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# CMS NOTES de la SMC

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No.  
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*From the President's Desk*

**Jacques Hurtubise, McGill University**

## In praise of disorder

By Jacques Hurtubise, CMS President



Pity the poor editorialist. He or she has to come up with an opinion, on something, every day of the week. My own charge as CMS president is somewhat lighter in this direction, and I only have to deliver a few times a year. Mind you, it should be related in some way to mathematics, and we have, as subjects go, a relatively controversy-free one. And I refuse to talk about NSERC one more time.

Instead, I have decided to share some thoughts about where we are going as a subject, thoughts provoked by the CMS study of our subject in Canada, as well as the evolution of my own department. Thus CMS presidents rush in where angels fear to tread. To those angels who object, all I can say is please join the Society- you could do wonders for our budget.

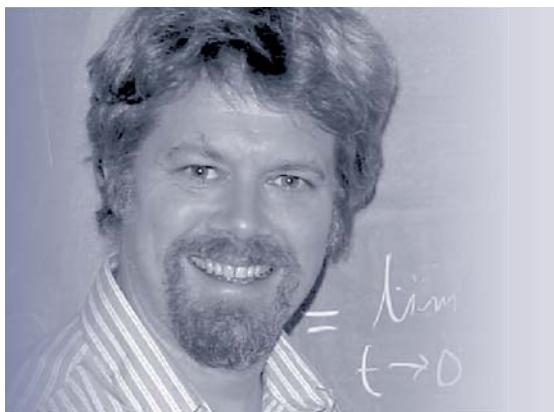
My main theme is: things are getting complicated. (Not a deep thinker, this one.)

If I can be allowed a dose of caricature, it was all so clean thirty years ago; analysis, algebra, logic, differential equations, geometry. Then things started happening. Let me start with my own neck of the woods, geometry. It had developed in a spectacular fashion over the preceding generation into a beautiful edifice. Then gauge theory came along, and of course,

again there was some deep and striking geometry that grew from it; but in parallel, there were these papers by physicists telling us what all the Betti numbers of our moduli spaces should be, what the intersection numbers were, and how many rational curves there were in varieties, all from quantum theoretic magic, and almost always turning out to be frustratingly correct when we caught up and were able to actually prove what they were saying. It was an invasion from outer space; some of us even learnt Martian.

Inside the tent, also, lines blurred. It got harder to do only algebraic number theory, or analytic number theory; success depended more and more on mastering some of both; and you really should have thrown in a bit of Lie theory, as well as a good deal of algebraic geometry. Likewise, you should have honed, even more, your pde as well as your topology if you are going to be a good geometer. Of course, people can, and do, stick to their area; but suddenly the bar was raised.

A big driver of all the complexity has been the development of applied mathematics. It really has grown enormously. The problems are interesting and difficult, the results are sometimes startling (did that pattern really come out of a variational problem in material science?), and they are governed by that difficult task-master, reality, which often says very nice, but we don't actually do things that way. Viewed from the outside of mathematics, a major turning-point was the role that mathematics and computing played in the human genome project; all of a sudden mathematical technique was seen to be useful in biological questions. One now sees, I think in consequence, a blooming of mathematical biology, not only in its discrete genomic manifestation, but also its more continuous modelling aspects.



## That Administrative Stuff

*By Robert Dawson, Saint Mary's University, Halifax*

Course planning may not be as exciting as research or teaching, but it has its interesting moments!

This year our department tried a new idea - at least for us, some universities have been doing this for years. We split our first year calculus course into a "physical sciences" course and a "life sciences" course. The first course was much the same as the one we had taught all along, ultimately calibrated by the needs of our two-year engineering program. The second covers fewer techniques and concentrates on a smaller palette of functions, but covers some subjects, like partial derivatives, that most physical science students will study in their second year.

The two courses also differ on the issue of preparation. We have retained the pretest for the physical sciences calculus, and still redirect the students who cannot pass it into a precalculus course. The life sciences course has no pretest, but there is a certain amount of review built into it.

So far, it seems to have worked astonishingly well. The number of really strong grades in both courses at Christmas was up significantly, and the number of failures was down. We are not sure why things improved so much in the physical sciences course, which is relatively little changed. Were the strong students really being slowed so much by weaker students? Of course, there is always the possibility that we're recruiting a different group of students this year; but if so, the pretest marks did not show it. The professors were good, but they were pretty much the same bunch who taught the single course in the past.

Meanwhile, just to keep things interesting, our administration has admitted a big cohort of engineering students at Christmas, so that our usually midsized "second chance first term" calculus course has swelled to more than twice its usual size. I think we've finally (mid-January) got all the room swaps and class creations out of the way.

**So, what's new for 2012 in your department?**

## Ces tracasseries administratives

*Par Robert Dawson, Saint Mary's University, à Halifax*

La planification d'un cours n'est peut-être pas aussi intéressant que de la recherche ou l'enseignement, mais ce n'est pas sans moments intéressants!

Cette année, notre département a tenté quelque chose de nouveau – ce l'était du moins pour nous. Certaines universités le font depuis des années. Nous avons scindé notre cours de calcul de première année en un cours de « sciences physiques » et un cours de « sciences de la vie ». Le premier ressemblait beaucoup au cours que nous enseignons normalement, mais calibré selon les besoins de notre programme de génie de deux ans. Le deuxième porte sur un nombre moins élevé de techniques et davantage sur une plus petite palette de fonctions, mais traite de certains sujets comme les dérivées partielles, matière que la plupart des étudiants en sciences physiques étudieront au cours de leur deuxième année.

Les deux cours sont différents aussi de par leur préparation. Nous avons conservé le test préalable pour le calcul en sciences physiques et dirigeons toujours les étudiants qui l'échouent vers un cours de pré-calcul. Le cours sur les sciences de la vie ne compte aucun test préalable, mais la révision en fait partie intégrante.

Jusqu'ici, tout semble avoir très bien fonctionné. Le nombre de notes très élevées dans les deux cours à Noël avait beaucoup augmenté et les échecs étaient à la baisse. Nous ne savons trop pourquoi les résultats se sont tant améliorés dans le cours des sciences physiques, cours qui a très peu changé en fait. Les étudiants très forts étaient-ils retenus à ce point par les étudiants plus faibles? Bien entendu, il est toujours possible que nous avons recruté un groupe différent d'étudiants cette année, mais si c'est le cas, les notes des tests préalables n'ont rien indiqué en ce sens. Les professeurs étaient bons, mais c'était en gros le même groupe qui avait enseigné le cours simple par le passé.

Pendant ce temps, juste pour rendre la vie intéressante, notre administration a admis un grand nombre d'étudiants en génie à Noël, de sorte que notre cours de calcul « deuxième chance au premier trimestre » qui est de taille moyenne habituellement a gonflé à plus de deux fois sa taille normale. Je pense que nous avons finalement réglé (à la mi-janvier) la question des échanges de pièces et de création de classes.

**Alors, qu'est-ce que 2012 réserve de neuf dans votre département?**

*Du bureau du président*  
**Jacques Hurtubise, McGill University**

## L'éloge du désordre

par Jacques Hurtubise, CMS President



**P**auvres éditorialistes. Ils doivent produire une opinion, sur quelque chose, tous les jours de la semaine. Ma propre tâche, comme président de la SMC, ne me force à en avoir que quelques fois par année. Ça doit porter sur les mathématiques, par contre, et nous avons un sujet relativement libre de controverses ; et je refuse de discuter encore une fois du CRSNG.

A la place, donc, je pensais partager quelques pensées sur notre direction comme sujet, réflexions provoquées par l'étude de la Société sur les mathématiques au Canada, ainsi que par l'évolution de mon propre département. Ainsi, citant de façon plutôt libre Alexander Pope, les présidents de la SMC s'empressent de foncer là où les anges craignent de mettre le pied ; aux anges qui s'objectent, je suggère de vous affilier à la Société ; ça fera des merveilles pour notre budget.

Le thème principal de ma pensée, citant cette fois-ci Sempé, est : tout se complique.

Si je peux me permettre un brin de caricature, c'était si simple il y a trente ans : analyse, algèbre, logique, équations différentielles, géométrie. Et puis, tranquillement... Laissez-moi commencer sur mon propre terrain, la géométrie, qui s'était au cours de la génération précédente développée en un édifice magnifique. Survient la théorie de jauge : il y a eu, encore, des résultats frappants, remarquables. Mais, en même temps, nous avons commencé à voir ces articles de physiciens nous disant quels étaient les nombres de Betti de nos espaces de modules, que devraient être les nombres d'intersection, combien il devraient y avoir de courbes rationnelles dans des variétés. Tout ceci venait comme par magie de la théorie quantique des champs, et tout ceci était presque invariablement correct quand nous avons pu rattraper le train et démontrer ce qu'ils nous disaient. C'était une invasion extra-terrestre ; certains d'entre nous ont même appris le martien.

A l'intérieur des murs, aussi, les démarcations sont devenues moins précises ; il est devenu plus difficile de s'en tenir à la théorie algébrique des nombres, ou à la théorie analytique ; on doit maintenant combiner, ajouter un peu de théorie de Lie, et une bonne dose de géométrie algébrique. De même, une maîtrise des edp, en plus de la topologie, est devenue de rigueur pour un géomètre. Bien sûr, on peut encore se spécialiser, mais la barre est plus haute.

Une source de toute cette nouvelle complexité est la montée des mathématiques appliquées. Celles-ci se sont développées

*Suite à la page 14*

## Letters to the Editors Lettres aux Rédacteurs

The Editors of the NOTES welcome letters in English or French on any subject of mathematical interest but reserve the right to condense them. Those accepted for publication will appear in the language of submission. Readers may reach us at [notes-letters@cms.math.ca](mailto:notes-letters@cms.math.ca) or at the Executive Office.

Les rédacteurs des NOTES acceptent les lettres en français ou anglais portant sur un sujet d'intérêt mathématique, mais ils se réservent le droit de les comprimer. Les lettres acceptées paraîtront dans la langue soumise. Les lecteurs peuvent nous joindre au bureau administratif de la SMC ou à l'adresse suivante : [notes-lettres@smc.math.ca](mailto:notes-lettres@smc.math.ca).

### NOTES DE LA SMC

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Note aux auteurs : indiquer la section choisie pour votre article et le faire parvenir au Notes de la SMC à l'adresse postale ou de courriel ci-dessous.

Les Notes de la SMC, les rédacteurs et la SMC ne peuvent être tenus responsables des opinions exprimées par les auteurs.

### CMS NOTES

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No responsibility for the views expressed by authors is assumed by the CMS Notes, the editors or the CMS.

