon that generating a solution to a problem often takes far longer than verifying that a given solution is correct. Similarly, if someone tells you that the number 13,717,421 can be written as the product of two smaller numbers, you might not know whether to believe him, but if he tells you that it can be factored as 3607 times 3803 then you can easily check that it is true using a hand calculator. One of the outstanding problems in logic and computer science is determining whether questions exist whose answer can be quickly checked (for example by computer), but which require a much longer time to solve from scratch (without knowing the answer). There certainly seem to be many such questions. But so far no one has proved that any of them really does require a long time to solve; it may be that we simply have not yet discovered how to solve them quickly. Stephen Cook formulated the P versus NP problem in 1971.

–Ian Stewart ans Stephen Cook

The Hodge Conjecture

In the twentieth century mathematicians discovered powerful ways to investigate the shapes of complicated objects. The basic idea is to ask to what extent we can approximate the shape of a given object by gluing together simple geometric building blocks of increasing dimension. This technique turned out to be so useful that it got generalized in many different ways, eventually leading to powerful tools that enabled mathematicians to make great progress in cataloging the variety of objects they encountered in their investigations. Unfortunately, the geometric origins of the procedure became obscured in this generalization. In some sense it was necessary to add pieces that did not have any geometric interpretation. The Hodge conjecture asserts that for particularly nice types of spaces called projective algebraic varieties, the pieces called Hodge cycles are actually (rational linear) combinations of geometric pieces called algebraic cycles.

–Pierre Deligne

The Poincaré Conjecture

If we stretch a rubber band around the surface of an apple, then we can shrink it down to a point by moving it slowly, without tearing it and without allowing it to leave the surface. On the other hand, if we imagine that the same rubber band has somehow been stretched in the appropriate direction around a doughnut, then there is no way of shrinking it to a point without breaking either the rubber band or the doughnut. We say the surface of the apple is "simply connected," but that the surface of the doughnut is not. Poincaré, almost a hundred years ago, knew that a two dimensional sphere is essentially characterized by this property of simple connectivity, and asked the corresponding question for the three dimensional sphere (the set of points in four dimensional space at unit distance from the origin). This question turned out to be extraordinarily difficult, and mathematicians have been struggling with it ever since.

–John Milnor

The Riemann Hypothesis

Some numbers have the special property that they cannot be expressed as the product of two smaller numbers, e.g., 2, 3, 5, 7, etc. Such numbers are called prime numbers, and they play an important role, both in pure mathematics and its applications. The distribution of such prime numbers among all natural numbers does not follow any regular pattern, however the German mathematician G.F.B. Riemann (1826 – 1866) observed that the frequency of prime numbers is very closely related to the behavior of an elaborate function $\zeta(s)$ called the Riemann Zeta function. The Riemann hypothesis asserts that all interesting solutions of the equation $\zeta(s) = 0$ lie on a straight line. This has been checked for the first 1,500,000,000 solutions. A proof that it is true for every interesting solution would shed light on many of the mysteries surrounding the distribution of prime numbers.

–Enrico Bombieri

Yang–Mills Existence and Mass Gap

he laws of quantum physics stand to the world of elementary particles in the way that Newton's laws of classical mechanics stand to the macroscopic world. Almost half a century ago, Yang and Mills discovered that quantum physics reveals a remarkable relationship between the physics of elementary particles and the mathematics of geometric objects. Predictions based on the Yang-Mills equation have been verified in high energy experiments performed at laboratories all over the world: Brookhaven, Stanford, CERN, and Tskuba. Nonetheless, there are no known solutions to their equations which both describe massive particles and are mathematically rigorous. In particular, the "mass gap" hypothesis, which most physicists take for granted and use in their explanation for the invisibility of "quarks," has never received a mathematically satisfactory justification. Progress on this problem will require the introduction of fundamental new ideas both in physics and in mathematics.

–Arthur Jaffe and Edward Witten

Navier–Stokes Existence and Smoothness

Waves follow our boat as we meander across the lake, and turbulent air currents follow our flight in a modern jet. Mathematicians and physicists believe that an explanation for and the prediction of both the breeze and the turbulence can be found through an understanding of solutions to the Navier–Stokes equations. Although these equations were written down in the 19th Century, our understanding of them remains minimal. The challenge is to make substantial progress toward a mathematical theory which will unlock the secrets hidden in the Navier–Stokes equations.

