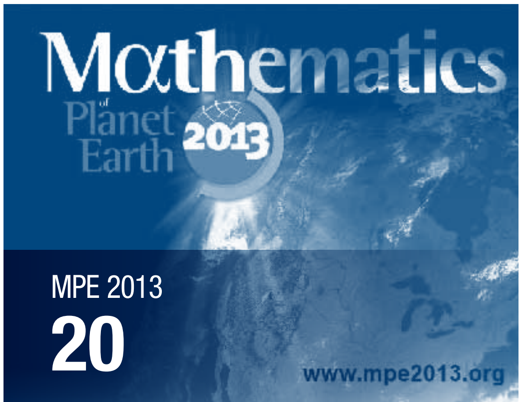

CMS NOTES de la SMC

Oct./Nov. 2012

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Canadian Mathematical Society
Société mathématique du Canada

From the Executive Director's Desk

Johan Rudnick, *CMS Executive Director*

The CMS as a Learning Society

Traditionally, learned societies promoted a field of academic pursuit by creating a formal community of members, by publishing scholarly journals, and by staging meetings. Today, most societies have learned from the communities they serve and have significantly evolved beyond their traditional roles. Most societies, including the CMS, have become multi-faceted service providers that engage all elements of their constituencies across a broad and diverse spectrum of interests. And it's both the services and the engagements that define the nature and character of the society. In my travels across Canada and at CMS meetings, I am always amazed that not everyone knows about or truly appreciates the breadth of what the CMS does on their behalf. Today, the CMS actually has learned to do quite a few things and for many reasons.

The CMS Represents Canadian Mathematics

As the national society, the CMS represents the Canadian mathematical community to various international and domestic bodies. On the international stage, the CMS represents Canadian mathematics at the ICIAM, the IMO, and working with the NRC, at the IMU. Domestically, the CMS represents the broad mathematical community to more specialized mathematical sciences. The CMS also represents the community to reciprocal societies, including the AMS and the MAA.

The CMS Showcases Canadian Mathematics

While mathematics may well be an international science, the Canadian community brings a unique perspective. The Canadian perspective is nurtured in the CMS journals by the CJM and CMB editor-in-chiefs as well as at CRUX. Furthermore, when the IMU holds its congress, the CMS stages a Canadian reception where the representatives from Fields, PIMS and CRM can host their international colleagues and highlight Canadian interests.

The CMS Celebrates Excellence

The CMS has a number of initiatives to recognize and to celebrate the excellence of the Canadian mathematical community. The CMS awards and prizes recognize excellence in such areas as best research article, best thesis, best teaching, and best poster presentation. The CMS also honours those who made a significant contribution to mathematics and the community. And, of course, each CMS math competition recognizes student excellence.

The CMS Advocates for Mathematics

Amid all the community angst and concern for the state of Discovery Grants, the CMS quietly held bilateral meetings with NSERC to explain the concerns from researchers and the unfortunate impact that NSERC changes were having on their research abilities. The CMS has also provided semi-annual forums where members can engage NSERC directly, including a presentation by the NSERC president. the CMS advocacy is not limited to pure research; the CMS, working with the SSC, has successfully advocated preserving educational outreach materials from Statistics Canada. Advocacy also encompasses students and our own community. The

FRANÇAIS PAGE 3

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Mathematics and Poetry

Robert Dawson, *Saint Mary's University*

Those of us who know mathematics understand the poetry of our subject. It is often an obscure, abstruse poetry, hard to explain to the uninitiated; but it is undeniably there. Astronomers may point out that, among the sciences, only their discipline had its own Muse, Urania; but geometry was traditionally the "second portfolio" of Erato, the Muse of erotic poetry. And when the ancient Greeks said "geometry", they basically meant "mathematics"; remember that Euclid's algorithm for greatest common factors was considered as geometry, and appeared in the *Elements* (VII.2).

John Donne (1572-1631) was undoubted aware of this link when he wrote "*A Valediction: forbidding Mourning.*" In this well-known poem, in which he takes leave of his love for a protracted absence, he not only manages to use the geometer's compasses as an extended metaphor for constancy at a distance ("Thy firmness makes my circle just / And makes me end, where I begun.") but manages to work in a phallic symbolism for the instrument as well!

Donne was not the first or the last to find material for poetry in mathematics. Sarah Glaz and JoAnne Growney point out in the introduction to their wonderful anthology *Strange Attractors: Poems of Love and Mathematics* (A W Peters, 2008) that

> the earliest poet who is known by name is also the earliest mathematician who is known by name: Enheduanna, chief priestess of the moon god Nanna in the city of Ur… (*op. cit.*, xi)

One of Enheduanna's surviving poems, a hymn to the grain goddess Nisaba, describes a woman performing astronomical and geodetic measurements.

Mathematics, of course, has often lurked in the structure of poetry through rhyme schemes and scansion patterns. Some verse forms are even more mathematical; the sestina, for instance, permutes six line-ending words by repeated applications of the permutation (1 2 3 4 5 6) -> (6 1 5 2 4 3). This particular permutation not only generates a maximal cyclic subgroup of S_6 but does as good a job as possible of avoiding repeated adjacencies. Poets have experimented with generalizations of the sestina form to other verse lengths.

There is poetry in mathematics; and mathematics in poetry.

Les mathématiques et la poésie

Robert Dawson, *Saint Mary's University*

Ceux et celles d'entre nous qui connaissons les mathématiques comprenons bien la poésie de notre sujet. Il s'agit bien souvent d'une poésie obscure et complexe, difficile à expliquer aux profanes; mais nul ne peut nier qu'elle existe. Les astronomes peuvent noter que, parmi toutes les sciences, seule leur discipline comptait sa propre muse, Uranie; mais la géométrie était traditionnellement le « deuxième portefeuille » d'Erato, la muse de la poésie érotique. Et quand les Grecs de l'Antiquité disaient « géométrie », ils parlaient en gros des « mathématiques ». Il ne faut pas oublier que l'algorithme d'Euclide pour les plus grands communs diviseurs était considéré comme un élément de « géométrie » et a figuré dans les *Éléments* (VII.2).

John Donne (1572-1631) était sans aucun doute conscient de ce lien lorsqu'il a écrit *Valédiction* : *des larmes*. Dans ce poème bien connu, où il prend congé de sa bien-aimée pour une absence prolongée, non seulement réussit-il à se servir du compas du géomètre comme métaphore étendue pour représenter la constance à distance (en anglais : « Thy firmness makes my circle just / And makes me end, where I begun »), mais il réussit aussi à associer un symbolisme phallique a' l'instrument!

Donne n'était pas le premier et ne sera pas le dernier à trouver de l'inspiration pour la poésie dans les mathématiques. Sarah Glaz et JoAnne Growney font remarquer dans l'introduction à leur merveilleuse anthologie, *Strange Attractors: Poems of Love and Mathematics* (A W Peters, 2008) que

> le plus ancien poète qu'on connaît de nom est aussi le plus ancien mathématicien qu'on connaît de nom : Enheduanna, grande prêtresse du dieu de la lune Nanna dans la ville d'Ur…(*op. cit.*, xi)

Un des poèmes d'Enheduanna qui a survécu, une ode à la déesse des céréales Nisaba, décrit une femme qui prend des mesures astronomiques et géodésiques.

Les mathématiques, bien entendu, ont bien souvent marqué la structure de la poésie grâce à des moyens rythmés et à des motifs de scansion. Certaines structures de strophes sont davantage axées sur les mathématiques : la sextine, par exemple, permute six mots de fin de ligne grâce à des applications répétées de la permutation (1 2 3 4 5 6) -> (6 1 5 2 4 3). Cette permutation particulière n'a pas seulement pour effet de créer un sous-groupe cyclique maximal de S_6, mais évite le mieux possible les contiguïtés répétées. Des poètes ont tenté des expériences avec des généralisations de la forme sextine à d'autres longueurs de strophes.