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## FEBRUARY 2012

**13–17** ICERM Workshop: Complex and  $p$ -adic Dynamics (Providence, RI)  
<http://icerm.brown.edu/sp-s12/workshop-1.php>

**16–18** Conference on Advances in Control and Optimization of Dynamical Systems (ISI, Bangalore, India)  
<http://www.acods.org>

**20–24** AIM Workshop: Stochastic Dynamics of small networks of neurons (Palo Alto, CA)  
<http://aimath.org/ARCC/workshops/neuronnetwork.html>

## MARCH 2012

**3–4** American Math Society Meeting (Univ. of Hawaii at Manoa, Honolulu)  
[www.ams.org/amsmtgs/sectional.html](http://www.ams.org/amsmtgs/sectional.html)

**12–16** Classifying fusion categories (Amer. Inst. of Math, Palo Alto, CA)  
<http://aimath.org/ARCC/workshops/fusioncat.html>

**17–18** American Math Society Meeting (George Washington Univ. Washington, Dist. Columbia)  
[www.ams.org/amsmtgs/sectional.html](http://www.ams.org/amsmtgs/sectional.html)

**19 – 23** Global Arithmetic Dynamics Workshop (Brown University, Providence RI)  
<http://icerm.brown.edu/sp-s12-w2>

**25–28** Partial Differential Equations and Applications (Hanoi, Vietnam)  
<http://www.amath.washington.edu/~kutz/vietnam/>

## APRIL 2012

**14** 69<sup>th</sup> Algebra Day (University of Ottawa)  
<http://mysite.science.uottawa.ca/neher/algebra/day2012.html>

**16–20** ICERM Workshop: Moduli Spaces associated to Dynamical Systems (Providence, RI)  
<http://icerm.brown.edu/sp-s12/workshop-3.php>

## APRIL (CONTINUED)

**19–22** Fields Institute workshop on Exceptional Algebras and Groups (Toronto)  
<http://www.fields.utoronto.ca/programs/scientific/11-12/exceptional/index.html>

**23–25** Conference on Analytical Approaches to Conflict (Royal Military Acad., Sandhurst, UK)  
[http://www.ima.org.uk/conferences/conferences\\_calendar/influence\\_and\\_conflict.cfm](http://www.ima.org.uk/conferences/conferences_calendar/influence_and_conflict.cfm)

**23 – 27** Workshop on  $p$ -adic Langlands Program (Fields Institute, Toronto, ON)  
[http://www.fields.utoronto.ca/programs/scientific/11-12/galoisrep/wksp\\_p-adic/index.html](http://www.fields.utoronto.ca/programs/scientific/11-12/galoisrep/wksp_p-adic/index.html)

## MAY 2012

**3** Nathan & Beatrice Keyfitz Lectures in Math & Social Sciences (Fields Institute, Toronto, ON)  
[http://www.fields.utoronto.ca/programs/scientific/keyfitz\\_lectures/fienberg.html](http://www.fields.utoronto.ca/programs/scientific/keyfitz_lectures/fienberg.html)

**7 – 11** From Dynamics to Complexity: conference celebrating work of Mike Shub (Fields Institute, Toronto, ON)  
<http://www.fields.utoronto.ca/programs/scientific/11-12/dynamics2complexity/index.html>

**20–27** European Conference on Elliptic and Parabolic Problems (Gaeta, Italy)  
<http://www.math.uzh.ch/gaeta2012>

**28 – June 3** Theory of Approximation of Functions and Applications (Kamianets-Podilsky, Ukraine)  
<http://www.imath.kiev.ua/~funct/stepconf2012/en/>

## JUNE 2012

**2–4** CMS Summer Meeting, University of Regina  
<http://www.cms.math.ca>

**24 – 28** Annual Meeting of CAIMS (Fields Institute & U.Toronto, ON)  
[http://www.fields.utoronto.ca/programs/scientific/11-12/CAIMS\\_SCMAI/index.html](http://www.fields.utoronto.ca/programs/scientific/11-12/CAIMS_SCMAI/index.html)

## JUNE (CONTINUED)

**24** – July 6 Séminaire de Mathématiques Supérieures: Graduate School on Probabilistic Combinatorics (CRM, Montréal, QC)  
<http://www.dms.umontreal.ca/~sms/2012/>

## JULY 2012

**2–6** 24<sup>th</sup> Conference on Operator Theory, (West Univ. Timisoara, Roumania)  
<http://www.imar.ro/~ot/>

**8 – 11** Trends in Set Theory (Stefan Banach International Mathematical Center, Warsaw, Poland)  
[http://www.impan.pl/~set\\_theory/Conference2012/](http://www.impan.pl/~set_theory/Conference2012/)

**9–15** 10<sup>th</sup> International Conference on Fixed Point Theory and Applications (Cluj-Napoca, Romania)  
<http://www.cs.ubbcluj.ro/~fptac/>

**16–20** HPM 2012 History and Pedagogy of Mathematics - The HPM Satellite Meeting of ICME-12 (Daejeon, Korea)  
<http://www.hpm2012.org>

**23–27** Algebraic Topology: applications and new directions (Stanford U, Palo Alto, CA)  
<http://people.maths.ox.ac.uk/tillmann/StanfordSymposium.html>

**29 – August 3** XVIII Brazilian Topology Meeting (XVIII Encontro Brasileiro de Topologia)  
[www.dm.ufscar.br/~ebt2012/](http://www.dm.ufscar.br/~ebt2012/)

## SEPTEMBER 2012

**20–22** Lie and Klein: the Erlangen program and its impact on mathematics and physics (Strasbourg, France)  
<http://www-irma.u-strasbg.fr/article1173.html>

## DECEMBER 2012

**8–10** CMS Winter Meeting, Montréal, QC  
<http://www.cms.math.ca>

## MATHEMATICAL CONGRESS OF THE AMERICAS

Guanajuato, Mexico, August 5-9, 2013



## CALL FOR NOMINATIONS

### FOR PRIZES TO BE AWARDED AT THE MCA2013

The following prizes winners will be chosen by the MCA Awards Committee which is appointed by the MCA2013 Steering Committee. In making their decisions the Awards Committee will be guided by the information in the nominating material and detailed insights about the nominees' professional accomplishments. It is important that the nominees most significant contributions and their impact be part of the nominating material.

The Awards Committee may, if it chooses to do so, make nominations.

#### ● THE MCA PRIZE:

Five prizes of \$1000 each will be awarded to mathematicians who are no more than 12 years past their PhD in August 2013.

Eligibility for consideration of nominees requires that they either received their graduate education or that they currently hold a position in one or more countries in the Americas.

The choice of the prize winners will be based on the documented mathematical achievements of the nominees.

The nominations must include a justifying statement of up to 2 pages, the CV of the nominee and one additional letter of support.

#### ● THE AMERICAS PRIZE:

One prize of \$5000 will be awarded to an individual or a group in recognition of their work to enhance collaboration and the development of research that links mathematicians in several countries in the Americas.

The nominations must include a description (up to 4 pages) of the work and any relevant citations that justify the award of the Americas Prize.

CVs of the nominees should be provided.

There must be four co-nominators from at least two different countries.

#### ● THE SOLOMON LEFSCHETZ MEDAL

Two medals with an award of \$5000 will be given to mathematicians in recognition of their excellence in research and their contributions to the development of mathematics in a country or countries in the Americas.

Nominations must include a justifying statement (up to 2 pages), and a brief paragraph that can be used in the announcement of the Medal, the CV of the nominee, and two additional supporting letters.

An individual may make up to two nominations in each category.

Self nominations will not be accepted in any category.

**Nominations and requests for information concerning the nominating process should be sent by e-mail to [mca2013.prizes@gmail.com](mailto:mca2013.prizes@gmail.com)**  
**The deadline for nominations is January 31, 2013.**



## CALL FOR SESSION PROPOSALS

Proposals to organize a special session at the MCA 2013 are welcomed by the Steering Committee. A proposal should include

- the names, affiliations and contact information of all the organizers
- a brief presentation of the topic and scope (up to one page)
- a preliminary list of the expected speakers

The topics should be broad and fairly well represented throughout the Americas. The list of organizers must include at least two mathematicians from different countries in the Americas. Preference will be given to proposals whose list of suggested speakers represents diversity in all aspects.

Each special session will consist of two 4-hour periods. We recommend that the organizers base their session on a total of 16 half hour time slots for their speakers.

**Proposals should be sent before July 31, 2012 to [mca2013.sessions@gmail.com](mailto:mca2013.sessions@gmail.com).**

## YOUNESS LAMZOURI

**The CMS Doctoral Prize recognizes outstanding performance by a doctoral student who graduated from a Canadian university in the preceding year. The first award was presented in 1997.**

Youness Lamzouri emerges from his doctoral studies as a fully fledged mathematician. He is a strong researcher, a very good writer of mathematics, and a clear effective teacher and lecturer who is popular with students at different levels.

Lamzouri's research is in the area of analytic number theory. His thesis provides a first good understanding of extreme values of the Riemann zeta-function (and of all L-functions) at the edge of the critical strip, an area involved in some of the most difficult and central problems in analytic number theory.

There was already a good understanding of the distribution of  $|\zeta(1+it)|$  in its full range, as  $t$  varies, but Lamzouri was able to give some idea of the distribution of  $\zeta(1+it)$  in the same range, showing that it is more dense near the real axis than had perhaps been expected.

Another striking aspect of Lamzouri's thesis work is his use of analytic techniques to understand questions on diophantine approximation (and thus settle a dispute as to the basis of the Lang-Waldschmidt conjecture on the limit of linear forms in logarithms); and in using diophantine approximation techniques (the Lang-Waldschmidt conjecture) to greatly extend the range of Fourier analysis involving  $ap^{it}$ 's.

*Youness Lamzouri obtained his PhD in mathematics from the University of Montreal in 2009. After graduation, he obtained an NSERC postdoctoral fellowship, and participated in the 2009-2010 special year on Analytic Number Theory at the Institute for Advanced Study in Princeton. He was the recipient of the 2004 Jean-Maranda Award for the best finishing undergraduate student in mathematics from the University of Montreal, and the 2006 Carl Herz Prize from the Institut des sciences mathématiques (ISM). Youness is currently a J. L. Doob Research Assistant Professor at the University of Illinois in Urbana-Champaign.*

**Le Prix de doctorat de la SMC a été créé pour récompenser le travail exceptionnel d'un étudiant au doctorat en mathématiques ayant obtenu un diplôme d'une université canadienne entre l'année précédente. Ce prix a été décerné pour la première fois en 1997.**

Youness Lamzouri termine ses études doctorales en mathématicien à part entière. Il excelle en recherche, il est doué en rédaction mathématique et il est un enseignant et un conférencier clair, efficace et populaire auprès des étudiants à divers niveaux.

Les recherches de M. Lamzouri est dans le domaine de la théorie analytique des nombres. Sa thèse nous donne pour la première fois une bonne compréhension des valeurs extrêmes de la fonction zêta de Riemann (et de toutes les fonctions L) au bord de la bande critique. C'est un domaine qui est relié à certains des problèmes les plus difficiles et les plus centraux de la théorie analytique des nombres.

On avait déjà une bonne compréhension de la distribution de  $|\zeta(1+it)|$  dans son domaine complet, à mesure que  $t$  varie, mais maintenant Lamzouri nous donne une certaine idée de la distribution de  $\zeta(1+it)$  dans le même domaine en prouvant qu'elle est plus dense que prévu près de l'axe des

réels. Un autre aspect frappant dans la thèse de Lamzouri est l'utilisation des techniques analytiques pour comprendre des questions d'approximation diophantienne (et régler ainsi un conflit concernant la conjecture de Lang-Waldschmidt sur la limite des formes linéaires dans les logarithmes) ; et l'emploi des techniques d'approximation diophantienne (la conjecture de Lang-Waldschmidt) pour étendre considérablement le domaine de l'analyse de Fourier impliquant les  $p^{it}$ .

*Youness Lamzouri a obtenu son doctorat en mathématiques de l'Université de Montréal en 2009. Il a par la suite obtenu une bourse de recherche postdoctorale du CRSNG et participé à l'année 2009-2010 en théorie analytique des nombres à l'Institut d'études avancées de Princeton. Il a reçu le prix Jean-Maranda 2004, décerné au finissant du baccalauréat en mathématiques de l'Université de Montréal ayant obtenu les meilleurs résultats, et le prix Carl Herz 2006 de l'Institut des sciences mathématiques. En ce moment, Youness détient une bourse de recherche et d'enseignement (J. L. Doob Research Assistant Professor) à l'Université de l'Illinois à Urbana-Champaign.*



## DAVID RODGERS

**The Distinguished Service Award was inaugurated to recognize individuals who have made sustained and significant contributions to the Canadian mathematical community and, in particular, to the Canadian Mathematical Society. The first award was presented in 1995.**

The Award was renamed the **Graham Wright Award for Distinguished Service** in 2008 in recognition of Graham Wright's 30 years of service to the Society as the Executive Director and Secretary.

David Rodgers is neither a Canadian nor a mathematician, yet he has shown incredible dedication to the CMS and its mission to promote the advancement, discovery, learning and application of mathematics in Canada.

David's involvement in the CMS began in 1997 when he was invited to sit on the Electronic Services Committee due to his expertise in electronic publishing. David's experience included serving as Director of Electronic Products and Services for the American Mathematical Society, where he helped to develop the website that has evolved into [www.ams.org](http://www.ams.org), and System Manager for *Mathematical Reviews* where

he developed the publication database that evolved into MathSciNet.

From this initial invitation, David quickly became an active CMS member. He served as a member of the Electronic Services Committee from 1997 to 2002 and chair of the committee from 2003 to 2007. He also served on the CMS Board of Directors for 2001 and 2002, and has acted as CMS treasurer since 2006.