–Charles Fefferman

The Birch and Swinnerton–Dyer Conjecture

Mathematicians have always been fascinated by the problem of describing all solutions in whole numbers x,y,z to algebraic equations like $x^2 + y^2 = z^2$. Euclid gave the complete solution for that equation, but for more complicated equations this becomes extremely difficult. Indeed, in 1970 Yu V. Matiyasevich showed that Hilbert's tenth problem is unsolvable, i.e., there is no general method for determining when such equations have a solution in whole numbers. But in special cases one can hope to say something. When the solutions are the points of an abelian variety, the Birch and Swinnerton-Dyer conjecture asserts that the size of the group of rational points is related to the behavior of an associated zeta function $\zeta(s)$ near the point s=1. In particular this amazing conjecture asserts that if $\zeta(1)$ is equal to 0, then there are an infinite number of rational points (solutions), and conversely, if $\zeta(1)$ is not equal to 0, then there is only a finite number of such points.

–Arthur Wiles

UNIVERSITY OF VICTORIA – VICTORIA, BRITISH COLUMBIA
DEPARTMENT OF MATHEMATICS AND STATISTICS
Position in Discrete Mathematics

The University of Victoria, Department of Mathematics and Statistics, invites applications for a tenure-track position in Discrete Mathematics. The competition is open to applicants at any level. The successful applicant will be sponsored for an NSERC University Faculty Award in the 2001-2002 competition, consequently only applications from candidates meeting the criteria for this program will be considered. We quote from the NSERC eligibility requirements: "you must be a Canadian citizen, or permanent resident of Canada, as of the nomination deadline date; and be a woman or Aboriginal person who holds a doctorate in one of the fields of research that NSERC supports; or expect to have completed all the requirements for such a degree, including your thesis defense, by the proposed date of appointment". Applicants should have a Ph.D. in Mathematics and a demonstrated record of, or the potential for, excellence in both research and teaching. The successful applicant will be expected to maintain a strong and active research program in Discrete Mathematics which includes the supervision of graduate students.
Applicants should send a curriculum vitae, and arrange for three confidential letters of recommendation to be sent to:

Discrete Mathematics Search
Department of Mathematics and Statistics
University of Victoria
PO Box 3045 STN CSC
Victoria BC V8W 3P4 Canada
Tel: (250) 721-7436
Fax: (250) 721-8962
e-mail: acme@math.uvic.ca
Website: http://www.math.uvic.ca/

The closing date for applications is **May 15, 2001.** The position may be taken up any time between April 1, 2002, and September 1, 2002.
For further information on NSERC University Faculty Awards, visit the NSERC website www.nserc.ca.
In accordance with the University's Equity Plan and pursuant to Section 42 of the B.C. Human Rights Code, the selection will be limited to women and aboriginal peoples. Candidates from these groups are encouraged to self-identify.
In accordance with Canadian immigration requirements, this advertisement is directed to Canadian citizens and permanent residents.

(EDITORIAL–continué de page 2)

S. S.: C'est difficile à comparer. Ça dépend de votre aptitude et de votre attitude. Je constate que vous avez beaucoup de facilité en mathématiques, même si vous réussissez très bien dans les deux matières. De plus, comme les étudiants sont nombreux à opter pour l'informatique, la concurrence sera donc plus féroce dans ce domaine.

Étudiant : L'informatique est très utile dans une société axée sur la technologie comme la nôtre, ce qui n'est pas le cas des mathématiques.

S. S.: L'utilité des mathématiques pour la société n'est généralement pas bien comprise. En fait, les mathématiques jouent un rôle considérable dans la société à plus d'un égard. Je vous donne quelques exemples. En février, presque exactement deux cents ans après la découverte du premier astéroïde, la sonde NEAR (Near Earth Asteroid Rendezvous), lancée de Cape Canaveral (Floride) en 1996, a fait un atterrissage forcé mais en douceur sur l'astéroïde Eros, en orbite à une vitesse vertigineuse, à 315 millions de kilomètres de la Terre. La sonde, qui s'est posée sur la surface accidentée de l'astéroïde, était commandée de la Terre par des scientifiques qui ont basé leurs calculs sur des processus mathématiques complexes. Par ailleurs, le public reconnaît davantage l'importance des mathématiques depuis qu'Andrew Wiles a réussi à prouver le dernier théorème de Fermat. De plus, la pièce de théâtre *Proof*, dont trois des quatre personnages étaient des mathématiciens, a été bien reçue à New York à l'automne 2000. La pièce a été suivie d'un symposium à l'Université de New York au cours duquel trois groupes de mathématiciens et de non-mathématiciens ont discuté des thèmes et des idées abordés dans la pièce. Et je pourrais en dire encore long sur l'utilité des mathématiques, mais j'imagine que ça suffit pour l'instant.