Voilà la poésie en mathématiques; et les mathématiques en poésie.

Du Bureau du directeur executive

Johan Rudnick, *Directeur exécutif de la SMC*

La SMC comme société d'apprentissage

Les sociétés savantes ont toujours fait la promotion d'un domaine d'intérêt pédagogique donné en créant une collectivité officielle de membres, en publiant des revues savantes et en organisant des réunions. Aujourd'hui, la plupart des sociétés ont tiré des leçons des collectivités à qui elles offrent leurs services et ont beaucoup évolué comparativement à leur rôle traditionnel. La plupart des sociétés, y compris la SMC, se sont transformées en des fournisseurs de services à facettes multiples faisant participer tous les éléments de la communauté et tenant compte du large éventail d'intérêts fort variés. Ce sont donc les services et les engagements qui définissent la nature et le caractère de la société. Au cours de mes voyages à travers le Canada et dans les réunions de la SMC, je suis toujours très surpris de constater que des gens ne connaissent pas ou n'apprécient pas à sa juste valeur l'ampleur des efforts que la SMC déploie pour eux. Aujourd'hui, la SMC fait bien des choses et les fait pour de nombreuses raisons.

La SMC représente les mathématiques au Canada

Comme société nationale, la SMC représente la communauté mathématique du Canada auprès de divers organismes internationaux et nationaux. Sur la scène internationale, la SMC représente les mathématiques canadiennes auprès de l'ICIAM et de l'OIM et collabore avec le NRC, à l'IMU. Au pays, la SMC représente la communauté mathématique générale par rapport à des domaines de sciences mathématiques plus spécialisés. La SMC représente également la communauté auprès de sociétés réciproques, y compris l'AMS et la MAA.

La SMC fait connaître les mathématiques canadiennes

Bien que les mathématiques soient un domaine scientifique international, la communauté canadienne y apporte une perspective unique. La perspective canadienne est cultivée dans des revues spécialisées de la SMC par les rédacteurs en chef du *Journal canadien de mathématiques* (JCM) et du *Bulletin canadien de mathématiques* (BCM) et dans CRUX. De plus, lorsque l'IMU organise son congrès, la SMC offre une réception canadienne où des représentants de Fields, de l'IPSM et du CRM peuvent accueillir leurs collègues internationaux et souligner les intérêts canadiens.

La SMC célèbre l'excellence

La SMC compte un certain nombre de projets qui visent à souligner et à célébrer l'excellence de la communauté mathématique du Canada. Les prix de la SMC soulignent l'excellence, notamment le meilleur article de recherche, la meilleure thèse, le meilleur enseignement et la meilleure présentation d'affiche. La SMC rend hommage aussi à ceux et celles qui apportent une grande contribution aux mathématiques et à la communauté. Et, bien entendu, chacun des concours de mathématiques de la SMC souligne l'excellence chez les étudiants.

Suite à la page 4

Letters to the Editors

Lettres aux Rédacteurs

The Editors of the NOTES welcome letters in English or French on any subject of mathematical interest but reserve the right to condense them. Those accepted for publication will appear in the language of submission. Readers may reach us at **notes-letters@cms.math.ca** or at the Executive Office.

Les rédacteurs des NOTES acceptent les lettres en français ou anglais portant sur un sujet d'intérêt mathématique, mais ils se réservent le droit de les comprimer. Les lettres acceptées paraîtront dans la langue soumise. Les lecteurs peuvent nous joindre au bureau administratif de la SMC ou à l'addresse suivante : **notes-lettres@smc.math.ca**.

NOTES DE LA SMC

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Les Notes de la SMC, les rédacteurs et la SMC ne peuvent être tenus responsables des opinions exprimées par les auteurs.

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La SMC à la défense des mathématiques

Dans le tourbillon qui jetait la tourmente au sein de la communauté et avivait les inquiétudes quant à l'état des bourses à la découverte, la SMC a tenu discrètement des réunions bilatérales avec le CRSNG dans le but d'expliquer les préoccupations des chercheurs et les effets malencontreux des changements apportés par le CRSNG sur leur capacité de recherche. La SMC a également offert des forums semi-annuels où les membres peuvent s'adresser directement au CRSNG, y compris une présentation par le président du CRSNG. Les efforts de promotion de la SMC ne se limitent pas à la recherche pure; la SMC, en collaboration avec la SSC, a réussi à faire conserver du matériel d'information pédagogique provenant de Statistique Canada. Les travaux de promotion s'étendent également aux étudiants et à notre propre communauté. Les efforts de promotion du comité étudiant de la SMC favorisent l'intérêt et l'engagement des étudiants. Les camps de mathématiques spécialisés de la SMC favorisent la compréhension et l'intérêt dans les communautés mal desservies. Le comité des femmes en mathématiques de la SMC a élaboré une formule pour un séminaire d'été national de mathématiques particulier pour les femmes, projet qui s'est concrétisé par la suite en Ontario.

La SMC favorise la recherche

Les revues savantes de la SMC jouissent d'une réputation enviable et comptent des lecteurs internationaux fidèles et diversifiés. Les réunions de recherche semi-annuelles de la SMC offrent aux chercheurs du Canada et de l'étranger l'occasion de participer directement aux projets actuels. La SMC appuie directement ou par l'intermédiaire du comité étudiant des ateliers de recherche étudiants ou des colloques tels que le Séminaire de mathématiques supérieures.

La SMC organise des rencontres scientifiques.

Elle organise et anime des réunions semi-annuelles et conjointes avec d'autres sociétés telles que la SCHPM et, sur la scène internationale, avec des sociétés de France et du Mexique. Outre les programmes de recherche et d'éducation, les réunions de la SMC sont l'endroit où l'on peut avoir des discussions en groupe sur des thèmes donnés, participer à des ateliers ou assister à des présentations spéciales et apprendre quels sont les des ouvrages qu'offrent actuellement les maisons d'édition.

La SMC appuie la communauté

La SMC met son infrastructure à la disposition d'autres sociétés. Elle a contribué à la publication des Comptes rendus mathématiques de la SRC, a aidé à animer une réunion de CanaDAM et à animer un concours Kangaroo et a tenu lieu de bureau pour la SSC. La SMC donne aussi son appuie à d'autres en contribuant financièrement à des concours provinciaux et régionaux et en versant des fonds à des particuliers qui s'occupent de projets qui seront bénéfiques pour la communauté.

La SMC fait participer les membres

La SMC constitue le forum national des membres des communautés canadiennes diversifiées des sciences mathématiques. L'adhésion à la SMC suscite la participation des particuliers, de même que des universités et d'autres sociétés. Bien que les membres de la SMC s'intéressent principalement à la recherche pure et aux intérêts pédagogiques, les perspectives d'adhésion de nouveaux membres s'améliorent pour l'ensemble des sciences mathématiques et à tous les niveaux de scolarité.

La SMC favorise l'apprentissage

Outre les ateliers organisés au cours des réunions semi-annuelles, la SMC anime un programme d'éducation en parallèle avec le programme de recherche. La SMC organise périodiquement un forum national sur l'enseignement des mathématiques. Outre les camps et les concours et la publication de CRUX, la SMC ajoute régulièrement des titres à la liste des ouvrages ATOM.

La SMC travaille à l'échelle internationale

À part représenter le Canada à l'étranger et à l'IMU, la SMC a collaboré avec quatre autres sociétés pour lancer le premier Congrès des mathématiques dans les Amériques en 2013. Elle organise, forme et encadre aussi l'Équipe mathématiques du Canada pour qu'elle participe à l'OIM. La SMC entretient des relations avec de nombreuses sociétés du monde entier. Et, pour élargir la perspective étudiante, elle SMC gère le programme Mathématiques à Moscou. Qui plus est, la SMC, au besoin, compte un agent des droits de la personne qui est chargé de défendre la liberté en matière d'études.