David's contributions to the CMS have played a significant role in the advancement of the Society. His expertise in the publishing field has contributed to many positive developments for CMS journals, including the adoption of an online editing system, the introduction of subscription agent and electronic-only pricing models, and the negotiation of a contract with Google to make back issues of the Canadian Journal of Mathematics and the Canadian Mathematical Bulletin from 1945-1996 freely available through Google Scholar.

As chair of the Electronic Services Committee, David oversaw many changes that positively impacted electronic services to CMS members as well as the Society as a whole, online registration system, the redesign of the CMS website, the linking of CMS journals through MathSciNet, and significant restructuring of the electronic

services workflow, allowing projects to be completed more efficiently.

David has also been instrumental in CMS fundraising initiatives. He helped to steer the 2007 fundraising efforts of the Society, developed a CMS fundraising database, and incorporated the U.S. non-profit Friends of the CMS, which allows US citizens to claim donations to the CMS on their income tax.

*David Rodgers received his M.S. in physical chemistry from the University of Michigan in 1972. He was Director of Enterprise Information Systems and IT Director for Professional Fee Billing for the University of Michigan Health System where he developed one of the first web-based systems for patient information and financial reporting. He was co-founder and first President for Arbortext and Executive Vice-President for Argus Associates, a web services company based in Ann Arbor, Michigan. He was Director of Academic Information Services and Member of the Administrative Board for the College of Literature, Sciences, and the Arts at the University of Michigan, where he developed a College-wide delivery system for academic information and a new model for College publications. He has been Adjunct Faculty for the School of Information at the University of Michigan and the Chemistry Department at the Florida Institute of Technology.*

**Créé en 1995, le Prix de la SMC pour service méritoire récompense les personnes qui contribuent de façon importante et soutenue à la communauté mathématique canadienne et, notamment, à la Société mathématique du Canada.**

En 2008, la Société a renommé cette récompense « **Prix Graham-Wright pour service méritoire** » en l'honneur du professeur Graham Wright, de l'Université d'Ottawa, qui a occupé avec dévouement pendant plus de 30 ans le poste de directeur administrative et secrétaire de la Société.



David Rodgers n'est ni un Canadien, ni un mathématicien, et pourtant, il est un apport exceptionnel à la SMC et à sa mission de promouvoir l'avancement, la découverte et l'apprentissage des mathématiques au Canada, ainsi que des applications qui en découlent.

La participation de David aux activités de la SMC remonte à 1997. On l'avait alors invité à siéger au Comité des services électroniques en raison de son expérience en publication électronique. En effet, il avait notamment été directeur des services et produits électroniques de l'AMS (American Mathematical Society), pour qui il a contribué à la création du site internet aujourd'hui logé au [www.ams.org](http://www.ams.org), et gestionnaire des systèmes pour les *Mathematical Reviews*, pour lesquels il a créé la base de données de publications connue aujourd'hui sous le nom de MathSciNet.

Après cette invitation, David est vite devenu un membre actif de la SMC. Il a siégé au Comité des services électroniques de 1997 à 2002 et présidé ce comité de 2003 à 2007. Il a aussi siégé au Conseil d'administration de la SMC en 2001 et 2002, et il occupe le poste de trésorier de la Société depuis 2006.

La contribution de David à la SMC a joué et continue de jouer un rôle capital dans l'avancement de la Société. Son expertise dans le domaine de la publication a engendré de nombreux développements positifs pour les revues de la SMC, notamment l'adoption d'un système d'édition en ligne, l'introduction d'un agent d'abonnement et d'un modèle de prix pour les versions électroniques seulement, et la négociation d'un contrat avec Google pour rendre les numéros du Journal canadien de mathématiques et du Bulletin canadien de mathématiques de 1945 à 1996 disponibles gratuitement dans Google Scholar.

En tant que président du Comité des services électroniques, David a supervisé les nombreux changements qui ont amélioré les services électroniques aux membres de la SMC, y compris la création d'une

base de données des Réunions et un système d'inscription en ligne, la refonte du site internet de la SMC, l'établissement de liens entre les revues de la SMC par l'intermédiaire de MathSciNet et une restructuration importante du déroulement des services électroniques qui permet de terminer les projets plus efficacement.

David a aussi joué un rôle clé dans les activités de financement de la SMC. Il a pris part à la direction de la campagne de financement de la Société en 2007, il a créé une base de données pour les activités de financement et incorporé un organisme à but non lucratif aux États-Unis appelé Friends of the CMS, grâce auquel les donateurs américains ont droit à des avantages fiscaux en retour d'un don à la SMC.

*David Rodgers a obtenu une maîtrise en chimie physique de l'Université du Michigan en 1972. Il a été directeur du service des systèmes d'information d'entreprise et directeur des technologies de l'information pour la facturation des honoraires professionnels du système de santé de l'Université du Michigan, où il a élaboré l'un des premiers systèmes web d'information sur les patients et de rapports financiers. Il a cofondé Arbortext et il en a été le premier président, puis il a été premier vice-président d'Argus Associates, entreprise de services web basée à Ann Arbor, au Michigan. Il a aussi été directeur des services d'information sur les études et membre du conseil d'administration du Collège de littérature, des sciences et des arts de l'Université du Michigan, où il a développé un système collégial de production de l'information sur les études et un nouveau modèle pour les publications du collège. Il a été professeur associé (adjunct faculty) à l'École d'information de l'Université du Michigan et au Département de chimie à l'Institut de technologie de la Floride.*

#### *In Praise of Disorder, continued from cover*

Computational power has also been a big factor. This has driven a major shift in our ability to test hypotheses, to do algebraic computations (throw it on Maple and see what happens), and this across the spectrum of mathematics; in the end it drives the way we do mathematics. Data sets also have become enormous, and now have all sorts of structures associated to them, and this leads to many interesting questions on the boundary of mathematics and statistics.

The pure-applied, mathematics-statistics boundaries are thus getting harder to patrol. In one of our departments the specialist in complex dynamical systems also has an active research programme in mathematical biology, and the number theorist has a vigorous (and quite successful) laboratory not only in cryptographic protocols, but also in questions such as data integrity. Elsewhere, there is suddenly an interest in applied algebraic topology, aiming to solve what is basically a data-mining question: what is the shape that cloud of billions of points in two hundred dimensions is trying to model, given experimental error? In yet another direction, algebraic geometry and statistics now have an intriguing, if still somewhat narrow, partnership in some data analysis questions.

With an empirical link comes different "levels" of truth: proof-truth (still the gold standard), physics truth (they see it, but we don't; maybe we should check...), experimental truth (I computed it in the following cases) and even, well, this doesn't work in theory, but in practice, look at those really neat pictures. This has always been with us to some degree, but they might become a bit more official. In some sense, this should not be a problem, as long as labels are clear; but they do tend to blur. I was amused over the holidays in reading a book about the history of quantum field theory: one of the important advances was blocked for a few years by the fact that it contradicted "a theorem". A theorem, yes, but of what type?

In any case the landscape has changed, I think in a healthy way. As usual, one of the main drivers of this is new people: one sees a different dynamic in our younger colleagues. I find the perspective a rather pleasant one. It is all rather fun, really, though of course, not terribly neat.



## IOSIF POLTEROVICH

**The Coxeter-James Prize was inaugurated to recognize young mathematicians who have made outstanding contributions to mathematical research. The first award was presented in 1978.**

Though still in the early years of his career, Iosif Polterovich has already become a leading name in the field of spectral geometry. This award recognizes Polterovich's accomplishments as a young researcher as well as his substantial contributions to mathematical research in general.

Polterovich's research is characterized by a new, fresh approach to long-standing problems. His research on heat invariants is a prime example. Heat invariants are among the oldest and most intensively studied spectral invariants, yet Polterovich found a completely new closed form expression for the entire sequence.

Other areas of study where Professor Polterovich has made fundamental contributions include isospectral domains with mixed boundary conditions, the asymptotics of eigenvalues of the Laplacian, and isoperimetric inequalities for eigenvalues.

*Professor Iosif Polterovich was born in Russia. He received his MSc from Moscow State University in 1995 and his PhD in 2000 from the Weizmann Institute of Science. He is presently Canada Research Chair (Tier II) at Université de Montréal. He has held postdoctoral and visiting positions at the Max Planck Institute for Mathematics in Bonn, the Mathematical Sciences Research Institute in Berkeley and in the Centre de recherches mathématiques in Montréal. His previous awards and honours include the 2008 G. de B. Robinson Award from the Canadian Mathematical Society, the 2006 André Aisenstadt Prize from the Centre de recherches mathématiques (CRM) and the J.F. Kennedy Prize from the Weizmann Institute of Science.*

**Le prix Coxeter-James rend hommage aux jeunes mathématiciens qui se sont distingués par l'excellence de leur contribution à la recherche mathématique. Il a été décerné pour la première fois en 1978.**

Bien qu'il n'en soit qu'au début de sa carrière, Iosif Polterovich est déjà bien connu dans le domaine de la géométrie spectrale. Ce prix souligne ses réalisations de jeune chercheur et sa contribution importante à la recherche en mathématiques en général.

La recherche de Polterovich est caractérisée par une nouvelle approche novatrice à des problèmes de longue date. Sa recherche sur les invariants de la chaleur en est un exemple parfait. Les invariants de la chaleur figurent parmi les plus anciens invariants spectraux à être étudiés et à faire l'objet d'une étude intensive; Polterovich a néanmoins trouvé une toute nouvelle expression de forme fermée pour la séquence entière.

On compte parmi les autres domaines d'étude où le professeur Polterovich a fourni une contribution fondamentale les domaines isospectraux avec conditions aux limites mixtes, l'asymptotique des valeurs propres du laplacien et les inégalités isopérimétriques pour des valeurs propres.

*Le professeur Iosif Polterovich est né en Russie. Il a obtenu sa maîtrise en sciences de la Moscow State University en 1995 et son doctorat en 2000 de la Weizmann Institute of Science. Il est actuellement chaire de recherche du Canada (Niveau II) à l'Université de Montréal. Il a occupé des postes postdoctoraux et de visiteur à la Max Planck Institute for Mathematics à Bonn, à la Mathematical Sciences Research Institute à Berkeley et au Centre de recherches mathématiques à Montréal. Il a été lauréat de prix et de mentions de mérite antérieures telles que le prix G. de B. Robinson 2008 de la Société mathématique du Canada, le prix André Aisenstadt 2006 du Centre de recherches mathématiques et le prix J.F. Kennedy de la Weizmann Institute of Science*

## HUGH THOMAS AND ALEXANDER YONG

**The G. de B. Robinson Award was inaugurated to recognize the publication of excellent papers in the *Canadian Journal of Mathematics* and the *Canadian Mathematical Bulletin* and to encourage the submission of the highest quality papers to these journals. The first award was presented for papers that appeared in the *Canadian Journal of Mathematics* in 1994-1995.**

Hugh Thomas and Alexander Yong are the recipients of the 2011 G. de B. Robinson Award for their paper “Multiplicity-Free Schubert Calculus”, published in the *Canadian Mathematical Bulletin* (53:1 2010, 171-186).

Grassmannians are fundamental objects in algebraic geometry and topology, and they play roles in representation theory, combinatorics, and some applications of mathematics. An old result of Schubert is that any subvariety of a Grassmannian is homologous to a unique nonnegative integer

linear combination of classes of Schubert varieties, which are the simplest subvarieties in a Grassmannian.

There are two natural candidates for the next simplest type of subvarieties. Multiplicity-free subvarieties are those whose linear combination has coefficients 0 or 1. A second are Richardson varieties, which are the intersection of two Schubert varieties, and whose linear combinations are related to tensor product decompositions in representation theory. In “Multiplicity-free Schubert Calculus”, Thomas and Yong give a classification of multiplicity-free Richardson varieties in a Grassmannian, answering a question of Fulton.

Previously, Stembridge solved the representation theoretic analog: classify multiplicity-free decompositions of tensor products of irreducible  $gl_n$ -representations. While related, it does not directly apply to Fulton’s question. Thomas and Yong first employ a simple reduction to certain basic Richardson varieties, and they show that basic Richardson varieties are multiplicity-free if and only if they fall into Stembridge’s classification.

