



# CMS NOTES de la SMC

## FROM THE VICE-PRESIDENT'S DESK

Ed Perkins  
UBC, North Vancouver

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### Kyosi Itô, Inaugural Recipient of the Gauss Prize 2006

Kyosi Itô, now 91 and one of the founders of modern probability, became the first recipient of the Gauss prize for applications of mathematics at the 2006 ICM in Madrid.

While working in relative isolation (to a degree this may be said of all probabilists at the time) at the Statistical Bureau of the Government of Japan, in 1942 Professor Itô published an article [I2] in a mathematics journal of Osaka University. It laid the foundations for much of the field for the next 40 years and contained the groundbreaking ideas for which he was awarded the Gauss Prize.

The relationship between the applied and pure ends of the mathematical spectrum has always been a symbiotic one and Itô's career is a wonderful demonstration of this. In [I4] he notes that as a student he loved both the beauty of pure mathematics but also the interactions between mathematics and mechanics and his introduction to probability was through statistical mechanics. One of his earliest papers (1943) was on turbulence. His groundbreaking work on stochastic integration, stochastic differential equations and stochastic calculus was carried out as a purely mathematical pathwise realization of the ideas of Feller and Kolmogorov. It has formed the basis of a vast range of stochastic models in

filtering theory, population genetics, mathematical finance and statistical physics. In Itô's words [Fu]:

*Because my own research on stochastic analysis is in pure mathematics, the fact that my work has been chosen for the Gauss Prize for applications of mathematics is truly unexpected and deeply gratifying. I hope therefore to share this great honor and joy with my family, teachers, colleagues, and students in mathematics, as well as with all those who took my work in stochastic analysis and extended it to areas far beyond my imagination.*

Prior to 1930, Bachelier (1900) proposed Brownian motion as a model for fluctuations of the stock market, Einstein (1905) won a Nobel prize for his work which used Brownian motion to give experimental confirmation of the atomic theory, and Wiener (1923) gave a mathematically rigorous construction of Brownian motion. (Having to be absolutely certain of every line, we mathematicians place last as usual, but are able to verify that Einstein did earn his Nobel.) Nonetheless, most probabilists would likely pick 1933 as the birth of modern probability. It is the year Kolmogorov's treatise on the foundations of the subject appeared. Itô studied at the University of Tokyo (1935-38) and read Kolmogorov [K], Feller [Fe], Lévy [L], and Doob [D]. Probability as a modern mathematical discipline was in its infancy. His thesis supervisor

at Tokyo was Shokichi Iyanaga. At a ceremony held at Kyoto U. in Sept. 2006 Prof. Itô wrote:

*Today my only regret is that I was not able to share the news of the Gauss prize with my teacher and mentor, Professor Shokichi Iyanaga, who passed away this June at the age of 100. As a mathematics student at the University of Tokyo in the 1930's, I would not have been able to continue my research interests in probability theory, had it not been for his kind and constant encouragements. Professor Iyanaga taught all of his students to pursue their own interests, whether or not these were popular at the time, and whether or not they had any visible potential practical applications.*

As a beginning researcher I had the pleasure of speaking to Prof. Itô at a 1983 Conference at Baton Rouge when Prof. Itô was lecturing on his recent work on SDE's in infinite-dimensional spaces. It was summer in Louisiana and we were staying in unairconditioned dormitory rooms. Many of the senior mathematicians had opted for hotels (as I would certainly do these days myself) but Itô stayed in the dorm rooms and in the evenings would discuss mathematics with many of the perspiring younger participants. When I mentioned that I had studied at U. Illinois and had taken courses on potential theory and measure theory from Prof. Doob, he immediately told me how Doob's works



## TOPOLOGY UP TO ISOMORPHISM?

Last month's editorial described the growing sense among mathematicians that we may have lost too much control over the publication of our work, and some of the initiatives that have been taken to regain that control. This was inspired by a discussion on a mathematical mailing list. During that discussion, it was mentioned that the entire editorial board of the influential journal *Topology* had resigned as a protest against the prices that the publisher (Elsevier) was charging. (As this editorial is written, Elsevier had not taken the sometime editors' names off their web page, apparently on the grounds that there are still articles to be released that they accepted for publication. This raises interesting ethical questions that we will not go into here.)

Last week (on January 17) the launch of the *Journal of Topology* was announced by the London Mathematical Society. The editorial board of the new journal consists largely of the group who resigned from *Topology*, with the addition of a few other eminent mathematicians. It appears that the new journal is designed to be isomorphic to the Elsevier one, but at about a third of the price. Manuscripts are being solicited now and the first issue is scheduled to appear in about a year.

It must be said that the price issue is not as simple as it might appear. While Elsevier's web pages give a price of \$1,665 (US) per year for a standard institutional subscription, an institution could reduce this to \$791 by combining immediate electronic access with year-end delivery of paper copies; and an individual subscription is listed at \$100.

The lower prices do not reflect massively lower costs, of course, but a marketing strategy familiar to airlines and supermarkets whereby the thrifty purchaser obtains a lower price on a "seat sale" ticket in return for fewer frequent flier points, or on a can of "no name" beans with a less exciting label. It is not an intrinsically dishonest practice, and can be a "win-win" situation when different customers have seriously different expectations. The CMS journals also have significant discounts for individual subscribers (and even more so for members).

Institutional subscriptions, of course, are the most significant ones, and the ones that must form the basis of any serious comparison. The new journal's price of \$570 (US) per year undercuts even Elsevier's lower price. However, the LMS has not, at the time of writing, announced any rate for individual subscribers; it appears that these may find the *Journal of Topology* more expensive.

It is gratifying to see more mathematical journals being published by professional societies and universities. Publication of research within the community can be a collective "right livelihood", providing societies with much-needed funds in an appropriate way, as well as a way to keep journal costs under control. We may hope that even the prices of commercially-produced journals can be kept within reason by more competition of this sort.

We wish the new journal good luck and unbounded issues (though in bound volumes.) It is not yet clear whether *Topology* is dead; but in any case, long live the *Journal of Topology*!

### NOTES DE LA SMC

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**TOPOLOGY SOUTIENDRA-T-ELLE LA COMPARAISON?**

Dans l'éditorial du mois dernier, je décrivais l'impression de plus en plus répandue chez les mathématiciens d'avoir perdu une bonne partie de leur droit de regard sur la publication de leurs travaux, et j'ai mentionné quelques initiatives ayant pour but de regagner un certain contrôle. Ce texte s'inspirait d'une discussion tirée d'une liste de diffusion mathématique. Durant cette discussion, quelqu'un a écrit que tout le conseil de rédaction de la prestigieuse revue *Topology* venait de démissionner en guise de protestation contre les prix demandés par l'éditeur (Elsevier). (Au moment où j'ai écrit ces lignes, Elsevier n'avait pas encore retiré de son site web le nom de certains de ces rédacteurs, apparemment parce qu'ils avaient encore des articles à publier qui avaient déjà été acceptés. Cette situation soulève des questions intéressantes sur le plan déontologique, que nous n'aborderons pas pour l'instant.)

La semaine dernière (le 17 janvier), la Société mathématique de Londres annonçait le lancement d'une nouvelle revue, le *Journal of Topology*. Le conseil de rédaction de la nouvelle revue se compose en grande partie de personnes qui ont démissionné de l'équipe de *Topology*, ainsi que de quelques autres éminents mathématiciens. Il semble que la nouvelle revue se veuille une réplique de la publication d'Elsevier, mais au tiers du prix. La sollicitation d'articles est en marche, et le premier numéro devrait paraître d'ici environ un an.

Il faut dire que la question du prix n'est pas si simple que cela. Même si Elsevier annonce sur son site web un prix de 1 665 \$US par année pour un abonnement institutionnel standard, un établissement peut réduire ce prix à 791 \$ en optant pour l'accès électronique immédiat avec livraison des revues papier à la fin de l'année, et que le tarif d'abonnement d'un particulier est de 100 \$.

Les prix les plus bas ne sont pas le résultat d'une baisse fulgurante du coût, bien sûr, mais plutôt une stratégie de marketing bien connue des compagnies aériennes et des supermarchés, qui permet aux consommateurs à l'affût des aubaines de dénicher un billet dans le cadre d'une promotion en obtenant moins de points aériens, ou d'acheter une boîte de haricots « sans nom », mais aussi sans éclat. Ce ne sont pas là des pratiques intrinsèquement malhonnêtes, et il arrive souvent que toutes les parties y trouvent leur compte, notamment lorsque divers consommateurs ont des attentes entièrement divergentes. La SMC offre elle-même des réductions considérables sur l'abonnement à ses revues aux particuliers (et encore plus à ses membres).

Les abonnements institutionnels, bien sûr, constituent la plus grande part du gâteau, et sont ceux qu'il convient d'utiliser à des fins de comparaison. Le prix de la nouvelle revue, soit 570 \$US par année, est inférieur au prix le plus bas d'Elsevier. Par contre, au moment où j'écrivais ces lignes, la Société mathématique de Londres n'avait pas encore annoncé son tarif d'abonnement pour les particuliers; il se pourrait que ce tarif soit plus élevé pour le *Journal of Topology* que pour *Topology*.

Il est stimulant de voir se multiplier le nombre de revues mathématiques publiées par des sociétés professionnelles et des universités. La publication de travaux de recherche dans notre milieu peut être une « mode de vie justen » collectif et procurer aux sociétés, de façon tout à fait correcte, les fonds dont elles ont grand besoin, ainsi qu'un bon moyen de freiner la flambée des tarifs d'abonnement. Il est permis d'espérer qu'une concurrence accrue de la sorte permettrait de maintenir même les revues publiées par des intérêts privés à des prix raisonnables.

Nous souhaitons numeros sans limite a' cette nouvelle revue. Sans savoir exactement ce qu'il advient de *Topology*, nous souhaitons néanmoins longue vie au *Journal of Topology*!

**Letters to the Editors  
Lettres aux Rédacteurs**

The Editors of the *NOTES* welcome letters in English or French on any subject of mathematical interest but reserve the right to condense them. Those accepted for publication will appear in the language of submission. Readers may reach us at [notes-letters@cms.math.ca](mailto:notes-letters@cms.math.ca) or at the Executive Office.

Les rédacteurs des *NOTES* acceptent les lettres en français ou anglais portant sur un sujet d'intérêt mathématique, mais ils se réservent le droit de les comprimer. Les lettres acceptées paraîtront dans la langue soumise. Les lecteurs peuvent nous joindre au bureau administratif de la SMC ou à l'adresse suivante : [notes-lettres@smc.math.ca](mailto:notes-lettres@smc.math.ca).

## Convexity Properties of Hamiltonian Group Actions

by Victor Guillemin and Reyer Sjamaar  
CRM Monograph Series 26  
AMS 2005, iv + 82 pp

It always gives a sense of satisfaction when sophisticated mathematical techniques lead to the answer to simply stated down to earth problems. One well-known question of this sort is: given two Hermitian matrices  $X, Y$  with known eigenvalues, what can you say about the eigenvalues of their sum  $X + Y$ ? To make this question more precise, consider the cone of ordered  $n$ -tuples

$$C = \{\lambda \in \mathbb{R}^n \mid \lambda_1 \leq \dots \leq \lambda_n\}$$

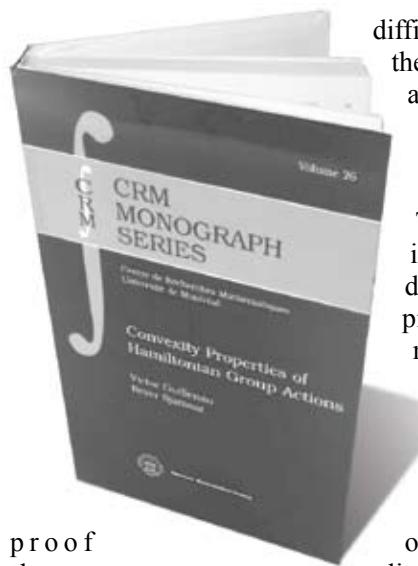
and let  $\mathcal{O}(\lambda)$  be the set of Hermitian  $n \times n$ -matrices with eigenvalues  $\lambda \in C$ . Given  $\mu, \gamma \in C$  we have

$$\mathcal{O}(\gamma) + \mathcal{O}(\mu) = \bigcup_{\lambda \in \Delta(\gamma, \mu)} \mathcal{O}(\lambda)$$

for a certain subset  $\Delta(\gamma, \mu) \subset C$ . What is this subset? A partial answer to this question, rather striking in itself, is that  $\Delta(\gamma, \mu)$  is always a convex polytope. That is, the possible eigenvalues of the sum of two matrices with prescribed eigenvalues  $\Delta(\gamma, \mu)$  are cut out by a finite number of inequalities. But what are these inequalities?

An interesting set of necessary inequalities for the polytope  $\Delta(\gamma, \mu)$  was found by Hermann Weyl in 1912, in his famous paper on the asymptotic distribution of eigenvalues of differential operators. In 1962, Alfred Horn made an ingenious guess for a *complete* set of inequalities, and he was able to prove his conjectures for  $n \leq 7$ . The full solution to Horn's conjecture was given in 1998 in the celebrated work of Alexander Klyachko. The breakthrough idea in that paper was to give a geometric interpretation of Horn's inequalities in terms of Schubert calculus, and the application of Mumford's geometric invariant theory. Since then, this result has been refined and extended in a number of directions: for instance, Agnihotri-Woodward have a similar result for *products* of unitary matrices (using *quantum* Schubert calculus), while Berenstein-Sjamaar extended Klyachko's result to sums of coadjoint orbits of a compact Lie group. The fascinating history of Klyachko's theorem and its relation to other problems in mathematics are surveyed in W. Fulton's article in Bull. A.M.S. 37 (2000), 209-249.

The fact that  $\Delta(\gamma, \mu)$  is a polytope may be viewed as a special case of the convexity properties of moment maps in symplectic geometry, due to Atiyah, Guillemin-Sternberg and Kirwan. The non-Abelian convexity theorem of Kirwan says that for a Hamiltonian action of a compact Lie group  $G$  on a closed symplectic manifold  $M$ , with moment map  $\Phi : M \rightarrow \mathfrak{g}^*$ , the set of co-adjoint orbits appearing in the moment map image form a convex polytope  $\Delta$ . The explicit computation of this polytope tends to be quite



difficult, in general. If  $G$  is the unitary group, and  $M$  is a product of the co-adjoint orbits corresponding to  $\gamma, \mu$ , then  $\Delta = \Delta(\gamma, \mu)$ .

The book under review is the first monograph devoted to convexity properties of moment maps for Hamiltonian group actions. The material is divided into five chapters.

Chapter I gives a detailed proof of the Abelian convexity theorem, discovered independently by Atiyah and Guillemin-Sternberg in 1981. A simple application of this result is the Schur-Horn theorem, describing the possible diagonal entries of a Hermitian matrix with prescribed eigenvalues: the image of the map  $\mathcal{O}(\lambda) \rightarrow \mathbb{R}^n$ , given by projection to the diagonal, is the convex hull of the set of points  $\lambda_\sigma = (\lambda_{\sigma(1)}, \dots, \lambda_{\sigma(n)})$  where  $\sigma$  ranges over the permutation group. The chapter also contains a sketched proof of Kirwan's non-Abelian convexity theorem, and discusses various generalizations of the convexity result, such as Lie-Poisson convexity theorems or convexity theorems for Hamiltonian loop group actions.

In Chapter II the authors discuss a constructive version of the non-Abelian convexity theorem. This theorem, due to Sjamaar, implies Kirwan's original result, but it also gives a recipe for computing the polytope 'locally'. Unfortunately, this local description is hard to implement in practice, and in particular it is not known how to derive Klyachko's theorem along these lines.

Chapter III gives concrete examples of convexity theorems, including the eigenvalue problem for sums of Hermitian matrices. This chapter contains one of the great highlights of the book: A beautifully simple proof of the *necessity* of Horn's inequalities, using Morse-theoretic techniques. The appearance of Schubert cells becomes quite natural from this perspective.

Chapter IV describes the role of convexity theorems in Kähler geometry. Suppose that the Hamiltonian  $G$ -space  $M$  carries a compatible complex structure preserved by the action. Then the action of  $G$  extends to a holomorphic action of the complexified group  $G^\mathbb{C}$ . A result of Atiyah, for the Abelian case, states that the moment map image of the closure of any  $G^\mathbb{C}$ -orbit is a convex polytope. Brion proved a similar statement for the non-Abelian case, and generalized further to images of invariant irreducible subvarieties  $N \subset M$ . Finally Guillemin-Sjamaar observed that it suffices for  $N$  to be invariant under the Borel subgroup of  $G^\mathbb{C}$ .

## BOOK REVIEW: THE HIGH-TECH APPROACH TO MATRICES HIGHLY RECOMMENDED *continued*

The final Chapter V deals with a number of applications of the convexity techniques. One of these applications concerns the ‘Kählerizability’ of multiplicity-free Hamiltonian  $G$ -spaces. In the early 1980’s, Delzant classified such spaces for the case that  $G$  is a torus, and proved in particular that all of these spaces carry a compatible invariant Kähler metric. It was widely expected that a similar result should hold for non-Abelian groups as well: after all, multiplicity free spaces are very special due to their huge symmetry group. This conjecture turned out to be false, and the first counterexample, due to

Tolman and Woodward, is an application of Atiyah’s Kähler-convexity result.

The book is a well-written account of some exciting new developments in mathematics. The entire theory is developed from scratch, and large parts of the book are accessible to students at the advanced undergraduate or beginning graduate level. It is highly recommended to anyone wishing to learn about convexity results in symplectic geometry and its down-to-earth applications.

### PROBLEM OF THE MONTH

*The following problem was submitted by  
Dr. Robert Dawson of St. Mary's University.*

#### Three Shadows, Two Bodies

An old math puzzle asks for the shape of a convex body that can cast the following shadows. Illuminated by rays parallel to the  $x$  axis, the shadow is a square; if the rays are parallel to the  $y$  axis the shadow is a circle; and if the rays are parallel to the  $z$  axis the shadow is an isosceles triangle.

Sources usually give a single solution. Here we ask - what is the volume of the largest possible solution; what is the volume of the smallest possible solution; and are these unique?

Send your own favorite problems to: [notes-editors@cms.math.ca](mailto:notes-editors@cms.math.ca)

Solution for February's problem: page 11

#### NSERC - CMS Math in Moscow Scholarships

The Natural Sciences and Engineering Research Council (NSERC) and the Canadian Mathematical Society (CMS) supports scholarships at \$9,000 each. Canadian students registered in a mathematics or computer science program are eligible.

The scholarships are to attend a semester at the small elite Moscow Independent University.

##### Math in Moscow Program

[www.mccme.ru/mathinmoscow/](http://www.mccme.ru/mathinmoscow/)

##### Application details

[www.cms.math.ca/bulletins/Moscow\\_web/](http://www.cms.math.ca/bulletins/Moscow_web/)

For additional information please see your department or call the CMS at 613-562-5702.

Two scholarships will be awarded in the spring competition. Deadline **March 30, 2007** to attend the Fall 2007 semester



#### Bourse CMS/CRSNG Math à Moscou

Le Conseil de Recherches en Sciences Naturelles et en Génie du Canada (CRSNG) et la Société mathématique du Canada (SMC) offrent des bourses de 9,000 \$ chacune. Les étudiantes ou étudiants du Canada inscrit(e)s à un programme de mathématiques ou d'informatique sont éligibles.

Les bourses servent à financer un trimestre d'études à la petite université d'élite Moscow Independent University.

##### Programme Math à Moscou

[www.mccme.ru/mathinmoscow/](http://www.mccme.ru/mathinmoscow/)

##### Détails de soumission

[www.cms.math.ca/bulletins/Moscou\\_web/](http://www.cms.math.ca/bulletins/Moscou_web/)

Pour plus de renseignements veuillez communiquer avec votre département ou la SMC au 613-562-5702.

Deux bourses seront attribuées au concours du printemps. Date limite le **30 mars 2007** pour le trimestre d'automne 2007



***Convexity and Well-Posed Problems***

by Roberto Lucchetti

CMS Books in Mathematics 22

AMS 2006, xvi + 305 pp, \$79.95

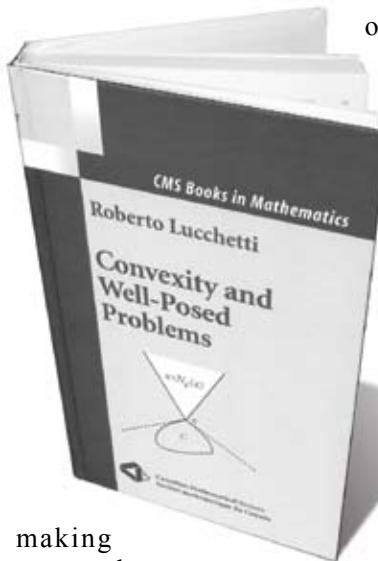
Convexity has played a central role in the development of variational analysis and optimization from the very beginning, and its fundamental importance is displayed already in Rockafellar's classical work [3]. Well-posedness has its roots in mathematical physics, but for variational problems it presents substantial challenges because normally such problems involve functions that are not differentiable, at least in the usual sense. These two areas are very much interconnected and have come to maturity at a time when it is increasingly important to reach out to a larger readership of students, researchers from other fields, and practitioners. The present book displays a synthesis of basic results on convexity and well-posedness in optimization. A nice addition to the fast-growing number of books in variational analysis, it is also self-contained, easy to read, and it can be used for teaching.