Étudiant : On me dit que les informaticiens font beaucoup d'argent. Est-ce qu'un mathématicien peut devenir riche?

S. S.: C'est vrai que les informaticiens sont mieux payés tant dans le milieu universitaire qu'en entreprise. Un mathématicien peut devenir millionnaire s'il parvient à résoudre l'un des problèmes de Clay. Problèmes à la page 18.

(EXEC–continued from page 1)

Office staff provides a lot of administrative support, the success of the meetings in Hamilton (June 2000) and Vancouver (December 2000) was also due to the extensive help received from the meeting directors, the local organizers and the numerous sessions organizers. The CMS is always indebted to the host university, to other participating universities and, through the National Program Committee, to the three research institutes for their indispensable support.

A highly successful meeting program together with ongoing expansion of our publications and educational programs contributed to a very productive year. The Society also organized and supported, via a grant from the CMS Endowment Fund, a number of special activities to help celebrate World Math Year 2000. These activities ranged from posters in the Montreal Transit System to museum exhibits in Sherbrooke and Regina, and from special lectures at provincial math association meetings to regional Math Camps. The CMS is grateful to all of those who helped make these activities a reality and who contributed largely to their success.

Our publication activities continue to be of a high standard and, in 2000, all periodicals were shipped on-time with issues available to subscribers on Camel approximately one week before the shipping date. With all CMS periodicals available online for both individual and institutional subscribers, more work is required of the Executive Office to ensure the necessary access information is provided to Camel in a timely and accurate manner. The Executive Office, in conjunction with those responsible for our web site, are continually trying to find ways to streamline the process, to provide subscribers with improved ways to update their information, and to improve the accounting system that provides electronic access to each subscriber.

The new CMS Book Series with Springer-Verlag is progressing well and the first four books in the Series appeared in 2000. The new agreement with the American Mathematical Society to publish the CMS Tracts in Mathematics has been signed and work commenced to attract and publish books to this new series. “A Taste of Mathematics” (ATOM) – a series of work booklets for high school students – continues to be well received and is a series that has a large market potential. All of the Society’s book and periodicals must be promoted more extensively so that each receives the necessary exposure and market potential.

Through the Canadian International Mathematical Olympiad team, the 2000 Canadian Mathematical Olympiad, the 2000 Canadian Open Mathematics Challenge, the CMS problem solving journal (CRUX with MAYHEM), special education sessions at our semi-annual meetings, public lectures, regional and national and Math Camps, and other activities, the CMS provides a wide array of educational enrichment activities. These activities are only possible because of the significant support received from provincial governments, corporations, foundations and CMS members. The 2000 Esso Math Camps program was extremely successful. Many of the students who participated have written to express their enthusiasm for the program and indicated how much they valued the opportunity to participate in this unique program. In 2001, it is hoped the program will include Math Camps at Simon Fraser University and Memorial University of Newfoundland. It is extremely encouraging that, in only three years, the program will have grown to provide a Math Camp in almost every province.

In June 2000, Richard Kane (Western) ended his term as CMS President and Jonathan Borwein (SFU) assumed the position as President. Richard’s leadership and direction contributed significantly to the development of the Society during his term. The continual growth of the Society impacts considerably on the work required by committee members, editors, organizers, directors, and particularly the President. The Society’s enviable national and international reputation would not be possible without the efforts and guidance of all of these individuals. Many thanks to all of those who helped make 2000 a most successful year.

NEWS FROM DEPARTMENTS

Université Laval, Québec, Québec

Appointment: Daniel Le Roux (assistant professor, applied mathematics, January 2001).

Retirement: Norbert Lacroix (July 2001).

GEORGE F. D. DUFF 1926-2001

The *CMS Notes* has been informed that Professor George Duff, a former president of the Society and a member of the University of Toronto faculty since 1952, passed away on Friday, March 2nd. Please see next month’s issue for an obituary and further information about his life and career.