The relation to representation theory and Stembridge’s work establishes one direction. For the other direction, given a basic Richardson variety not in Stembridge’s classification, they give a different reduction which decreases the coefficients, and then use combinatorics to show that some coefficients exceed 1. The beauty of this result is the elegant application of these two simple reductions.

*Hugh Thomas was born and raised in Winnipeg. He did his undergraduate work at the University of Toronto, and his Ph.D. at the University of Chicago, under the direction of William Fulton. In 2004, after holding postdoctoral fellowships at the University of Western Ontario and the Fields Institute, he joined the faculty of the University of New Brunswick. He is interested in a variety of topics from algebra and combinatorics, including cluster algebras, representation theory of hereditary algebras, and Schubert calculus. Alexander Yong attended the University of Waterloo, receiving a B.Math degree in 1998 and an M.Math in 1999. He obtained a doctorate from the University of Michigan in 2003, under the direction of*



*Sergey Fomin. He held postdoctoral positions at the University of California, Berkeley, the Fields Institute and the University of Minnesota. He joined the University of Illinois at Urbana-Champaign in 2008 where he is presently an Assistant Professor in the Department of Mathematics. His research is in algebraic combinatorics.*

**“There are two natural candidates for the next simplest type of subvarieties.”**

**Le prix G. de B. Robinson rend hommage aux mathématiciens qui se sont distingués par l'excellence de leurs articles parus dans le *Journal canadien de mathématiques* et le *Bulletin canadien de mathématiques*, et vise à encourager la présentation d'articles de première qualité pour ces revues. Il a été décerné pour la première fois pour des articles qui sont apparus dans le *Journal canadien de mathématiques* en 1994-1995.**

Hugh Thomas et Alexander Yong sont les récipiendaires de son prix G. de B. Robinson 2011 pour leur article « Multiplicity-Free Schubert Calculus », publié dans le *Bulletin canadien de mathématiques* (53:1 2010, 171-186).

Les Grassmanniennes sont des objets fondamentaux dans la géométrie algébrique et la topologie, et elles jouent des rôles dans la théorie de représentation, la combinatoire, et quelques applications des mathématiques. Par un vieux résultat de Schubert on sait que n'importe quelle sous-variété d'une grassmannienne est homologue

à une combinaison linéaire unique avec des coefficients entiers non négatifs des classes des variétés de Schubert, qui sont les sous-variétés les plus simples dans une Grassmannienne.

Il y a deux candidats naturels pour le prochain type le plus simple de sous-variétés. Le premier est le type des sous-variétés simples, celles qui sont des combinaisons Z-linéaires n'ayant que des coefficients égaux à 0 ou 1. Le deuxième est le type des variétés de Richardson, qui sont des intersections de deux variétés de Schubert et dont les combinaisons linéaires sont reliées à la décomposition du produit tensoriel de certaines représentations. Dans l'article “Multiplicity-Free Schubert Calculus”, Thomas et Young donnent une classification des variétés de Richardson simples dans les variétés Grassmanniennes, ce qui répond à une question de Fulton.

Stembridge a déjà résolu la question analogue de la théorie de représentation: classifier les décompositions simples des produits tensoriels des représentations irréductibles de  $gl_n$ . La question n'est pas directement reliée à celle de Fulton, malgré la connexion entre les deux. Thomas et Yong utilisent d'abord une réduction simple à certaines variétés de base de Richardson, et ils montrent que les variétés de base de Richardson sont simples si et seulement si elles tombent dans la classification de Stembridge.

La relation avec la théorie de représentation et au travail de Stembridge établit une direction. Pour l'autre direction, ils donnent une réduction différente qui diminue les coefficients et puis ils utilisent la combinatoire pour montrer que quelques coefficients dépassent 1, pour une variété de base de Richardson qui ne tombe pas dans la classification de Stembridge. La beauté de ce résultat est l'application élégante de ces deux réductions simples.

*Hugh Thomas est né et a agrandi à Winnipeg. Il a fait ses études de premier cycle à l'Université de Toronto, et son Ph.D. à l'Université de Chicago sous la direction de William Fulton. En 2004, après des mandats de post-doctorat à l'Université de Western Ontario et l'institut Fields, il s'est*

*joint à l'Université du Nouveau Brunswick. Ses intérêts de recherche incluent plusieurs domaines comme l'algèbre et la combinatoire, incluant les algèbres amassées et la théorie de représentation des algèbres héréditaires et le calculus de Schubert. Alexandre Yong a fait ses études postsecondaires à l'Université de Waterloo, où il a obtenu un baccalauréat en Math en 1998 et une maîtrise en Math en 1999. Il a obtenu un doctorat de l'Université du Michigan en 2003, sous la direction de Sergey Fomin. Il a tenu des postes postdoctorales à l'Université de Californie, à Berkeley ainsi qu'à l'institut Fields et à l'Université du Minnesota. Il s'est joint l'Université d'Illinois et l'Urbana-Champagne en 2008 où il est actuellement un professeur adjoint au département des mathématiques. Sa recherche est dans le domaine de la combinatoire algébrique.*

**“Il y a deux candidats naturels pour le prochain type le plus simple de sous-variétés.”**



*Editors: John Grant McLoughlin (University of New Brunswick) and Jennifer Hyndman (University of Northern British Columbia)*

In this issue we look at how students learn and the difference between novice and expert learners. If you have ever wondered why your students did not answer a test question as well as you hoped, the explanation might be below.

## USING COGNITIVE LOAD THEORY TO BETTER ASSESS STUDENTS' LEARNING

by Djun Kim, Skylight Research Associate, Dept. of Mathematics, UBC [djun.kim@math.ubc.ca](mailto:djun.kim@math.ubc.ca) and Joanne Nakonechny, Director and Research Associate, Skylight, UBC, current e-mail address: [jnakone@gmail.com](mailto:jnakone@gmail.com)

Assessing and evaluating students' mathematical knowledge is an inherently difficult task for instructors as they are

challenged with constructing problems that correctly assess students' ability at a given point. In this column, we explore how understanding the differences between experts' and novices' problem solving heuristics and Cognitive Load Theory can better inform assessment question construction.

### Example: The easy exam question that wasn't

Question: Find a function  $f$  defined on the interval  $[-1, 1]$ , which satisfies  $f(-1) < 0$  and  $f(1) > 0$ , for which the value  $f(c) \neq 0$  for all  $c$  between  $-1$  and  $1$  inclusive.

The above question was used on a first year calculus mid-term exam to test students' understanding of the intermediate value theorem (IVT). The three instructors who vetted the question were surprised when students did not do well on this question. But, on analyzing the question and student responses, we began to understand the

difficulty students had in terms of expert versus novice problem solving, and cognitive load.

### Expert and Novice Problem Solvers

Expectations about problem solving ability are often grounded in personal experience. Unfortunately, instructors' personal experiences are those of experts which can cause difficulty in setting appropriate assessment problems. Experts and novices problem solve in fundamentally different ways and a literature review (Duke ARC) details five major variations:

1. Experts classify problems based on deep structure while novices use surface structure details.
2. Expert knowledge is organized around basic principles while novice knowledge is more randomly arranged.
3. Experts start with general equations while novices start with specific ones.
4. Experts see problem solving as a process while novices see it as a recall task.
5. Experts use qualitative representations extensively while novices do not.

These major dissimilarities often create a context where instructors do not recognize the depth of separation between what they think they are asking students to do and what they are actually asking students to do.

### Cognitive Load Theory

Cognitive Load Theory (Paas et al. 2003) broadens our understanding of learning by focusing on three main learning factors:

1. intrinsic cognitive load - refers to the demands made on working memory in the execution of a task as a result of the task itself;
2. extraneous cognitive load – refers to material that is not necessary for performing a task;
3. germane or effective cognitive load – refers to enhancement of learning through schema acquisition and automation.

Both extraneous and germane load can be controlled by the instructor while intrinsic load cannot as it is subject based.

## Applying Expert vs. Novice Problem Solving and Cognitive Load Theory

The novice learner does not have the same access to well structured, schematized information as does an expert. While an expert can handle more extraneous cognitive loading as s/he can more quickly parse and exclude it, the novice learner cannot and begins to slow down or cease to problem-solve effectively.

We hypothesized the given exam question was too high on the expert/novice continuum, and that it presented excessive cognitive load for the learners. We decided to try to re-structure the question to reduce both the expert/novice spread and extraneous cognitive load, and also increase the germane load.

We re-wrote the question as three parts (see below). We wrote the first part to assess recall, using a fill-in-the-blank text (cloze form) in which students complete key elements left blank in the text. The second part asked students to interpret/transform the definition they completed in the previous question, by giving an example in the form of an appropriately labelled graph (increasing germane load through scaffolding<sup>1</sup> from the first part of the question and decreasing expert/novice spread as well). The final part asked students to provide the example asked for in the original question.

1. State the Intermediate Value Theorem by filling in the following blanks: Suppose  $f$  is continuous on the interval \_\_\_\_\_ and  $L$  is a number between  $f(a)$  and  $f(b)$ . Then there is at least one number \_\_\_\_\_ in  $(a, b)$  satisfying  $f(c) = L$ .
2. Sketch the graph of a continuous function  $f$  illustrating the Intermediate Value Theorem as stated in part (a). Make sure to include the labels  $a$ ,  $b$ ,  $c$ ,  $f(a)$ ,  $f(b)$  and  $L$ .

<sup>1</sup> Cazden defines three important features of scaffolds: "they make it possible for the novice to participate in the mature task from the very beginning; and they do this by providing support that is both adjustable and temporary." (p. 107)

3. Give an example of a function  $f$  defined on the interval  $[-1, 1]$ , satisfying  $f(-1) = -1$  and  $f(1) = 1$ , such that  $f(x) \neq 0$  for any  $x$  in  $[-1, 1]$ .

When correctly answered the blanks in part a) would be filled in with  $[a, b]$  and  $c$ . An example accepted in part c) would not be continuous.

## Results and Discussion

The original question, presented as one of two parts in a midterm exam to three sections of a course with a total of 280 students, had a combined mean score of 59%. We did not record marks for the separate parts, but the second part appeared to present more of a challenge to students than the first part, which asked for an example of a function which satisfied the IVT.

The re-written question, presented the subsequent year in the same course, had mean scores as follows: part a) 70.5%, part b) 60.9%, and part c) 37.3%, with a combined mean for the three parts of 56.2%.

We were pleased to see a significant correct completion of the IVT cloze text and even in the second part of the question, more than half the students were successful. Student marks dropped significantly on the final part. We view the question parts as testing recall, understanding, and analysis, respectively. As these signify progressively higher level knowledge, we expected a decline in mean scores across the three parts. However, we query whether part c) could be further clarified to reduce the drop in scores from part b) to part c).

## Writing questions

To work with the above presented key ideas, we found asking the following types of questions helped us to establish appropriate question levels for students while also providing information on the level of cognitive load students may be experiencing:

What aspect of student understanding do I want to assess? e.g. Is it the student's ability to recall a formula/theorem? Is it their technical skill?

Why is it important for students to be able to do it? e.g. Why is it important that students be able to state the IVT....

Does the question test the stated learning objectives for the course? e.g. Does the question appropriately examine the students' knowledge in sufficient breadth and depth?

## Conclusion

In conclusion, keeping in mind the differences between novice and expert problem solvers and referring to Cognitive Load Theory can enhance and clarify the types of questions set to assess students' learning. When the questions are within the capacity of novice problem solvers with normal cognitive loading, students' performance can then inform us as to what difficulties they are encountering when learning the material and what material they have actually learned.

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### Notices of some upcoming meetings in 2012

There are several upcoming conferences on how students learn mathematics, both national and international.

### Psychology of Mathematics Education (PME)

The focus of PME is how students learn mathematics, how instructors teach mathematics, and how mathematicians, teachers and students do mathematics. The 2012 conference will take place in Taipei, Taiwan, from July 18-22. Details on PME can be found at <http://www.igpme.org/>.

The organization also has a North American chapter, *PME-NA*. The *PME-NA* 2012 meeting is being held at Western Michigan University in Kalamazoo from November 1 to 4, 2012. Details on the meeting can be found at <http://pmena.org/2012/>.