La SMC sensibilise le public

La SMC organise une conférence publique à chacune des réunions semi-annuelles. En collaboration avec un certain nombre d'universités, la SMC organisera une série de conférences à travers le Canada dans le but de souligner le projet international des Mathématiques de la planète Terre 2013 et d'en faire la promotion. La SMC collabore avec divers médias afin de sensibiliser les gens et de mieux faire comprendre l'importance des mathématiques. Cette collaboration améliorera la présentation de comptes rendus au public.

La SMC forme des partenariats avec d'autres

La SMC fournit un cadre de partenariat national afin de partager des pratiques exemplaires et de développer des projets collectivement. Elle compte des réseaux de partenariat pour les camps mathématiques et des concours. La SMC collabore avec des universités à travers le Canada afin d'organiser les conférences pour les Mathématiques de la planète Terre 2013. De plus, la SMC collabore avec des organisations pour entreprendre des travaux qui font avancer les intérêts communs. La SMC a eu la chance d'obtenir l'appui financier du gouvernement fédéral, de pratiquement tous les gouvernements provinciaux et territoriaux, de même que de l'ensemble de nos merveilleux instituts.

Notre nature savante

La nature et les caractéristiques de la SMC sont fonction de ses programmes et de ses projets, qui sont inspirés et régis par les membres de la SMC et les membres de la communauté des mathématiques du Canada. La SMC a appris de ses membres, de ses partenaires, de ses partisans et de tous ceux et celles avec qui elle entretient des liens – elle est bien plus qu'une simple organisation de membres chargée de publications et organisatrice de réunions. Grâce au soutien d'une communauté engagée, la SMC continuera d'apprendre quels sont les moyens de mieux bâtir l'avenir et de devenir une voix nationale plus importante pour les mathématiques au Canada.

OCTOBER 2012

24-26 International Conference on Number Theory and Applications (Bangkok, Thailand)
http://maths.sci.ku.ac.th/icna2012

NOVEMBER 2012

17 Info-Metrics & Nonparametric Inference (UC Riverside, CA)
www.america.edu/cas/economics/info-metrics/orkshop-2012-november.cfm

16-18 Special Functions, PDEs and Harmonic Analysis, Conference in honor of C. P.Calderon (Chicago, IL)
http://www.roosevelt.edu/calderon

19-20 Seminar on History of Mathematics etc (Delhi, India)
http://www.indianshm.com

DECEMBER 2012

7 Canadian Launch of Mathematics of Planet Earth 2013
Lancement canadien de Mathématiques de la planète Terre 2013
www.mpe2013.org

8-10 CMS Winter Meeting Fairmont Queen Elizabeth, Montreal, Quebec Scientific program: CRM
www.cms.math.ca/winter12

10-14 Reproducibility in Computational & Experimental Maths. (Providence, RI)
http://icerm.brown.edu/tw12-5-rcem

17-21 International Conference on Theory, Methods & Applications of Nonlinear Equations, (Kingsville, TX)
www.tamuk.edu/artsci/math/conference_2012/html

19-22 Conference on Commutative Rings, Interger-valued Polynomials, etc (Graz, Austria)
www/integer-valued.org/conf2012

JANUARY 2013

31-Jan 11 Recent Advances in Operator Theory & Algebras (Bangalore, India)
http://www.isibang.ac.in/~jay/rota.html

1-9 Workshop & Conference on Limit Theorems in Probability (Bangalore, India)
http://www.isibang.ac.in/.html

4-8 iCERM Workshop on Whittaker Fns, Schubert Calculus & Crystals (Providence, RI)
http://icerm.brown.edu/sp-s13-w2

20-22 Discrete Geometry for Computer Imagery (Seville, Spain)
http://dgci2013.us.es/

FEBRUARY 2013

11-15 Sage Days: Multiple Dirichlet Series, Combinatorics, And Representation Theory (Providence, RI)
http://www.icerm.brown.edu/sp-s13-wl

MARCH 2013

4-8 Whittaker Functions, Schubert Calculus & Crystals (Providence, RI)
http://www.icerm.brown.edu/sp-s13-w2

APRIL 2013

15-19 Multiple Dirichlet Series, Combinatorics, And Analytic Number Theory (Providence, RI)
http://www.icerm.brown.edu/sp-s13-w3

JUNE 2013

16-23 51st International Symposium on Functional Equations (Rzeszów, Poland)
tabor@univ.rzeszow.pl

JULY 2013

1-5 Erdös Centennial (Budapest, Hungary)
http://www.renyi.hu/conferences/erdos100/index.html

CMS student committee outreach promotes student interest and engagement. Specialty the CMS math camps promote understanding and interest within underserved communities. The CMS Women in Mathematics committee developed the notion of a special national math summer seminar for women that was subsequently staged in Ontario.

The CMS Promotes Research

The CMS scholarly journals have established a solid reputation and have garnered a diverse international following. The CMS semi-annual research meetings provide researchers from across Canada and abroad with an opportunity to be engaged directly with current initiatives. The CMS directly or through the student committee also supports student research workshops or colloquia, such as the Séminaire de Mathématiques Supérieures.

The CMS Hosts Scientific Meetings

The CMS organizes and stages semi-annual as well as joint meetings with other societies, such as CSHPM, and internationally with societies from France and Mexico. In addition to research and education programs, The CMS meetings provide topical panel discussions, workshops, special presentations, and exposure to current publisher offerings.

The CMS Supports Community

The CMS makes its infrastructure available to other societies. The CMS has helped publish the RSC Math Reports, helped stage a CanaDAM meeting, helped run a Kangaroo competition, and operate as the office for the SCC. The CMS also supports others by providing funding to provincial and regional competitions and to individuals for projects benefiting the community.

The CMS Engages Members

The CMS provides the national forum for membership across the broad and diverse Canadian communities of mathematical sciences. The CMS membership engages individuals as well as universities and other societies. And while the CMS membership has largely focussed on pure research and education interests, the membership outlook is expanding across the full spectrum of mathematical sciences and across education levels.

The CMS Promotes Learning

In addition to workshops at semi-annual meetings, the CMS holds an education program in parallel to the research program. The CMS periodically stages a national education forum. In addition to the camps and competitions and publishing CRUX, the CMS regularly adds to the growing list of ATOM books.

The CMS Works Internationally

In addition to representing Canada abroad and at the IMU, the CMS has worked with four other societies to launch the inaugural Mathematical Congress of Americas in 2013. The CMS also stages, trains, and coaches Math Team Canada to compete at the IMO. The CMS maintains relationships with numerous societies from around the world. And, to broaden the student perspective, the CMS operates the Math in Moscow program. Furthermore, as needed, the CMS has a Human Rights Officer position to defend academic freedom.

The CMS Raises Public Awareness

The CMS stages a public lecture at each semi-annual meeting. Working with a number of universities, the CMS will be staging a series of lectures across Canada to highlight and raise awareness about the international Mathematics of Planet Earth 2013 (MPE2013) initiative. The CMS is working with various media outlets to raise awareness and understanding of the importance of mathematics and to improve reporting to the public.

The CMS Partners With Others

The CMS provides a national partnership framework to share best practices and collectively develop initiatives. The CMS has partnership networks for math camps as well as competitions. The CMS is working with universities across Canada to stage the MPE2013 lectures. And, the CMS works in partnership with organizations to undertake work to advance common interests. The CMS has been fortunate to receive support from the federal government, almost all provincial governments and territories, and from every one of our wonderful institutes.

Our Learned Nature

The nature and character of the CMS is shaped by its programs and initiatives which draw their creation and direction from CMS members and Canadian mathematical constituencies. The CMS has learned from its members, partners, supporters, and everyone it engages – it has expanded well beyond simple membership, publications, and meetings. With the support of an engaged community, the CMS will continue learning how best to help build the future and become a stronger national voice for Canadian mathematics.