DU BUREAU DU DIRECTEUR ADMINISTRATIF

Rapport annuel 2000

En mars 1998, sous la gouverne de la présidente de l'époque, Katherine Heinrich, la Société a entrepris un examen approfondi de ses activités et de son fonctionnement. Richard Kane, président de 1998 à 2000, et Jonathan Borwein, le président actuel, ont à tour de rôle supervisé cet important exercice de planification stratégique. Les rapports finals de sept groupes de travail (Budget et politique, Représentation au Conseil, Fonds de dotation de la SMC, Publications, Finances et collecte de fonds, Soutien à la communauté mathématique et Stratégies administratives) et d'un comité spécial (Services électroniques et Camel) sont désormais publiés sur Camel (www.cms.math.ca/Projects/).

On a ensuite demandé au Comité exécutif d'étudier tous les rapports et les recommandations qu'ils contenaient, puis d'élaborer une structure et une stratégie d'ensemble pour les années à venir. Au cours de l'année 2000, chacun des comités permanents, ainsi que l'ensemble des membres de la Société, ont eu l'occasion de commenter les rapports et les recommandations. Le Comité exécutif a déjà enclenché l'examen des commentaires et présentera ses conclusions au Conseil d'administration en juin 2001. Nous avons consacré beaucoup d'énergie à cet exercice de planification stratégique, et même si le tout n'est pas encore terminé, plusieurs changements ont déjà été apportés. À la fin du processus, la SMC se sera donné les outils nécessaires pour mener ses activités et offrir des services de la façon la plus efficace, efficiente et rentable possible.

Comme on le constate à la lecture des rapports annuels 2000 des comités permanents, la Société soutient une vaste gamme d'activités éducatives et scientifiques, et tout un programme de publication. Toutes ces activités sont rendues possibles grâce à l'apport remarquable des membres de la Société et d'autres personnes de l'extérieur.

Au cours des dernières années, le nombre de séances tenues à nos Réunions semestrielles a considérablement augmenté, tout comme le nombre de participants à ces rencontres. Nous devons sans contredit le succès des Réunions de Hamilton (juin 2000) et de Vancouver (décembre 2000) au personnel du bureau administratif, qui joue un important rôle de soutien, mais également au travail monstre accompli par les directeurs des Réunions, les organisateurs locaux et les nombreux responsables de séances. La SMC doit aussi une fière chandelle aux universités hôtes, aux autres universités participantes et, par l'entremise du Comité du programme national, aux trois instituts de recherche, pour leur soutien inestimable.

En somme, comme en font foi le vif succès remporté par nos Réunions et l'intensification continuelle de nos programmes éducatifs et de nos activités de publication, la SMC aura connu une année très productive. La Société a par ailleurs organisé et financé, par l'entremise d'une subvention de son Fonds de dotation, un certain nombre d'activités spéciales dans le cadre de l'année internationale des mathématiques. Au nombre de ces activités, mentionnons la réalisation d'affiches destinées au réseau de transport de la communauté urbaine de Montréal; des expositions dans des musées de Sherbrooke et de Regina, ainsi que des conférences spéciales présentées à l'occasion de congrès d'associations provinciales de mathématiques et des camps de mathématiques régionaux. La SMC tient à remercier toutes les personnes qui ont contribué à la concrétisation et au succès de ces entreprises.

Côté publications, nos productions sont toujours d'excellente qualité. De surcroît, tous nos périodiques ont été envoyés aux dates prévues cette année, et les abonnés aux versions en ligne ont eu accès à leurs numéros environ une semaine avant la date d'expédition des versions papier. Toutefois, depuis que tous nos périodiques sont accessibles en ligne, tant pour les abonnés à titre individuel que les établissements, la tâche de travail du personnel du bureau administratif a sensiblement augmenté. Il faut en effet voir à ce que les abonnés aient tous les renseignements nécessaires pour accéder à Camel, et cela de la façon la plus efficace possible. Le bureau administratif, en collaboration avec les responsables du site Web, tente toujours de trouver des moyens d'alléger le processus, de faciliter aux abonnés la mise à jour de leur compte et d'améliorer le système qui donne accès aux publications en ligne à chaque abonné.