### Canadian Mathematics Education Study Group (CMESG)

The 2012 Meeting is scheduled for May 25 – 29 at Université Laval, Québec. From their website: “The aims of the Study Group are the following: to advance education by organizing and coordinating national conferences and seminars to study and improve the theories of the study of mathematics or any other aspects of mathematics education in Canada at all levels; and to undertake research in mathematics education and to disseminate the results of this research.” Information on past working groups and conferences is available at: <http://publish.edu.uwo.ca/cmesg/>.

### 12<sup>th</sup> International Congress on Mathematical Education (ICME)

This international conference is held every four years with the next conference July 8 – 15, 2012 in Seoul, Korea. Details concerning the scientific programs, events, topic groups, and general information can be found at <http://www.icme12.org/>.

### L'éloge du désordre suite de la page 3

énormément. Les problèmes sont intéressants et difficiles, les résultats parfois étonnantes (est-ce que ce patron est vraiment obtenu à partir d'une équation variationnelle en science des matériaux ?) et elles sont régies par cette dure contrainte, la réalité, qui dit souvent merci, c'est bien beau, mais on ne fait pas les choses de cette façon. Vu de l'extérieur des mathématiques, un point tournant majeur a été le rôle des mathématiques et de l'informatique dans le projet du génome humain ; tout à coup, la technique mathématique s'avérait utile pour nos biologistes. On a vu, je crois en conséquence, une éclosion des mathématiques de la biologie, non seulement sous sa forme discrète et génomique, mais aussi sa version de modélisation continue.

Les frontières entre le pur et l'appliqué, et entre les mathématiques et la statistique devient donc plus difficile à patrouiller. Dans un de nos départements le spécialiste en dynamique complexe fait aussi de la biologie mathématique, et le théoricien des nombres a un laboratoire vigoureux et bien financé non seulement en protocoles cryptographiques, mais aussi sur des questions en intégrité de données. Ailleurs, on a soudain un intérêt en topologie appliquée, visant essentiellement une question d'inférence : quel forme est-ce que ce nuage de milliards de points en deux

cent dimensions essaie de représenter, étant donné l'erreur expérimentale ? Dans une autre direction, on a maintenant des applications de la géométrie algébrique en statistique.

Avec le lien empirique, on voit différents « niveaux » de vérité : la preuve, toujours le plus haut niveau, mais aussi la vérité des physiciens (ils le voient, on devrait peut-être vérifier...), la vérité expérimentale (je l'ai vérifié dans les cas suivants) et même, c'est vrai qu'en théorie ça ne marche pas, mais regarde ces belles images... Du moment que les étiquettes sont claires, ceci ne devrait pas poser de problème ; mais le flou s'installe toujours. J'ai été amusé pendant les vacances de constater, en lisant un livre sur l'histoire de la théorie quantique des champs, qu'une percée importante a été bloquée pendant quelques années parce qu'elle contredisait un « théorème ». Un théorème, bien sûr ; mais de quelle sorte ?

En tout cas, le paysage change, et je crois de façon saine. Comme d'habitude, un des moteurs principaux de ceci est l'arrivée de nouvelles personnes. On constate une dynamique différente chez nos jeunes collègues. Je trouve la perspective plutôt plaisante, bien que ce ne soit pas vraiment très ordonné.

## LETTER TO THE EDITOR

To the Editor:

Canadian mathematicians are justly proud of our connection with the Fields Medal, long considered to be the highest honour any mathematician can receive.

The September 2011 issue of the EMS Newsletter includes an interview with John Milnor, the winner of the 2011 Abel Prize, and as long ago as 1962, the Fields Medal. Near the end of this very interesting interview, asked what he thinks of receiving the Abel Prize, on top of all the other distinctions he has already received, Milnor replies in part that “it is surely the most important one”.

This comes as a shock, especially as the Abel Prize was created only in 2002. It causes one to wonder what might be done to defend the status of the Fields Medal. Certainly Milnor's views should be sought, as well as those of the individuals behind the creation of the Abel Prize. Canadian institutions such as the CMS and the Fields Institute should take the lead here.

**Peter Fillmore**

**Professor Emeritus**

**Dalhousie University**

# A Course on the Web Graph

by Anthony Bonato

AMS Graduate Studies in Mathematics, Vol. 89  
ISBN 973-0-8218-4467-0

Reviewed by  
Jeannette Janssen,  
Dalhousie University

## A mathematician's view of the new science of networks

**T**here is no doubt that the advent of the World Wide Web has made us look at information differently. Nowadays, we often find our facts in complex information networks, rather than in a hierarchically organized library. The new view on the networked nature of information even reaches beyond the Web, to data sets from biology, social science and ecology. Some researchers have started to speak of a "new science of networks".

For mathematicians, the study of networks is known as graph theory. It is part of the folklore of mathematics that graph theory originates with a question posed by the idle burghers of Königsberg to the great mathematician Euler: whether it was possible to traverse all seven bridges of the town on a Sunday stroll, without crossing any bridge twice. For a long time, graph theory remained associated with its playful origins, and was seen as a pursuit of mathematicians taking a break from their "serious" research.

The advent of the computer gave the impulse that propelled graph theory into the mathematical mainstream. Graphs form a natural environment for developing efficient algorithms, and for studying computationally intractable problems. Graph theory flourished; mathematical research is always invigorated when new applications are discovered. Canada's own Bill Tutte was instrumental in developing the field, and graph theory remains one of our mathematical strengths.

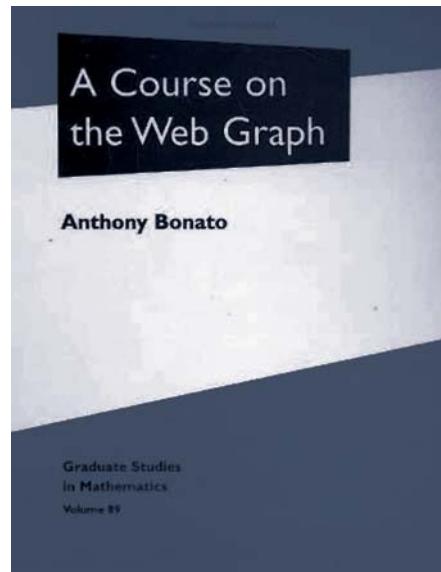
The idea of studying the graph formed by Web pages and hyperlinks cropped up after the creation of Google. The seismic shift in Web

search that led to Google's success was based on the idea that the Web graph can be used to rank search results. Researchers soon took notice that the structure of the Web graph could reveal unknown aspects of the Web. A decade ago, a research team from Altavista and one from Notre Dame University, first explored the properties of the Web graph. Graph theorists took note, and found a wealth of interesting research problems.

The book "A course on the Web graph" by Anthony Bonato gives an excellent introduction to various aspects of the Web graph and the mathematical techniques that can be applied to their exploration. In this field, the state of the art is a moving target, so this book does not pretend to give a complete overview. Rather, it chooses a number of major areas, and gives a clear exposition of each. The book is centered on the Web graph, but, as is discussed in Chapter 2, the techniques apply to a variety of complex networks.

*"... graph theory originates with a question posed by the idle burghers of Königsberg to the great mathematician Euler ..."*

The first major topic is stochastic graph models, treated in Chapter 4. Such models aim to explain the emergence of specific graph properties by proposing a mechanism which guides the formation of the graph. The chapter extensively explores the principle of preferential attachment: graphs grow over time, and vertices that already have high degree are more likely to receive more edges. This principle leads to the heavy tail degree distribution that is observed in real life, and is generally considered an essential ingredient of models for many types of networks. Some other models are also discussed, most notably a model where new vertices imperfectly copy the edges of an existing vertex. The analysis of these models involves advanced knowledge of probabilistic techniques such as martingales; these techniques are introduced in Chapter 3. The second topic is PageRank, the ranking function first used by Google. PageRank values are the limiting state probabilities of



a random walk on the Web graph (modified to guarantee convergence). Thus, the analysis of the PageRank algorithm requires the study of Markov Chains. A number of different ranking techniques, based on other graph-derived Markov chains, are also discussed in Chapter 5.

The last topic, the Infinite Web, gives a perspective on graph models that is a bit further from the mainstream. It represents one of the research interests of the author. A common tool in the study of time processes is to study the behaviour as time goes to infinity. For most graph models, the size of the graph increases over time, so in the limit this results in an infinite graph. The infinite graphs that occur as limits somehow represent the essential properties of the model.

This book was the result of a course taught by the author at a summer school of the Atlantic Association for Research in the Mathematical Sciences (AARMS). I have recommended the book to my graduate students as an introduction to research on graph models, and found that the students appreciate its expository style. The book contains extensive problem sets, making it a good teaching tool. The book is largely self-contained, and lends itself well for a "Topics" course in discrete mathematics. To my knowledge, this book is the best introduction for mathematics students to the exciting research that started with the study of the Web graph.

# Monoidal Functors, Species and Hopf Algebras

by Marcelo Aguiar and Swapneel Mahajan

CRM Monograph Series n. 29, American Mathematical Society, Providence, Rhode Island USA, 2010. ISBN: 978-0-8218-4776-3

Reviewed by Mitja Mastnak,  
Saint Mary's University

The nicest mathematics always happens as a result of synergy between different, often seemingly unrelated, areas of mathematics. This research monograph is a great case in point. It connects ideas from algebra, category theory, combinatorics, and geometry. The monograph contains an embarrassment of riches: it simplifies and unifies known theories, it introduces many new exciting results, and it will serve as a road-map for years of future research. The book is very well written, with a lot of detail

and examples, making it a good candidate for a textbook for a learning seminar or for an advanced graduate course.

Hopf algebras are ubiquitous algebraic structures that arise in many branches of mathematics. They first appeared in topology as cohomology of connected Lie groups (and later H-spaces) in the 1940's. Hopf algebras and quantum groups in particular, having supplanted the role of groups as carriers of symmetry, have now been playing a very prominent role in algebra as well as in physics for almost three decades. In combinatorics Hopf algebras encode assembly and disassembly of combinatorial structures. The monograph provides, among other things, a unified framework that allows us to construct and study Hopf algebras arising from all of the areas discussed above.

The text is organized in three parts. Part I reviews the background material from category theory. It focuses on monoidal and braided monoidal categories. It also examines higher monoidal categories. Special attention is dedicated to the notion of a bilax monoidal functor which plays a central role in this work. These are functors  $F$  between braided monoidal categories that are lax (that is, there

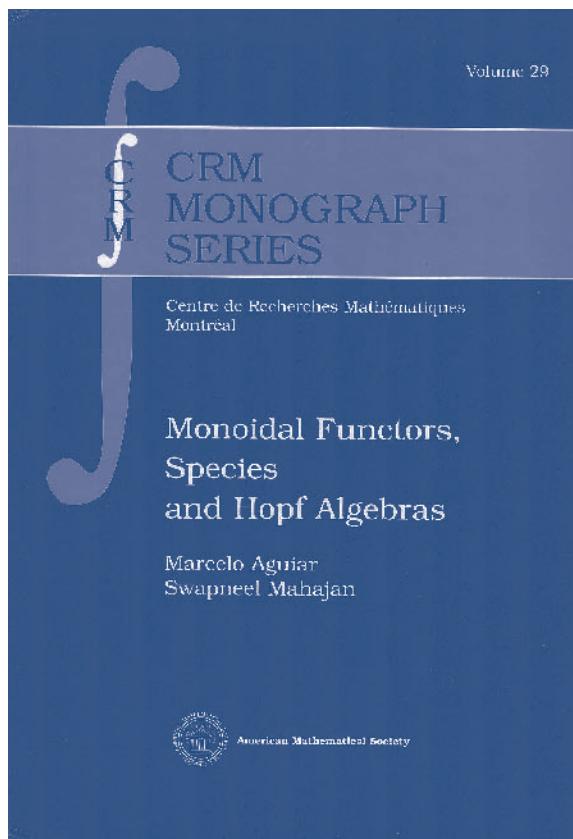
is a unital and associative, but not necessarily invertible, and not necessarily braided, natural transformation from  $F \otimes F$  to  $F(- \otimes -)$ ) and colax (there is a dual natural transformation from  $F(- \otimes -)$  to  $F \otimes F$ ) in such a way that the two natural transformations satisfy a braiding compatibility relation (roughly speaking, the two obvious natural transformations from  $F(-_1 \otimes -_2) \otimes F(-_3 \otimes -_4)$  to  $F(-_1 \otimes -_3)F(-_2 \otimes -_4)$  coincide). One example of such a functor is the chain complex functor, where the lax and colax structures are given by the Eilenberg-Zilber and the Alexander-Whitney maps. This example is somewhat special as its lax structure is braided. In general, there is a subtle, yet a very important distinction between bilax, braided lax, and

braided colax functors which disappears in the strong case where all of these notions coincide (as proven in the monograph).