The book starts with an introduction, in Chapter 1, to basic definitions and properties of convex sets and convex functions. Then, Chapter 2 is devoted to an important class of functions, with domains in a Banach space, that are convex, lower semicontinuous, and whose epigraphs are nonempty sets that do not contain vertical lines. A fundamental characterization of such a function is shown: it is the pointwise supremum of all affine functions minorizing it. Chapter 3 deals with differentiating convex functions and, in particular, with properties of the (convex) subgradient. In Chapter 4, minimization of convex functions is discussed, with a special emphasis on Ekeland's principle as a tool for obtaining generic existence of minima.

The relationships between a convex function and its Fenchel conjugate are described in Chapter 5, with more details regarding subdifferentials of these functions, as well as the form of the conjugate of the sum. This is used in Chapter 6, where the Lagrange duality theory is outlined and illustrated with examples from convex programming and the calculus of variations. Chapter 7 is devoted to duality in linear programming from the perspective of the theory of two-player zero-sum games. The material presented in this section may be very useful in economics and business courses that cover linear programming.

The rest of the book might be a bit more challenging for beginners. In Chapter 8 the author presents various notions of set convergence and relations among them. The material in this chapter complements the book [1].

Associated notions of convergence of functions and their subdifferentials are shown in Chapter 9. Well-posed



making  
research,  
various papers, accessible in a self-contained book  
form.

optimization problems are treated in the final Chapter 10, where Tikhonov, Levitin-Polyak, and strong well-posedness are introduced, along with related concepts of stability, marking the progress that has been made since the publication of the book [2]. This rather long chapter also introduces the reader to porosity and generic well-posedness, these new directions of which had been scattered in

a self-contained book form.

Quite a few exercises, some of them challenging, are given throughout the book. More detailed hints would have been helpful for a novice in the field. A set of appendices provides background material in analysis and topology. Finally, a list of notations, definitions, and some important theorems, as well as a modest bibliography (which is in fact sufficient in the age of MathSciNet) and index, end the book.

In summary, this book has several nice features. The selection of material and the level of difficulty of some parts can nicely fit into any more traditional course on variational analysis and optimization. The fundamental concepts and results presented make the book a valuable reference for anyone interested in optimization. And finally, the book gives the author's view of recent developments in the area, which, together with some new results, will certainly be of interest to researchers.

## References:

1. G. Beer, *Topologies on closed and closed convex sets*. Mathematics and its Applications, 268. Kluwer Academic Publishers Group, Dordrecht, 1993.
2. A. L. Dontchev, T. Zolezzi, *Well-posed optimization problems*, Lecture Notes in Mathematics, 1543. Springer-Verlag, Berlin, 1993.
3. R. T. Rockafellar, *Convex Analysis*, Princeton Mathematical Series, No. 28 Princeton University Press, Princeton, N.J. 1970.

**Measures, Integrals and Martingales**

by René Schilling

Cambridge 2005 x + 381 pp, US \$90.00, \$50.00(pbk)

The first fifteen chapters of this text cover the basic material on measures, measurable functions, integration, convergence,  $L^p$ -spaces, product measures and uniform integrability. The remaining eight chapters deal with martingales, Radon-Nikodým theorem, applications of martingales, inner product spaces, conditional expectations, orthonormal systems and convergence behaviour. Five appendixes summarize the basic prerequisite material. Undergraduate calculus and analysis of reals are the only essential prerequisites. Numerous illustrations and exercises are included, and these are not merely drill problems but are there to consolidate what has already been learnt and to discover variants, diversions and extensions of the main material. Hints and solutions are made available on the internet.

**Nonlinear Optimization**

by Andrzej Ruszczyński

Princeton 2006, xii + 446pp, US \$ 59.50

Optimization is one of the most important areas of modern applied mathematics, with applications in engineering, economics, finance, statistics, management science and medicine. The author of this book is a leading expert in the optimization of nonlinear stochastic systems. He presents a comprehensive treatment that would enable graduate students and researchers to understand with a reasonable period the salient ideas and various aspects of nonlinear optimization. The chapters cover convex analysis, optimality conditions, duality, numerical methods for problems involving nondifferentiable functions, semidefinite programming, metric regularity, stability theory of set-constrained systems and sensitivity analysis of optimization problems. The book can be used profitably for graduate courses or independent study. It has many examples, figures and exercises illustrating the relevance and applications of theoretical concepts.

**On Quaternions and Octonions  
Their Geometry, Arithmetic, and Symmetry**

by John H. Conway and Derek A. Smith

A. K. Peters, MA 2003, xii + 159pp, \$29.00 US

This book is about the geometry of the quaternion and octonion algebras. Following an historical introduction, the special properties of three and four dimensional Euclidean spaces are exemplified using quaternions leading to enumerations of the corresponding finite groups of symmetries. The second part of the book discusses the less familiar octonion algebra, concentrating on its remarkable ‘triviality symmetry’ after an appropriate study of Moufang loops. The arithmetics of the

quaternions and octonions are also described and the book concludes with a new theory of octonion factorization.

**First Steps for Math Olympians, Using the American Mathematics Competitions**

by J. Douglas Faires

MAA 2006, xxi + 305pp

A major aspect of mathematical training and its benefit to society is the ability to use logic to solve problems. In this book problems from past fifty years of American Mathematics Competitions are given with solutions. The problems have been edited to conform to the modern mathematical practice that is used in current competitions. Multiple solutions are included to show the reader that there is generally more than one way to approach the solution to a problem. The goal of the book is to promote interest in mathematics by providing the reader with the tools to attack problems that occur in usually in problem solving exams. Three examples are given in each chapter to illustrate the range of topics and difficulty. These are followed by ten exercises arranged in increasing order of difficulty. Solutions to these exercises are provided in a separate chapter at the end of the book. The first four chapters are elementary. Chapters 5, 6 and 7 deal with geometry. Chapters 8 and 9 concern counting techniques and probability. Number theoretic problems are given in chapters 10 and 11. Chapter 11 is on sequences and series. Statistics is dealt with in Chapter 13. The final four chapters are concerned with trigonometry, logarithm functions, three dimensional geometry and complex numbers. The book will be useful for self study and also for teachers of problem-solving sessions.

**Classical and Quantum Orthogonal Polynomials in One Variable**

by Mourad E. H. Ismail

Encyclopedia of Mathematics and its Applications 98

Cambridge 2005 xviii + 705 pp, US \$140.00

This is the first modern treatment of orthogonal polynomials from the point of view of special functions. The coverage is encyclopedic, including classical topics such as Jacobi, Hermite, Laguerre, Hahn, Charlier and Meixner polynomials as well as those, e.g. Askey-Wilson and Al-Salam-Chihara polynomial systems, discovered over the last 50 years: multiple orthogonal polynomials are discussed for the first time in book form. Modern applications of the subject are dealt with, including birth and death processes, integrable systems, combinatorics, and physical models. A chapter on open research problems and conjectures is designed to stimulate further research on the subject. Exercises of varying degrees of difficulty are included. A comprehensive bibliography rounds off the work, which will be valued as an authoritative reference and for graduate teaching.

## Mathematics books for young children

The editorial office of these *NOTES* recently received a fine book that introduces middle school pupils to coding, a topic that surely would have immediate appeal for many of them. Many readers might have leafed through this book on display in the Peters' booth at the December CMS meeting in Toronto.

Janet Beissinger & Vera Pless, *The Cryptoclub: using mathematics to make and break secret codes*

A.K. Peters, Wellesley, MA, 2006 xvi+199 pages

ISBN-13: 978-1-56881-223-6; -10: i-56881-223-X

List price: US\$35.00

The authors, both at the University of Illinois in Chicago, the first at the Institute for Mathematics and Science Education and the second a professor of cryptography in the Department of Mathematics, Statistics and Computer Science, state in their preface that there is much in the mathematical theory of coding within the mathematical reach of a young adolescent. Indeed, “the Vigenère Cipher, ..., once believed to be unbreakable, can actually be cracked by today’s middle-grade students (as long as the key isn’t too long) by common factors of certain numbers.” A sequence of seven units takes the students from the ciphers of Julius Caesar to the RSA public key encryption made popular by Ronald Rivest, Adi Shamir and Leonard Adleman in 1977.

Despite its challenging mathematical content, there is nothing dry about this book. All the techniques employed for modern children’s books are used here to good effect: a story line to connect the topics, with coloured illustrations, boxes to set of examples and exercises, as well as a number of historical anecdotes set on special pages. While the book is designed to be used by a class or club led by a teacher, a pupil or small group could work through it.

The first two units treat various sorts of substitution ciphers, including ones that use a keyword, along with techniques for deciphering them. In particular, one can analyze letter frequencies, which addresses topics in data management. A complicated form of substitution cipher is due to Vigenère, in which the shift changes with each letter according to a matching letter in a keyword which codes the letter A. The study of this occupies almost 50 pages, as the pupils must master factorization techniques required to discover the length of the key word. The remaining four units of the book provide the fundamentals of modular arithmetic leading up to

multiplicative and affine ciphers, and finally the determination of primes and raising of powers modulo a prime that will be used in the RSA public key cryptosystem.

It is worth briefly summarizing the prime number theory that the book covers. The children learn how to list primes using the sieve of Eratosthenes and to check for primality by dividing numbers by primes not exceeding their square roots. In the process, they learn about twin primes, Sophie Germaine primes ( $p$  for which  $2p+1$  is also prime), Mersenne primes and the Great Internet Mersenne Prime Search. (The largest prime determined up to February, 2005 is  $2^{25,964,951} - 1$ .) They find out under what conditions an integer has an inverse modulo  $n$  and invited to experiment, and finally are told how to find the RSA decryption key.

An important message conveyed in the book is that mathematics is alive and growing. While it is premature to develop much of the theory involved, the pupils have through the exercises a good opportunity to get a feel for the underlying mathematical structures.

The book just reviewed is the latest in a growing list of volumes suitable for presecondary students. Mitsumasa Anno has a number of charming books for the young, *Anno’s Counting Book*, *Anno’s Magic Seeds*, *Anno’s Mysterious Multiplying Jar* and *Anno’s Math Games*. My favourite Anno book *Anno’s Hat Tricks*, coauthored with Akihiro Nozaki, introduces logical reasoning through puzzles in which children, seeing coloured hats on the heads of other children, have to deduce the colour of their own hats.

*The Man Who Counted*, by Malba Tahan, is the story of Beremiz Samir, told in *Arabian Nights* style, in which a shepherd boy develops a talent with numbers and passes through a succession of adventures involving solutions of mathematical problems to achieve both fame and fortune. The chapters can be read independently.

*The Number Devil: a Mathematical Adventure* by Hans Magnus Enzensberger is a deservedly popular tale of a twelve-year-old boy who hates mathematics and, in his dreams, meets the “number devil” who takes him on many mathematical adventures.

*The Math Curse*, by Jon Scieszka, affects a young girl who is constrained to see a mathematics problem in everything around her.

Theoni Pappas is a prolific mathematical expositor for children whose publications include *Fractals, Googols and other Mathematical Tales* and *Math for Kids & Other People Too!*.

### Meditations on the attainments of matriculating students

When university mathematicians are critical of the preparation of secondary students for university work, their comments are often interpreted as meaning that there should be more material in the syllabus. The issue is more subtle than this; I would like to report on some recent experiences that highlight what many of us are looking for in our first year students.

In December, I was at the University of Waterloo to join the marking team for the Canadian Open Mathematics Challenge. Questions B2 and B3 were both on geometry. I thought them excellent as they epitomized the sort of thing that every student graduating from Grade 12 mathematics should be capable of, if they are planning to use that mathematics in advanced study. Unfortunately, most of them did not do a good job on the questions, and many who did solve the problems were hardly fluent in managing even the most straightforward computations. The two questions were as follows:

**B2.** The circle  $x^2 + y^2 = 25$  intersects the  $x$ -axis at points  $A(-5, 0)$  and  $B(5, 0)$ . The line  $x = 11$  intersects the  $x$ -axis at point  $C$ . Point  $P$  moves along the line  $x = 11$  above the  $x$ -axis and  $AP$  intersects the circle at  $Q$ .

(a) Determine the coordinates of  $P$  when triangle  $AQB$  has maximum area. Justify your answer.

(b) Determine the coordinates of  $P$  when  $Q$  is the midpoint of  $AP$ . Justify your answer.

(c) Determine the coordinates of  $P$  when the area of triangle  $AQB$  is  $\frac{1}{4}$  of the area of triangle  $APC$ . Justify your answer.

**B3.** (a) The trapezoid  $ABCD$  has parallel sides  $AB$  and  $DC$  of lengths 10 and 20, respectively. Also, the length of  $AD$  is 6 and the length of  $BC$  is 8. Determine the area of trapezoid  $ABCD$ .

(b)  $PQRS$  is a rectangle and  $T$  is the midpoint of  $RS$ . The inscribed circles of triangles  $PTS$  and  $RTQ$  each have radius 3. The inscribed circle of triangle  $QPT$  has radius 4. Determine the dimensions of rectangle  $PQRS$ .

Some of the solutions to problem B2 (a) exemplified the adage that a little knowledge is a dangerous thing; the students saw the word “maximum” and looked for a derivative to take. Once

students recognized that the area is maximized with the vertical height, and that the line  $AP$  would pass through  $(0, 5)$ , then it is simply a matter of identifying the intersection of the lines  $y = x + 5$  and  $x = 11$ . Part (b) is a matter of recognizing that the abscissa of the midpoint of  $AP$  is 3; if they remember and can use the pythagorean triple  $(3, 4, 5)$  to locate  $Q$ , then the answer is immediate. All that is needed is an internalization of proportionality and similar figures. A couple of students observed that  $BQ$  was the right bisector of  $AP$ , so that  $|BP| = |AB| = 10$  and triangle  $BCP$  is a 6–8–10 right triangle. Most of the successful solutions to (c) were painfully pedestrian. However, a couple noticed that triangles  $AQB$  and  $ACP$  were similar with factor  $1/2$ . This led quickly to  $|AQ| = \frac{1}{2}|AC| = 8$ . Since  $|AB| = 10$ , we find that  $|BP| = 6$  and  $|CP| = 2|BP| = 12$ . There is no background here that cannot be reasonably expected on any secondary syllabus. The issues are less content and more mathematical fluency and structural insight.

Question B3 is another question for which straightforward but tedious solutions exist, but become easy with the right perspective. The best solution given for (a) arose out of the realization that connecting  $A$  and  $B$  to the midpoint of  $CD$  partitioned the trapezoid into three congruent 6–8–10 right triangles whose areas (by the base-height formula, if you look at the triangles sideways) are each 24. The best solution given for (b) relied on the insight that triangles  $TIU$  and  $VJP$  were similar with factor  $3/4$ , where  $I$  and  $J$  are the respective incentres of triangles  $PTS$  and  $PQT$ ,  $ST$  is tangent to the smaller incircle at  $U$  and  $PQ$  is tangent to the larger 3 incircle at  $V$ . If we take  $|PV| = 4x$ , then  $|UT| = 3x$ , whence  $4x = 3 + 3x$  and  $x = 3$ . If  $|PS| = 3 + 3y$ , then an application of Pythagoras’ theorem on triangle  $PTS$  yields the equation  $(3x + 3y)^2 = (3x + 3)^2 + (3y + 3)^2$  which reduces to  $xy = x + y + 1$  or  $(x - 1)(y - 1) = 2$ .

Let us look more closely at the ingredients in the solution for (b). There are three theoretical results needed. The first is to note that  $TP$  is a transversal of two parallels, so that the alternate angles  $STP$  and  $TPV$  are equal. The second is that the tangent rays from an external points to a circle are equal. The third is that the incentre of a triangle lies on the bisector of any of its angles. The last two can be convincingly established by appealing to the reflection of the configuration in the diameter of the circle passing through the external point. However, we need the insight that two crucial triangles are similar. Finally, for ease of dénouement, it helps to set the manipulations up in a way that delivers the result efficiently. While it might be unreasonable to expect every student to produce such a solution, exercises that they are regularly exposed to

should be sufficiently rich that each student can on occasion produce solutions that indicate the assimilation of important mathematical values.

Another question that was interesting is this regard was Problem **B1** on the 2006 Putnam competition:

**B1.** Show that the curve  $x^3 + 3xy + y^3 = 1$  contains only one set of three distinct points  $A$ ,  $B$ , and  $C$ , which are the vertices of an equilateral triangle, and find its area.

Of course, we will not know until November, 2007 how the candidates fared with this question. However, while it was given to tertiary students, it is definitely not out of line for secondary students to be asked to attempt it. The expression on the left side of the equation should ring some bells, evoking the expansion of  $(x + y)^3$ . At this point, a rather sophisticated bit of algebraic insight comes in handy: if we make the restriction that  $x + y = 1$ , we can put the factor  $(x + y)$  with  $3xy$  to transform the equation into the form  $(x + y)^3 = 1$ . Thus,  $x + y = 1$  is consistent with the equation, so that the line is part of the locus of the equation. This also means that  $x + y - 1$  should be a factor of  $x^3 + 3xy + y^3 - 1$ . Indeed,

$$x^3 + 3xy + y^3 - 1 = (x + y - 1)(x^2 + y^2 - xy + x + y + 1).$$

Setting the second factor equal to zero apparently gives the equation of a conic section, which, because of the symmetry in  $x$  and  $y$  should have an axis along the line  $x = y$ . It is quickly found that the points  $(\frac{1}{2}, \frac{1}{2})$  and  $(-1, -1)$  lie on the locus. This suggest that the expressions  $(x + 1)$  and  $(y + 1)$  and  $(x - y)$  might possibly bear on the second factor. In fact,

$$2(x^2 + y^2 - xy + x + y + 1) = (x - y)^2 + (x + 1)^2 + (y + 1)^2.$$

The locus of the original equation is thus a straight line and a single point of the line, and it is now straightforward to answer the question.

Is **B1** an appropriate question for a test or entrance examination? No. But it, and a diet of other questions of similar ilk, should certainly be given on homework assignments and used in group work. Such exercises promote the powers of recognition, analysis and exploitation of basic facts that constitute an indispensable part of student progress in even the most mundane of university mathematics courses.

Another example is a problem that was brought to the COMC marking from one of the teachers, who found

it as a bonus question on a grade 11 test. She told me that the students had not studied logarithms, so that the solution could only use the laws of exponents. Given that  $60^a = 3$  and  $60^b = 5$ , determine

$$12^{\frac{1-a-b}{2(1-b)}}.$$

You might want to try it before proceeding. The solution does not appear to be obvious, and it flummoxed me at first. However, a reasonably direct approach delivers the goods. From the second condition, one finds that  $5 = 12^{b/(1-b)}$ . Write the first equation as  $2^2 \cdot 5^a \cdot 12^a = 12$  so that  $2^2 = 12^{1-a} \cdot 5^{-a}$ ; substitute for 5 in terms of 12 and roll to a successful conclusion. While one might wonder about this as a test question, even for a bonus, it is a useful challenge to post on a bulletin board in the classroom. Its artificiality should not detract from the challenge of finding the simple answer that the proposer had in mind. Indeed, one might pause to pay homage to the creator of this question.

While it might be wished that this or that piece of mathematics were better known, most thoughtful observers see the problem of student preparation more with the use that students make of what is already on the curriculum and the analysis and connections that they might or might not be able to make. None of the foregoing problems discussed above involves material not already on nor easily incorporated into the existing syllabus. But a student who approaches them with a cookbook of procedures and formulae is unable to make much progress; some insight into the underlying structure along with a strategic approach and knitting together of knowledge from different sources is needed.

My comments should be seen as supplementing rather than countering the recommendations of Peter Taylor of Queen's University. In his address [see p. 19, this issue] at the recent CMS Toronto meeting, where he received the Adrien Pouliot Award, he presented two nice examples of the sort of classroom investigations he promotes. In the first instance, students had to measure the fall in air pressure of an inflated tire after it had been suddenly punctured and find a function that modelled it. In the second, students experimented to find the relative frequency  $f(x)$  of success in throwing a beanbag into a box located a distance  $x$  meters away, and then decide on the distance that would maximize a "reward function"  $x^2f(x)$ .

The first of these leads students to an understanding of the situations that give rise to exponential decay and how such situations can be described and analyzed mathematically. The

second leads to an appreciation of the properties that a function,  $f(x)$ , can be expected to have and how one balances a situation with contradictory aspects, here the interaction of a function  $x^2$  that increases with  $x$  against one  $f(x)$  that decreases. Unlike the contest problems, the initial focus is not on the mathematical description but on the intuitive grasp of a situation which can then be mathematized appropriately.

All discussions of student preparation lead back to the

characteristics of their teachers – how much mathematics they know, how they know it, the experiences they have as practising mathematicians, their confidence in handling whatever students may bring to an unprogrammed situation and their ability to interpret student responses and take up advantages and disadvantages of different approaches to a situation.

## CALL FOR NEWS FROM THE DEPARTMENTS

This is a request for news items to appear in the next issue of the NOTES. Reply to notes-editors@cms.math.ca by the deadline indicated at the back of this issue. Our intention is to circulate this reminder at least once per term and to run this column in all 8 issues (Sep, Oct, Nov, Dec, Feb, Mar, Apr, May).

We hope that departments will submit news at least once per term. Thank you for your cooperation.

Please use the format given below.