PIMS

Correction Notice

In the article titled "**Canadian Undergraduate Mathematics Conference 2012**" which appeared in the September 2012 issue of CMS Notes (Volume 44 No. 4), the Pacific Institute for the Mathematical Sciences was omitted as an event sponsor.

Modern approaches to the invariant-subspace problem

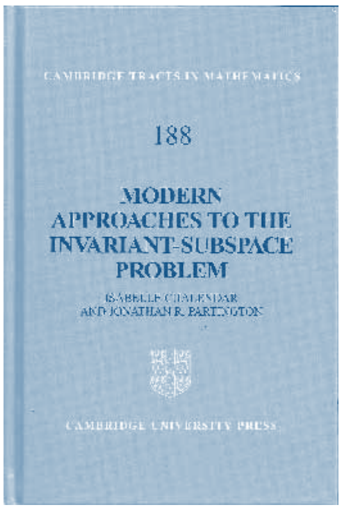
by Isabelle Chalendar and Jonathan R. Partington.

Cambridge Tracts in Mathematics

Cambridge University Press, 2011.

Reviewed by ***Heydar Radjavi****,*
The University of Waterloo, Waterloo ON, Canada.

ISBN 978-1-107-01051-2.



The well-known Invariant-Subspace problem asks the question: If T is a continuous linear operator from a complex Banach space X into itself, does X have a nontrivial closed subspace Y which is taken into itself by T? "Nontrivial" means that Y is different from $\{0\}$ and X. If X is "small enough," i.e., finite-dimensional, or "large enough," i.e., nonseparable, then the answer is an easy yes; otherwise, the problem is still unsettled for the general T on the general X. Negative answers are known only for some Banach spaces. But no operator without an invariant subspace in known on a reflexive space–even on a Hillbert space, the simplest and "most symmetric" of them all.

There have been valiant attempts by many to solve the problem at least for particular spaces or particular operators. For example, normal operators on a Hilbert space and compact operators on any Banach space are known to have plenty of invariant subspaces. An interesting recent development gave an affirmative answer to the problem on a certain X by solving another problem: Argyros and Haydon constructed a new Banach space X on which every operator is a compact perturbation of a scalar multiple of the identity operator, and thus has an invariant subspace.

The present volume presents the necessary background in the brief first chapter. The functional-analytic, measure-theoretic, and operator-theoretic prerequisites are all given here, mostly without proof, of course, but with appropriate references. Among the results listed here are Stone-Weierstrass theorem, Banach-Steinhaus theorem, Riesz representation thorems, Radon-Nikodym, Riesz-Dunford functional calculus, Beurling and Wiener theorems, and Carlson's interpolation theorem. These preliminaries out of the way, the authors start dealing in depth with a wealth of topics, every one of them eventually leading to the existence of invariant subspaces for various classes of operators. By the end of Chapter 3, for example, they have given a complete proof of the existence of invariant subspaces for subnormal operators. A subnormal operator is, by definition, the restriction of a normal operator N to an invariant subspace of N. Although normal operators are known to have an abundance of invariant subspaces, it is a fact of life in infinite dimensions that a restriction of a normal operator to an invariant subspace is not necessarily normal. So the proof for subnormal operators took a major effort.

Polynomially bounded operators and Bishop operators are discussed in Chapters 4 and 5, respectively. Certain subclasses of these two classes have been proven to have invariant subspaces. All the proofs are given in full detail. In the latter class, by the way, lie certain prime candidates for experiments in the hope of negative results. Applications of fixed-point theorems are discussed in Chapter 6, leading to such results as the existence of an invariant subspace for a compact perturbation of a self-adjoint operator–this time on a real Hilbert space; the answer in not yet known in the complex case.

Universal operators are discussed in Chapter 8. These are operators whose very existence may surprise the uninitiated: A single operator U on a Hilbert space H, say, such that every operator on H is essentially a "part" of U. More precisely put, we are asking every operator T to be similar to a scalar multiple of a restriction of U to one of the invariant subspaces of U. Such universal "models" exist. To solve the invariant subspace problem for Hilbert spaces, then, it suffices to concentrate on such a single model U and ask whether every infinite-dimensional invariant subspace of it properly contains another one. Tempting, isn't it? But no luck so far.

The other chapters include presentations of minimal vectors, moments sequences, and positive operators. Let me mention one more affirmative result from Chapter 9, which follows from theorems on moment sequences. The result is more general, but I'll mention a special case of it: assume an operator T on the Hilbert space of square-summable sequences can be represented by a tridiagonal matrix; that is, all the entries of the matrix of T are zero except on the main diagonal and the two diagonals adjacent to it. If the adjacent diagonals have nonnegative entries, then T has a nontrivial invariant subspace.

The authors have managed to gather an impressive volume of topics in this volume of under 300 pages. The book can be enthusiastically recommended for graduate-level topics courses or learning seminars. There is enough material here for at least two semesters

The physical appearance of the volume is pleasant. There are very few typographical errors. Just to prove that I have gone through the book, I mention what I found:

Page 32, line 14 : the second "K" should be in lower case.

36, -9 : "equation" should be "inequality".

101, -8 : "Chapter 3" should be "Chapter 4".

213, 7 : "invariant" should be inserted before "subspaces".

Elliptic curves and big Galois representations

by Daniel Delbourgo

London Mathematical Society Lecture Notes Series

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Reviewed by ***Ram Murty****, Queen's University, Kingston ON, Canada.*

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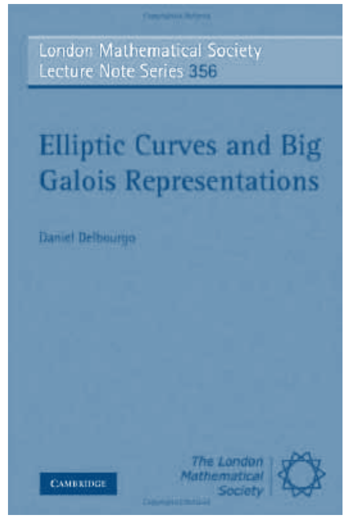

Perhaps one of the prettiest results of mathematics is the elegant formula

$$1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\cdots=\frac{\pi^2}{6},$$

proved by Euler in 1735. Later, Euler extended his theorem by giving special values of the Riemann zeta function at positive even arguments:

$$\zeta(2k) := \sum_{n=1}^{\infty}\frac{1}{n^{2k}} = -\frac{B_{2k}(2\pi i)^{2k}}{2(2k)!},$$

where B_n denotes the n-th Bernoulli number. By the functional equation for the Riemann zeta function, this is equivalent to the simpler formula,

$$\zeta(1-k) = -\frac{B_k}{k}.$$

In the middle of the 19th century, Kummer made the important connection between Bernoulli numbers and orders of ideal class groups of the ring of integers of cyclotomic fields. In addition, he proved a fundamental congruence, now called Kummer's congruence, that if p is a prime, and $m \equiv \mathrm{n} \neq 0$ (mod $(p-1)$) with m, n even, then

$$\frac{B_m}{m} \equiv \frac{B_n}{n} \pmod{p}.$$

In other words, $\zeta(1-n) \equiv \zeta(1-m)$ (mod p). In all of these investigations, Kummer was motivated by Fermat's last theorem and was attempting to prove it. In this endeavour, he was partially successful. Precisely, he showed that for an odd prime p, the equation $x^p + y^p = z^p$ has no non-trivial integer solutions if p does not divide the numerator of any of the Bernoulli numbers B_{2j} for $j = 1, 2, \ldots, (p-3)/2$. Such primes are called regular primes and it is still unknown if there are infinitely many such primes. Yet, Kummer's result was the most important contribution to Fermat's last theorem before Wiles's proof in 1996 that approached the question by a totally different method. However, in spite of falling short of settling Fermat's last theorem, Kummer's work opened the way for a new theme connecting special values of zeta and L-functions to the arithmetic and algebraic structure of number fields and varieties and this theme has been a dominant one in number theory ever since. That is, special values of analytic functions like the Riemann zeta function and more generally, Dirichlet L-functions, or the Hasse-Weil L-functions, are related to arithmetic objects such as orders of ideal class groups, Selmer groups and more generally Chow groups. The book under review focuses on one aspect of this universal theme, namely that of L-functions attached to elliptic curves and "big" Galois representations and relating them to orders of Selmer groups.