La nouvelle collection d'ouvrages de la SMC chez Springer-Verlag progresse rapidement. Les quatre premiers ouvrages sont d'ailleurs parus en 2000. Une nouvelle entente avec la American Mathematical Society concernant la publication de la collection Traités de mathématiques de la SMC a été signée, et nous avons déjà entrepris les démarches pour attirer les auteurs et publier des ouvrages dans cette nouvelle collection. Quant à notre collection ATOM - Aime-t-on les mathématiques, livrets destinés aux élèves du secondaire, elle continue de recevoir des éloges. Cette collection a d'ailleurs un fort potentiel commercial. Ajoutons que tous les ouvrages et périodiques de la Société devront faire l'objet d'une promotion accrue si l'on souhaite les faire connaître davantage à tous les clients potentiels.

La SMC offre par ailleurs une vaste gamme d'activités d'enrichissement. L'équipe canadienne qui participe à l'Olympiade internationale de mathématiques, l'Olympiade de mathématiques du Canada 2000, le Défi ouvert canadien de mathématiques 2000, le journal de résolution de problèmes de la SMC (CRUX with MAYHEM), des séances spéciales portant sur l'éducation dans le cadre des Réunions semestrielles, des conférences publiques ainsi que des camps mathématiques régionaux et nationaux sont au nombre des activités qu'elle appuie ou qu'elle organise. Encore une fois, ces activités doivent leur réalisation à l'appui considérable des gouvernements provinciaux, de sociétés privées, de fondations et des membres de la SMC. Mentionnons aussi que les camps de mathématiques Esso 2000 ont connu un vif succès. Un grand nombre de participants nous ont écrit pour nous faire part de leur enthousiasme envers ce programme unique et ont souligné à quel point ils était heureux d'avoir eu la chance d'y prendre part. En 2001, on espère élargir le programme et offrir des camps aux universités Simon Fraser et Memorial (Terre-Neuve).Il est très stimulant de constater qu'après trois ans seulement, un camp de mathématiques sera offert dans presque chacune des provinces.

En juin 2000, Richard Kane (Western) a terminé son mandant à la présidence de la SMC, et Jonathan Borwein (Simon Fraser) a pris la relève. Sous la direction de Richard, la Société a fait de grands pas. Évidemment, la croissance continue de la SMC se répercute sur la charge de travail des comités, des équipes de rédaction, des organisateurs, des dirigeants et, en particulier, du président. La Société n'aurait pas la réputation enviable dont elle jouit maintenant tant au pays qu'à l'étranger sans la contribution et l'appui de toutes ces personnes. Un grand merci à tous ceux et celles qui ont fait de l'an 2000 une année si réussie!

CALL FOR NOMINATIONS / APPEL DE CANDIDATURES

Coxeter-James / Jeffery-Williams / Krieger-Nelson Prize Lectureships

Prix de conférence Coxeter-James / Jeffery-Williams / Krieger-Nelson

The CMS Research Committee is inviting nominations for three prize lectureships.

The Coxeter-James Prize Lectureship recognizes outstanding young research mathematicians in Canada. The selected candidate will deliver the prize lecture at the Winter 2001 Meeting in Toronto, Ontario. Nomination letters should include at least three names of suggested referees.

The Jeffery-Williams Prize Lectureship recognizes outstanding leaders in mathematics in a Canadian context. The prize lecture will be delivered at the Summer 2002 Meeting in Québec, Québec. Nomination letters should include three names of suggested referees.

The Krieger-Nelson Prize Lectureship recognizes outstanding female mathematicians. The prize lecture will be delivered at the Summer 2002 Meeting in Québec, Québec. Nomination letters should include three names of suggested referees.

The deadline for nominations is **September 1, 2001**. Letters of nomination should be sent to the address below:

*****

Le Comité de recherche de la SMC invite les mises en candidatures pour les trois prix de conférence de la Société, la Conférence Coxeter-James, la Conférence Jeffery-Williams et la Conférence Krieger-Nelson.

Le prix Coxeter-James rend hommage à l'apport exceptionnel des jeunes mathématiciens au Canada. Le candidat choisi présentera sa conférence lors de la réunion d'hiver 2001 à Toronto (Ontario). Les lettres de mises en candidatures devraient inclure les noms d'au moins trois répondants possibles.