Appointments (rank, date, field):

Promotions (rank, date):

Retirements (rank, date):

Resignations:

Death (rank, date):

Awards/Distinctions:

Visitors (name, country, area, date):

Other News:

## Solution to February's Problem

Yes, Alice can test whether the ten coins have equal weight in three weighings. She weighs coins  $\{1,2,3,4,5\}$  against  $\{6,7,8,9,10\}$ ; coins  $\{1,2,3,4\}$  against  $\{5,7,8,9\}$ ; and coins  $\{5,6,10\}$  against  $\{7,8,9\}$ .

$A$  = number of heavy coins in  $\{1,2,3,4\}$

$B$  = number of heavy coins in  $\{5\}$

$C$  = number of heavy coins in  $\{7,8,9\}$

$D$  = number of heavy coins in  $\{6,10\}$

If the three weighings balance we have  $A + B = C + D$ ,  
 $A = B + C$ ,  $C = B + D$  and thus  $2B + C = C + D$ ,  $2B = D$ ,  $3B = C$ ,  $4B =$

A. The only solutions with  $B$  in  $\{0,1\}$  are  $(0,0,0,0)$  and  $(4,1,3,2)$ .

This surprising result was discovered in 1997 by Kozlov and Vu (see "Coins and Cones", JCTA 78 (1997) 1-14).

How many coins can you test with four weighings? Five?

It is not entirely obvious, by the way, that the number of weighings needed is a monotone function of the number of coins. A proof of this would be nice, a counterexample amazing!

### Correction

The Call for the Research Prizes, in the February issue, contained a few inconsistencies in the French text. Please find the revised text on page 32

### Erratum

Quelques erreurs se sont glissées dans la version française de l'appel de candidatures pour les prix de recherche, publié dans les Notes de février. Vous trouverez la version corrigée à la page 32.

### Correction

The 2007 Elections Slate contained incorrect information concerning the spelling of a candidate's name. The correct spelling of the candidate's name is Xiaoqiang Zhou (Memorial).

### Erratum

Un nom a été mal écrit dans la liste des candidats aux élections 2007. Il aurait fallu lire Xiaoqiang Zhou (Memorial).

on continuous parameter stochastic processes were pivotal in his decision to study probability. Kolmogorov had constructed Markov transition functions as the fundamental solutions of the forward equation associated with a second order differential operator (described here in one spatial dimension)  $\mathcal{L} = a(x) \frac{d^2}{dx^2} + b(x) \frac{d}{dx}$ . He would then use his general existence theorem to construct a Markov process associated with this transition kernel. Itô felt much of this work was fine analysis but somehow the probability was secondary (the title suggests as much). Lévy offered up something more exciting in a direct manipulation with the sample paths of a process with stationary and independent increments (now called Lévy processes). The Fourier transforms at a fixed time of such processes were characterized by Lévy and Khinchin, but it is a formula which only comes to life with Lévy's pathwise construction of the stochastic process as a "Poisson sum of compensated jumps".<sup>1</sup> Lévy's pathwise approach had excited Prof. Itô - pathwise arguments are what draw many of us to the subject. Many of Lévy's arguments were non-rigorous, however (although almost always correct). In [I4] Itô writes "I had a hard time following Lévy's argument because of his unique intrinsic description". It was only when he read Doob's works on modifications of continuous parameter processes that Itô realized how to carry out Lévy's construction rigorously. The construction was written in [I1], formed the basis of his thesis, and is now called the Lévy-Itô construction. This pathwise perspective has stayed with Itô

throughout his career.

Much of the above work was written while Itô was at the Statistical Bureau (1939-43). M. Fukushima indicated to me that government agencies and private companies would then often hire bright young students as a future investment and let them pursue their own interests as well. Itô writes [I4] "I was given sufficient time for my own study thanks to the kindness of Mr. T. Kawashima, Head of the Bureau." This kindness in wartime Japan also led to the seminal work [I2] which shaped what many of us have been doing during our entire careers. As described in the Introduction and Foreword in [I4], his goal here was to construct the diffusion processes of Kolmogorov [K] by a direct manipulation of the sample paths in the spirit of Lévy. (Feller [Fe] had extended Kolmogorov's construction to allow jump kernels and it was in this general setting Itô was working, but these extensions were published later in his 1951 AMS Memoir.) His observation was that instantaneously at  $x$  Kolmogorov's processes were performing Brownian motion with drift  $b(x)$  and standard deviation  $\sigma(x) = \sqrt{a(x)}$ . Kolmogorov carries this out at the level of probability distributions of the underlying random motion; Itô did it directly at the level of sample paths by building the random process  $X(t)$  as the solution of the stochastic differential equation

(SDE)

$$dX(t) = \sigma(X(t))dB(t) + b(X(t))dt.$$

Here  $B$  is a Brownian motion and so has

sample paths of unbounded variation on every interval of positive length almost surely. So, to make sense of the first term in (SDE) Itô had to interpret the integral, not sample path by sample path, but rather process by process. One puts in a stochastic process  $f(s, \cdot)$  (i.e., a measurable function of the ever-suppressed  $\omega$ ) satisfying certain integrability and measurability conditions, and one produces a stochastic process  $\int_0^t f(s, \cdot) dB(s)$  satisfying certain integrability and measurability conditions. To get a useful integral with a dominated convergence theorem, one must restrict the class of integrands  $f(s, \omega)$  to those that depend on past information, say of the Brownian motion  $B$ . It is a condition which is satisfied for the integrand arising in (SDE), Itô. Having made sense of (SDE) Itô then connected it up with Kolmogorov's process (and the operator  $\mathcal{L}$ ) through the formula that bears his name

(IF)

$$\begin{aligned} df(X(t)) &= f'(X(t))dX(t) + \frac{1}{2}f''(X(t))\sigma^2(X(t))dt \\ &= f'(X(t))\sigma(X(t))dB(t) + \mathcal{L}f(X(t))dt. \end{aligned}$$

The intuition is clear enough with hindsight.  $B$  has infinite variation but has quadratic variation on  $[0, t]$  equal to  $t$  in a certain sense. One must use a second order Taylor expansion for  $X(t)$  locally and sum over the increments to derive (IF). Itô's integral had a precursor - as Stroock and Varadhan put it, "Itô is the Lebesgue of this branch of integration theory; Paley and Wiener were its Riemann." Paley and Wiener (1934) had constructed the stochastic integral for square integrable deterministic integrands which would not help one in interpreting (SDE).<sup>2</sup>

<sup>1</sup>The idea in [L] and [I1] is to construct the process by adding the sum of its jumps: for compound Poisson processes (jumps occur at a finite rate according to some common distribution) this works fine. In general, small jumps occur at an infinite rate so one must truncate them at  $\epsilon$  and take a limit. The delicate issue here is that, in general, the sample paths have unbounded variation and the sum of jumps only converges conditionally. To get convergence in the above scheme one must subtract off the mean of the jumps to get a difference of two terms, each approaching infinity, which converges to the limiting value of the process. To do this at all times simultaneously one introduces a sea of potential jumps indexed by size and time at which they occur - the space - time Poisson point process of jumps.

<sup>2</sup> It appears, however, that Itô did not know of this in his 1942 paper, although he mentions in [I4] that he read it carefully after moving to Nagoya in 1943, and cites it in a 1944 paper on the integral where its influence can be seen.

Like many significant breakthroughs, Itô's construction and associated analysis was ahead of its time. Itô writes [I4]: "I do not know anyone who read this paper [I2] thoroughly when it appeared except my friend G. Maruyama." Maruyama told Fukushima that he read it repeatedly under the light of a gatekeeper's box while in a military camp where he had been drafted. Itô moved to Nagoya U. (1943-52) and submitted an extended version of his work (in English) on stochastic d.e.'s including jumps to the Memoirs of the AMS. Itô credits Doob with assisting with the publication in the US. Itô's original work was done without the continuous parameter martingale theory which Doob was just developing at the same time on the other side of the ocean. It is the approach which greatly simplifies and extends Itô's original construction (and lack thereof makes the original construction all the more impressive). Doob immediately appreciated what Itô had done and started to extend Itô's stochastic integral and calculus to general martingales, using, of course, martingale theory. Further extensions were carried out by Itô's former students H. Kunita and S. Watanabe (1967) and P.A. Meyer and the Strasbourg school, culminating in the most general theory of stochastic integration with respect to semimartingales - that is a theorem not an opinion. Stepping back a bit, it was really in the 60's that the area of stochastic integration and stochastic differential equations seemed to reignite. I learned the theory from two excellent sources which appeared then: Itô's Tata Institute Notes [I3] and H. McKean's deceptively slim book, Stochastic Integrals [McK]. The latter is an effective way to learn stochastic analysis as each sentence is an exercise left for the reader (Itô describes it as "compact but full of interesting material"). There is a tricky step in the definition of Itô's integral where one proves denseness of a certain class of integrands. It is easy if one assumes the modern notions of predictability or

progressive measurability but Itô originally worked with a larger class of adapted and jointly measurable integrands and many of the (then) "modern" proofs seemed to get it wrong. Moving backward in time I discovered the correct and somewhat trickier proof in [I3].

Prof. Itô moved to Kyoto University in 1952 and spent 27 years there, half of which he spent abroad. It gave him the opportunity to work at many of the leading centres in the world (Princeton, Stanford, Cornell, Aarhus) and also bring his ideas to the rest of the world. It is hard to imagine any university today being so far-sighted and generous with such a valuable resource. It clearly enabled Itô to embrace some new and rich theories and in turn exert a positive influence on the development of probability in Japan as he would frequently return to Kyoto U. and lecture on new areas of probability. He arrived in Princeton in 1954 shortly after Feller had completed his analytical classification of one-dimensional diffusions (continuous strong Markov processes). Again the challenge of a pathwise construction of these processes was natural for Itô. He teamed up with Feller's then student, McKean, and 10 years later they completed *the book* on the subject [IM]. Itô describes a key interaction with Feller in [I5]:

*"McKean tried to explain to Feller my work on the stochastic differential equations... It seemed to me that Feller did not fully understand its significance, but when I explained Lévy's local time to Feller, he immediately appreciated its relevance to the study of the one dimensional diffusion. Indeed Feller immediately gave us a conjecture... which was eventually substantiated in my joint paper with McKean (1963)."*

Lévy's local time which Itô had read as a student in [L] turned out to be the critical ingredient in the Itô-McKean pathwise construction of general one-dimensional

continuous strong Markov processes. It provides the means to a random change of time of a Brownian path, which, together with a deterministic change of scale, gave the general result (those who have actually read [IM] will know this is a simplification but a justifiable one). Although the class of processes Itô constructed as solutions of stochastic differential equations form the central and canonical examples, it is curious that they are barely mentioned in their book. Although they both start with a Brownian path, the two constructions are in fact completely different. For one-dimensional diffusions the time and scale change technique proved to be the most general and so it is natural that the Kyoto U. seminar in the later 50's and early 60's would try to extend some of these ideas to general Markov processes. Feller's one-dimensional diffusions were symmetric Markov processes with respect to his speed measure which governed the rate of change of the aforementioned random time change. Beurling and Deny had already introduced the study of Dirichlet forms for general symmetric Markov processes. Masatoshi Fukushima, one of Itô's graduate students then, extended the above time-change ideas to the study of general symmetric Markov processes and with M. Silverstein became one of the two leaders in a subject of continuing importance. Martin boundary theory as developed by Doob and Dynkin, as well as Wentzell's boundary conditions for  $d$ -dimensional diffusions were other topics of study. To this day there is no general classification of  $d$ -dimensional continuous Markov processes. A host of examples (eg. of Tom Salisbury (York U.) (1986)) suggest this question is not going to be so tractable. On the other hand the theory of  $d$ -dimensional Markov processes is still dominated by Itô's stochastic differential equations and extensions thereof. His methods of stochastic analysis run through most of the infinite-dimensional theories as well, where Itô has also done influential

work. The multiparameter extension of Itô's stochastic integration pioneered by Renzo Cairoli and John Walsh (UBC) play an important role in the modern theory of stochastic partial differential equations. The theory of Dawson-Watanabe superprocesses developed by Itô's student Shinzo Watanabe and Don Dawson (Carleton) is one celebrated class of infinite-dimensional examples in which Itô's stochastic analysis plays a central role. He also played a more direct role. During Itô's tenure as a professor at Stanford (1961-64) he became interested in branching processes through proximity to Harris, Karlin and others. M. Fukushima recalls a resulting series of stimulating lectures on branching processes given by Itô during a trip back to Kyoto U. This and subsequent discussions led to the Ikeda-Nagasawa-Watanabe work on branching Markov processes and S. Watanabe's celebrated paper (1968) on continuous state branching processes. This paper has shaped many of my own research contributions.

I will not try to make this an encyclopedic description of Itô's contributions, as that would take a book. Fortunately the book has been written [I4]. I should, however, mention Itô's groundbreaking work on excursion theory (1970) which decomposes a general Markov process into excursions from a recurrent point and the process up to the recurrent point. The complicating issue is that typically the process returns to the point instantaneously. The solution is an infinite dimensional version of the same Poisson point process ideas Itô used in his thesis to prove the Lévy-Itô decomposition of a Lévy process. Instead of individual jumps taking values in the line, the Poisson points being glued together are now excursion paths taking values in an appropriate path space. Itô also suggested the converse construction of building a Markov process up from its excursions and, under suitable conditions, this synthesis was carried out by Tom Salisbury (York U.) and Chris Rogers.

The Gauss Prize in Madrid seemed to be part of a general recognition that the field of probability had come of age as the work of Fields medallists Okounkov and Werner both highlight probability. Canadians played a role in the celebration as Okounkov's work with Rick Kenyon (UBC) played a significant role in his citation, and Don Dawson (Carleton) was on the Fields Medal Committee. (The mathematics community in Canada had already anticipated this emergence when they elected three probabilists to the current CMS Executive.)

I have always felt fortunate to be part of a field where the leaders are so supportive of younger researchers. The kindness and interest Prof. Itô showed in my work on that hot Baton Rouge evening remains with me. His mentoring established a whole generation of probabilists in Japan - his students include such influential figures as T. Sirao, T. Hida, N. Ikeda, M. Nisio, S. Watanabe, H. Kunita, M. Fukushima and T. Yamada. Many more were active participants in Itô's Probability Seminar at Kyoto U. including M. Motoo, T. Watanabe, T. Ueno, H. Tanaka and K. Sato. Their students in turn are the current leaders in today's flourishing Japanese probability community, and collectively represent one of Prof. Itô's greatest legacies.

**Acknowledgement:** I would like to thank Don Dawson for making certain materials available to me and to Hans Föllmer for making his article on Itô's work for the 2006 ICM volume [Fo] available to me before publication. Special thanks go to Masatoshi Fukushima for sending me his lovely article [Fu] and for answering a number of queries. This article owes much to his written article and patience in answering a number of queries. Both [Fo] and [Fu] give a more detailed account of Itô's mathematics and are highly recommended reading.

## References

- [D] J.L. Doob, Stochastic processes depending on a continuous parameter, *Trans. Amer. Math. Soc.* **42**, 107-140 (1937).
- [Fe] W. Feller, Zur Theorie der stochastischen Prozesse (Existenz und Eindeutigkeits-sätze), *Math. Ann.* **113**, 113-160 (1936).
- [Fo] H. Föllmer, On Kiyosi Itô's work and its impact, to appear, *Proc. 2006 ICM, Madrid* (2007).
- [Fu] M. Fukushima, On the works of Kiyosi Itô and stochastic analysis, to appear in *Jap. J. Math.* (2007).
- [I1] K. Itô, On stochastic processes (infinitely divisible laws of probability), *Japan. Jour. Math.* **XVIII**, 261-301 (1942).
- [I2] K. Itô, Differential equations determining a Markov Process (in Japanese), *Journ. Pan-Japan Math. Coll.* No. 1077, 1352-1400 (1942); (in English) in *Kiyosi Itô, Selected Papers*, D. W. Stroock and S.R.S. Varadhan (ed.'s), 42-75 (1986).
- [I3] K. Itô, *Lectures on Stochastic Processes*, Tata Institute of Fundamental Research, Bombay (1960).
- [I4] Itô, *Kiyosi Itô, Selected Papers*, D. W. Stroock and S.R.S. Varadhan (ed.'s), Springer-Verlag, 1986.
- [I5] K. Itô, Memoirs of my research in stochastic analysis, in *Proc. Abel Symp. 2005, Stochastic Analysis and Applications-A Symposium in Honor of K. Itô*, 1-5, Springer (2006).
- [IM] K. Itô and H. P. McKean Jr., *Diffusion Processes and Their Sample Paths*, Springer-Verlag, 1965.
- [K] A. Kolmogorov, Über die analytischen Methoden in Wahrscheinlichkeitsrechnung, *Math. Ann.* **104**, 415-458 (1931).
- [L] P. Lévy, *Théorie de l'Addition des Variables Aléatoires*, Gauthier-Villars, Paris, 1937.
- [McK] H.P. McKean Jr., *Stochastic Integrals*, Academic Press, NY and London, 1969.

## Call for Applications - CMS Executive Director and Secretary Appel de candidatures - Directeur administratif et secrétaire de la SMC

The Canadian Mathematical Society (CMS), one of the leading mathematics organizations in Canada, seeks applications for the position of Executive Director and Secretary. This position offers a unique and exciting opportunity for an individual with energy, drive, initiative and enthusiasm to make a difference on the national stage.

The CMS works to enhance Canada's capacity to innovate and compete globally by promoting the discovery, learning and application of mathematics in Canada. With approximately 1000 members from across Canada and beyond, the CMS's active volunteers support efforts that identify and develop young mathematicians through its math competitions, math camps and other educational activities. The CMS enhances the practice of mathematics in Canada through national conferences, and by publishing research journals, books, and newsletters in both print and electronic formats.

The work of the CMS is carried out by a large number of dedicated and enthusiastic volunteers, together with a superb and experienced staff of eight at the Executive Office in Ottawa. The Executive Director and Secretary must be an effective and experienced administrator, able to address differing points of view with the tact and courtesy appropriate to a largely volunteer organization. The position is full-time and requires sound judgment, independence, travel, and flexibility in the scheduling of working hours.

The Executive Director and Secretary is appointed by the Board of Directors of the CMS, reports to the CMS President and represents the CMS to universities, governments, the corporate sector, institutes, and to other societies and officials. He/she is responsible to the Executive Committee and the Board of Directors of the CMS for the operations of the CMS Executive Office and for the other administrative offices throughout Canada. He/she works in close contact with the CMS President, with responsibility for aspects of the Society's publications, meetings, fundraising, web site, projects, and operations. Some restructuring of the current duties in the position is possible, to align them with the skills of an outstanding applicant. The position starts July 1, 2008 (but the ability to assume some duties on a part-time basis prior to that date is an asset).

### QUALIFICATIONS

A doctoral degree in mathematics or experience in an academic and research environment in mathematics is preferred. The candidate should have: prior administrative or managerial experience; excellent organizational skills; excellent interpersonal skills; excellent analytical and problem solving skills; excellent and proven communication skills (preferably in both official languages); experience in setting and managing budgets; and the ability to develop and implement policies that support and promote the work and programs of the CMS. Experience in fundraising and promotion is an advantage. The candidate will work principally out of the CMS Ottawa office.

The deadline for applications is **April 30, 2007, 4pm**. Please submit applications, including a resume, cover letter, and the names of at least three references by fax, email or mail to:

La Société mathématique du Canada (SMC), l'un des principaux regroupements mathématiques du pays, cherche à pourvoir le poste de directeur administratif et secrétaire. C'est un poste unique et stimulant, idéal pour une personne énergique, dynamique, enthousiaste et ayant l'esprit d'initiative qui souhaite se distinguer sur la scène nationale.

La SMC a pour mission de rehausser la capacité du Canada d'innover et d'être concurrentiel à l'échelle mondiale en favorisant la découverte et l'apprentissage des mathématiques, et les applications qui en découlent, au Canada. Forte de ses quelque 1000 membres du Canada et d'ailleurs, et grâce au travail bénévole de nombreux membres actifs, la SMC dépiste et forme de jeunes mathématiciens par l'entremise de concours, de camps et d'autres activités mathématiques éducatives. La SMC rehausse l'activité mathématique au pays par l'organisation de congrès nationaux et la publication de revues, de livres et de bulletins, en format papier et électronique.

Les activités de la SMC sont menées par un grand nombre de bénévoles dévoués et enthousiastes, appuyés par un personnel hors pair et compétent de huit personnes au bureau administratif d'Ottawa. Le directeur administratif et secrétaire doit être une personne efficace et expérimentée, capable de gérer des points de vue divergents avec le doigté et la courtoisie nécessaire dans un organisme en grande partie bénévole. Le ou la titulaire de ce poste à plein temps doit posséder un bon jugement, être autonome, pouvoir se déplacer fréquemment et être souple quant à son horaire de travail.

Le directeur administratif et secrétaire est nommé par le conseil d'administration de la SMC, relève du président de la SMC et représente la SMC auprès des universités, des gouvernements, du secteur privé, des instituts et d'autres sociétés et dirigeants. Cette personne est redevable au comité exécutif et au conseil d'administration de la SMC pour ce qui est du fonctionnement du bureau administratif d'Ottawa et des autres bureaux de la SMC Canada. Elle travaille en étroite collaboration avec le président de la SMC, et s'occupe de divers aspects des publications, des congrès, des activités de financement, du site internet, des projets et du fonctionnement. Une certaine restructuration des tâches actuelles du titulaire du poste est possible, de manière à tenir compte des compétences d'un candidat ou d'une candidate remarquable. La personne choisie entrera en fonction le 1er juillet 2008 (la possibilité d'assumer certaines tâches à temps partiel avant cette date est toutefois un atout).