In retrospect, one can view the work of Wiles as an extension of Kummer's work in that Wiles also makes use of special values of zeta functions attached to the symmetric squares of elliptic curves. But even though Wiles made fundamental advances in the field, there are still many open questions involving special values of L-series attached to elliptic curves such as the Birch and Swinnerton-Dyer conjecture, which is listed as one of the "Million-Dollar" Millenium Problems by the Clay Mathematics Institute. A fundamental tool in these investigations has been Iwasawa theory alluded to below.

In 1964, Kubota and Leopoldt re-interpreted Kummer's congruence for the Bernoulli numbers leading to the discovery of a new object: the p-adic analogue of the Riemann zeta function. They also constructed p-adic analogues of Dirichlet's L-functions. Later, Iwasawa formulated a conjecture that these p-adic zeta and L-functions are the characteristic polynomial of the Frobenius automorphism attached to the prime p, acting on a module built out of ideal class groups of the tower of cyclotomic fields $\mathbb{Q}(\zeta_{p^n})$ for varying n. This so-called Iwasawa main conjecture was proved by Mazur and Wiles in 1984. This was later extended by Wiles with $\mathbb{Q}$ replaced by a totally real field and by Rubin, for imaginary quadratic fields. Thus emerged a general theory, now called Iwasawa theory, to study the structure of ideal class groups of a tower of number fields. In subsequent decades, efforts were made to extend this theory to the realm of elliptic curves.

An elliptic curve E over a number field F is an equation of the form $y^2 = f(x)$ with $f(x)$ a cubic with coefficients in F. For each prime ideal v, one can count the number of solutions, N_v (say) of the equation $y^2 = f(x)$ (mod v). Including the "point at infinity", we can write $\mathrm{N}_v = \mathrm{N}(v) + 1 - a_v$ where $N(v)$ is the norm of the ideal v and a_v satisfies Hasse's inequality $|a_v| \le 2\sqrt{N(v)}$. The Hasse-Weil L-function attached to E is (essentially)

$$L(E,s) = \prod_v \left(1 - \frac{a_v}{N(v)^s} + \frac{1}{N(v)^{2s-1}}\right)^{-1}.$$

Because of Hasse's inequality, this function converges absolutely for $\Re(s) > 3/2$ and thus defines an analytic function in this region. It is conjectured that $L(E, s)$ extends to an entire function and satisfies a functional equation relating s to $2 - s$ in analogy with the Riemann zeta function. When $F = \mathbb{Q}$, this is a theorem of Wiles (in the semistable case) and Breuil, Conrad, Diamond and Taylor (in the general case). In particular, one can speak of the special value $L(E, 1)$ and this is the focus of attention for the Birch and Swinnerton-Dyer conjecture. Indeed, by a celebrated theorem of Mordell and Weil, the set of points (x, y) with $x, y \in F$ and satisfying $y^2 = f(x)$ (together with the "point at infinity") has the structure of a finitely generated abelian group. The rank of this group, denoted r, is called the rank of the elliptic curve. The Birch and Swinnerton-Dyer conjecture is that $L(E, s)$ has a zero of order r at $s = 1$ and

$$\lim_{s\to 1} (s-1)^{-r} L(E, s) = \frac{hR\Omega r}{w},$$

where h is the order of a certain group called the Tate-Shafarevich group $Ш_E$ (conjectured to be finite and analogous to the ideal class group of algebraic number fields), R is the discriminant of the "height pairing," Ω is the transcendental period of E (analogous to π in the case of the Riemann zeta function), τ is the "Tamagawa factor" and w is the square of the order of the torsion part of $E(F)$. This should be viewed as the elliptic analogue of the class number formula in algebraic number theory.

The analogue of Iwasawa theory for elliptic curves was initiated by Mazur and the corrresponding "main conjecture" was formulated by him and is now essentially proved. This allows us to formulate a p-adic analogue of the Birch and Swinnerton-Dyer conjecture.

In the meantime, the Birch and Swinnerton-Dyer conjecture has been substantially generalized by Bloch and Kato. Given a cuspidal modular form of weight k with Fourier expansion

$$f_k(z) = \sum_{n=1}^{\infty} a_n e^{2\pi i n z},$$

one can associate (with Hecke) an L-series

$$L(f_k, s) = \sum_{n=1}^{\infty} \frac{a_n}{n^8}.$$

By the Ramanujan-Petersson conjecture (proved by P. Deligne in 1974), this L-series converges for $\Re(s) > k/2+1$. However, as Hecke showed in the 1930's, the L-function admits an analytic continuation to the entire complex plane and satisfies a functional equation relating s to $k/2-s$. If s_0 is an integer with $1 \le s_0 \le k-1$, then Bloch and Kato predict that the value of $L(f_k, s_0)$ is related to the order of a group $Ш_k$ which they define cohomologically. In the special case $k = 2$, this group is the Tate-Shafarevich group. Beilinson and then Kato showed that these critical values are governed by certain cohomology classes, now called Kato-Beilinson zeta elements.

The book under review, consisting of ten chapters and three appendices, is aimed at graduate students and researchers interested in learning the basic material to enter into the sanctum sanctorum of this mysterious and profound area of research. It begins with an informative introduction giving the basic background of the theory of elliptic curves, with copious references to the relevant literature. The second chapter reviews the p-adic L-functions and in particular Kato's theory linking K-theory to the construction of p-adic L-functions analogous to the role played by Euler systems in earlier versions of the theory. In subsequent chapters, the author generalizes the method of Beilinson and Kato's Euler system to a two-variable version. In this context, he reviews the relevant theory of Hida in Chapter 4. The remaining chapters are highly technical with Chapter 5 developing a two-variable interpolation theory for modular symbols and Chapter 6 leading to a two-variable Iwasawa theory for modular forms. The remaining chapters are devoted to exploring the consequences of this work. In particular, in Chapter 10, the author evaluates certain special values as well as formulations of the "main conjecture" in this two-variable setting. There is also a listing of some examples and open problems at the end of this chapter.

Finally, there are three appendices proving several technical results, with the last appendix being written by Paul Smith. In summary, this is a research monograph and targeted for graduate students engaged in this vibrant research area that is at the forefront of recent developments. It promises to be a valuable resource for many working in the field.

PIMS-Central relocates to state-of-the-art facility

The UBC headquarters for the Pacific Institute for the Mathematical Sciences (PIMS) has moved to a new state-of-the-art, centrally located facility on the 4th floor of the brand new Earth Sciences Building (ESB) at UBC.

This impressive building is centrally located at UBC's Point Grey Campus and PIMS' new space includes a reception area, offices for administration, offices for up to 24 visitors and postdoctoral fellows and lab workspace for ten graduate students. It also boasts a dedicated videoconferencing room and a spacious lounge and an array of classrooms throughout the building are available for PIMS seminars and workshops.

CMS Notes
Volume 44 No. 5, October / November 2012

Jennifer Hyndman, *University of Northern British Columbia*
John Grant McLoughlin, *University of New Brunswick*

The mandate of the Pacific Institute for the Mathematical Sciences (PIMS) "is to promote excellent research and applications of the mathematical sciences, to facilitate the training of highly qualified personnel, to enrich public awareness of and education in mathematics and to create partnerships with similar organizations around the world."