Le prix Jeffery-Williams rend hommage à l'apport exceptionnel des mathématiciens d'expérience au Canada. La Conférence sera présentée lors de la réunion d'été 2002 au Québec, (Québec). Les lettres de mises en candidature devraient inclure les noms d'au moins trois répondants possibles.

Le prix Krieger-Nelson rend hommage à l'apport exceptionnel des mathématiciennes au Canada. La Conférence sera présentée lors de la réunion d'été 2002 au Québec, (Québec). Les lettres de mises en candidatures devraient inclure les noms d'au moins trois répondants possibles.

La date limite pour les mises en candidatures est **le 1 septembre 2001**. Les lettres de mises en candidatures devraient être envoyées à :

Douglas Stinson, CMS Research Committee / Comité de recherche de la SMC
Department of Pure Mathematics,
University of Waterloo
200 University Ave West, Waterloo,ON
Canada N2L 3G1

CMS Summer Meeting 2001
University of Saskatchewan
Saskatoon, Saskatchewan
June 2-4, 2001

Programme Update

The most up-to-date information concerning the programmes, including scheduling, and electronic registration is available at the following world wide web address:

http://www.cms.math.ca/Events/summer01

Meeting registration forms and hotel accommodation forms can be found in the February 2001 issue of the *CMS Notes* and are also available on the website, along with on-line forms for registration and submission of abstracts.

Updates on Symposia Speakers

There have been a number of additions to the list of invited speakers. Please refer to the web site for the most up-to-date information.

Abstracts will also appear on the web site as they become available.

Réunion d'été 2001 de la SMC
Université de la Saskatchewan
Saskatoon (Saskatchewan)
2-4 juin 2001

Mise à jour du programme

Vous trouverez l'information la plus récente sur les programmes, y compris les horaires et le formulaire d'inscription éléctronique, à l'adresse Web suivante :

http://www.cms.math.ca/Events/summer01

Les formulaires d'inscription et de réservation d'hôtel seront aussi publiés dans le numéro de février 2001 des *Notes de la SMC*. Vous les trouverez également sur notre site web, ainsi que les formulaires de résumés de conférences.

Liste de conférenciers

Il y a eu quelques additions à la liste de conférenciers. Veuillez consulter le site Web pour l'information la plus récente.

Les résumés de conférences paraîtront sur le site dès que nous les recevrons.

Coffee will be provided during the official coffee breaks and will be staggered throughout the rest of the programme.
Du café sera servi durant les pauses à l'horaire et de temps à autres pendant le reste du programme.

#	Session	Room / Salle
	Registration and Exhibits / Inscription et expositions	Rooms TBA
1	Abstract Harmonic Analysis / Analyse harmonique abstraite	
2	Geometric Topology / Topologie géometrique	
3	Graph Theory / Théorie des graphes	
4	Infinite dimensional Lie theory and representation theory / Théorie de Lie en dimension infinie et théorie des représentations	
5	Mathematical Education Cognition in Mathematics / Enseignement des mathématiques Cognition et mathématiques	
6	Matrix Analysis / Analyse matricielle	
7	Model theoretic algebra / Algèbre en théorie des modèles	
8	Number Theory - in honour of David Boyd / Théorie des nombres - en l'honneur de David Boyd	
9	Rigorous studies in the statistical mechanics of lattice models / Études rigoureuses dans la mécanique statistique des modèles de réseaux	
10	Scattering theory and integrable systems / Diffusion inverse et systèmes intégrables	
11	Contributed Papers / Communications libres	

CMS Summer Meeting 2001 - Réunion d'été 2001 de la SMC

University of Saskatchewan, Saskatoon, Saskatchewan

For updates, see our website / Pour l'info. plus récente, consulter notre site Web. http://www.camel.math.ca/CMS/Events/summer01/