### COMPÉTENCES

La préférence sera accordée à une personne qui possède un doctorat en mathématiques ou de l'expérience en milieu universitaire et en recherche dans le domaine des mathématiques. Cette personne possédera : de l'expérience en gestion; un sens aigu de l'organisation; un excellent sens des relations humaines; une excellente capacité d'analyse et de résolution de problèmes; des compétences linguistiques exceptionnelles et éprouvées (préféablement dans les deux langues officielles); de l'expérience en planification et en gestion budgétaire; des aptitudes pour l'élaboration et l'application de politiques qui soutiennent et stimulent les activités de la SMC. Une bonne connaissance des campagnes de financement et de promotion constituera un avantage. Le bureau administratif de la SMC à Ottawa est le lieu de travail principal.

Les personnes intéressées ont jusqu'au **30 avril 2007 à 16 h** pour poser leur candidature. Elles doivent faire parvenir leur demande, accompagnée d'un curriculum vitae, d'une lettre de présentation et d'au moins trois références, par fax, par courriel ou par la poste à :

#### Executive Director Search / Concours pour le poste de directeur administratif

Canadian Mathematical Society / Société mathématique du Canada

577 King Edward

Ottawa, Ontario Canada K1N 6N5

Attn: Dr Thomas Salisbury, CMS President / président de la SMC

Tel: 613.562.5702 FAX: 613.565.1539

e-mail: president@cms.math.ca / courriel : president@smc.math.ca

May 31 – June 3 / du 31 mai au 3 juin  
Delta Hotel, Winnipeg, Manitoba / Hôtel Delta, Winnipeg (Manitoba)  
Host: University of Manitoba / Hôte : Université du Manitoba

## Sessions

### Algebraic Varieties with Group Actions

#### Variétés algébriques avec actions de groupes

Org: Jaydeep Chipalkatti (Manitoba)

Megumi Harada (MacMaster), Colin Ingalls (UNB), Kiumars Kaveh (Toronto),  
Hugh Thomas (UNB), Matthieu Willems (Toronto)

### Banach Algebras and Abstract Harmonic Analysis

#### Algèbre de Banach et analyse harmonique abstraite

Org: Yong Zhang (Manitoba)

Constantin Costara (Laval), Antoine Derighetti (Lausanne, Switzerland), Brian Forrest (Waterloo), Colin C. Graham (UBC), Niels Grønbæk (Copenhagen, Denmark), Zhiguo Hu (Windsor), Zinaida Lykova (Newcastle, UK), Tianxuan Miao (Lakehead), Matthias Neufang (Carleton), Chi-Keung Ng (Chern Inst. of Mathematics, Nankai Univ., China), Thomas Ransford (Laval), Zhong-Jin Ruan (Illinois - Urbana-Champaign), Nico Spronk (Waterloo), Ross Stokke (Winnipeg), Thomas V. Tonev (Montana-Missoula), Ali Ülger (Koc Univ., Turkey), Michael C. White (Newcastle, UK), Yong Zhang (Manitoba)

### Complex Function Theory

#### Théorie des fonctions complexes

Org: Ian Graham (Toronto), Eric Schippers (Manitoba)

Roger Barnard (Texas Tech), Tom Bloom (Toronto), Maritza Branker (Niagara), Richard Fournier (CRM), David Herron (Cincinnati), Gabriela Kohr (Babes-Bolyai, Romania), Daniela Kraus (Würzburg, Germany), David Minda (Cincinnati), Jerry Muir Jr. (Scranton), Rajesh Pereira (Saskatchewan), David Radnell (American University of Sharjah, UAE), Thomas Ransford (Laval), Oliver Roth (Würzburg, Germany), Stephan Ruscheweyh (Würzburg, Germany), Alex Solynin (Texas Tech), Ted Suffridge (Kentucky), Dror Varolin (Stony Brook), Brock Williams (Texas Tech)

### Computer Algebra and Computer Algebra Systems

#### L'algèbre computationnelle et systèmes d'algèbre computationnelle

Org: Michael Monagan (SFU)

### Finite Combinatorics

#### Combinatoire finie

Org: Robert Craigen (Manitoba), David Gunderson (Manitoba)

Richard Guy (Calgary), Jonathan Jedwab (SFU), Hadi Kharaghani (Lethbridge), William Kocay (Manitoba), Ben Li (Manitoba), Vaclav Linek (Winnipeg), Wendy Myrvold (Victoria), Ortrud Oellerman (Winnipeg), Ranganathan Padmanabhan (Manitoba), Bruce Reed (McGill); Douglas Stinson (Waterloo)

### Mathematical Algorithms for Medical Imaging

#### Algorithmes mathématiques pour l'imagerie médicale

Org: Sima Noghanian (Manitoba)

Jorge Alpuche (CancerCare Manitoba), Ali Ashtari (Manitoba), Homa Fashandi (Manitoba), Colin Gilmore (Manitoba), Ian Jeffry (Manitoba), Amir Meghdadi

(Manitoba), Cameron Melvin (Manitoba), Puyan Mojabi (Manitoba), Malcolm Ng (Manitoba), Barbara Pawlak (Manitoba), Abas Sabouni (Manitoba), Gabriel Thomas (Manitoba), Niranjan Venugopal (CancerCare Manitoba)

### Mathematical Biology

#### Biologie mathématique

Org: Gerda de Vries (Alberta), Frithjof Lutscher (Ottawa)

Caroline Bampfylde (Alberta), Jacques Belair (Montreal), Luciano Buono (UOIT), Eric Cytrynbaum (UBC), David Earn (McMaster), Frank Hilker (Alberta), Anmar Khadra (UBC), Frithjof Lutscher (Ottawa), Alex Potapov (Alberta), Robert Smith (Ottawa), Philippe Tracqui (IMAG, Grenoble, France), Rebecca Tyson (UBC-Okanagan), Allan Wilms (Guelph)

### Mathematical / Computational Finance

#### Finance mathématique et computationnelle

Org: Ruppa K. Thulasiram (Manitoba)

Joe Campolieti (Wilfred Laurier), Tahir Choulli (Alberta), Ulrich Horst (UBC), Ali Lazrak (UBC), Alex Paseka (Manitoba), Luis Seco (Risk Lab), Ruppa K. Thulasiram (Manitoba), LiQun Wang (Manitoba)

### Mathematical Immunology

#### Mathématiques en Immunologie

Org: Robert Smith (Ottawa), Beni M. Sahai (Cadham Provincial Laboratory)

Rafi Ahmed (Emory), Rustom Antia (Emory), Daniel Coombs (UBC) Denise Kirschner (Michigan State), Robert Smith (Ottawa), Dominik Wodarz (California – Irvine)

### Mathematical Physics

#### Physique mathématique

Org: Richard Froese (UBC), Tom Osborn (Manitoba)

Twareque Ali (Concordia), Marco Bertola (Concordia), Rainer Dick (Saskatchewan), Stephan Gustavson (UBC), John Harnad (CRM), Lisa Jeffrey (Toronto), Nan-Kuo Ho (National Cheng Kung Univ., Taiwan), Marco Merkli (McGill), Rob Milson (Dalhousie), David Rowe (Toronto)

### Mathematics Education

#### L'éducation mathématique

Org: Abba Gumel (Manitoba), Randall Pyke (SFU)

### Mathematics of Infectious Diseases

#### Modélisation mathématique des maladies infectieuses

Org: Abba Gumel (Manitoba)

Christopher Bowman (Inst. for Biodiagnostics), Gerardo Chowell-Puente (Los Alamos National Laboratory), Elamin Elbasha (Merck Inc.), Horacio Gomez-Acevedo (Tougaloo College), Hongbin Guo (Alberta), Mudassar Imran (Arizona State), Edward Lungu (Botswana), Tufail Malik (Arizona State), Ronald Mickens (Clark Atlanta), Helen Moore (American Inst. of Mathematics), Miriam Nuno (Harvard), Chandra Podder (Manitoba), Oluwaseun Sharomi (Manitoba), Baojun Song (Montclair State), Lin Wan (UBC), James Watmough (UNB), Matthias Winter (Brunel, UK), Huaiping Zhu (York)

# CMS-MITACS Joint Conference 2007 / CONGRÈS CONJOINT MITACS-SMC 2007

## Model Theory and its Applications

### Théorie des modèles et ses applications

Org: Bradd Hart (McMaster), Thomas Kucera (Manitoba), Rahim Moosa (Waterloo)

John Baldwin (Illinois – Chicago), Paul Bankston (Marquette University), Alf Dolich (Illinois – Chicago), Alina Duca (Manitoba), Dragos Ghica (McMaster), Bradd Hart (McMaster), Dierdre Haskell (McMaster), Ivo Herzog (Ohio State; Lima), Thomas Kucera (Manitoba), Salma Kuhlmann (Saskatchewan), Chris Laskowski (Maryland), Rahim Moosa (Waterloo), Philipp Rothmaler (CUNY-Bronx), Thomas Scanlon (California – Berkeley), Patrick Speissegger (McMaster), Charles Steinhorn (Vassar College), Yevgeniy Vasilyev (Windsor)

## Network Algorithms

### Algorithmes des réseaux

Org: Evangelos Kranakis (Carleton)

Stefan Dobrev (Ottawa), Yiqiang Q. Zhao (Carleton)

## Nonlinear Methods in Computational Mathematics

### Méthodes nonlinéaires en mathématiques computationnelles

Org: Kirill Kopotun (Manitoba)

A. Bass Bagayogo (Univ. College of Saint-Boniface), Oleg Davydov (Strathclyde, UK), Francisco-Javier Muñoz-Delgado (Universidad de Jaén, Spain), Zeev Ditzian (Alberta), Jacek Gilewicz (Centre de Physique Théorique, France), Tom Hogan (The Boeing Company), Yingkang Hu (Georgia Southern), Bojan Popov (Texas A&M), Andriy Prymak (Alberta), Ping Zhou (St. Francis Xavier)

## Quantum Information Theory

### Théorie de l'information quantique

Org: Richard Cleve (Waterloo)

Gilad Gour (Calgary) , Patrick Hayden (McGill), Barry Sanders (Calgary), Alain Tapp (Montréal)

## Representations of Finite and Algebraic Groups

### Représentations des groupes finis et des groupes algébriques

Org: Gerald Cliff (Alberta), Anna Stokke (Winnipeg)

Peter Campbell (Bristol, UK), Xueqing Chen (Wisconsin - Whitewater), Jaydeep Chipalkatti (Manitoba), Greg Lee (Lakehead), Allen Herman (Regina), David McNeilly (Alberta), Fernando Szechtman (Regina), Qianglong Wen (Alberta)

## Resource Allocation Optimization

### Optimisation d'allocation de ressources

Org: Binay Bhattacharya (SFU) Abraham Punnen (SFU)

Robert Benkoczi (Queen's ), Prosenjit Bose (Carleton), Harvey Greenberg

(Colorado), David Kirkpatrick (UBC), Pat Morin (Carleton), Katta G. Murty (Michigan), Godfried Toussaint (McGill)

## Statistical Learning

### Apprentissage statistique

Org: Yoshua Bengio (Montreal)

## Contributed Papers

### Communications libres

Org: Ross Stokke (Winnipeg)

Papers of 20 minutes duration are invited. For an abstract to be eligible, the abstract must be submitted online ([www.cms.math.ca/Events](http://www.cms.math.ca/Events) or [www.mitacs.ca/AC07](http://www.mitacs.ca/AC07)) before April 15, 2007. The abstract must be accompanied by its contributor's registration form and payment of the appropriate fees. To better assist the organizers, please include the Primary (2000) AMS Classification (<http://www.ams.org/msc/>).

Nous lançons un appel de communications libres de 20 minutes chacune. Les résumés devront respecter nous parvenir au plus tard le 15 avril (Veuillez utiliser le formulaire électronique à [www.cms.math.ca/Reunions](http://www.cms.math.ca/Reunions) ou [www.mitacs.ca/AC07](http://www.mitacs.ca/AC07)).

Nous demandons à chacun de joindre au résumé le formulaire d'inscription et le règlement des frais pertinents. Pour faciliter la tâche des organisateurs, veuillez préciser la classification de sujets AMS 2000 (<http://www.ams.org/msc/>).

## Poster Session

### Session d'Affiches

We encourage students and postdoctoral fellows to display posters to present their recent work and results. This gives a chance for media, professors, and students of all levels to gain an appreciation for the type of projects being undertaken in the field of mathematical sciences. Posters are judged during the conference, and prizes are awarded for the top ten posters.

Nous encourageons les étudiants et chercheurs postdoctoraux à présenter des affiches de même que leurs plus récents travaux et résultats de recherche. Ces présentations offrent la possibilité aux médias, aux professeurs et aux étudiants de tous les niveaux de se familiariser avec les types de projets de recherche dans le domaine des mathématiques. Les affiches seront jugées pendant la conférence, et des prix seront attribués pour les dix meilleures affiches.

## Scientific Directors / Directeurs du Congrès

Don Dawson (Carleton)  
[ddawson@math.carleton.ca](mailto:ddawson@math.carleton.ca)

Fereidoun Ghahramani (Manitoba)  
[fereidou@cc.umanitoba.ca](mailto:fereidou@cc.umanitoba.ca)



## Local Arrangements / Logistique locale:

Abba Gumel (Manitoba)

**THE YUEH-GIN GUNG AND DR. CHARLES Y. HU DISTINGUISHED SERVICED TO MATHEMATICS AWARD  
CONFERRED JANUARY 6, 2007 AT THE JOINT MATHEMATICS MEETINGS, NEW ORLEANS**

The Gung and Hu Award for Distinguished Service to Mathematics, first presented in 1990, is the endowed successor to the Association's Award for Distinguished Service to Mathematics, first presented in 1962. This award is intended to be the most prestigious award for service offered by the Association. It honors distinguished contributions to mathematics and mathematical education - in one particular aspect or many, and in a short period or over a career. The initial endowment was contributed by husband and wife Dr. Charles Y. Hu and Yueh-Gin Gung. It is worth noting that Dr. Hu and Yueh-Gin Gung were not mathematicians, but rather a professor of geography at the University of Maryland and a librarian at the University of Chicago, respectively. They contributed generously to our discipline because, as they wrote, "We always have high regard and great respect for the intellectual agility and high quality of mind of mathematicians and consider mathematics as the most vital field of study in the technological age we are living in."

### **2007 Recipient**

Lee Lorch's mathematical research has been in the areas of analysis, differential equations, and special functions. His teaching positions have included the City College of New York, Pennsylvania State University, Fisk University, Philander Smith College, the University of Alberta, Howard University, Royal Institute of Technology (Stockholm) and Aarhus University. He was at York from 1968 until retirement in 1985 and remains active in the mathematical community.

His scholarship has been recognized by election to Fellowship in the Royal Society of Canada; appointment to committees of the Research Council of Canada; election to the Councils of the American Mathematical Society, the Canadian Mathematical Society, and the Royal Society of Canada; and by many invitations to lecture.

Lee Lorch is a remarkable teacher of mathematics and an inspiration to his students. Among those he guided were Etta Falconer, Gloria Hewitt, Vivienne Malone Mayes, and Charles Costley. He has recruited into graduate work and mathematical careers many students who would not have otherwise considered such a path. [See V. Mayes, American Mathematical Monthly, 1976, pp708-711; and P. Kenschaft, Change Is Possible,

American Mathematical Society, 2005.]

During the early organization of the Association for Women in Mathematics, Lee gave sage advice about the value of inclusiveness in supporting effective advocacy. He is responsible for the appearance of the preposition "for" in place of the initially proposed "of" in the name of the AWM.

Throughout his career he has been a vocal advocate and energetic worker for human rights and educational opportunities. His interventions, especially in the 1950's, led to changes in the policies and practices of the AMS and the MAA that ensured that all mathematicians could participate in the official events of these organizations. While his actions have not solved all the problems he addressed, surely his energy has contributed to much progress.

As an example, we cite events surrounding a meeting in 1951 held in Nashville. Lee Lorch, the chair of the mathematics department at Fisk University, and three black colleagues, Evelyn Boyd (now Granville), Walter Brown, and H. M. Holloway came to the meeting and were able to attend the scientific sessions. However, the organizer for the closing banquet refused to honor the reservations of these four mathematicians. (Letters in Science, August 10, 1951, pp. 161-162 spell out the details). Lorch and his colleagues wrote to the governing bodies of the AMS and MAA seeking bylaws against discrimination. Bylaws were not changed, but non-discriminatory policies were established and have been strictly observed since then.

For his life-long contributions to mathematics, his continued dedication to inclusiveness, equity, and human rights for mathematicians, and especially his profound influence on the lives of minority and women mathematicians who have benefited from his efforts, the MAA presents this Yueh-Gin Gung and Charles Y. Hu Award for Distinguished Service to Mathematics to Lee Lorch.

Lee Lorch, FRSC, is Professor Emeritus at York University in Toronto. Born in New York, his undergraduate studies were at Cornell. He holds a Ph.D. from the University of Cincinnati, mentored by Otto Szasz.

While in the U.S. army during the war, and shortly before going overseas, he

married Grace Lonergan, a Boston school teacher. She was dismissed for committing matrimony and became the first Boston teacher to contest that policy, but lost. A plaque commemorating her pioneering struggle and celebrating her subsequent civil rights activities now adorns the entrance to a Boston public school. Their participation in the struggle against housing discrimination cost Lorch two jobs in quick succession. Moving south, their efforts to speed the end of segregation in public education, as mandated by the Supreme Court (1954), cost Lorch the last two posts he was able to obtain in the U.S. He was summoned before the House Committee on Un-American Activities and cited for "contempt" for refusing to say whether he had ever been a member of the Communist Party. He was acquitted. Grace Lorch was called before the Senate Subcommittee on Internal Security, where she also refused to answer political questions. Years later, Lorch received honorary degrees from two of the institutions that had dismissed him. In 1959 the couple moved to Canada. Both have received awards for their civil rights contributions.

### **Response from Lee Lorch**

While this award honors me, it gives me even greater satisfaction that, by making it, the MAA emphasizes its support for equity.

There are all too many proofs that this fight is far from over. One surrounds us here: Katrina and post-Katrina New Orleans. Why was New Orleans left so vulnerable? Why was flood control, so urgently and obviously needed, set aside? Its low-lying areas, overwhelmingly African-American, seed-beds of world famous African-American music, are ruined, their residents scattered and disheartened, their communities in peril of dissolution.

Even the AMS home page tells us only of Tulane - not of the several afflicted HBCUs. Perhaps no one in these institutions has submitted a report. Maybe they do not feel really part of the mathematical community. Why not? What is being done about it?

"The struggle continues." Happily, this award is a sign of which side the MAA is on.

Thank you, thank you very much!

## THE ADRIEN-POULIOT AWARD LECTURE

### Introduction to Adrien Pouliot Award Lecture

Peter Taylor at Queen's University in Kingston was the winner of the 2006 Adrien Pouliot Award for significant and sustained contribution to mathematics education. As noted in the citation, "Peter's work is grounded in an innovative and evolving curriculum philosophy and an approach to mathematics which is fundamentally aesthetic". The depth of Peter's approach is wonderfully exemplified in the homily he delivered at last December's Winter Meeting in Toronto, which is reproduced here.

The reference for the citation is

<http://www.cms.math.ca/Prizes/citations/ap2006.pdf>.

by Ed Barbeau

### The Structure of a Mathematics Curriculum

It is an honour to receive the 2006 Pouliot Award for contributions to Mathematics Education. The many past winners of this award have made a great impact on mathematics education in Canada, and I'm proud to be among them. In my talk I will review the main concerns and projects of 40 years of work.

We are privileged to study an extraordinary subject of enormous beauty and power. Pure mathematicians and physicists know this well, but even in my work in mathematical biology I am often blown away by the elegance of many of the results.

But this is a well kept secret. I assure you that hardly any high school students, and precious few university students, know about this. We are diligent in keeping this a secret from all but the privileged few.

That's a tragedy—a tragedy for the subject because it's so awesome, a tragedy for our students because they miss something they need at a deep level, and it is bad news for the future of the world.

The book I am reading right now is called Heat: how to stop the planet from burning. Malcolm Griffin lent it to me and told me it was the climate change book not to miss. The author, George Monbiot, declares at the beginning: Nothing here is as it seems. He research for this book has involved me in a long series of surprises. What I have sought to do throughout the text is to start from first principles, to believe nothing until it is demonstrated, to junk any technology, however pleasing it may be, which does not work. Well that's what mathematics does, and that's one reason the world needs it so badly.

But not enough kids are studying engineering, physical science and math. Most find mathematics boring and irrelevant. Many are intimidated by it and find that it fills them with anxiety. Certainly almost no one finds school mathematics exciting or engaging. We need to try to understand why this is the case.

On many occasions over the past 8 years I have worked with the Ministry of Education in Ontario in the process of writing the policy documents for the senior school math curriculum. It's a challenging process as we must navigate around a number of different views on content and emphasis. The final product is pedestrian and might be compared to a manual for assembling a vacuum cleaner—something you do need to study at one or two points in your life, but otherwise it just stays on the shelf. parts of it are devoted to a discussion of the importance of process elements—engaging the student in investigation and inquiry, but most of it is a list of the technical skills we want our students to obtain at that level. In designing a curriculum, that kind of information is of course crucial.