PIMS is deeply involved in educational initiatives. Recently some recognition came through CMS as its BC education coordinator, Melania Alvarez, was named the 2012 recipient of the Adrien Pouliot Award. There is much more about the activities and ideas of PIMS on the website ***www.pims.math.ca***

Here the references to PIMS are being offered as background to this issue's feature. Malgorzata Dubiel (Simon Fraser University) shared an unpublished article with people involved in the BC math community. We thought it appropriate to provide an edited and shortened version for the readership here.

The article was in fact a discussion paper that Malgorzata Dubiel prepared in response to a request from PIMS. Its purpose was to be sent to the BC Ministry of Education as representation of the positions and concerns of PIMS with respect to issues in mathematics education including curriculum, resources, and directions. It is likely that the many players in the field of mathematics ranging from teachers to students, ministry officials to mathematicians, and others would have varied responses to the actual content of the piece. It should be pointed out that the complete article contains more BC and/or PIMS references, along with considerably greater length. An effort has been made by the editors to honour the spirit of Malgorzata's paper while adapting it for the Education Notes. Feedback on this article or other contributions can be sent directly to ***johngm@unb.ca*** *or* ***jennifer.hyndman@unbc.ca****.*

PIMS and Mathematics Education in BC

Adapted from a paper written by **Malgorzata Dubiel**, *Simon Fraser University*

PIMS has a direct impact on mathematical training and education at the post-secondary level: undergraduate, graduate and post-graduate. However, it is at the elementary and secondary level that the attitudes towards mathematics and science are formed, as well as confidence in one's ability to do mathematics. Students' attitudes may be changed in later grades, but the chances for a positive change diminish every year. Therefore, if we want to improve math and science literacy and have more students prepared for math, science and technology careers, we have to focus attention on our elementary, middle and high schools.

It follows that if we want to ensure that our schools prepare students for future challenges, we need to allow those experts with deep mathematical knowledge and experience to participate in mapping the directions for our schools. The universities and PIMS are heavily involved in activities promoting math and science. But math and science education within the school system are the key to improving the big picture. Those who are turned off math and sciences in schools will likely ignore the promotional activities of PIMS and the universities.

PIMS and the BC universities are interested in working with the school system on improving this situation, if given an opportunity to do so. While some educators may be reluctant to consider cooperation with mathematicians because of the echo of Californian "Math Wars", this is not the Canadian way. There have been examples of successful cooperation between mathematicians and ministries of education in Canada. Most notably, in 1997-98 the Ontario Ministry of Education invited institutions to submit proposals when revising the high school curriculum in mathematics. Subsequently, a proposal from the Fields Institute for the Mathematical Sciences (a sister institute to PIMS) was accepted, and the mathematicians from Fields wrote the curriculum papers. This was the beginning of Fields' involvement in educational issues.

1. School curriculum.

The School curriculum should prepare students for their future studies, careers and roles as productive and knowledgeable members of the society. This includes - for many - proper preparation for post secondary studies. What we need to realize here is that earlier grades teach – or ought to teach – concepts that are essential to understanding mathematics at university level, and that building strong foundations in elementary grades is at least as important as the last two years of high school. Indeed, a sound understanding of the mathematical concepts that should be presented in elementary school is necessary if students are to be successful in high school mathematics.

Curriculum documents are important resources for teachers and therefore need to be written in a way that helps them to understand what and how to teach, and how what they teach fits into a "big picture" of mathematics. The documents need to be written in a way that helps teachers whose math and science preparation is often inadequate and who may lack knowledge, time, or resources to fill in the gaps and correct errors. We don't want students to learn concepts that they will need to "unlearn" later, or not learn concepts that are needed for understanding mathematics at higher levels.

New curricula have improved methodologies and approaches to teaching math and sciences. However, even the best methodologies will not make up for insufficient attention paid to subject content, consistency and continuity throughout the documents, or to incorrect definitions and terminology.

While the recent BC mathematics curriculum is an improvement over the previous ones, there are still problems. For example: removal of mathematical algorithms from elementary school will affect those who need to understand the nature and use of algorithms in computer science and computer programming; also, not teaching the long division algorithm makes it difficult for students to understand how to divide polynomials in precalculus and calculus. The algorithms, which have been used for hundreds of years, were removed supposedly because children had difficulties in understanding them. But – was the problem with the algorithms, or with the way they were taught? (For comments on this point – which is not unique to BC - see: **www.aft.org/pdfs/americaneducator/fall2009/wu.pdf**)

Interestingly, the new curriculum requires the use of manipulatives, often as a replacement for algorithms. While manipulatives can help students understand new concepts, their use in the new curriculum (and the textbooks based on this curriculum) is misguided, for several reasons. First, they are often introduced in a rigid and prescriptive way (case in point: tokens that are used to define multiplication and division of integers), devoid of intuition, and hence, force students to learn the algorithm given in the book on how to use these manipulatives. Second, manipulatives are treated as something one is required to master and then to demonstrate that mastery on provincial exams, rather than being used simply as an aid in understanding certain concepts and techniques. (e.g., algebra tiles, used to teach factoring of quadratic expressions).

Many problems with the curriculum arise not from its content, but from the way the curriculum documents are written. The BC curriculum is based on the WNCP Common Curriculum Framework; however, WNCP documents have been rewritten for BC use. Unfortunately the BC edition is definitely not an improvement on the original, in spite of providing six pages of references. While the Prescribed Learning Outcomes part is reasonable (though the language is, in places, ambiguous and confusing), the Suggested Achievement Indicators, which "translate" the curriculum for the teachers, are written in a way that suggest a lack of understanding of the relative importance of facts and concepts, of differences between definitions and conclusions, and what details need to be included in teaching materials used by students. Sometimes, there is no clear connection between listed learning outcomes and corresponding achievement indicators. There are places where the language is not mathematically correct. More seriously, the document fails to provide a sense that there are important ideas in mathematics, and remains focused on the skills.

When learning mathematics, students need to explore mathematical ideas within different contexts in order to increase their capacity for abstraction and understanding. The number of concepts that are being taught per grade was decreased in response to the criticism that the curriculum was a mile long and an inch deep; in many other countries students learn fewer concepts per grade but learn them more deeply. However the connections between concepts within the curriculum have not been sufficiently developed. So a critical aspect of learning mathematics has been neglected.

A few examples (*quoted sections in italics*) are provided as snapshots from the Achievement Indicators in BC K - 7 Curriculum – for details see **http://www.bced.gov.bc.ca/irp/pdfs/mathematics/2007mathk7.pdf**:

Grade 3, PLI C3: one of the Achievement Indicators:
Show that 100 centimetres is equivalent to 1 metre by using concrete materials.

Which materials? A ruler divided into centimetres?

Grade 4, PLI A8: one of the Achievement Indicators
Provide examples of when two identical fractions may not represent the same quantity (e.g., half of a large apple is not equivalent to half of a small apple; half of ten cloudberries is not equivalent to half of sixteen cloudberries).

Note that this statement confuses the concept of fraction as a number, and as part of a quantity. You cannot say "two identical fractions" when you mean fractions of different quantities. Language like this confuses students and teachers, and contributes to students' difficulties with understanding fractions.

Grade 5, PLI A5: one of the Achievement Indicators
Describe a solution procedure for determining the product of two given 2-digit factors using a pictorial representation, such as an area model.

The model is for understanding the meaning, not a "solution procedure"!

Grade 6, PLI C5: one of the Achievement Indicators:
Demonstrate that the sides of a regular polygon are of the same length and that the angles of a regular polygon are of the same measure.
Why would you demonstrate a definition?

Grade 7, PLI A4: one of the Achievement Indicators:
Provide an example where the decimal representation of a fraction is an approximation of its exact value.
This is incorrect as stated; "the" decimal representation is an approximation. This is an example where a good idea is worded incorrectly, becoming confusing and/or wrong.