Time	Thursday / jeudi May 31 mai	Friday / vendredi June 1 juin	Saturday / samedi June 2 juin	Sunday / dimanche June 3 juin	Monday / lundi June 4 juin
8:00am - 5:00pm			Registration / Inscription Exhibits / Expositions	Registration / Inscription Exhibits / Expositions	Registration / Inscription
8:30			Opening / Overture	8:30-11:00 SESSIONS	8:30-9:30 SESSIONS
9:00			GEORGIA BENKART		
9:30					COFFEE/PAUSE CAFÉ
10:00			COFFEE/PAUSE CAFÉ		BARRY SIMON
10:30			10:30-12:00 SESSIONS		
11:00		11am - 1pm CMS Development Group Luncheon (Delta Terrace Lounge)		GEOFFREY GRIMMETT	ZOE CHATZIDAKIS
11:30					
12:00			12:00-2:00 Delegates' Luncheon Lunch des participants	12:00-1:30 Lunch provided / Dîner fourni	12:00-1:30 Lunch provided / Dîner fourni
12:30					
1:00				CMS General Meeting / Assemblée Générale de la SMC	
1:30		1:30-6:30 CMS Board of Directors Meeting / Réunion du Conseil d'administration de la SMC (Delta Battleford Room)		LISA JEFFREY Krieger-Nelson Lecture	1:30-5:30 SESSIONS
2:00			DAVID BOYD Jeffery-Williams Lecture		
2:30				2:30-5:30 SESSIONS	
3:00			3:00-5:30 SESSIONS		
3:30					
4:00					
4:30					
5:00					
5:30					
6:00	6:00-9:00 CMS Executive Committee Meeting / Réunion du Comité éxecutif de la SMC (Delta Terrace Lounge)				
6:30				6:30-7:30 Reception / Réception (Delta Adam Ballroom Foyer)	
7:00		7:00-9:00 Registration/ Inscription Reception / Réception (Delta Terrace Lounge)			
7:30			7:30-8:30 DE WITT SUMNERS Public Lecture	7:30 pm Banquet (Delta Adam Ballroom)	
8:00					
8:30			8:30-10:00 Reception and Graduate Student Poster Session / Présentations des étudiants diplômés		
9:00					
9:30					
10:00					

CALENDAR OF EVENTS / CALENDRIER DES ÉVÉNEMENTS

APRIL 2001 AVRIL 2001

25–26 Workshop on Mathematical Formalisms for RNA Structure, (CRM, Montréal)
http://www.CRM.UMontreal.CA/biomath/

MAY 2001 MAI 2001

18–20 ISM Gradute Student Conference / Colloque ISM des Etudiants Avancés (McGIll University, Montréal)
www.math.uqam.ca/ISM/francais/colloque2001.html

25–26 2001 Seaway Number Theory Conference (Carleton University, Ottawa)
http://www.math.carleton.ca (see Upcoming Events)

25–29 Annual Meeting of the Canadian Mathematics Education Study Group (University of Alberta, Edmonton)
http://cmesg.math.ca

25–29 Workshop on Groups and 3-Manifolds, (CRM, Univ. de Montréal, Québec)
Organizer: S. Boyer (UQAM) boyer@math.uquam.ca

25–27 Annual meeting and special session on French mathematics, Canadian Society for History and Philosophy of Mathematics / Société canadienne d'histoire et de philosophie des mathématiques (Université Laval, Québec)
http://www.cshpm.org

JUNE 2001 JUIN 2001

2–4 CMS Summer Meeting / Réunion d'été de la SMC (University of Saskatchewan, Saskatoon, Saskatchewan)
http://www.cms.math.ca/CMS/Events/summer01

2–5 One Hundred Years of Russell's Paradox (Munich)
http://www.lrz-muenchen.de/ godeherd.link/russell1101.html, Ulrich.Albert @lrz.uni.muenchen.de

4–8 International Conference on Computational Harmonic Analysis (City University of Hong Kong)
malam@cityu.edu.hk

4–13 Hamiltonian Group Actions and Quantization, in the Symplectic Topology, Geometry, and Gauge Theory Program (Fields Institute, Toronto and CRM, Montréal)
http://www.fields.utoronto.ca/symplectic.html

12–17 8th Annual Canadian Undergraduate Mathematics Conference 8e conférence canadienne des étudiants en mathématiques (Laval University, Québec
http://cumc.math.ca or http://ccem.math.ca

JULY 2001 JUILLET 2001

1–14 42nd International Mathematical Olympiad (Washington D.C., USA)
imo2001.usa.unl.edu

9–13 Workshop on Geometric Group Theory, (CRM, Univ. de Montréal, Québec)
Organizer: D. Wise (Brandeis & McGill Univ.) daniwise@brandeis.edu

9–20 Séminaire de mathématiques supérieures NATO Advanced Study Group (Université de Montréal)
http://www.dms.umontreal.ca/sms