**“The book of Aguiar and Mahajan is a quantum leap toward the mathematics of the future. I strongly recommend it to all researchers in algebra, topology, and combinatorics.”**

Part II is dedicated to the study of Hopf monoids in the Joyal's category of species. Roughly speaking these are vector spaces graded by sets. More precisely, a vector species is a functor from the category of finite sets with bijections to the category of vector spaces. The monoidal category of species is a very natural place for encoding combinatorial as well as geometric structures and various operations (such as, for example, join and break) on them.

Part III illustrates how ideas from Parts I and II give rise to a unified approach to Hopf algebras. The main step is the construction of Fock functors from species to graded vector spaces. These functors are bilax monoidal and transport Hopf monoids from the category of species to the category of graded vector spaces. This produces not only a multitude of Hopf algebras from combinatorics and geometry, but also accounts for quantum groups such as, for example, quantum linear spaces. It seems that discussion about the latter is cut rather short, but hopefully it will be investigated in more detail in some future work. If you are still not convinced, then look for more information in a very detailed and informative introduction to the monograph. Let me end the review with a quote from the foreword written by André Joyal: “The book of Aguiar and Mahajan is a quantum leap toward the mathematics of the future. I strongly recommend it to all researchers in algebra, topology, and combinatorics.”



**BRIEF BOOK REVIEWS**

by Renzo A. Piccinini, Dalhousie University

**FEARLESS SYMMETRY –  
Exposing the hidden  
patterns of numbers***by Avner Ash and Robert Gross, Princeton  
University Press, Princeton and Oxford 2006***ISBN-13 : 978-0-691-12492-6**

This is a book intended for readers who have no particular mathematical experience, but are interested in some basic problems of mathematics. No theorems are proved, but all the relevant definitions are given and explained in simple terms. The book starts with a description of what is a “representation” in Mathematics and touches on group theory, modular arithmetic, varieties, reciprocity laws, Galois theory, and ends with remarks about the proof of Fermat’s Last Theorem. This is a gigantic feat in view of the audience to which the book is intended. As Barry Mazur says in the Foreword, “recent successes[in mathematics] have advertised to a wide audience that math remains humanity’s grand work-in-progress...” The authors have shown this in an admirable way.

**NIST Handbook of  
Mathematical Functions***Edited by Frank W. Olver, Daniel W. Lozier,  
Ronald F. Boisvert and Charles W. Clark –  
NIST and Cambridge University Press, 2010***ISBN 978-0-521-19225-5 (Hardback)****ISBN 978-0-521-14063-8 (Paperback)**

Researchers in Pure and Applied Mathematics, the Physical Sciences, Engineering and elsewhere often encounter special functions as part of their work. To use these functions effectively it is necessary to have ready access to their properties. The National Institute of Standards and Technology (NIST) Handbook is an invaluable reference book for special functions and their properties. It is divided in 36 chapters written by an impressive array of internationally well known scientists, ranging from Bessel functions, Legendre and related functions, Gamma functions, Zeta functions, Bernoulli and Euler polynomials, etc., and onto Integrals with coalescing saddles.

**MODELS, LOGICS, AND  
HIGHER-DIMENSIONAL  
CATEGORIES, CRM***Proceedings and Lecture  
Notes vol. 53.**Edited by Bradd Hart, Thomas G. Kucera,  
Anand Pillay, Philip J. Scott and Robert  
A.G. Seely, American Mathematical  
Society 2001.***ISBN 978-0-8218-7281-9.**

The volume is a tribute to the work of Mihály Makkai; it contains the text of 17 talks presented at a meeting held at the Centre des Recherches Mathématiques at the Université de Montréal, on June 8-10, 2009, in Makkai’s honour. The meeting focused on the main themes of Mihály Makkai’s research career: model theory, categorical model theory and logics, and higher-dimensional category theory. The papers are preceded by an interesting and moving short paper by Thomas G. Kucera: “Mihály Makkai: A Biographical Note, with Reminiscences”.

**GEOMETRY, RIGIDITY,  
AND GROUP ACTIONS***Edited by Benson Farb and David Fisher.  
Chicago Lectures in Mathematics Series.  
The University of Chicago Press 2011***ISBN-13: 978-0-226-23788-6.**

From the Preface: “In September 2007, a conference was held at the University of Chicago in honor of the sixtieth birthday of Robert J. Zimmer. The conference was a testament to and a celebration of Zimmer’s continuing and lasting influence on the fields of geometry, rigidity, and group actions. The wide variety of papers submitted to this volume are just one indication of that influence. “ The volume contains survey and research papers; there are 19 papers in all, divided in four parts: 1) Group Actions on Manifolds; 2) Analytic, Ergodic, and Measurable Group Theory; 3) Geometric Group Theory, and 4) Group Actions on Representations Varieties.

**SPECTRUM AND DYNAMICS**  
Proceedings of the Workshop  
held in Montréal, QC, April 7 –  
11, 2008 at the CRM*Edited by Dmitry Jakobson, Stéphane  
Nonnenmacher and Josif Polterovich  
American Mathematical Society, 2010.***ISBN 978-0-8218-4778-7**

From the Introduction: “The goal of [the] meeting was to bring together experts working in two different and yet interrelated areas of mathematics: geometric spectral theory and dynamical systems.” The volume presents ten papers, to wit: “Notes on the minicourse Entropy of Chaotic Eigenstates”, by Stéphane Nonnenmacher; “Geometry of the High Energy Limit of Differential Operators of Vector Bundles”, by Alexander Strohmaier; “Classical and Quantum Dynamics in Transverse Geometry of Riemannian Foliations”, by Yuri A. Kordyukov; “Eigenvalue Variations and Semiclassical Concentration”, by Luc Hillairet; “ A Remainder Estimate for Weyl’s Law on Liouville Tori”, by Hughes Lapointe; “ Embedded Eigenvalues for Cartan-Hadamard Manifolds”, by Harold Donnelly; “On Minimal Partitions: New Properties and Applications to the Disk”, by Bernard Helffer and Thomas Hoffmann-Ostenhof; “Asymptotic Vertex Growth for Graphs”, by Mark Pollicott; “Nearest  $\lambda q$ -Multiple Fractions”, by Dieter Mayer and Tobias Mühlnerbruch; “Comparing Length Functions on Free Groups”, by Richard Sharp.

# Outer Functions and the Real World

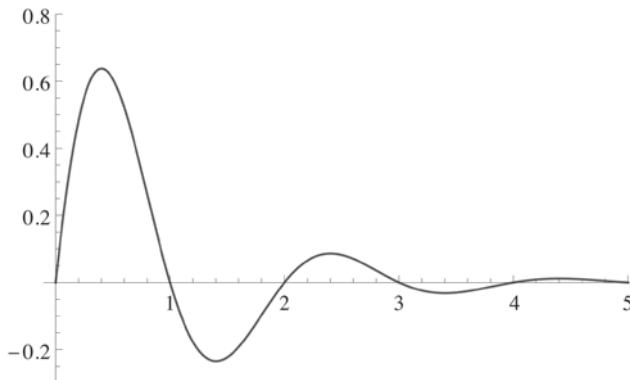
MICHAEL P. LAMOUREUX, DEPARTMENT OF MATHEMATICS AND STATISTICS, UNIVERSITY OF CALGARY

*Editor: Florin Diacu (University of Victoria)*

**I**t was a dark and stormy night. Suddenly, a shot rang out! A door slammed. The maid screamed.<sup>1</sup>

A mathematician observes that our ears perceive each of these sounds as a collection of sinusoids, with our cochlear hair cells responding to the amplitude of each of the individual frequencies that make up the sound. But our ears also respond to the direction in time of a sound. A gunshot, played as reversed in time, sounds very different to us. Instead of the initial burst of energy, we would first hear the whoosh and reverberations of the bullet, followed by a loud snap at the end.

Certain explosive sounds, like a gunshot or a door slam, begin with a burst of energy – in a concrete sense, the sound is front-loaded, with most of the energy near the beginning. This characterization can be made mathematically precise.



Outer functions have a similar front-loaded property. An analytic function  $F$  on the open unit disk  $\mathbb{D}$  in the complex plane has a series expansion  $F(z) = \sum_{n=0}^{\infty} a_n z^n$  for all  $z \in \mathbb{D}$ . The function uniquely determines the coefficients and vice-versa. The energy of the function is defined as  $\|F\|^2 = \sum_0^{\infty} |a_n|^2$ , which, when finite, places  $F$  in the Hardy-Hilbert space  $H^2(\mathbb{D})$ . For  $F \in H^2(\mathbb{D})$ , there is a natural extension to the closure of the disk whose values  $F(e^{i\theta})$  correspond to the Fourier transform of the sequence  $(a_0, a_1, a_2, \dots)$ , with entries recovered from the spectrum as  $a_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} F(e^{i\theta}) e^{-in\theta} d\theta$ . By Jensen's inequality, it follows that

$$(1) \quad \log |a_0| \leq \frac{1}{2\pi} \int_{-\pi}^{\pi} \log |F(e^{i\theta})| d\theta$$

Outer functions are the analytic functions satisfying (1) with exact equality. Of all functions in  $H^2(\mathbb{D})$  with a given amplitude spectrum  $A(\theta) = |F(e^{i\theta})|$ , the outer function is the one that maximizes energy in the first  $a_0$ . We can say that  $F \in H^2(\mathbb{D})$  is outer if

$$F(z) = \lambda \cdot \exp \left( \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{(e^{i\theta}) + z}{(e^{i\theta}) - z} k(\theta) d\theta \right)$$

for some real-valued integrable function  $k$ , and unit scalar  $\lambda$ . It follows that  $k(\theta)$  is the log spectrum of  $F$ ,  $k(\theta) = \log |F(e^{i\theta})|$ . Moreover, writing  $F(z) = \exp(u(z) + iv(z))$  for real harmonic functions  $u, v$ , the resulting boundary values  $u(e^{i\theta}), v(e^{i\theta})$  are Hilbert transform pairs. This is the well-known Kramer-Kronig relation in physics used to characterize minimum phase systems.

Four standard results follow from this formula: Given any function  $f \in H^2(\mathbb{D})$ ,

- (1) there is a unique (up to scalar) outer function  $F \in H^2(\mathbb{D})$  with the same amplitude spectrum,  $|f(e^{i\theta})| = |F(e^{i\theta})|$ , for all  $\theta$ ;
- (2) there is an inner function  $G \in H^2(\mathbb{D})$  yielding the factorization  $f(z) = F(z)G(z)$ ; for all  $z \in \mathbb{D}$ ;
- (3) the outer function dominates the original, with  $|f(z)| \leq |F(z)|$ , for all  $z \in \mathbb{D}$ ;
- (4) for  $f(z) = \sum b_n z^n$ ,  $F(z) = \sum a_n z^n$ , the outer function's coefficients also dominate, with

$$\sum_0^N |b_n|^2 \leq \sum_0^N |a_n|^2, \text{ for all } N$$

This is the mathematical insight into characterizing explosive signals. A sound, sampled in discrete time, is represented by a sequence of real numbers  $(a_0, a_1, a_2, \dots)$ . Its Fourier spectrum  $F(e^{i\theta}) = \sum a_n e^{in\theta}$  is given by the boundary values of  $F(z) = \sum a_n z^n$  on the unit disk. The sound is front-loaded if it has more energy in the initial coefficients,  $\sum_0^N |b_n|^2 \leq \sum_0^N |a_n|^2$ , for all  $N$ ; compared to any other sampled sound  $(b_0, b_1, b_2, \dots)$  with the same amplitude Fourier spectrum  $|f(e^{i\theta})| = |F(e^{i\theta})|$ . This front-loading is equivalent to stating that  $F$  is an outer function (see [5] for more details).