So it's an important document but it's a policy document, not a

curriculum. It was never intended to be used as a curriculum. The problem is that that's precisely what it has been used for. To be more precise, the problem is that it has been used as the structural framework for the curriculum.

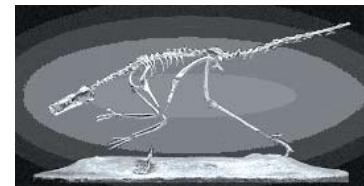
The problem then is one of structure. The structure that serves the policy document is not meant for the curriculum. If so used it bestows the kiss of death.

The reasons for this are complex and no doubt partly known to most readers. It's deemed important (even essential) to cover all the material prescribed in the policy document and the simplest way to ensure this is to follow its logical structure. Even more, it typically appears impossible to cover everything unless it is closely followed. Indeed there is so much in the document, that following it is guaranteed to occupy your full class time. I know this because teachers frequently come to me and say that they like my investigative problems but there's never enough time to work with them properly.

Well since investigation and engagement are key components of learning, a natural response is to try to cut the curriculum back a bit to make room for more of this. But this is inevitably problematic as we find that nothing really should be cut. Everything seems equally important.

The irony is that there is a way to cover the policy document and do the investigative examples, but it requires letting go of the structure of the policy document. It requires putting the entire document aside and basing the curriculum on an entirely different structure.

I find the following analogy helpful. Suppose you wanted to build an animal that could fly and you were given as a starting point the structure at the right. Let's call this the advanced functions animal. Others are possible—we could have had the geometry or discrete math animal. But this advanced functions animal, which is the heaviest of all the beasts, is the one deemed most relevant to the needs of the world; its job in life is to forage and dominate the environment and kill and eat the other less fortunate beasts. [You can see where math anxiety comes from.]



So that's your starting point. You see right away that the structure is too heavy to ever get airborne, and must be cut down. But an interesting and embarrassing thing happens. You discover that's almost impossible to cut anything out. The harder you try, the more you see that everything is actually needed. Protection requires big horns, and feeding requires big jaws. You've got to eat after all and the less fortunate beasts are not so easy to crush. So cutting down is not ultimately going to get you there.

[In 1992 I was at an NSF reform calculus conference in San Antonio and at one point, to make some room for the "new" problems, Andy Gleason sent us all off in small groups each to take a portion of the calculus curriculum and see what we could jettison. Guess what?—everything was deemed to be necessary.]

What you need is a completely different starting point. A different structure. Something more like the one on the right. I can imagine building wings on this guy (in fact that's exactly what evolution decided to do). I can imagine using this guy as the basis of a curriculum.



A traditional curriculum is based on a partition of mathematics into different fields (geometry, algebra, functions, probability, discrete math) and these sub-fields are typically built up logically and sequentially. But that's not how mathematicians work. For them, the starting point is a problem (which often belongs to a subfield) and the techniques they use and develop are the ones that have a chance of serving the problem. That's a more active design and more intense and engaging. It also happens to fit rather well the operational mode of today's young people.

For me, a curriculum is a network (a better term than "sequence") of investigations, exhibits, works of art, demonstrations, playgrounds, microworlds, grains of sand, whatever you want to call them, which uncover the secrets of the material and provide opportunities for conceptual growth and technical mastery. The surprise is that a good network can actually cover all the significant material in the policy document at the same time as it implements its many process-oriented objectives. In fact the coverage is even better because it's in a natural context.

However the curriculum we construct in this way will often, particularly at the beginning, seem quite disjoint from the policy document and we worry that we won't cover it all and we find this difficult to handle. A big job of "letting go" is needed here, of trusting teachers and students, of trusting the material itself to work its magic. [By the way, there's a lot to be gained from giving the teachers a strong message that we trust them.] The paradoxical truth is that the only way to be true to the policy document is to let it go. And I am convinced (after many years of trying different things at different levels and giving problems to different kinds of students and observing their struggles and their successes and failures) that this approach can cover the ground every bit as well as five loaves and two well-chosen small fishes can feed a multitude. But it is a long-term approach rather than short-term. It accomplishes its objective over many years.

As an illustration of this issue, a recent dilemma in the design of the senior math curriculum in Ontario, we had the problem of what to put in the final course for university-bound science majors. Should it be geometry, discrete math, vectors, or calculus, or maybe we could fit two of the four? But why not have all of them and none of them, and call it something like optimization (to take an idea of Ed Barbeau) and just solve a bunch of wonderful max-min problems? I bet we'd cover some important ideas in all four of those areas and learn a lot more real mathematics while we were at it. One could hardly imagine such a course being accepted under the current curriculum philosophy. But if it was, who would be harmed?

Well, maybe that's our work cut out for us over the next 10 years.

### Notes:

A number of examples of the investigative problems I work with are found on my website at:

<http://www.mast.queensu.ca/~peter/investigations/index.html>

In the Pouliot talk itself, I highlighted the Tire pressure model and Throwing balls into boxes.

### Picture credits:

1. National Oceanic & Atmospheric Administration (NOAA), NOAA Central Library.

[http://www.history.noaa.gov/art/noaa\\_gallery/nart0007.html](http://www.history.noaa.gov/art/noaa_gallery/nart0007.html)

2. National Science Foundation Directorate for Biological Sciences.

[http://www.nsf.gov/news/news\\_summ.jsp?cntn\\_id=104498&org=EF](http://www.nsf.gov/news/news_summ.jsp?cntn_id=104498&org=EF)

## CALL FOR NOMINATIONS

### CJM/CMB - Associate Editors

The Publications Committee of the CMS solicits nominations for Associate Editors for the Canadian Journal of Mathematics (CJM) and the Canadian Mathematical Bulletin (CMB). The appointment will be for five years beginning January 1, 2007. The continuing members (with their end of term) are below.

The deadline for the submission of nominations is April 15, 2007.

Nominations, containing a curriculum vitae and the candidate's agreement to serve should be sent to the address below.

Address for Nominations / Adresse de mise en candidatures:

**Juris Steprans, Chair / Président**

CMS Publications Committee / Comité des publications de la SMC

Department of Mathematics, York University

N520 Ross, 4700 Keele Street

Toronto, Ontario M3J 1P3

[chair-pubc@cms.math.ca](mailto:chair-pubc@cms.math.ca)

## APPEL DE MISES EN CANDIDATURE

### JCM/BCM - Rédacteurs associés

Le comité des publications de la SMC sollicite des mises en candidatures pour des rédacteurs associés du Journal canadien de mathématiques (JCM) et Bulletin canadien de mathématiques (BCM). Le mandat sera de cinq ans et débutera le 1 janvier 2007. La liste des éditeurs qui sont en cours de mandat se trouve ci-dessous..

L'échéance pour proposer des candidats est le 15 avril 2007.

Les mises en candidature, accompagnées d'un curriculum vitae ainsi que du consentement du candidat(e), devrait être envoyées à l'adresse ci-dessous.

## CURRENT MEMBERS / MEMBRES ACTUELS

### CJM Editors-in-Chief / Rédacteurs-en-chef du JCM

H. Kim (Toronto) 12/2011; R. McCann (Toronto) 12/2011.

### CMB Editors-in-Chief / Rédacteurs-en-chef du BCM

Nantel Bergeron (York) 12/2010; Jianhong Wu (York) 12/2010.

### Associate Editors / Rédacteurs associés

K. Bezdek (Calgary) 12/2011; Steven Boyer (UQAM) 12/2008; J.

Colliander 12/2011; W. Craig (McMaster) 12/2007; Luc Devroye (McGill)

12/2009; Alan Dow (York) 12/2010; George Elliott (Toronto) 12/2010;

Pengfei Guan (McMaster) 12/2008; K. Hare (Waterloo) 12/2011; Stephen

Kudla 12/2008; Thomas Ransford (Laval) 12/2009; Ravi Vakil (Stanford

University) 12/2009.

## CMS AWARDS ANNOUNCEMENT/LAURÉATS DES PRIX DE LA SMC

### 2006 Adrien Pouliot Award Prix Adrien-Pouliot

**The Adrien Pouliot Award is for individuals, or teams of individuals, who have made significant and sustained contributions to mathematics education in Canada.**

The 2006 Adrien Pouliot Award is awarded to Peter D. Taylor (Queen's) for his outstanding contributions to the teaching and learning of mathematics in Canada. Peter's work is grounded in an innovative and evolving curriculum philosophy and an approach to mathematics which is fundamentally aesthetic. His passion for revealing the aesthetics in mathematics is perhaps best illustrated by the course Mathematics and Poetry that he teaches jointly with a colleague in the English Department at Queen's. In this course Peter immerses students in beautiful problems to reveal qualities shared by mathematics and poetry.

Peter Taylor is a professor in the Department of Mathematics and Statistics at Queen's University, cross-appointed to the Department of Biology and the Faculty of Education. During his career Peter has taught and published in all three areas including two semesters in high school to prepare for the extensive curriculum writing work he continues to do with the Ontario Ministry of Education. A central thrust of his curriculum work involves the construction of problems which are investigative in nature but at the same time deliver the key ideas and techniques of the standard curriculum, particularly calculus and linear algebra. He has produced a number of books of investigative problems which are in wide circulation in the school system. He was a founding member of the Canadian Math Education Study Group (CMESG), served as chair of the CMS Education Committee from 1983 to 1987, and is a regular participant in the activities of the Fields Institute Mathematics Education Forum.

Peter has presented his innovative approach to mathematics education at many meetings of educators. These include a plenary lecture at a CMESG meeting, a plenary talk at the PIMS Changing the Culture Conference and education sessions at CMS meetings. Of particular note is a joint lecture, Reinventing the Teacher, with one of his graduate students, Nathalie Sinclair, at the 2000 ICME conference in Tokyo - one of two lectures singled out on the front page of the final conference newsletter. His reputation as a teacher has been recognized by the Queens' Arts and Science Teaching Award (1986), a MAA Distinguished Teaching Award (1992), and a 3-M Teaching Fellowship (1994).



Peter Taylor, Harley Weston, and Tom Salisbury

**Le prix Adrien-Pouliot rend hommage aux personnes ou aux groupes qui ont fait une contribution importante et soutenue à l'éducation mathématique au Canada.**

Le prix Adrien-Pouliot 2006 est décerné à Peter D. Taylor (Queen's) pour sa contribution exceptionnelle à l'enseignement des mathématiques au Canada. Les travaux de Peter reposent sur sa conception d'un programme innovateur et évolutif, ainsi qu'une approche mathématique fondamentalement esthétique. Sa passion pour l'esthétisme des mathématiques ne ressort nulle part mieux que dans le cours Mathématiques et poésie qu'il donne avec un collègue du Département d'anglais de Queen's. Dans ce cours, Peter présente de superbes problèmes qui révèlent des qualités communes aux mathématiques et à la poésie.

Peter Taylor est professeur au Département de mathématiques et de statistique de l'Université Queen's tout en étant aussi affecté au Département de biologie et à la Faculté d'éducation. Durant sa carrière, il a enseigné et publié dans les trois domaines. Il a notamment enseigné deux semestres dans une école secondaire en guise de préparation à l'élaboration de programmes pour le ministère de l'Éducation de l'Ontario. Il s'attache surtout à construire des problèmes qui font appel à l'investigation tout en véhiculant les principaux concepts et techniques du programme-cadre, particulièrement en calcul différentiel et intégral et en algèbre linéaire. Il a en outre publié un certain nombre d'ouvrages sur la résolution de problèmes qui sont bien connus dans le système scolaire. Il est un membre fondateur du Groupe canadien d'étude en didactique des mathématiques (GCEDM), il a présidé le Comité d'éducation de la SMC de 1983 à 1987 et il participe activement aux activités du forum sur l'enseignement des mathématiques de l'Institut Fields.

Peter a présenté sa démarche novatrice à l'occasion de nombreuses rencontres d'éducateurs. Il a donné une conférence principale lors d'un congrès du GCEDM, une conférence plénière dans le cadre du congrès « Changing the Culture » du PIMS, et des communications dans les sessions sur l'éducation des Réunions de la SMC. Il a notamment donné une conférence intitulée Reinventing the Teacher avec l'une de ses étudiantes, Nathalie Sinclair, lors du congrès ICME 2000 à Tokyo, l'une des deux conférences qui ont fait la une du bulletin final du congrès. Sa réputation d'enseignant n'est plus à faire. Il a d'ailleurs reçu le Queens Arts and Science Teaching Award (1986), le Distinguished Teaching Award de la MAA (1992) et le Prix 3M pour l'excellence en enseignement (1994).

### CMS Prize Lectureships and Awards Programmes - Prix et bourses de la SMC

**The most up-to-date information concerning all CMS Prize Lectureships & Awards programmes, including complete lists of recipients, can be found at: [www.cms.math.ca/Prizes/](http://www.cms.math.ca/Prizes/)**

**Vous trouverez l'information la plus récente sur les prix et bourses de la SMC, y compris les listes de lauréats, sur le site Web suivant : [www.smc.math.ca/Prizes/](http://www.smc.math.ca/Prizes/)**

## 2006 Jeffery-Williams Prize Prix Jeffery-Williams

**The Jeffery-Williams Prize recognizes mathematicians who have made outstanding contributions to mathematical research.**

Dr. Andrew Granville is recognized world-wide as a leading analytic number theorist. He has contributed to various areas of number theory, and in each one, he has left his mark with striking results and solutions to long standing problems.

A sample of his first rate work has to include his proof with Alford and Pomerance of the infinitude of Carmichael numbers with the important practical consequence that many commercially available primality tests falsely certified many composite numbers as prime. One should also mention his papers with Friedlander, and very recently with Soundararajan, on the irregularities of the distribution of primes, and more generally, of arithmetic sequences in arithmetic progressions; his work with Bombieri and Pintz on squares in arithmetic progressions; his series of papers with Soundararajan on the distributions of character and exponential sums; and his interesting paper with Stark on the abc conjecture and Siegel zeros of L-functions of imaginary quadratic fields.

The above papers contain major breakthroughs and constitute fundamental advances in analytic number theory. Accordingly, they have appeared in the top tier of mathematical journals. Dr. Granville's contributions have been recognized by an invitation to speak at the International Congress of Mathematicians in Zurich 1994, as well as numerous prizes and honours.

An excellent expositor, Dr. Granville is highly sought after as a plenary speaker or to explain sophisticated mathematical concepts and facts in an intriguing and understandable way. A typical example of his mastery of the art of exposition is his very recent comprehensive article "It is easy to determine whether a given integer is prime" in the Bulletin of the American Mathematical Society.

The Canadian mathematical community is extremely fortunate that Dr. Granville returned to Canada in 2002, where he earlier undertook graduate studies and a postdoctoral fellowship. While not in Canada, he maintained strong ties through the organization of sessions, service to NSERC, the Fields Institute, as well as to the Canadian Number Theory Association.

Dr. Andrew Granville received a Bachelor of Arts (Honours) degree in 1983 and a Certificate of Advanced Studies (Distinction) in 1984, both at Trinity College, Cambridge University. He went to Queen's University where, under the supervision of Paulo Ribenboim, he completed his Ph. D. in 1987 (which included what were then some of the best results known on Fermat's Last Theorem).

He was a postdoctoral fellow at the University of Toronto (1987-1989) and a member of the Institute for Advanced Study, Princeton (1989-1991). In 1991, he took up a position as Assistant Professor



Tom Salisbury and Andrew Granville

at the University of Georgia, where he attained the rank of Associate Professor in 1993 and became Full Professor and holder of the David C. Barrow Chair of Mathematics in 1995. In 2002, he returned to Canada, accepting a Canada Research Chair at the Université de Montréal.

His numerous honours include an Alfred P. Sloan Research Fellowship (1992-1995), a Presidential Faculty Fellowship (awarded by President Clinton) from 1994 to 1999, the 1995 Hasse Prize of the Mathematical Association of America, and the 1999 Ribenboim

Prize of the Canadian Number Theory Association.

He served on NSERC's Grant Selection Committee from 1995 to 1998, on the NSERC Membership Subcommittee from 1996 to 1997, and the Computation Subcommittee from 1997 to 1998. For the United States National Science Foundation (NSF), he was a member of the CAREER Panel in 1996, the POWRE Panel in 2000, and the "Committee of Visitors" in 2001.

Dr. Granville has served on the AMS Editorial Boards Committee (1996-1999), the AMS Conference Program Selection Committee (1998-2000), serving as Chair from 1999 to 2000, and is currently a member of the CMS Research Committee. He has also served on the Editorial Boards of close to a dozen other journals. He has supervised many graduate students and postdoctoral fellows.

***Le prix Jeffery-Williams rend hommage aux mathématiciens ayant fait une contribution exceptionnelle à la recherche mathématique.***

Andrew Granville est une sommité mondiale de la théorie analytique des nombres. Il a contribué à plusieurs domaines de la théorie des nombres, et il a laissé sa marque dans chaque cas, en obtenant des résultats étonnantes et en proposant des solutions impressionnantes à des problèmes de longue date.

Entre autres travaux exceptionnels, mentionnons la preuve qu'il a formulée avec Alford et Pomerance de l'infinité des nombres de Carmichael, qui a comme importante incidence pratique, que beaucoup de tests commerciaux disponibles sur le marché donnent beaucoup de nombres composés comme premiers. Sans oublier les articles publiés en collaboration avec Friedlander, et tout récemment avec Soundararajan, sur les irrégularités de la distribution des nombres premiers et, plus généralement, des séquences arithmétiques dans les progressions arithmétiques; ses travaux avec Bombieri et Pintz sur les carrés dans les progressions arithmétiques; ses séries d'articles avec Soundararajan sur les distributions de caractères et de sommes exponentielles, et un article intéressant en collaboration avec Stark sur la conjecture abc et les zéro de Siegel des L-fonctions de corps quadratiques imaginaires.

Ces articles constituent des percées importantes et des progrès fondamentaux en théorie analytique des nombres. De fait, ils ont

### 2006 Jeffery-Williams Prize Prix Jeffery-Williams suite

paru dans les meilleures revues mathématiques. M. Granville a en outre été invité comme conférencier au Congrès international des mathématiciens de Zurich, en 1994, en plus de recevoir de nombreux prix et distinctions.

Excellent orateur, Andrew Granville est un conférencier très sollicité. Il sait expliquer des concepts et des faits mathématiques complexes d'une façon intéressante et compréhensible. On retrouve un exemple type de sa maîtrise de l'art de l'exposition dans son tout récent grand article intitulé "It is easy to determine whether a given integer is prime" publié dans le Bulletin de l'American Mathematical Society.

La communauté mathématique canadienne est extrêmement choyée qu'Andrew Granville soit rentré au Canada en 2002, où il avait entrepris ses études supérieures et obtenu une bourse de recherche. Durant son séjour à l'étranger, il a conservé des liens étroits avec le pays en organisant des séances pour le CRSNG, l'Institut Fields et l'Association canadienne de théorie des nombres.

Andrew Granville a obtenu son baccalauréat ès arts (avec spécialisation) en 1983 ainsi qu'un certificat d'études supérieures (avec mention) en 1984 au Trinity College (Cambridge). Il a ensuite poursuivi ses études à l'Université Queen's, où il a terminé son doctorat en 1987 sous la direction de Paulo Ribenboim (il avait déjà obtenu certains des meilleurs résultats sur le dernier théorème de Fermat connus à ce jour).

Il a reçu une bourse postdoctorale de l'Université de Toronto (1987-1989) et il a été membre de l'Institute for Advanced Study-Princeton (1989-1991). En 1991, il a accepté un poste de profes-

seur associé à l'Université de Géorgie, où est devenu professeur agrégé en 1993, puis professeur titulaire et titulaire de la chaire de mathématiques David C. Barrow en 1995. En 2002, il est revenu au Canada, acceptant une Chaire de recherche du Canada à l'Université de Montréal.

Au nombre de ses distinctions, mentionnons la bourse de recherche Alfred P. Sloan (1992-1995), la Presidential Faculty Fellowship (décernée par le président Clinton) de 1994 à 1999, le prix Hasse 1995 de la Mathematical Association of America et le prix Ribenboim 1999 de l'Association canadienne de théorie des nombres.

Il a siégé à plusieurs comités du CRSNG : Comité de sélection des subventions (1995-1998), sous-comité de l'adhésion (1996-1997) et sous-comité de l'informatique (1997-1998). Pour la National Science Foundation (NSF), aux États-Unis, il a siégé aux comités CAREER (1996) et POWRE en 2000, et au "comité des visiteurs" en 2001.

Andrew Granville a également siégé au Conseil des comités de rédaction de l'AMS (1996-1999), au Comité d'organisation du programme du congrès de l'AMS (1998-2000) - dont il a été président de 1999 à 2000 - et il est actuellement membre du Comité de recherche de la SMC. Il a de plus siégé au comité de rédaction d'une bonne dizaine d'autres revues. Il a dirigé de nombreux étudiants aux cycles supérieurs et boursiers postdoctoraux.

### 2006 Doctoral Prize Prix de doctorat

**The CMS Doctoral Prize recognizes outstanding performance by a doctoral student who graduated from a Canadian university.**

As a graduate student of Professor Christopher Godsil, University of Waterloo, Michael Newman wrote an outstanding dissertation which presents extensions and applications of the Delsarte-Hoffman bound on the size of independent sets in graphs. The thesis interweaves the solutions of three intriguing yet ostensibly unrelated problems into a unified tapestry by virtue of their common methodological treatment. The results obtained are important and the exposition first-rate.