While one could argue that most examples given above are not serious enough to be concerned, they are of concern for two reasons: first, many teachers lack knowledge and/or confidence to realize what the problem is, and, second, they can lead to serious problems in textbooks based on those documents. A case in point is that the second example (above) "inspired" considerable incorrect and/or misguided writing in a Grade 4 textbook in the leading series, *Math Makes Sense*.

Note that many of those problems could have been corrected easily, given the opportunity, and many mathematicians would be happy to assist with this. Subject specialists (mathematicians and scientists from BC post secondary institutions) have not been involved sufficiently in discussions about, or design of, the school curriculum in mathematics. Discussions have been limited, restricted to grades 11 and 12 only using a poorly designed and cumbersome questionnaire based on the previous curriculum. With the so-called "Precalculus Stream", designed for students who intend to study sciences, computing science or engineering, an attempt emerged to focus narrowly on skills needed for calculus. Hence, we have created a narrow path from arithmetic through algebra towards calculus, ignoring many aspects of geometry – which, ironically, are essential to understanding calculus. And, students without sufficient 3D intuition and geometrical knowledge have serious difficulties with multivariable calculus. Again, this is not just a BC problem, as described in the editorial **"Endless Algebra - the Deadly Pathway from High School to College",** by the NCTM President J. Michalel Shaughnessy – see: **http://www.nctm.org/about/content.aspx?id=28195**

2. School Textbooks.

Even with the use of web based and other electronic resources increasing rapidly in recent years, school textbooks (and accompanying teacher resources prepared by textbook publishers) serve as the key resource for teachers. School textbooks are based on the curriculum. They are written by practicing teachers, many lacking sufficient mathematical knowledge and understanding of the "big picture" to ensure the quality and correctness of mathematical concepts and terms, as well as a proper development and continuity throughout K–12. These books often magnify curriculum problems and add several new ones. This is very clear when we look through the most popular K–9 textbook series in BC, *Math Makes Sense*, by Pearson Education.

The books are based on sound pedagogy. However, there are so many problems with the content including: mathematical errors; incorrect terminology; word problems that are an insult to the students' intelligence and common sense; lack of coherence and focus throughout the series; and investigations of patterns and sequences and an introduction to algebraic thinking that reinforce the perception that mathematics is a set of rules that only a teacher knows rather than a result of creative thought.

A few examples of such errors in the books are provided here:

(i) Terminology

Grade 5 textbook: The glossary uses the word *"object"* to describe what we would call a polyhedron. While a polyhedron may be considered a difficult word, the same book uses the word *"polygon"*, which is similar in difficulty and unfamiliarity. At the same time, the word *"object"* has an everyday meaning that makes such substitution confusing – even in the same glossary, when it defines speed as a measure of how fast an **object** is moving …

Grade 7 textbook: An *"average"* is defined as "the number that represents all numbers in a set" and which, as they claim later and even have a Reflect exercise to reinforce this, may or may not be equal to a mean. However, in statistics, average is the same as mean!

(ii) Errors

Note that the errors below are not just typos. Further, each has a potential of introducing a misconception that will create difficulty for the students in future learning. The list of such errors is too long to produce here. A couple of examples are provided here:

The **Grade 7** textbook asks the question:

Can you draw a circle with a circumference of exactly 33 cm?

The answer says:

No, because π never terminates or repeats. So, the circumference will never be a whole number.

The **Grade 8** textbook, in an answer to a question asked of students, claims that 13.999… is not equal to 14 and uses this fact as part of the reasoning in the problem - even though other books in the series prove that, as we know, this is true.

(iii) Ridiculous problems

Grade 6 textbook, Unit 3, Lesson 1, Numbers to Thousandths and Beyond.

Here we see all kinds of senseless examples to serve as vehicles for learning about decimal numbers. Two such statements follow:

i. The ostrich is the world's largest living bird. It can have a mass of 156.489 kg.

ii. Four baseballs have a total mass of 575.94 g. Find the mass of 1 baseball.

Would we weigh 4 baseballs to the nearest hundredth of a gram, or an ostrich to a nearest thousandth of a kilogram? And if we weighed 4 different baseballs, would we get the same result?

Math makes sense? Hardly.

The alternative series, *Math Links*, is plagued with similar errors and lack of understanding of the big picture we want students to catch a glimpse of.

3. Preparation of teachers.

Research shows that good, knowledgeable and confident teachers can work even without proper resources. But teachers who lack a sufficient knowledge and/or confidence in the subject will likely not do a good job, even if they are trying hard.

The BC College of Teachers is the body which decides on the level of knowledge required for teachers in BC, and the professional teachers programs at BC universities are built on this basis - students (future teachers) seldom want to take courses in subjects considered difficult if such courses are not required for their certificate. While a vast amount of research done in the US and Canada (and other countries) in recent years demonstrates the need for deep understanding of mathematics in elementary and middle school teachers, the College of Teachers requires only one mathematics course, and not even a mathematics methods course. This is far below what is required in most Canadian provinces and US states, not to mention other countries.

The Math Task Force created by the Ministry of Education in 1999 came up with a recommendation (supported by the BCCUPMS) for at least two mathematics content courses and two mathematics methods course being required for elementary and middle school teacher preparation. This recommendation has been ignored. Note that the recent US task force recommendations suggest an even higher level of math preparation – see the recent publication of the Conference Board of the Mathematical Sciences in the US: **http://www.cbmsweb.org/MET_Document/index.htm**

4. Proliferation of "distributed learning" high school courses.

In recent years, the BC government has been actively soliciting creation of online courses, so called "distributed learning" courses, which have been created without much (if any) quality control. These courses are increasingly popular since they provide a way for students to get good grades with little effort - and usually with very little benefit. This, together with discontinuation of grade 12 provincial exams in mathematics and therefore no quality check, has already started to impact the universities - and the students. Students are coming to universities and colleges with good grades, but inadequate preparation for university level courses. Universities are increasingly faced with the need to remedy the situation, which means unloading school work on universities. This is doubly unfair to students, who have easier access to universities but are paying dearly for this once they get there. This also requires additional resources at the post secondary level – likely far more costly than what is saved by remedying the problem in the first place.

5. Conclusion

This document briefly outlines problems with the way mathematics is taught in BC. The situation is serious, and needs to be treated as such if we want to build a better future for young people in our province. The only way to correct many of the problems is, first, by acknowledging that the problem exists, and, second, by allowing people who know mathematics – the mathematicians – to be partners in any discussions on curriculum, textbooks, and education of teachers.

On normal forms in classical dynamics

Cristina Stoica, *Department of Mathematics, Wilfrid Laurier University*

Given an equivalence relation on a set $\mathcal{X}$, an element of $\mathcal{X}$ is in normal form if it is the "simplest" within its class. For example, similarity is an equivalence relation on square matrices, and we agree that the simplest objects within each equivalence class are the matrices in Jordan canonical form. Any matrix can be brought to this form by a change of coordinates.

In differentiable dynamics, $\mathcal{X}$ is the space of smooth vector fields on some finite-dimensional smooth manifold M such that each vector field $X \in \mathcal{X}$ uniquely defines a complete flow $\phi_t^X : M \to M$, where $\phi_t^X(z) = x(t; z)$ is the integral curve of X with initial condition z. A change of coordinates is the action of a diffeomorphism h on the set of flows by conjugation, $(h, \phi_t^X) \longrightarrow h \circ \phi_t^X \circ h^{-1}$, and the corresponding action of h on vector fields, $(h, X) \longrightarrow Dh \circ X \circ h^{-1}$. These actions partition the set of flows and the set of vector fields into equivalence classes. It remains to say what the "simplest" flows (or vector fields) are. As we will see, a complete definition is difficult even at a local level (near a point z). This note discusses *local* normal forms, first for general vector fields and then for Lie equivariant and Hamiltonian vector fields.