16–21 COCOA VII - The Seventh International Conference on Computational Commutative Algebra (Queen's University, Kingston)
A. Geramita (tony@mast.queensu.ca)
http://cocoa.dima.unige.it/

17–21 First Joint International Meeting between AMS and Société Math. de France, History of Math. special session, Tom Archibald (Acadia Univ.)
http:///www.ams.org/meetings/

22–25 International Symposium on Symbolic and Algebraic Computation, (University of Western Ontario, London, Ontario)
http://www.orcca.on.ca/issac2001/

23–Aug.3 Combinatorics and Matrix Theory, (Laramie, Wyoming)
sfallat@math.wm.edu, http://math.uwyo.edu/

AUGUST 2001 AOÛT 2001

7–9 Nordic Conference on Topology and its applications, NORDTOP 2001 (Sophus Lie Centre at Nordfjordeid, Norway)
nordtop2001@mail.mathatlas.yorku.ca

7–10 The 4th Conference on Information Fusion, (CRM, Univ. de Montréal, Québec)
communications@crm.umontreal.ca

12–18 Thirty-ninth International Symposium on Functional Equations (Sandjberg, Denmark, organized by Aarhus University) *Henrik Stetkaer: stetkaer@imf.au.dk*
http://www.imf.au.dk/isfe39

13–15 13th Canadian Conference on Computational Geometry, (University of Waterloo)
http://compgeo.math.uwaterloo.ca/ cccg01

13–15 Second Gilles Fournier Memorial Conference / Seconde Conférence à la mémoire de Gilles Fournier (Université de Sherbrooke, Sherbrooke, Québec)
http://www.dmi.usherb.ca/evenements

15–18 Second Workshop on the Conley Index and related topics / Deuxième atelier sur l'indice de Conley et sujets connexes (Université de Sherbrooke, Sherbrooke, Québec)
http://www.dmi.usherb.ca/evenements

20–23 Second Canada-China Mathematics Congress (Vancouver)
http://www.pims.math.ca/science/2001/canada-china/

SEPTEMBER 2001 SEPTEMBRE 2001

22–26 Applications of Discrete Mathematics, Australian Mathematical Society (Australian National University, Canberra) *Ian Roberts: iroberts@darwin.ntu.edu.au*
or Lynn Batten: lmbatten@deakin.edu.au

DECEMBER 2001 DÉCEMBRE 2001

8–10 CMS Winter Meeting / Réunion d'hiver de la SMC (Toronto Colony Hotel, Toronto, Ontario)
http://www.cms.math.ca/CMS/Events/winter01

MAY 2002 MAI 2002

3–5 AMS Eastern Section Meeting (CRM, Université de Montréal)
http://www.ams.math.org/meetings/

JUNE 2002 JUIN 2002

6–8 CAIMS 2002 (University of Calgary)
Samuel Shen: shen@maildrop.srv.ualberta.ca

15– 17 CMS Summer Meeting / Réunion d'été de la SMC (Université Laval, Québec, Québec)
Monique Bouchard: meetings@cms.math.ca

24–28 Special Activity in Analytic Number Theory (Max Planck Institute, Bonn) *moroz@mpim-bonn.mpg.de*

JULY 2002 JUILLET 2002

22–30 44rd International Mathematical Olympiad (University of Strathclyde, Glasgow, UK0

AUGUST 2002 AOÛT 2002

20–28 International Congress of Mathematicians (Beijing, China) *http://icm2002.org.cn/*

DECEMBER 2002 DÉCEMBRE 2002

8–10 CMS Winter Meeting / Réunion d'hiver de la SMC (University of Ottawa / Université d'Ottawa, Ottawa, Ontario)
Monique Bouchard: meetings@cms.math.ca

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Full Page	$ 215	$ 405	$ 540
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Des suppléments sont applicables pour des places de choix - communiquer avec notes-ads@smc.math.ca			

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December/décembre	October 15 octobre
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Inside page/page intérieure: 7.5 x 10 in/pouces	

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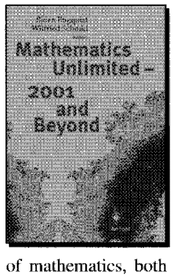

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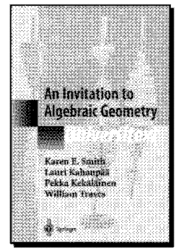
AN INVITATION TO ALGEBRAIC GEOMETRY



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