Michael Newman received his B.Math. from the University of Waterloo in 1992 and his M.Sc. from the University of Manitoba in 2000. He completed his Ph.D. in 2005 and, since then he has held an NSERC postdoctoral fellowship at Queen Mary College in London, England.



Michael Newman and Tom Salisbury

**Le Prix de doctorat de la SMC récompense le travail exceptionnel d'un étudiant ou d'une étudiante au doctorat ayant obtenu un diplôme d'une université canadienne.**

Durant ses études supérieures sous la supervision du professeur Christopher Godsil de l'Université de Waterloo, Michael Newman a rédigé une dissertation exceptionnelle présentant des extensions et des applications de la borne de Delsarte-Hoffman sur la taille des ensembles indépendants dans les graphes. Sa thèse intègre les solutions de trois problèmes intrigants, mais aussi visiblement non reliés, en un seul tableau grâce à une méthodologie commune. Les résultats obtenus sont importants, et la démonstration, exceptionnelle.

Michael Newman a obtenu un baccalauréat en mathématiques de l'Université de Waterloo en 1992 et une maîtrise en sciences de l'Université du Manitoba en 2000. Il a obtenu son doctorat en 2005 et poursuit depuis des recherches postdoctorales au collège Queen Mary à Londres grâce à une bourse du CRSNG.

## CMS AWARDS ANNOUNCEMENT/LAURÉATS DES PRIX DE LA SMC

### 2006 G. de B. Robinson Award Prix G. de B. Robinson

The G. de B. Robinson Award was inaugurated to recognize the publication of excellent papers in the Canadian Journal of Mathematics and the Canadian Mathematical Bulletin and to encourage the submission of the highest quality papers to these journals.

The 2006 G. de B. Robinson Award is presented to Dr. Malcolm Harper for his paper entitled “ $Z[\sqrt{14}]$  is Euclidean” published in the Canadian Journal of Mathematics, Volume 56 (2004), no. 1, pp. 55-70.

This paper resolves a long-standing question initially posed by Pierre Samuel. In a fundamental paper written in 1971, Samuel raised numerous questions about Euclidean rings, the most celebrated one being whether  $Z[\sqrt{14}]$  is Euclidean. It is well-known that this ring is not Euclidean for the norm map, so Samuel's question is if another map exists making the ring Euclidean. Shortly after Samuel's paper, Weinberger showed that if we assume the generalized Riemann hypothesis (GRH), then the ring is Euclidean, albeit for some strange Euclidean function. In a series of papers written in the 1980's, Rajiv Gupta, Kumar Murty and Ram Murty devised new techniques to study Euclidean rings in an attempt to remove the use of the GRH from Weinberger's work. Their work ultimately led David Clark and Ram Murty to show that  $Z[\sqrt{14}, 1/p]$  is Euclidean for the prime  $p=1298852237$ , without the use of GRH. In his doctoral thesis, Harper showed that the result of Clark and Ram Murty holds for any prime  $p$ . Later, by an ingenious use of the large sieve method, he removed the use of the auxiliary prime and established Samuel's conjecture.

Malcolm Harper completed his bachelor's degree (with distinction) in physics and his master's degree in mathematics at the University of Regina in 1994. He then moved to McGill University and obtained his Ph.D. under the direction of M. Ram Murty in 2000. The paper for which Harper is given the Robinson Award was based on his doctoral thesis.



Malcolm Harper  
and Juris Steprans

Le prix G. de B. Robinson rend hommage aux mathématiciens qui se sont distingués par l'excellence de leurs articles parus dans le Journal canadien de mathématiques et le Bulletin canadien de mathématiques, et vise à encourager la présentation d'articles de première qualité pour ces revues.

La SMC décerne son prix G. de B. Robinson 2006 à Malcolm Harper pour son article intitulé «  $Z[\sqrt{14}]$  is Euclidean » publié dans le Journal canadien de mathématiques, volume 56 (2004), no 1, pp. 55-70.

Cet article résout une question posée il y a longtemps par Pierre Samuel. Dans un article fondamental écrit en 1971, Samuel soulevait plusieurs questions à propos des anneaux euclidiens, la plus célèbre étant «  $Z[\sqrt{14}]$  est-il euclidien? ». Il est bien connu que cet anneau n'est pas euclidien par rapport à la fonction « valeur absolue »; Samuel demande donc si l'anneau serait euclidien s'il existait une autre fonction. Peu après la publication de l'article de Samuel, Weinberger a montré que si l'on tient compte de l'hypothèse de Riemann généralisée, l'anneau serait bel et bien euclidien, bien qu'il s'agisse d'une étrange fonction euclidienne. Dans une série d'articles écrits dans les années 1980, Rajiv Gupta, Kumar Murty et Ram Murty ont élaboré de nouvelles techniques pour l'étude des anneaux euclidiens dans le but d'empêcher l'utilisation de l'hypothèse de Riemann généralisée avancée par Weinberger. Ces travaux ont abouti lorsque David Clark et Ram Murty ont démontré que  $Z[\sqrt{14}, 1/p]$  était euclidien pour le nombre premier  $p=1298852237$ , sans recourir à l'hypothèse de Riemann généralisée. Dans sa thèse de doctorat, Malcolm Harper a montré que le résultat de Clark et de Ram Murty s'appliquait à tout nombre premier  $p$ . Plus tard, par un emploi ingénieux de la méthode du crible, il a éliminé l'emploi du nombre premier auxiliaire et a établi la conjecture de Samuel.

Malcolm Harper a obtenu un baccalauréat en physique (avec distinction) et une maîtrise en mathématiques de l'Université de Regina en 1994. Il a poursuivi au doctorat à l'Université McGill sous la direction de Ram Murty et a terminé ses études en 2000. L'article pour lequel Malcolm Harper obtient le prix G. de B. Robinson découle de sa thèse de doctorat.

### 2006 Distinguished Service Award (DSA) & The Inaugural David Borwein Distinguished Career Award

The CMS Distinguished Service Award was created in 1995 to recognize individuals who have made sustained and significant contributions to the Canadian mathematical community and, in particular, to the Canadian Mathematical Society.

The David Borwein Distinguished Career Award was created in 2004 to recognize individuals who have made an exceptional, broad, and continued contribution to Canadian mathematics. The award is named for David Borwein, a former President of the CMS and an eminent Canadian mathematician.

Dr. Richard Kane has profoundly influenced the course of mathematics in Canada through his long service to the profession, to his university, and to the CMS. He is a distinguished researcher



Richard Kane and Tom Salisbury

and a dedicated teacher. His exemplary career, taken as a whole, is eminently deserving of recognition through the receipt of the inaugural Borwein award, as well as the 2006 CMS Distinguished Service Award.

Dr. Kane received his BA from the University of Toronto in 1967 and his PhD from the University of Waterloo in 1973. He has been a member of the Department of Mathematics at the University of Western Ontario since 1980 and a full professor there since 1983, serving twice as Chair of the Department. Prior to coming to Western he taught at the University of Alberta, and held postdoctoral fellowships at Oxford and MIT. He has held visiting positions at the Institute for Advanced Study, (Princeton), the Centre de Recerca Matematica (Barcelona), the Max-Planck-Institut für Mathematik (Bonn), the University of Aberdeen, the University of California at

2006 Distinguished Service Award (DSA) & David Borwein Distinguished Career Award continued

2006 Prix de la SMC pour service méritoire (PSM) &

David-Borwein de mathématicien émérite pour l'ensemble d'une carrière suite

San Diego, and the University of Sydney.

His research is in the area of algebraic topology, particularly the homology theory of Lie groups, an area in which he has authored four monographs and numerous journal articles. He has supervised four PhD theses. He was a lead organizer of the 1996 Fields Institute thematic program in homotopy theory, and has organized several other workshops and conferences. His research contributions were recognized by his election to the Royal Society of Canada in 1988. Other honours include being named a Fields Institute Fellow in 2002 and receiving a University of Waterloo Faculty of Mathematics Alumni Achievement Medal in 2003.

Dr. Kane has served the CMS in a number of roles, most notably as President (1998-2000), but also as Vice President, and as chair of the research, finance, and international affairs committees. He has served his university, the Royal Society, the Natural Sciences and Engineering Research Council (NSERC), and the Fields Institute in a number of roles. But his most singular contribution has been his leadership on the national stage, and in particular his contributions to building a strong research infrastructure for Canadian mathematics. He was the scientific convener of the 1996 NSERC Review of Canadian mathematics, a review prompted by a prior negative evaluation. In response, Dr. Kane led the efforts to examine the strengths and weaknesses of mathematics in Canada. Following an international evaluation this led to government recognition of the excellence of Canadian research in mathematics and of its importance to Canada, as well as to a concrete plan to build the infrastructure needed to raise this research to new heights. Continuing these efforts, Dr. Kane served as chair of the Mathematics Steering Committee for the NSERC reallocations exercises in 1997 and 2001, and chaired the NSERC Liaison Committee for the mathematical sciences in 2005. The direct impact of these efforts was secure and enhanced funding for mathematical research in Canada. This benefited both individual researchers, and also helped to build or enhance the infrastructure that is enabling Canada to play an increasingly significant role on the world stage - the three Canadian mathematics research institutes, the Banff International Research Station, the MITACS National Centre of Excellence (NCE), the NSERC leadership support program, and other initiatives. The success of these efforts stems from the collaborative and unified vision established by the Canadian mathematical community, a vision made possible in large part by the diplomacy, integrity, wisdom, and leadership of Dr. Richard Kane.

**Créé en 1995, le Prix de la SMC pour service méritoire sert à récompenser les personnes qui contribuent de façon importante et soutenue à la communauté mathématique canadienne et, notamment, à la Société mathématique du Canada.**

**Le prix David-Borwein de mathématicien émérite pour l'ensemble d'une carrière rend hommage à un mathématicien qui a fait une contribution exceptionnelle et soutenue aux mathématiques canadiennes. Ce prix a été nommé en l'honneur de David Borwein, ancien président de la SMC et éminent mathématicien canadien.**

Richard Kane joue depuis toujours un rôle primordial dans la communauté mathématique canadienne à travers son travail soutenu pour la profession, pour son université et pour la SMC. Brillant chercheur et enseignant dévoué, il mène une carrière exemplaire qui mérite éminemment la distinction qui accompagne le premier prix David-Borwein, ainsi que le Prix pour service méritoire, pour 2006.

Richard Kane a obtenu son baccalauréat de l'Université de Toronto en 1967 et son doctorat de l'Université de Waterloo en 1973. Membre du Département de mathématiques de Western Ontario depuis 1980 et professeur titulaire depuis 1983, il a en outre assuré la direction du département à deux reprises. Avant d'arriver à Western, il a enseigné à l'Université de l'Alberta et fait des stages de recherche postdoctorale à Oxford et au MIT. Il a été professeur invité par l'Institute for Advanced Study, (Princeton), le Centre de Recerca Matematica (Barcelone), le Max-Planck-Institut für Mathematik (Bonn), ainsi que les universités d'Aberdeen, de la Californie à San Diego et de Sydney.

Ses recherches portent sur la topologie algébrique, en particulier sur l'homologie des groupes de Lie, domaine dans lequel il a publié quatre monographies et de nombreux articles. Il a également dirigé quatre thèses de doctorat. Il a de plus été l'organisateur principal du programme thématique de l'Institut Fields en 1996 sur la théorie homotopique et il a organisé plusieurs autres ateliers et congrès. La Société royale du Canada a d'ailleurs reconnu sa contribution à la recherche en le faisant membre en 1988. Entre autres honneurs et récompenses, il est devenu membre de l'Institut Fields en 2002 et il a reçu la médaille des anciens de la Faculté de mathématiques de l'Université de Waterloo en 2003.

Richard Kane a occupé divers postes à la SMC, en particulier celui de président (1998-2000), mais aussi de vice-président et de président des comités de la recherche, des finances et des affaires internationales. Il a également joué divers rôles dans son université, à la Société royale, au Conseil de recherches en sciences naturelles et en génie (CRSNG) et à l'Institut Fields. C'est toutefois son leadership sur la scène nationale qui aura le plus marqué son œuvre, en particulier ses démarches pour la création d'une infrastructure de recherche mathématique solide au Canada. Il a été responsable scientifique de l'Examen des mathématiques au Canada mené par le CRSNG en 1996, examen déclenché par une évaluation négative. Richard Kane a réagi en dirigeant l'examen des forces et des faiblesses des mathématiques au Canada. Suite à une évaluation internationale, le gouvernement a reconnu l'excellence de la recherche mathématique qui se faisait au Canada et son importance pour le pays, et a pris des mesures concrètes pour mettre en place l'infrastructure nécessaire qui permettrait de porter la recherche vers de nouveaux sommets. Dans la même foulée, M. Kane a présidé le Comité de direction en mathématiques lors des exercices de réaffectation du CRSNG de 1997 et de 2001, ainsi que le Comité de liaison du CRSNG en sciences mathématiques en 2005. Les conséquences directes de ces efforts ont mené à l'obtention de garanties et à des hausses du financement consacré à la recherche mathématique au Canada. Très bénéfiques pour les chercheurs, ces mesures ont aussi contribué à bâtir et à améliorer l'infrastructure qui permet au Canada de jouer un rôle de plus en plus important sur la scène mondiale - les trois instituts de recherche mathématique canadiens, la Station de recherche internationale de Banff, le Réseau de centres d'excellence MITACS (RCE), le programme de soutien au leadership du CRSNG et d'autres initiatives. Le succès de ces initiatives tient à la vision unifiée et collaborative de la communauté mathématique canadienne, vision rendue possible en grande partie grâce à la diplomatie, à l'intégrité, à la sagesse et au leadership de Richard Kane.

## THE DAVID BORWEIN DISTINGUISHED CAREER AWARD SCULPTURE: CREATED BY HELAMON FERGUSON

Helaman Ferguson studied both art and mathematics at Hamilton College, New York and then enrolled in a doctoral program in mathematics at the University of Wisconsin, Madison. Ferguson dropped out of school for a couple of years to work as a computer programmer, then resumed his math studies, obtaining his master's degree in mathematics at Brigham Young University (BYU) in Provo and a doctorate in group representations at the University of Washington, Seattle. In 1971, he became a professor at BYU.

As a mathematician, Ferguson is perhaps best known for the PSLQ algorithm he developed with BYU colleague Rodney Forcade in the late 1970's. This algorithm, finds mathematical relations among seemingly unrelated real numbers. Among many other applications, PSLQ provided an efficient way of computing isolated digits within pi and blazed a path for modeling hard-to-calculate particle interactions in quantum physics. In 2000, the journal Computing in Science and Engineering named it one of the top 10 algorithms of the 20th century.

Ferguson left BYU in 1988 and now devotes most of his time to his art. He's done commissions for the Maryland Science and Technology Center, the University of California, Berkeley, the University of St. Thomas in St. Paul, and many other institutions. He has also designed small sculptures for awards presented by the Clay Mathematics Institute in Cambridge, Massachusetts,



Sculpture of the David Borwein Distinguished Career Award created by Helaman Ferguson

the Association for Computing Machinery in New York City and most recently the Canadian Mathematical Society.

Ferguson's admirers say his artwork goes far beyond academic exercises. David Broadhurst, a quantum field theorist at the Open University in Milton Keynes, U.K., learned about Ferguson's sculpture after using the PSLQ algorithm in his research in quantum mechanics. He compares Ferguson's artistic renderings of math to

Fournier playing the Bach cello suites,

"giving expression to abstract forms, whose beauty is preexistent to the interpretation, yet recreated in a widely accessible medium."

The David Borwein Award sculpture is based upon the mathematics of Madelung's constant, M, which measures the electrochemical stability of salt. Precisely M is the alternating sum of  $\sum_{n,m,p} \frac{(-1)^{n+m+p}}{\sqrt{n^2 + m^2 + p^2}}$  over all non-zero lattice points in three-dimensional space. This sum is highly conditional and both its computation and its theory rely on understanding the geometry of Jacobi theta functions and their Mellin transforms. See, for example, <http://mathworld.wolfram.com/MadelungConstants.html>.

This constant thus straddles number theory and analysis and leads into a study of divergent series - a subject which David is especially expert in. The geometry of elliptic/theta functions forms the basis for the physical sculpture.

## WANTED: Books for Review RECHERCHEÉS : Livres pour critiques littéraires

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Paul Halmos, one of the most influential mathematicians of the last half of the twentieth century, died on October 2, 2006 at the age of ninety.

Paul wrote “To be a mathematician you must love mathematics more than family, religion, money, comfort, pleasure, glory.” Paul did love mathematics. He loved thinking about it, talking about it, giving lectures and writing articles and books. Paul also loved

language, almost as much as he loved mathematics. That is why his books and expository articles are so wonderful. Paul took Hardy’s famous dictum that “there is no permanent pace in the world for ugly mathematics” very seriously: he reformulated and polished all the mathematics that he wrote and lectured about, and presented it in the most beautiful way.

Irving Kaplansky, the great Canadian mathematician who also last year at the age of eighty-nine, wrote “Paul Halmos is the wittiest person I know.” Many quotations from Paul’s writing illustrating Kaplansky’s statement can be found on the internet (just Google “Paul Halmos quotations”). Here are some others:

*You can’t be perfect, but if you don’t try, you won’t be good enough.*

*If you have to ask, you shouldn’t even ask.*

*Once the problem is solved, its repetitive application has as much to do with mathematics as the work of a Western Union messenger boy has to do with Marconi’s genius.*

*The criterion for quality is beauty, intricacy, neatness, elegance, satisfaction, appropriateness – all subjective, but somehow mysteriously shared by all.*

*There is no Berlitz course for the language of mathematics; apparently the only way to learn it is to live with it for years.*

*The recommendations I have been making are based partly on what I do, more on what I regret not having done, and most on what I wish others had done for me.*

*Almost everybody’s answer to “What to publish?” can be expressed in either one word – “less” – or two words – “good stuff”.*

Man-Duen Choi put together a number of titles of Halmos’s writings to form a cute narrative - see “A Postscript” on page 799 of volume 103 (1996) of the American Mathematical Monthly.

Paul liked to be provocative. He wrote, for example, “The best way to learn is to do; the worst way to teach is to talk.” He did follow this with “Having stated this extreme position, I’ll rescind it immediately. I know that it is extreme, and I don’t really mean it – but I wanted to be very emphatic about not going along with the view that learning means going to lectures and reading books.” However, his explanation did not mollify some people who were very proud of their ability to lecture.

Perhaps Paul’s most provocative statement in print (those who had the pleasure of participating in discussions with him heard even more provocative comments) was his title for an article published in 1981: “Applied Mathematics is Bad Mathematics.” Although Paul began the article with “It isn’t really (applied mathematics, that is, isn’t really bad mathematics), but it’s different,” the title angered many applied mathematicians.

Paul made fundamental contributions to ergodic theory and measure theory (his classic books “Lectures on Ergodic Theory”

and “Measure Theory”, and many papers) and to algebraic logic (see “Algebraic Logic” and, with S. Givant, “Logic as Algebra”). His book “Naïve Set Theory” is a beautiful exposition of axiomatic (Zermelo-Fraenkel) set theory, in spite of its “naïve” title. But Paul’s most important contributions to research in mathematics, at least from our prejudiced point of view, were to the theory of operators on Hilbert space.

Paul created and led a vigorous school of operator theory. He introduced central concepts such as unitary dilations, subnormal operators and quasitriangularizability, and proved the fundamental theorems about them. These, and other concepts he developed, became major subjects of research; there is now a large body of knowledge about each of these topics.

Paul had extraordinary ability to discover the central questions concerning a large number of different aspects of operator theory. In particular, his famous article “Ten problems in Hilbert space” shaped a great deal of subsequent research in operator theory and in  $C^*$  algebras.

As Berkeley mathematician Don Sarason (who competes with Erret Bishop for the title “Most-distinguished of Halmos’s twenty-one Ph.D. students”) wrote, in his introduction to Paul’s selected works, “Halmos embodies the ideal mixture of researcher and teacher. In him, each role is indistinguishable from the other. Perhaps that is the key to his remarkable influence.”

Paul wrote what he termed “an automathography”, a fascinating book entitled “I Want to be a Mathematician.” This is a mathematical autobiography, and contains much advice that is very useful to all mathematicians and to all those who aspire to be mathematicians. Towards the end of that book, Paul evaluates his career as follows: “I was, in I think decreasing order of quality, a writer, an editor, a teacher, and a research mathematician.”

Paul’s self-evaluation may be quite accurate, but it is important to understand how high a standard he was setting for himself. He was certainly as good a mathematical writer as ever existed. As an editor, Paul played a fundamental role in developing several of the series of mathematics books published by Springer-Verlag, as well as in editing several journals. Virtually everyone who ever heard him lecture will testify that his lectures were maximally interesting, clear and inspiring. (Luckily, several videotapes of Paul’s lectures can be purchased from the A.M.S. and the M.A.A.). Moreover, Paul’s total contribution to research in mathematics is very impressive.

As Paul wrote, “it takes a long time to learn to live – by the time you learn your time is gone.” However, one can learn much about living from others, and Paul taught many mathematicians a huge amount, about mathematics and about life. For instance, Paul wrote “I like to start every course I teach with a problem.” Those who wish to follow Paul’s example in this respect can use Paul’s book “Problems for Mathematicians Young and Old”; it contains a surprising variety of beautiful problems from a variety of areas of mathematics.