When a gunshot is heard from a distance, it may sound attenuated from traveling that distance, muffled from passing through a door, and with extra reverberations from echoing off a wall. But it never flips around backwards. Whatever happens to the signal, it remains front-loaded as it propagates through space. This is an experimental observation, which has been carefully measured in physical situations, such as recording the sound of a dynamite shot as it travels through the earth as part of a seismic exploration experiment.

<sup>1</sup> Snoopy's take on Bulwer-Lytton's classic line, in "Peanuts" by Charles M. Schulz (1965).

When modelling signal propagation, one asks what are the linear operators that map front-loaded signals to front-loaded signals? More generally, since a signal could start at arbitrary times, we seek linear operators that map front-loaded signals of the form

$(0, 0, 0, \dots, 0, a_n, a_{n+1}, \dots)$ , with  $a_n \neq 0$  maximized,

to other relatively front-loaded signals of the form

$(0, 0, 0, \dots, 0, b_m, b_{m+1}, \dots)$ , with  $b_m \neq 0$  maximized.

Recast as a question of linear operators on Hardy-Hilbert space, we obtain:

**Theorem 1.** (Thm 4 in [4]) *Let  $A : H^2(\mathbb{D}) \rightarrow H^2(\mathbb{D})$  be a bounded linear operator that preserves the set of shifted outer functions  $\{z^n F(z) \in H^2(\mathbb{D}) : F \text{ outer}, n \geq 0\}$ . Then there exists integers  $p, q \geq 0$  and outer functions  $\phi, \psi$ , with  $\|\phi\|_\infty \leq 1$ , such that*

$$(Af)(z) = z^q \psi(z) f(z^p \phi(z)), \text{ for all } f \in H^2(\mathbb{D}), \text{ all } z \in \mathbb{D}.$$

Thus, there is not a lot of freedom in choosing such an operator to model propagation of signals. The function  $\phi$  is physically the most interesting as it results in an exponentially decaying factor in the attenuation through the channel. This is experimentally observed as  $Q$ -attenuation in seismic signals, for instance, which results in a time and frequency dependent decay of the form  $e^{-\pi t\omega/Q(\omega)}$ , where  $t$  is the time in the transmission channel,  $\omega$  is the frequency parameter and  $Q = Q(\omega)$  is a frequency dependent attenuation parameter. The function  $\psi$  determines a stationary filter effect that is time independent, while the integers  $p, q$  relate to relative time delays and upsampling in the propagated signal.

For polynomials,  $f(z) = 1+2z$  is not outer, but  $F(z) = 2+z$  is outer, as it concentrates energy in the coefficient 2. Since  $|1 + 2e^{i\theta}| = |2 + e^{i\theta}|$  for all  $\theta \in [-\pi, \pi]$ ,  $f$  and  $F$  have the same amplitude spectrum. The “outerness” of any polynomial  $F(z) = \sum_{n=0}^N a_n z^n$  is characterized by the location of its roots, which must lie outside the open unit disk.

The celebrated Pólya-Schur problem (see [1], [2]) asks which linear operators on polynomials preserve  $\Omega$ -stability, i.e. preserve the set of polynomials with no zeros in some fixed region  $\Omega \subset \mathbb{C}$ . We thus obtain a general solution for Pólya-Schur:

**Theorem 2.** (Thm. 3 in [3]) *Suppose  $\Omega_1 \subset \mathbb{C}$  is bounded,  $\Omega_2 \subset \mathbb{C}$  has non-empty interior, and let  $P(\Omega_j)$  denote the set of polynomials in  $\mathbb{C}[z]$  that do not vanish on  $\Omega_j$ . A linear map  $A : \mathbb{C}[z] \rightarrow \mathbb{C}[z]$  has the property that  $A(P(\Omega_1)) \subset P(\Omega_2) \cup \{0\}$  if and only if either*

1. *there exists a linear functional  $v : \mathbb{C}[z] \rightarrow \mathbb{C}$  and a polynomial  $\psi \in P(\Omega_2)$ , such that  $A(f) = v(f)\psi$ , for all  $f \in \mathbb{C}[z]$ ; or*
2. *there exists  $\psi \in P(\Omega_2)$  and a non-constant polynomial  $\phi$  with  $\phi(P(\Omega_2)) \subset \Omega_1$ , such that  $A(f) = (f \circ \phi)\psi$ , for all  $f \in \mathbb{C}[z]$ .*

That is, in the non-trivial case, the  $\Omega$ -stability preserving linear map is just composition with polynomial  $\phi$  followed by multiplication with polynomial  $\psi$ . One can contrast this constructive solution to Pólya-Schur with the abstract characterization reported in [1], [2].

The work discussed here, including the Pólya-Schur solution, was born from an analysis of Gabor deconvolution in seismic imaging. The mathematics of Gabor deconvolution is described in [6], a non-stationary generalization of Wiener deconvolution used in geophysics to accurately model  $Q$ -attenuation of seismic wave propagation in the earth.

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# Cardio-Mathematics

JOHN W. CAIN\*

*Editor: Florin Diacu (University of Victoria)*

Modern toilets and cardiac cells have been famously likened to one another because of what they have in common: excitability. In the absence of electrical stimulus currents, the voltage  $v$  across a cell membrane equilibrates to an asymptotically stable resting potential  $v_{\text{rest}}$ , much as the toilet remains in a resting state if its handle is not pressed. If either system is stimulated too weakly (sub-threshold electrical current or pressing the handle too lightly), the system relaxes rapidly to its rest state. Super-threshold stimulation causes a dramatic response known as *action potential* for cardiac cells and *flushing* in the toilet. Both systems eventually return to rest, but require time to recover excitability – briefly pressing the toilet handle (no matter how hard) during a flush does not elicit a noticeable response.

Understanding connections between the cardiac action potential and actual heart rhythm is not too difficult. Models of the action potential in a single cell take the form

$$C_m \frac{dv}{dt} + F(v, w) + I_{\text{stim}}(t) = 0, \quad \frac{dw}{dt} = G(v, w), \quad (1)$$

where  $C_m$  is the capacitance of the cell membrane and  $F : \mathbb{R} \times \mathbb{R}^n \rightarrow \mathbb{R}$ ,  $(v, w) \rightarrow F(v, w)$ , represents a sum of the various currents which flow across the membrane. The vector  $w \in \mathbb{R}^n$  describes the cell membrane's (variable) permeability to inward or outward current flow as modeled by  $G : \mathbb{R} \times \mathbb{R}^n \rightarrow \mathbb{R}^n$ ,  $(v, w) \rightarrow G(v, w)$ . The term  $I_{\text{stim}}$  corresponds to an impulsive stimulus current supplied by the heart's native beating activity (more below). Choosing the functions  $F$  and  $G$  such that the system (1) is excitable and reproduces experimental data is an ongoing challenge. At the very least, there must exist an asymptotically stable rest state  $(v_{\text{rest}}, w^*)$  to which the cells would equilibrate, were it not for the occasional stimuli supplied by  $I_{\text{stim}}$ . Moreover, the equation  $F(v, w) = 0$  should implicitly define a manifold  $\Sigma$  in phase space forming an "excitability boundary" (see Figure 1).

The stimulus current can be thought of as a linear combination of  $\delta$ -distributions

$$I_{\text{stim}}(t) = \sum_j \mu_j \delta(t - t_j), \quad (2)$$

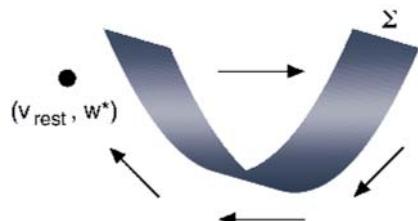


Figure 1: Schematic diagram of phase space for a model of an excitable medium.

where  $\mu_j$  is a stimulus strength (measured in microamps, no variation from beat-to-beat) and  $t_j$  is the arrival time of the  $j$ th stimulus (i.e. the beginning of the  $j$ th heartbeat). Substituting (2) into (1) results in a system of impulsively forced differential equations. For information on impulsive DEs in a similar context see, e.g., [1]. The relationship between dynamics of (1) and the inter-beat intervals  $\Delta t_j = t_j - t_{j-1}$  is of critical importance.

The stability of heart rhythm (or lack thereof) can be extracted from the impulsive DE model of the action potential. Let  $v = v_{\text{thr}}$  denote a voltage above which we regard tissue as being "excited." The hyperplane  $v = v_{\text{thr}}$  partitions  $(v, w)$  phase space into two regions, and a solution trajectory of the DEs crosses  $v = v_{\text{thr}}$  twice during a typical beat: once as the cell is becoming excited ( $\frac{dv}{dt} > 0$ ) and once as the cell is recovering excitability ( $\frac{dv}{dt} < 0$ ). The difference between these two crossing times measures the action potential duration (APD), and we shall denote the APD of the  $j$ th beat by  $A_j$ . Using the plane  $v = v_{\text{thr}}$  much as we would use a Poincaré section to define a first return map, there exists a mapping  $A_{j+1} = \phi(A_j, \Delta t_j)$  which recursively determines the sequence of APDs. This recurrence (known as the *restitution mapping*) and its higher-dimensional generalizations, can be used to study heart rhythm. In particular, if the stimulus times  $t_i$  are evenly spaced with period  $\tau$ , the mapping  $A_{j+1} = \phi(A_j, \tau)$  can suffer period-doubling bifurcations as (the heart rate) is varied, leading to abnormal oscillations in APD.

Research at the interface of mathematics and cardiology can be doubly rewarding in the sense that problems of high clinical importance tend to spawn questions of independent mathematical interest, such as:

- How do the statistical properties of the sequence of stimulus times  $t_i$  affect the dynamics of the DEs or the mapping  $\phi$ ? Which, if any, cardiac abnormalities can be predicted or diagnosed by analysis of these sequences?
- What is the "best" way for a medical device to intervene by perturbing the sequence  $t_i$  in such a way that normal rhythm is preserved/recovered?
- Beyond the cardiac context, can we prove new results about the dynamics of impulsively-forced excitable systems?

The article [2] surveys these and other research problems in "mathematical cardiology," and the textbook [3] is an excellent starting point for mathematicians who are interested in applications to physiology.

**Acknowledgments.** Support of the National Institutes of Health under grant T15 HL088517-02 is gratefully acknowledged.

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- [1] A. Cattia, D.G. Schaeffer, T.P. Witelski, E.E. Monson, and A.L. Lin, On Spiking Models for Synaptic Activity and Impulsively Forced Differential Equations, SIAM Review 50, 3 (2008), 553-569.
- [2] J.W. Cain, Taking Math to Heart: Mathematical Challenges in Cardiac Electrophysiology, Notices Amer. Math. Soc. 58, 4 (2011), 542-549.
- [3] J.P. Keener and J. Sneyd, Mathematical Physiology, Springer-Verlag, New York, 1998.

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## NSERC-CMS Math in Moscow Scholarships

The Natural Sciences and Engineering Research Council (NSERC) and the Canadian Mathematical Society (CMS) support scholarships at \$9,000 each. Canadian students registered in a mathematics or computer science program are eligible.

The scholarships are to attend a semester at the small elite Moscow Independent University.

Math in Moscow Program  
[www.mccme.ru/mathinmoscow](http://www.mccme.ru/mathinmoscow)

### APPLICATION DETAILS

[www.cms.math.ca/Scholarships/Moscow](http://www.cms.math.ca/Scholarships/Moscow)

**Deadline March 31, 2012 to attend the Fall 2012 semester.**

## Bourse CRSNG-SMC Math à Moscou

Le Conseil de Recherches en Sciences Naturelles et en Génie du Canada (CRSNG) et la Société mathématique du Canada (SMC) offrent des bourses de 9,000 \$ chacune. Les étudiantes ou étudiants du Canada inscrit(e)s à un programme de mathématiques ou d'informatique sont éligibles.

Les bourses servent à financer un trimestre d'études à la petite université d'élite Moscow Independent University.