Paul advised efficiency in all tasks: if a letter has to be answered, or a review has to be written for mathematical reviews, do it right away, rather than thinking for months “I’d better get to that.” Good advice but, we have to confess, we failed to follow it in the writing of this obituary. It doesn’t take a psychoanalyst to figure out why we failed: we wanted to postpone this last goodbye to Paul Halmos. But the time has now come. Goodbye, Paul; thanks very much for so much.

**Kiyosi Itô, récipiendaire inaugural du Prix Gauss 2006**

Kiyosi Itô, maintenant 91 et un des fondateurs de la probabilité moderne, est devenu le premier récipiendaire du prix de Gauss pour les applications des mathématiques au CIM à Madrid.

Pendant qu'il travaillait dans l'isolement relatif (à un certain degré, ceci était le cas de tous les probabilistes de l'époque) au bureau des statistiques du gouvernement japonais, le professeur Itô publiait en 1942 un article [ 12 ] dans un journal de mathématiques de l'université d'Osaka. Cet article a posé les fondations de base de ce domaine pour les 40 années à venir et contenait les idées innovatives pour lesquelles il a été attribué le prix de Gauss.

Le rapport entre l'aspect appliqué et l'aspect pur des mathématiques a toujours été symbolique et la carrière d'Itô en est une merveilleuse démonstration. Dans [ 14 ] il note que comme étudiant, il aimait la beauté des mathématiques pures mais également les interactions entre les mathématiques et la mécanique et son introduction à la probabilité était à travers la mécanique statistique. Un de ses papiers de 1943 était sur la turbulence. Son travail innovateur sur l'intégration stochastique, les équations différentielles stochastiques et le calcul stochastique a été mené comme une réalisation purement mathématique des idées de Feller et de Kolmogorov. Ce travail a formé la base d'une vaste gamme des modèles stochastiques dans la théorie du filtrage, la génétique de population, les finances mathématiques et la physique statistique. Dans les propres mots d'Itô [ Fu ] :

*Comme ma propre recherche sur l'analyse stochastique est en mathématiques pures, le fait que mon travail a été choisi pour le prix de Gauss pour les applications des mathématiques est vraiment inattendu et profondément satisfaisant. J'espère donc partager ce grand honneur et joie avec ma*

*famille, les enseignants, les collègues et les étudiants en mathématiques, aussi bien avec tous ceux qui ont pris mon travail dans l'analyse stochastique et l'ont étendu à des horizons au-delà de mon imagination*

Avant 1930, Bachelier (1900) a proposé le mouvement brownien comme modèle pour les fluctuations du marché boursier, Einstein (1905) a gagné le prix Nobel pour son travail qui a employé le mouvement brownien pour donner une confirmation expérimentale de la théorie atomique, et Wiener (1923) a donné une construction mathématiquement rigoureuse du mouvement brownien. (Comme nous aimons être absolument certains de chaque ligne, nous les mathématiciens sommes placés derniers comme d'habitude, mais nous pouvons vérifier qu'Einstein a, en effet, gagné son Nobel.) Néanmoins, la plupart des probabilistes sélectionneraient probablement 1933 comme l'année de naissance de la probabilité moderne. C'est l'année où le traité du Kolmogorov sur les bases du sujet est apparu. Itô a étudié à l'université de Tokyo (1935–38) et a lu des travaux de Kolmogorov [ K ], Feller [ Fe ], L'evy [ L ], et Doob [ D ]. La probabilité comme discipline mathématique moderne était dans sa petite enfance. Son directeur de thèse à Tokyo était Shokichi Iyanaga. À une cérémonie tenue à l'université de Kyoto en septembre 2006, professeur Itô a écrit :

*Aujourd'hui mon seul regret est que je ne pouvais pas partager les nouvelles du prix de Gauss avec mon professeur et mentor, professeur Shokichi Iyanaga, qui est décédé ce mois de juin à l'âge de 100. En tant qu'étudiant des mathématiques à l'université de Tokyo dans les années 30, je n'aurais pas pu continuer mes intérêts de recherche dans la théorie des probabilités, sans ses encouragements aimables et constants. Professeur Iyanaga a enseigné*

*tous ses étudiants à poursuivre leurs propres intérêts, indépendamment de la popularité de ces intérêts à l'époque ou de leurs applications pratiques potentielles.*

Comme un chercheur débutant, j'avais le plaisir de parler à Prof. Itô dans une Conférence en 1983 au Bâton Rouge où il donnait une présentation sur son travail recent sur les EDS dans des espaces à dimensions infinis. C'était l'été à Louisiana et nous restions dans des chambres de résidence sans air clématisé. Beaucoup des mathématiciens professionnels avaient opté pour des hôtels (comme je le ferais certainement moi-même dans nos jours) mais Itô restait dans des chambres de résidence et le soir, il discuterait les mathématiques avec beaucoup des jeunes participants en sueur. Quand J'ai mentionné que j'avais étudié à l'université d'Illinois et avais pris des cours sur la théorie potentielle et la théorie de la mesure de prof. Doob, il m'a immédiatement dit que les travaux de Doob sur les processus stochastiques à paramètre continu étaient pivotaux dans sa décision d'étudier la probabilité. Kolmogorov avait construit des fonctions de transition de Markov comme les solutions fondamentales de l'équation du futur liée à un opérateur différentiel du second degré (décris ici dans une dimension spatiale)  $\mathcal{L} = a(x) \frac{d^2}{dx^2} + b(x) \frac{d}{dx}$ .

Il utiliserait ensuite son théorème général d'existence pour construire un processus de Markov associé à ce noyau de transition. Itô jugeait qu'une bonne partie de ce travail était de l'analyse fine et que la probabilité était, à un certain degré, secondaire (le titre suggère ceci). Lévy offrait quelque chose encore plus intéressante dans une manipulation directe avec les trajectoires d'un processus à accroissements indépendants et stationnaires (qu'on appelle maintenant les processus de Lévy). Les transformations de Fourier à un temps fixé de tels processus ont été caractérisés par Lévy et Khinchin, mais c'est une formule qui se contrôlise seulement avec

la méthode trajectorielle (i.e. le système est étudié trajectoire par trajectoire) de Lévy du processus stochastique comme une “ somme de Poisson de sauts compensés ”.<sup>1</sup> L’approche trajectorielle de Lévy a stimulé l’intérêt du prof. Itô—Pour plusieurs d’entre nous, les arguments trajectoriels sont derrière nos attractions au sujet. Toutefois, Plusieurs des arguments de Lévy n’étaient pas rigoureux (bien que presque toujours corrects). Dans [I4], Itô écrit ” J’ai eu de la difficulté à suivre l’argument de Lévy à cause de sa description intrinsèque unique ”. C’était seulement après qu’il a lu les travaux de Doob sur les modifications des processus à paramètres continus qu’Itô a réalisé comment effectuer la construction de Lévy rigoureusement. Cette construction décrite dans [I1], a formé la base de sa thèse, et s’appelle maintenant la construction de Lévy-Itô. Cette perspective trajectorielle est restée avec Itô pour toute sa carrière.

Une grande partie du travail ci-dessus a été écrit pendant qu’Itô était au bureau des statistiques (1939-43). M. Fukushima m’a indiqué que les organismes gouvernementaux et les entreprises privés souvent embauchaient de jeunes étudiants brillants comme investissement dans leurs futurs tout en leur laissant la liberté de poursuivre leurs propres intérêts de recherche. Itô écrit [I4] “ Grâce à la bienveillance de M. T. Kawashima, directeur du bureau, j’avais suffisement du temps pour mes propres études ”. Cette bienveillance en temps de guerre au Japon a également mené au travail séminal [I2] qui a déterminé ce que plusieurs d’entre nous avaient fait pendant

toutes leurs carrières. Comme décrit dans l’introduction et l’avant-propos de [I4], son but ici était de construire les processus de diffusion de Kolmogorov [K] par une manipulation directe des trajectoires dans le sens de Lévy. (Feller [Fe] avait étendu la construction de Kolmogorov pour permettre des noyaux de sauts et Itô travaillait dans ce cadre général mais ces extensions ont été publiées plus tard dans son mémoire de 1951 (SMA). Son observation était qu’instantanément en  $x$ , les processus de Kolmogorov effectuaient un mouvement brownien avec une déviation  $b(x)$  et un écart type  $\sigma(x) = \sqrt{a(x)}$ . Kolmogorov applique ceci au niveau des distributions de probabilité du mouvement aléatoire fondamental ; Itô l’a fait directement au niveau des trajectoires en établissant le processus aléatoire  $X(t)$  comme solution de l’équation stochastique

(EDS)

$$dX(t) = \sigma(X(t))dB(t) + b(X(t))dt.$$

Ici  $B$  est un mouvement brownien et par suite il admet, prèsque certainement, des trajectoires avec une variation illimitée sur chaque intervalle de longueur positive. Ainsi, pour bien interpréter le premier terme dans (EDS), Itô devait interpréter l’intégrale, pas trajectoriellement, mais plutôt processus par processus. On met  $f(s, \cdot)$  dans un processus stochastique ( $c.-\rightarrow -d.$ , une fonction mesurable de la variable  $\omega$  qu’on supprime prèsque toujours) satisfaisant certaines conditions de mesurabilité et d’intégrabilité et on produit un processus stochastique  $\int_0^t f(s, \cdot) dB(s)$  satisfaisant certaines conditions de

mesurabilité et d’intégrabilité. Pour obtenir une intégrale utile avec un théorème de convergence dominé, on doit limiter la classe des intégrands  $f(s, \omega)$  à celles qui dépendent sur le mouvement brownien  $B$ . C’est une condition qui est satisfaite pour l’intégrand qui figure dans (EDS). Itô faisait ensuite la connection entre (EDS) et le processus de Kolmogorov (et l’opérateur  $\mathcal{L}$ ) par la formule qui porte son nom

(IF)

$$\begin{aligned} df(X(t)) &= f'(X(t))dX(t) + \frac{1}{2}f''(X(t))\sigma^2(X(t))dt \\ &= f'(X(t))\sigma(X(t))dB(t) + \mathcal{L}f(X(t))dt. \end{aligned}$$

L’intuition est assez claire rétrospectivement.  $B$  a une variation infinie mais admet une variation quadratique sur  $[0, t]$  égale à  $t$  dans un certain sens. On doit utiliser un développement de Taylor du second ordre pour  $X(t)$  localement et prendre la somme sur les incrément pour dériver (IF). L’intégrale d’Itô avait un précurseur – comme Stroock et Varadhan disaient, “ Itô est Lebesgue de cette branche de la théorie d’intégration ; Paley et Wiener étaient son Riemann. ” Paley et Wiener (1934) avaient construit l’intégrale stochastique pour les intégrands déterministes de carré intégrables qui n’aidaient pas dans l’interprétation de (EDS). <sup>2</sup> Comme beaucoup de percées significatives, la construction et l’analyse associée d’Itô sont venues en avant de leurs temps. Itô écrit [I4] : “ je ne connais personne qui a lu cet article [I2] complètement quand il est apparu à l’exception de mon ami G. Maruyama. ” Maruyama a indiqué à Fukushima qu’il l’a lu à plusieurs reprises sous la lumière d’une maison de garde-barrière dans un camp militaire où il faisait son service militaire.

<sup>1</sup> L’idée dans [L] et [I1] est de construire le processus en ajoutant la somme de ses sauts –pour des processus composés de Poisson (les sauts se produisent à un taux fini selon une certaine distribution commune) ceci fonctionne très bien. En général, les petits sauts se produisent à un taux infini, donc on doit les tronquer à  $\epsilon$  et prendre une limite. La sensibilité de cette technique réside dans le fait que les trajectoires ont une variation illimitée en général et la somme des sauts converge seulement conditionnellement. Pour obtenir la convergence dans la technique ci-dessus, on doit soustraire le moyen des sauts pour obtenir une différence de deux termes, chacun tendant vers l’infini, qui converge à la valeur limite du processus. Pour faire ceci à tout moment simultanément, on introduit un ensemble des sauts potentiels indexés par la grandeur et le temps où ils se produisent – le processus spatio-temporel ponctuel des sauts de Poisson.

<sup>2</sup> Il semble, cependant, qu’Itô ne connaissait pas ceci dans son article de 1942 bien qu’il mentionnait dans [I4] qu’il l’avait lu soigneusement après son déménagement à Nagoya en 1943, et le mentionnait dans un papier de 1944 sur l’intégrale où leur influence était claire.

Itô a déménagé à l'université de Nagoya (1943-52) et a soumis aux mémoires de la SMA une version détaillée de son travail (en anglais) sur les équations différentielles stochastiques incluant les sauts. Itô attribue à Doob l'aide avec la publication aux ÉU. Le travail original d'Itô était accompli sans la théorie des martingales à paramètre continu que Doob développait en même temps à l'autre côté de l'océan.

C'est l'approche qui a considérablement simplifié et étendu la construction originale d'Itô (le manque de cette approche dans la construction originale la rend plus impressionnante). Doob a immédiatement apprécié ce qu' Itô avait fait et avait commencé à étendre l'intégrale stochastique et le calcul d'Itô aux martingales générales, en utilisant, naturellement, la théorie des martingales. D'autres extensions ont été effectuées par les anciens étudiants d'Itô's H. Kunita et S. Watanabe (1967) et P.A. Meyer et l'école de Strasbourg, menant à la théorie la plus générale d'intégration stochastique par rapport aux semimartingales – c'est un théorème et non pas une opinion. C'était vraiment dans les années 60 que le domaine de l'intégration stochastique et des équations stochastiques semblait relancer. J'ai appris la théorie de deux excellentes sources: Notes d'institut Tata d'Itô [ I3 ] et le livre trompeusement mince de H. McKean's, Intégrales Stochastiques [ Mk ]. Ce livre est une source efficace d'apprentissage de l'analyse stochastique comme chaque phrase est un exercice laissé pour le lecteur (Itô le décrit en tant que " compact mais plein des matériaux intéressants "). Il y a une étape délicate dans la définition de l'intégrale d'Itô où on prouve le fait q'une certaine classe des intégrands est dense. Il est facile d'assumer les notions modernes de la prévisibilité ou de la mesurabilité progressive mais Itô a travaillé initialement avec une plus grande classe des intégrands adaptés et conjointement mesurables et plusieurs des preuves " modernes " (à l'époque) ne semblaient pas l'avoir correctement. En reculant dans le temps, j'ai découvert la preuve correcte et légèrement plus compliquée dans [ I3 ].

Prof. Itô a déménagé à l'université de Kyoto en 1952 et a passé 27 ans là-bas – il a passé la moitié de ces années à l'étranger. Ceci lui a donné l'occasion de

travailler à plusieurs des principaux centres de recherche dans le monde (Princeton, Stanford, Cornell, Aarhus) et d'apporter également ses idées au reste du monde. Il est difficile d'imaginer n'importe quelle université aujourd'hui d'être si prévoyante et généreuse avec une ressource si valable. L'université de Kyoto a clairement permis à Itô d'embrasser quelques nouvelles et riches théories et alternativement d'exercer une influence positive sur le développement de la probabilité au Japon car il reviendrait fréquemment à l'université de Kyoto et à la conférence sur les nouveaux domaines de la probabilité. Il est arrivé à Princeton en 1954 juste après que Feller avait accompli sa classification analytique des diffusions unidimensionnelles(processus forts continus de Markov). De nouveau, le défi d'une construction trajectorielle de ces processus était normal pour It Prof. Itô a déménagé à l'université de Kyoto en 1952 et a passé 27 ans là-bas – il a passé la moitié de ces années à l'étranger. Ceci lui a donné l'occasion de travailler à plusieurs des principaux centres de recherche dans le monde (Princeton, Stanford, Cornell, Aarhus) et d'apporter également ses idées au reste du monde. Il est difficile d'imaginer n'importe quelle université aujourd'hui d'être si prévoyante et généreuse avec une ressource si valable. L'université de Kyoto a clairement permis à Itô d'embrasser quelques nouvelles et riches théories et alternativement d'exercer une influence positive sur le développement de la probabilité au Japon car il reviendrait fréquemment à l'université de Kyoto et à la conférence sur les nouveaux domaines de la probabilité. Il est arrivé à Princeton en 1954. Il joignait ses efforts avec McKen, l'étudiant de Feller à l'époque, et 10 ans plus tard ils ont accompli le livre sur le sujet [ IM ]. Itô décrit une interaction principale avec Feller dans [ I5 ] : " McKean a essayé d'expliquer à Feller mon travail sur les équations stochastiques... j'ai eu l'impression que Feller n'a pas entièrement compris sa signification, mais lorsque j'ai expliqué le temps local de Lévy à Feller, il a immédiatement apprécié sa pertinence avec l'étude de la diffusion unidimensionnelle. En effet Feller nous a immédiatement proposée une conjecture... qui a été par la suite démontrée dans mon papier conjoint avec McKean (1963). "

Le temps local de Lévy qu'Itô avait

lu en tant qu'étudiant dans [ L ] s'est avéré être l'ingrédient essentiel dans la construction trajectorielle d'Itô-McKean des processus généraux forts, continus et unidimensionnels de Markov. Il fournit les moyens à un changement aléatoire dans le temps d'une trajectoire brownienne qui, avec un changement déterministe d'échelle, a donné le résultat général (ceux qui ont réellement lu [ IM ] sauront que c'est une simplification mais justifiable). Bien que la classe des processus qu'Itô a construit comme solutions des équations stochastiques fournit les exemples centraux et canoniques, il est intéressant de savoir qu'ils sont à peine mentionnés dans leur livre. Bien que les deux commencent par une trajectoire brownienne, les deux constructions sont en fait complètement différentes. Pour des diffusions unidimensionnelles, la technique de changement du temps et de l'échelle s'est avérée comme étant la plus générale; ainsi il était normal que le séminaire de l'université de Kyoto à la fin des années 50 et au début des années 60 essayeraient d'endre certaines de ces idées aux processus généraux de Markov. Les diffusions unidimensionnelles de Feller étaient des processus symétriques de Markov par rapport à sa mesure de vitesse qui a régi le taux de variation du changement du temps aléatoire mentionné ci-dessus. Beurling et Deny avaient déjà présenté l'étude des formes de Dirichlet pour des processus symétriques généraux de Markov. Masatoshi Fukushima, un des étudiants gradués d'Itô à l'époque, a étendu l'idée du changement dans le temps mentionnée ci-dessus à l'étude des processus symétriques généraux de Markov et à côté de M. Silverstein, il est devenu l'un des deux experts dans un sujet d'importance continue. La théorie de la frontière de Martin telle que développée par Doob et Dynkin ainsi que les conditions de frontière de Wentzell pour des diffusions d-dimensionnelles étaient d'autres matières d'étude. À ce jour, il n'y a aucune classification générale des processus de Markov d-dimensionnels continus. Une série d'exemples (eg de Tom Salisbury (York U.) (1986)) suggèrent que cette question ne sera pas si malléable. D'autre part la théorie de processus d-dimensional de Markov est toujours dominée par les équations stochastiques et les extensions d'Itô. Ses méthodes d'analyse stochastique fonctionnent aussi bien pour la plupart des

théories infini-dimensionnelles, où Itô a également effectué un travail influent. Les travaux originaux de Renzo Cairoli et John Walsh (UBC) dans l'extension à plusieurs paramètres de l'intégration stochastique d'Itô jouent un rôle important dans la théorie moderne des équations partielles stochastiques. La théorie de processus supérieurs de Dawson-Watanabe développée par l'étudiant d'Itô Shinzo Watanabe et Don Dawson (Carleton) est une classe connue des exemples infini-dimensionnels dans lesquels l'analyse stochastique d'Itô joue un rôle central. Il a aussi joué un rôle plus direct.

Pendant qu'Itô était un professeur à Stanford (1961 - 64) il s'est intéressé aux processus de branchement par la proximité à Harris, Karlin et d'autres. M. Fukushima se rappelle d'une série des présentations stimulantes sur les processus de branchement données par Itô pendant un retour à l'université de Kyoto. Ceci et des discussions ultérieures ont mené au travail d'Ikeda-Nagasawa-Watanabe sur les processus de branchement de Markov et le fameux article de S. Watanabe (1968) sur les processus de branchement continus. Cet article a influencé beaucoup de mes propres contributions à la recherche.

Je n'essayerai pas de faire de cet article une description encyclopédique des contributions d'Itô, comme ceci prendrait un livre. Heureusement, le livre a été écrit [ I4 ]. Je devrais, cependant, mentionner le travail innovateur d'Itô sur la théorie des excursions (1970) qui décompose un processus général de Markov en des excursions d'un point recurrent jusqu'un point recurrent. La complexité réside dans le fait que typiquement le processus revient au point instantanément. La solution est une version infini-dimensionnelle des mêmes idées des processus ponctuels de Poisson employées par Itô dans sa thèse pour prouver la décomposition de Lévy-Itô d'un processus de Lévy. Au lieu des sauts individuels qui prennent des valeurs dans la ligne, les points de Poisson étant collés ensemble sont maintenant des trajectoires d'excursion prenant des valeurs dans un espace de trajectoires approprié. Itô a également suggéré la construction inverse d'établir un processus de Markov à partir de ses excursions et, dans des conditions

appropriées, cette synthèse a été effectuée par Tom Salisbury (York U.) et Chris Rogers.

Le prix de Gauss à Madrid semblait faire partie d'une reconnaissance générale du fait que le champ de la probabilité était considéré comme le travail des lauréats de la médaille Fields Okounkov et Werner tous les deux accentuent la probabilité. Les Canadiens jouaient un rôle dans la célébration pendant que le travail d'Okounkov avec Rick Kenyon (UBC) jouait un rôle significatif dans sa citation, et Dawson (Carleton) était sur le Comité de médailles de Fields. (la Communauté mathématique du Canada avait déjà prévu cette apparition quand elle a élu trois probabilistes à l'exécutif actuel de la SMC.)

Je me suis toujours considéré chanceux pour faire partie d'un discipline où les experts sont aussi supportifs des chercheurs plus jeunes. La gentillesse et l'intérêt que prof. Itô a montré à l'égard de mon travail cette soirée chaude à Bâton rouge reste avec moi. Son mentorat a établi une génération entière des probabilistes au Japon – ses étudiants incluent des figures influentes telles que T. Sirao, T. Hida, N. Ikeda, M. Nisio, S. Watanabe, H. Kunita, M. Fukushima et T. Yamada. Beaucoup plus étaient des participants actifs à la conférence de la probabilité d'Itô à l'université de Kyoto comprenant M. Motoo, T. Watanabe, T. Ueno, H. Tanaka et K. Sato. Leurs étudiants sont à leurs tours les experts dans la communauté épanouissante des probabilistes d'aujourd'hui du Japon, et représentent collectivement un des plus grands patrimoines de prof. Itô's.

Remerciement: Je voudrais remercier Don Dawson pour rendre certains matériaux disponibles pour moi et Hans Föllmer pour me donner accès à son article sur le travail d'Itô pour CIM 2006 volume [Fo] avant sa publication. Un remerciement spécial à Masatoshi Fukushima pour m'envoyer son merveilleux article [ Fu ] et pour répondre à un certain nombre de questions. Cet article doit beaucoup à son article écrit et à sa patience en répondant à un certain nombre d'enquêtes. [ Fo ] et [ Fu ] donnent ensemble un exposé plus détaillé des mathématiques d'Itô et ils sont fortement recommandés pour une lecture.

## Références

- [ D ] J.L. Doob, Stochastic processes depending on a continuous parameter, *Trans. Amer. Math. Soc.* **42**, 107-140 (1937).
- [ Fe ] W. Feller, Zur theorie der stochastischen prozesse (existenz und eindeutigkeitssatz), *Math. Ann.* **113**, 113-160 (1936).
- [ Fo ] H. Föllmer, On Kiyosi Itô's work and its impact, to appear, *Proc. 2006 ICM, Madrid* (2007).
- [ Fu ] M. Fukushima, On the works of Kiyosi Itô and stochastic analysis, to appear in *Jap. J. Math.* (2007).
- [ I1 ] K. Itô, On stochastic processes (infinitely divisible laws of probability, *Japan. Jour. Math.* **XVIII**, 261-301 (1942).
- [ I2 ] K. Itô, Differential equations determining a Markov Process (in Japanese), *Journ. Pan-Japan Math. Coll.* No. 1077, 1352-1440 (1942); (in English) in *Kiyosi Itô, Selected Papers*, D.W. Stroock and S.R.S. Varadhan (ed.'s), 42-75 (1986).
- [ I3 ] K. Itô, *Lectures on Stochastic Processes*, Tata Institute of Fundamental Research, Bombay (1960).
- [ I4 ] K. Itô, *Kiyosi Itô, Selected Papers*, D. W. Stroock and S.R.S. Varadhan (ed.'s), Springer-Verlag, 1986.
- [ I5 ] K. Itô, Memoirs of my research in stochastic analysis, in *Proc. Abel Symp. 2005, Stochastic Analysis and Applications-A Symposium in Honor of K. Itô*, 1-5, Springer (2006).
- [ IM ] K. Itô and H. P. McKean Jr., *Diffusion Processes and Their Sample Paths*, Springer-Verlag, 1965.
- [ K ] A. Kolmogorov, Über die analytischen methoden in wahrscheinlichkeitsrechnung, *Math. Ann.* **104**, 415-458 (1931).
- [ L ] P. Lévy, *Théorie de l'Addition des Variables Aléatoires*, Gauthier-Villars, Paris, 1937.
- [ McK ] H.P. McKean Jr., *Stochastic Integrals*, Academic Press, NY and London, 1969.

# CALL FOR NOMINATIONS / APPEL DE MISES EN CANDIDATURE

*The CMS Research Committee is inviting nominations for three prize lectureships. These prize lectureships are intended to recognize members of the Canadian mathematical community.*

*Le Comité de recherche de la SMC lance un appel de mises en candidatures pour trois de ses prix de conférence. Ces prix ont tous pour objectif de souligner l'excellence de membres de la communauté mathématique canadienne.*

## Prix Coxeter-James Prize Lectureship

2008

The Coxeter-James Prize Lectureship recognizes young mathematicians who have made outstanding contributions to mathematical research. The selected candidate will deliver the prize lecture at the Winter Meeting.

The recipient shall be a member of the Canadian mathematical community. Nominations may be made up to ten years from the candidate's Ph.D: researchers having their PhD degrees conferred in 1997 or later will be eligible for nomination in 2007 for the 2008 Coxeter-James prize. A nomination can be updated and will remain active for a second year unless the original nomination is made in the tenth year from the candidate's Ph.D.

Le prix Coxeter-James rend hommage aux jeunes mathématiciens qui se sont distingués par l'excellence de leur contribution à la recherche mathématique. La personne choisie prononcera sa conférence à la Réunion d'hiver.

Cette personne doit être membre de la communauté mathématique canadienne. Les candidats sont admissibles jusqu'à dix ans après l'obtention de leur doctorat : ceux qui ont obtenu leur doctorat en 1997 ou après seront admissibles en 2007 pour le prix Coxeter-James 2008. Toute mise en candidature est modifiable et demeurera active l'année suivante, à moins que la mise en candidature originale ait été faite la 10e année suivant l'obtention du doctorat.

## Prix Jeffery-Williams Prize Lectureship

2009

The Jeffery-Williams Prize Lectureship recognizes mathematicians who have made outstanding contributions to mathematical research. The prize lecture will be delivered at the Summer Meeting. The recipient shall be a member of the Canadian mathematical community. A nomination can be updated and will remain active for three years.

Le prix Jeffery-Williams rend hommage aux mathématiciens ayant fait une contribution exceptionnelle à la recherche mathématique. La personne choisie prononcera sa conférence à la Réunion d'été. Cette personne doit être membre de la communauté mathématique canadienne. Toute mise en candidature est modifiable et demeurera active pendant trois ans.

## Prix Krieger-Nelson Prize Lectureship

2009

The Krieger-Nelson Prize Lectureship recognizes outstanding research by a female mathematician. The prize lecture will be delivered at the Summer Meeting. The recipient shall be a member of the Canadian mathematical community. A nomination can be updated and will remain active for two years.

Le prix Krieger-Nelson rend hommage aux mathématiciennes qui se sont distinguées par l'excellence de leur contribution à la recherche mathématique. La lauréate prononcera sa conférence à la Réunion d'été. La lauréate doit être membre de la communauté mathématique canadienne. Toute mise en candidature est modifiable et demeurera active pendant deux ans.

The deadline for nominations is June 30, 2007. Letters of nomination should be sent to the address below.

La date limite de mises en candidature est le 30 juin 2007. Veuillez faire parvenir les dossiers de candidature à l'adresse ci-dessous.

Nominators should ask at least three referees to submit letters directly to the Chair of the CMS Research Committee by September 30, 2007. Some arms length referees are strongly encouraged. Nomination letters should list the chosen referees, and should include a recent curriculum vitae for the nominee, if available.

Les proposants doivent faire parvenir trois lettres de référence au président du Comité de recherche de la SMC au plus tard le 30 septembre 2007. Nous vous incitons fortement à fournir des références indépendantes. Le dossier de candidature doit comprendre le nom des personnes données à titre de référence ainsi qu'un curriculum vitae récent du candidat ou de la candidate, dans la mesure du possible.

J.F. Jardine, Chair / Président  
CMS Research Committee / Comité de recherches de la SMC  
Department of Mathematics  
The University of Western Ontario  
London, Ontario N6A 5B7 Canada

The 2007 Krieger-Nelson and Jeffrey-Williams Prizes will be presented at the CMS-MITACS Joint Conference 2007 in Winnipeg, Manitoba, May 31 to June 3. Les prix Krieger-Nelson et Jeffrey-Williams 2007 seront présentés à la Congrès conjoint MITACS-SMC 2007 à Winnipeg (Manitoba) du 31 mai au 3 juin.

# CALL FOR NOMINATIONS / APPEL DE MISES EN CANDIDATURE

## Prix Adrien-Pouliot Prize Lectureship

2007

Nous sollicitons la candidature de personnes ou de groupe de personnes ayant contribué de façon importante et soutenue à des activités mathématiques éducatives au Canada. Le terme « contributions » s'emploie ici au sens large; les candidats pourront être associés à une activité de sensibilisation, un nouveau programme adapté au milieu scolaire ou à l'industrie, des activités promotionnelles de vulgarisation des mathématiques, des initiatives, spéciales, des conférences ou des concours à l'intention des étudiants, etc.

Les candidatures doivent nous être transmises via le « Formulaire de mise en candidature » disponible au site Web de la SMC : [www.cms.math.ca/Prix/info/ap](http://www.cms.math.ca/Prix/info/ap). Pour garantir l'uniformité du processus de sélection, veuillez suivre les instructions à la lettre. Toute documentation excédant les limites prescrites ne sera pas considérée par le comité de sélection.

Il est possible de renouveler une mise en candidature présentée l'an dernier, pourvu que l'on en manifeste le désir avant la date limite. Dans ce cas, le présentateur n'a qu'à soumettre des documents de mise à jour puisque le dossier original a été conservé. Les mises en candidature doivent parvenir au bureau de la SMC avant le **30 avril 2007**. Veuillez faire parvenir vos mises en candidature en six exemplaires à l'adresse ci-dessous :

Nominations of individuals or teams of individuals who have made significant and sustained contributions to mathematics education in Canada are solicited. Such contributions are to be interpreted in the broadest possible sense and might include: community outreach programmes, the development of a new program in either an academic or industrial setting, publicizing mathematics so as to make mathematics accessible to the general public, developing mathematics displays, establishing and supporting mathematics conferences and competitions for students, etc.

Nominations must be submitted using the Nomination Form available from the CMS Web site at: [www.cms.math.ca/Prizes/info/ap](http://www.cms.math.ca/Prizes/info/ap). To assure uniformity in the selection process, please follow the instructions precisely. Documentation exceeding the prescribed limits will not be considered by the Selection Committee.

Individuals who made a nomination in 2006 can renew this nomination by simply indicating their wish to do so by the deadline date. Only materials updating the 2006 Nomination need be provided as the original has been retained. Nominations must be received by the CMS Office no later **April 30, 2007**. Please send six copies of each nomination to the address given below.

The Adrien Pouliot Award / Le Prix Adrien-Pouliot  
Canadian Mathematical Society / Société mathématique du Canada  
577 King Edward  
Ottawa, Ontario K1N 6N5

## Distinguished Service Award / Prix de la SMC pour service méritoire

2007

In 1995, the Society established this award to recognize individuals who have made sustained and significant contributions to the Canadian mathematical community and, in particular, to the Canadian Mathematical Society.

Nominations should include a reasonably detailed rationale and be submitted by **March 31, 2007**, to the address below.

En 1995, la Société mathématique du Canada a créé un prix pour récompenser les personnes qui contribuent de façon importante et soutenue à la communauté mathématique canadienne et, notamment, à la SMC.

Pour les mises en candidature prière de présenter des dossiers avec une argumentation convaincante et de les faire parvenir, **le 31 mars 2007** au plus tard, à l'adresse ci-dessous :

Selection Committee / Comité de sélection  
Distinguished Service Award / Prix pour service méritoire  
Canadian Mathematical Society / Société mathématique du Canada  
577 King Edward  
Ottawa, Ontario K1N 6N5

The 2007 Adrien-Pouliot and Distinguished Service Awards will be presented at the CMS Winter 2007 Meeting in London, ON, December 8 to 10.  
Les prix pour service méritoire et Adrien-Pouliot seront présentés à la Réunion d'hiver 2007 de la SMC à London (Ontario), du 8 au 10 décembre.

# CALENDAR OF EVENTS / CALENDRIER DES ÉVÉNEMENTS

## MARCH

## 2007

## MARS

- 4-8** Twelfth International Conference on Approximation Theory (Menger Hotel, San Antonio, TX)  
[www.math.vanderbilt.edu/~at07/at07.html](http://www.math.vanderbilt.edu/~at07/at07.html)
- 
- 10-13** TComplex Cobordism in Homotopy Theory: its impact and prospects (Johns Hopkins University, Baltimore, MD)  
[www.lehigh.edu/~at07/at07.html](http://www.lehigh.edu/~at07/at07.html)
- 
- 15-17** "Seminar on Stochastic Processes", Fields Institute. See  
[www.fields.utoronto.ca/programs/scientific/06-07/ssp2007/](http://www.fields.utoronto.ca/programs/scientific/06-07/ssp2007/)
- 
- 19-23** Representation of Surface Groups (AIM Research Conference Center, Palo Alto, CA)  
[www.aimath.org/ARCC/workshops/surfacegroups.html](http://www.aimath.org/ARCC/workshops/surfacegroups.html)
- 
- 19-23** "Motives and Algebraic Cycles" dedicated to the mathematical heritage of Spencer J. Bloch. Fields Institute (co-organized with the Clay Mathematics Institute)
- 
- 29-31** The Forty-First Spring Topology and Dynamics Conference 2007 (University of Missouri-Rolla MO)  
<http://web.umr.edu/~stdc2007/>

## APRIL

## 2007

## AVRIL

- 14-15** AMS Regional Meeting (Stevens Institute of Technology, Hoboken, NJ)  
[www.ams.math.org/meetings/](http://www.ams.math.org/meetings/)
- 
- 22-27** The Mathematics of Electricity Supply and Pricing: Industry workshop plus short course(Surfers Paradise, Queensland Australia)  
[www.amsi.org.au/electricity.php](http://www.amsi.org.au/electricity.php)

## MAY

## 2007

## MAY

- 7-9** "Adaptive Dynamics in Theory and Practice", Fields Institute Workshop, University of Ottawa:  
[www.mathstat.uottawa.ca/%7Efluts037/FIELDS/fieldsworkshop.html](http://www.mathstat.uottawa.ca/%7Efluts037/FIELDS/fieldsworkshop.html)
- 
- 14-18** "Stacks in geometry and topology", Fields Institute Thematic Program; workshop
- 
- 18-20** The 2007 Midwest Geometry Conference (MGC 2007) (University of Iowa, Iowa City, IA)  
[www.emis.de/journals/SIGMA/](http://www.emis.de/journals/SIGMA/)
- 
- 20-24** The CAIMS Annual Meeting (Banff Conference Centre)

- 26-30** "Homotopy theory of schemes", Fields Institute Thematic Program; workshop

- 21-23** Applications of Analysis to Mathematical Biology (Duke University, Durham, NC)  
[www.math.duke.edu/conference/AAM07](http://www.math.duke.edu/conference/AAM07)

- 22-26** Extremal problems in complex and real analysis (Peoples Friendship University of Russia, Moscow, Russia)  
[www.albany.edu/~pb6916/](http://www.albany.edu/~pb6916/), stessin@math.albany.edu

- 29-Jun1** The Fourth International Conference on Mathematical Biology (Wuyishan City, Fujian, P.R. China)  
[www.csmb.org.cn/](http://www.csmb.org.cn/)

- 30-Jun2** Fifth International Conference of Dynamic Systems and Applications (Morehouse College, Atlanta, GA)  
[www.dynamicpublishers.com/icdsa5.htm](http://www.dynamicpublishers.com/icdsa5.htm),  
*icdsa5@yahoo.com*

- 31-Jun3** CMS-MITACS Joint Conference 2007, Host: University of Manitoba; Delta Hotel, Winnipeg, Manitoba  
[www.cms.math.ca/events](http://www.cms.math.ca/events), [meetings@cms.math.ca](mailto:meetings@cms.math.ca)

## JUNE

## 2007

## JUIN

- 18-23** Combinatorics and Optimization 40th Anniversary Conference (University of Waterloo, Waterloo, ON)  
[www.math.uwaterloo.ca/Cand\\_Dept/Conference/40thConference.shtml](http://www.math.uwaterloo.ca/Cand_Dept/Conference/40thConference.shtml)

- 24-Jul1** 45th International Symposium on Functional Equations (Bielsko-Biala, Poland)  
[romanger@us.edu.pl](mailto:romanger@us.edu.pl), [knikodem@ath.bielsko.pl](mailto:knikodem@ath.bielsko.pl)

## JULY

## 2007

## JUILLET

- 2-6** Design Theory of Alex Rosa, a meeting in celebration of Alex Rosa's 70th birthday (Bratislava, Slovakia)  
[www.dumr.edu/~dfroncek/alex/index.htm](http://www.dumr.edu/~dfroncek/alex/index.htm)

- 4-8** International Conference on Nonlinear Operators, Differential Equations and Applications (ICNODEA 2007) (Bolyai University, Cluj-Napoca, Romania)  
[www.dumr.edu/~dfroncek/alex/index.htm](http://www.dumr.edu/~dfroncek/alex/index.htm)

- 10-14** The Twenty-Second IEEE Symposium on Logic in Computer Science (LICS 2007) (Wroclaw, Poland)  
[www.dumr.edu/~dfroncek/alex/index.htm](http://www.dumr.edu/~dfroncek/alex/index.htm)

- 15-17** Seminar on Stochastic Processes (Fields Institute, Toronto)  
[www.fields.utoronto.ca/programs/scientific/06-07/ssp2007/](http://www.fields.utoronto.ca/programs/scientific/06-07/ssp2007/)

- 16-20** 6th International Congress on Industrial and Applied Mathematics

# CALENDAR OF EVENTS / CALENDRIER DES ÉVÉNEMENTS

(Zurich, Switzerland)  
[www.iciam07.ch](http://www.iciam07.ch)

- 16-22** The 8th International Conference on Fixed Point Theory and its Applications (Ching Mai University, Thailand)  
[www.math.science.cmu.ac.th/ICFPTA2007/](http://www.math.science.cmu.ac.th/ICFPTA2007/)
- 19-23** Motives and Algebraic Cycles, A Conference Dedicated to the Mathematical Heritage of Spencer J. Bloch (Fields Institute, Toronto) Co-organized by the Clay Mathematics Institute  
[www.fields.utoronto.ca/programs/scientific/06-07/homotopy/](http://www.fields.utoronto.ca/programs/scientific/06-07/homotopy/)
- 26-30** Workshop on Homotopy Theory of Schemes (Fields Institute, Toronto)  
[www.fields.utoronto.ca/programs/scientific/06-07/homotopy/](http://www.fields.utoronto.ca/programs/scientific/06-07/homotopy/)
- 27-29** 4th Joint meeting of the Canadian Society for History and Philosophy of Mathematics/Société canadienne d'histoire et de philosophie des mathématiques and the British Society for the History of Mathematics (Concordia University) [www.csphm.org](http://www.csphm.org)
- 31-Aug 3** First Joint International Meeting between the AMS and the Polish Mathematical Society (Warsaw, Poland)  
[www.ams.org/amsmtgs/internmtgs.html](http://www.ams.org/amsmtgs/internmtgs.html)

SEPTEMBER	2007	SEPTEMBRE
<b>17-21</b> "Free Probability, Random Matrices, and Planar Algebras" (Fields Institute workshop) <a href="http://www.fields.utoronto.ca/programs/scientific/07-08/operator_algebras/">www.fields.utoronto.ca/programs/scientific/07-08/operator_algebras/</a>		
OCTOBER	2007	OCTOBRE
<b>29-Nov 2</b> "Von Neumann Algebras" (Fields Institute workshop) <a href="http://www.fields.utoronto.ca/programs/scientific/07-08/operator_algebras/">www.fields.utoronto.ca/programs/scientific/07-08/operator_algebras/</a>		
NOVEMBER	2007	NOVEMBRE
<b>12-16</b> "Structure of C*-Algebras" (Fields Institute workshop) <a href="http://www.fields.u">www.fields.u</a>		
DECEMBER	2007	DÉCEMBRE
<b>12-15</b> First Joint International Meeting between the AMS and the New Zealand Mathematical Society (NZMS) (Wellington, New Zealand) <a href="http://www.ams.org/amsmtgs/internmtgs.html">www.ams.org/amsmtgs/internmtgs.html</a>		
<b>8-10</b> CMS Winter 2007 Meeting, Host: University of Western Ontario; Hilton Hotel, London, Ontario <a href="http://www.cms.math.ca/events_meetings@cms.math.ca">www.cms.math.ca/events, meetings@cms.math.ca</a>		

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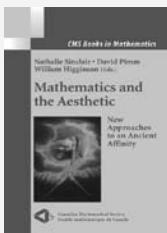
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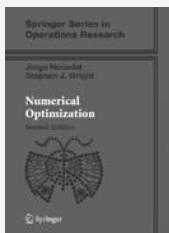
2007. 288 pp. 78 illus. (CMS Books in Mathematics)  
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J.H. Silverman, Brown University, Rhode Island

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Jorge Nocedal, Northwestern University, Illinois and Stephen Wright, University of Wisconsin

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