Programme Math à Moscou  
[www.mccme.ru/mathinmoscow](http://www.mccme.ru/mathinmoscow)

### DÉTAILS DE SOUMISSION

[www.smc.math.ca/Bourses/Moscou](http://www.smc.math.ca/Bourses/Moscou)

**Date limite le 31 mars 2012 pour le trimestre d'automne 2012.**



## 2012 CMS Summer Meeting

June 2-4, 2012, Regina Inn and Ramada Hotel (Regina, SK)  
Host: University of Regina

## Réunion d'été SMC 2012

2-4 juin, hotels Regina Inn et Ramada (Regina, SK)  
Hôte : Université de Regina

### Prizes | Prix

- Krieger-Nelson Prize | Prix Krieger-Nelson  
to be announced | à être annoncé
- Coxeter-James Prize | Prix Coxeter-James  
to be announced | à être annoncé
- Excellence in Teaching Award  
Prix d'excellence en enseignement  
to be announced | à être annoncé
- Adrien Pouliot Award 2011 Prix Adrien-Pouliot  
Malgorzata Dubiel (SFU)

### Plenary Speakers | Conférences plénierées

- Dror Bar-Natan (Toronto)
- Lisa Jeffrey (Toronto)
- Marius Junge (Illinois-Urbana Champaign)
- Ulrike Tillmann (Oxford)

### Scientific Directors | Directeurs scientifiques

- Doug Farenick - [douglas.farenick@uregina.ca](mailto:douglas.farenick@uregina.ca)  
T: 306-585-4425
- Don Stanley - [stanley@math.uregina.ca](mailto:stanley@math.uregina.ca)  
T: 306-585-4343

### Public Lecture | Conférence publique

John Fyfe (Canadian Centre for Climate Modelling  
and Analysis; Victoria)

Friday   Vendredi June 1 juin	Saturday   Samedi June 2 juin	Sunday   Dimanche June 3 juin	Monday   lundi June 4 juin
	8:00 – 16:00 – Registration   Inscription 9:30 – 16:00 - Exhibits   Expositions 9:30 - 16:00- Poster Session d'affiches		8:00 – 14:00 Registration   Inscription
	8:15 – 8:30 <b>Opening   Ouverture</b>	8:00 – 10:00 <b>Scientific Sessions</b> Session scientifiques	8:00 – 10:00 <b>Scientific Sessions</b> Session scientifiques
	8:30 – 9:20 <b>Plenary Lecture</b> Conférence plenaire		
	9:30 – 10:00 Break   Pause	10:00 – 10:30 Break   Pause	10:00 – 10:30 Break   Pause
	10:00 – 11:30 <b>Scientific Sessions</b> Session scientifiques	10:30 – 11:15 <b>Plenary Lecture</b> Conférence plenaire	10:30 – 11:20 <b>Plenary Lecture</b> Conférence plenaire
11:00 AM – 13:00 CMS Development Group (Regina Inn, Wascana B)	11:30 – 12:20 <b>Excellence in Teaching A.</b>	11:30 – 12:20 <b>Krieger-Nelson Prize Lecture</b>	11:30 – 12:20 <b>Coxeter-James Prize Lecture</b>
13:00 – 17:00 CMS Board of Directors Meeting (Regina Inn, Wascana B)	12:30 – 14:00 - Break <b>CMS Annual General Meeting</b>	12:30 – 14:00 Break   Pause	12:30 – 14:00 Break   Pause
	14:00 – 15:00 <b>Scientific Sessions</b> Session scientifiques	14:00 – 15:00 <b>Scientific Sessions</b> Session scientifiques	14:00 – 16:00 <b>Scientific Sessions</b> Session scientifiques
	15:00 – 15:50 <b>Plenary Lecture</b> Conférence plenaire	15:00 – 15:50 <b>Plenary Lecture</b> Conférence plenaire	
18:00-19:00 John Fyfe Public Lecture Conférence publique	16:00 – 16:15 Break   Pause	16:00 – 16:15 Break   Pause	
	16:15 – 17:45 <b>Scientific Sessions</b> Session scientifiques	16:15 – 17:45 <b>Scientific Sessions</b> Session scientifiques	
19:00-20:30 Welcome Reception Réception de bienvenue	18:00-19:30 Malgorzata Dubiel 2011 Prix Adrien Pouliot Award		18:30 - 19:15 Reception   Réception 19:15 – 22:00 Banquet
	20:00 – 22:00 Student Social (TBD)		updated   mise à jour January 23 janvier

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## Réunion d'été SMC 2012

2-4 juin, hotels Regina Inn et Ramada (Regina, SK)  
Hôte : Université de Regina

**Sessions****Cluster Algebras and Related Topics****Algèbres amassées et sujets reliés**

Org: Ralf Schiffler (Connecticut), Hugh Thomas (UNB)

**Combinatorics | Combinatoire**

Org: Karen Meagher (Regina), Marni Mishna (SFU)

**Complex Geometry and Related Fields****Géométrie complexe et domaines reliés**

Org: Tatyana Barron (Western), Eric Schippers (Manitoba)

**Computation of Analytical Operators in Applied and Industrial Mathematics | Calcul des opérateurs analytiques en mathématiques appliquées et industrielles**

Org: Peter Gibson (York), Michael Lamoureux (Calgary)

**Connections in Mathematics Education****Connexions dans l'enseignement des mathématiques**

Org: Roberta La Haye (Mount Royal), Patrick Maidorn (Regina), Kathy Nolan (Regina)

**Free Probability Theory: New Developments and Applications | Théorie des probabilités libres: applications et développements récents**

Org: Serban Belinschi (Saskatchewan), Benoît Collins (Ottawa)

**Geometric Topology | Topologie géométrique**

Org: Steve Boyer (UQAM), Ryan Budney (Victoria), Dale Rolfsen (UBC)

**Geometry and Topology of Lie Transformation Groups | Géométrie et topologie des groupes de transformation de Lie**

Org: Lisa Jeffrey (Toronto), Liviu Mare (Regina)

**Harmonic Analysis and Operator Spaces****Analyse harmonique et espaces d'opérateurs**

Org: Yemon Choi (Saskatchewan), Ebrahim Samei (Saskatchewan)

**Homotopy Theory | Théorie de l'homotopie**

Org: Kristine Bauer (Calgary), Marcy Robertson (Western)

**Interactions Between Algebraic Geometry and Commutative Algebra | Intérations entre la géométrie algébrique et l'algèbre commutative**

Org: Susan Cooper (Central Michigan), Sean Sather-Wagstaff (North Dakota State)

**Number Theory | Théorie des nombres**

Org: Mark Bauer (Calgary), Richard McIntosh (Regina), Eric Roettger (Mount Royal)

**Operator Algebras | Algèbres des opérateurs**

Org: Martin Argerami (Regina), Juliana Erljman (Regina), Remus Floricel (Regina)

**Perspectives in Mathematical Physics****Perspectives en physique mathématique**

Org: Yvan Saint-Aubin (Montréal), Luc Vinet (Montréal)

**Representation Theory of Groups, Lie Algebras, and Hopf Algebras | Théorie de représentation des groupes, des algèbres de Lie et de Hopf**

Org: Allen Herman (Regina), Fernando Szechtman (Regina)

**Total Positivity | Positivité totale**

Org: Shaun Fallat (Regina), Michael Gekhtman (Notre Dame)

**Contributed Papers | Communications libres**

Org: Edward Doolittle (First Nations University), Fotini Labropulu (Regina)

**AARMS-CMS Student Poster Session****Présentations par affiches pour étudiants**

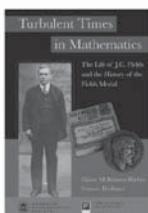
Org: Bahman Ahmadi (Regina), Ruhi Ahmadi (Regina), Yong Liu (Regina)

**Workshops, May 29 - June 1, University of Regina**  
**Ateliers, 29 mai - 1 juin, Université de Regina (en anglais)**

Workshop on Algebra and Geometry

Workshop on Groups, Lie Algebras, and Hopf Algebras

## § TITLES FROM THE AMS §



### TURBULENT TIMES IN MATHEMATICS THE LIFE OF J.C. FIELDS AND THE HISTORY OF THE FIELDS MEDAL

Elaine McKinnon Richm and Frances Hoffman

*Drawing on a wide array of archival sources, Richm and Hoffman provide a vivid account of Fields' life and his part in the founding of the highest award in mathematics. Filled with intriguing detail—from a childhood on the shores of Lake Ontario, through the mathematics seminars of late 19th century Berlin, to the post-WWI years of the fragmented international mathematical community—it is a richly textured story engagingly and sympathetically told. Read this book and you will understand why Fields never wanted the medal to bear his name and yet why, quite rightly, it does.*

—June Barrow-Green, Open University, Milton Keynes, United Kingdom

J. C. Fields, the foremost Canadian mathematician of his time, was educated in Canada, the United States, and Germany, and championed an international spirit of cooperation to further the frontiers of mathematics. It was during the awkward post-war period that J. C. Fields established the Fields Medal, an international prize for outstanding research, which soon became the highest award in mathematics. J. C. Fields intended it to be an international medal, and a glance at the varying backgrounds of the fifty-two Fields medallists shows it to be so.

Who was Fields? What carried him from Hamilton, Canada West, where he was born in 1863, into the middle of this turbulent era of international scientific politics? A modest mathematician, he was an unassuming man. This biography outlines Fields' life and times and the difficult circumstances in which he created the Fields Medal. It is the first such published study.

A co-publication of the AMS and Fields Institute.

2011; approximately 256 pages; Softcover; ISBN: 978-0-8218-6914-7; List US\$45; AMS members US\$36; Order code MBK/80

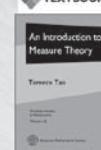
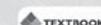


### THE GAME OF COPS AND ROBBERS ON GRAPHS

Anthony Bonato, Ryerson University, Toronto, ON, Canada, and Richard J. Nowakowski, Dalhousie University, Halifax, NS, Canada

This book is the first and only one of its kind on the topic of Cops and Robbers games, and more generally, on the field of vertex pursuit games on graphs. The reader will gain insight into all the main directions of research in the field and will be exposed to a number of open problems.

**Student Mathematical Library**, Volume 61; 2011; 276 pages; Softcover; ISBN: 978-0-8218-5347-4; List US\$45; AMS members US\$36; Order code STML/61



### AN INTRODUCTION TO MEASURE THEORY

Terence Tao, University of California, Los Angeles, CA

This is a graduate text introducing the fundamentals of measure theory and integration theory, which is the foundation of modern real analysis. The material is intended to cover a quarter or semester's worth of material for a first graduate course in real analysis.

There is an emphasis in the text on tying together the abstract and the concrete sides of the subject, using the latter to illustrate and motivate the former. There are a large number of exercises throughout that develop key aspects of the theory, and are thus an integral component of the text.

**Graduate Studies in Mathematics**, Volume 126; 2011; 206 pages; Hardcover; ISBN: 978-0-8218-6919-2; List US\$53; AMS members US\$42.40; Order code GSM/126



### RENORMALIZATION AND EFFECTIVE FIELD THEORY

Kevin Costello, Northwestern University, Evanston, IL

This book makes perturbative quantum field theory accessible to mathematicians with no prior experience in the subject. The author's insightful use of the renormalization group and effective field theory could help to launch a wider use of the topics. The book also assists physicists in learning the powerful methodology of mathematical structure.

**Mathematical Surveys and Monographs**, Volume 170; 2011; 251 pages; Hardcover; ISBN: 978-0-8218-5288-0; List US\$84; AMS members US\$67.20; Order code SURV/170

### Also of interest

#### INTRODUCTION TO ORTHOGONAL, SYMPLECTIC AND UNITARY REPRESENTATIONS OF FINITE GROUPS

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#### ALGEBRAIC CURVES AND CRYPTOGRAPHY

V. Kumar Murty

Fields Institute Communications, Volume 58; 2010; 133 pages; Hardcover; ISBN: 978-0-8218-4311-6; List US\$79; AMS members US\$63.20; Order code FIC/58

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CRM Proceedings & Lecture Notes, Volume 52; 2010; 207 pages; Softcover; ISBN: 978-0-8218-4778-7; List US\$99; AMS members US\$79.20; Order code CRMP/52

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