Near regular points z, where $X(z) \neq 0$, the simplest flows are linear, corresponding to constant vector fields. The Flow Box Theorem guarantees that the flow near a regular point is conjugate to a linear flow. Near points where the vector field vanishes, i.e. at equilibria, linear vector fields are the simplest. Building on Poincaré's ideas, Sternberg, [6], proved that most vector fields near equilibria are locally C^∞ conjugate to linear ones. The non-generic cases appear at resonances (see below), where the definition must be amended as follows.

Let z_0 be an equilibrium of X. For simplicity, fix a local coordinate system around z_0, which is taken to be the origin. Then we write $X(z) = Lz + f^{(2)}(z) + f^{(3)}(z) + \ldots$, where L is the linearization of X at 0 and $f^{(k)}(z)$, $k \geq 2$, are in P_k, the space of homogeneous polynomials of degree k in z. Construct a sequence $h_k \in \mathrm{Diff}(M)$, $k = 2, 3 \ldots$, such that, if $X^{(k+1)} := (Dh_k)^{-1} \circ X^{(k)} \circ h_k$, with $X^{(2)} := X$, the linear part of $X^{(k+1)}$ is L and, ideally, each $X^{(k+1)}$ has no non-zero terms of orders 2 to k. Since we deal with Taylor expansions of vector fields, h_k can be taken as flows of polynomial vector fields. The change of coordinates generated by a polynomial vector field P is described by the Lie series theorem: for $\varepsilon > 0$ we have

$$\left(\left(\phi_\varepsilon^P\right)^{-1}, X\right) \to X^{\text{new}} = \left(D\phi_\varepsilon^P\right)^{-1} \circ X \circ \phi_\varepsilon^P = X - \varepsilon\, \mathrm{ad}pX + \frac{\varepsilon^3}{2!}\, \mathrm{ad}p(\mathrm{ad}pX) + \ldots$$

where $\mathrm{ad}pX = [P, X] = DX \cdot P - DP \cdot X$ is the Jacobi-Lie bracket of P and X. Assuming that at $\varepsilon = 1$ the above expansion is valid, it turns out that at each step we seek a homogeneous polynomial $P_k \in P_k$ so that the *homological* equation $f^{(k)} + \mathrm{ad}_L(P_k) = 0$ is satisfied. Since $\mathrm{Im}(\mathrm{ad}_L)$ may have a non-trivial complement G_k in $\mathrm{P}k$, we may be left with non-removable *resonant* terms $\hat{f}^{(k)} \in \mathrm{G}_k$. So, for any $m \geq 1$, there is a sequence of changes of coordinates such that the jet of order m of the transformed vector field $\hat{X}$ is $j^m\hat{X} = L + \hat{f}^{(2)} + \hat{f}^{(3)} \ldots + \ldots + \hat{f}^{(m)}$ with+ $\hat{f}^{(k)} \in \mathrm{G}_k$, $k = 2, \ldots, m$. Near the equilibrium, the jet $j^m\hat{X}$ of order m is called the *truncated normal form* of X of order m, and if the series is convergent, the vector field $\hat{X}$ is called the *normal form* of X.

We can now highlight a key application of normal forms: if the truncated normal form is taken as the "unperturbed" system, persistence arguments can be used to show that its dynamical features continue in the non-truncated (full) system. It can be shown that P_k may be endowed with an inner product structure so that $\mathrm{Im}(\mathrm{ad}_L) \oplus\perp \mathrm{Ker}(\mathrm{ad}_L)^* = P_k$. Then $j_m X$ commutes with $\exp(tL^T)$, $t \in R$, implying that for large classes of systems the truncated normal form acquires a toroidal symmetry with respect to the original vector field. This extra symmetry is one of the main properties used to understand local dynamics phenomena, such as branching patterns of equilibria and periodic solutions.

The same ideas apply to Hamiltonian systems on symplectic, or more generally Poisson manifolds, with the additional requirement that the transformations applied are symplectic, respectively Poisson, so that the truncated vector field preserves its structure. A Poisson structure on a manifold M is a bilinear operation $\{\ ,\ \}$ on $C^\infty(M)$ so that $(C^\infty(M), \{\ ,\ \})$ is a Lie algebra and $\{\ ,\ \}$ is a derivation on each factor. Given a function $h \in C^\infty(M)$, there is a unique vector field X_h on M, called the *Hamiltonian* vector field of h, such that $X_h[g] = \{g, h\}$ for all $g \in C^\infty(M)$, where $X_h[g]$ is the Lie derivative of g along X_h. A standard example is the canonical (symplectic) bracket on $\mathbb{R}^{2n}$ for which Hamitonian systems take the form $\dot{q} = h_p(q, p), \dot{p} = -h_q(q, p)$.

We apply the previous theory with changes of coordinates given by time-1 flows of a Hamiltonian polynomial vector field X_f, and find that X_h is transformed to another Hamiltonian vector field, $X_{h^{new}}$, where $h^{\text{new}} = h + \{f, h\} + (1/2!)\{f, \{f, h\}\} + \ldots$. This is a consequence of the antihomomorphism between the Lie algebras $C^\infty(M)$ and $\mathcal{X}(M)$, that is, $\mathrm{ad}_{X_h} X_k = -X_{\{h,k\}}$ for any two functions h, k. Thus the normal form computations can be done entirely at the level of Hamiltonians. For canonical systems, the resulting change of coordinates transforms h into $\hat{h}$ such that $j^m\hat{h} = \hat{h}^{(2)} + \hat{h}^{(3)} + \ldots \hat{h}^{(m)}$, where $X_{\hat{h}^{(2)}}$ is the semisimple part of the linear part of $X_{\hat{h}}$, and $\{\hat{h}^{(2)}, \hat{h}^{(k)}\} = 0$ for any $k \geq 2$. Normal form theory for Hamiltonians on Poisson manifolds with linear Poisson structure corresponding to a semisimple algebra was recently developed by Monnier and Zung, [2].

Originating in the work of Poincaré and Birkhoff, normal forms for generic and canonical Hamiltonian systems is a well developed classical subject and an extensive body of literature is available, see, e.g, [3] and references therein. In contrast, normal form theory for Lie equivariant (symmetric) vector fields is a relatively new direction of research.

A vector field X is equivariant under the action of a Lie group G if $X(gz) = Dg(z)X(z)$ for all $g \in \mathrm{G}$. A relative equilibrium is a dynamical solution $z(t)$ that is also a one-parameter subgroup orbit of G, i.e. $z(t) = \exp(t\omega)z_0$ for some $\omega \in \mathfrak{g}$, the Lie algebra of G, and $z_0 \in M$. Relative equilibria play an important rôle for the dynamics, pretty much as equilibria play for generic vector fields. When studying symmetric systems, it is useful to pass to a coordinate system where the directions along and transversal to the group are distinct. Such systems of coordinates are provided by *slice* theorems. In short, in a sufficiently small neighborhood of an orbit Gz_0 with isotropy subgroup $K := \{g \in G : gz_0 = z_0\}$, the phase space is isomorphic to the slice bundle $G \times_K S = (G \times \mathrm{S})/K$, where S, called the *slice*, is a subspace of the tangent space at z_0 that is transversal to Gz_0. The quotient by K corresponds to the identifications $(g, z) = (gk^{-1}, ks)$ for all $k \in K$. In these coordinates, a relative equilibrium $z(t)$ corresponds to $(\exp(\mathrm{t}\omega), 0)$ and the dynamics takes the form $\dot{g} = gf_G(s), \dot{s} = f_S(s)$, where $f_S : S \longrightarrow S$ and $f_G : S \longrightarrow \mathfrak{g}$ are K-equivariant, i.e., $f_S(ks) = kf_S(s)$ and $fG(ks) = kf_G(s)k^{-1}$ for all $k \in K$ and $s \in S$. Thus, locally, the dynamics in the slice $\dot{s} = f_S(s)$ drives the dynamics in the group (or "drift") directions $\dot{g} = gf_G(s)$. For proper actions, a characterisation of normal forms near relative equilibria in terms of additional equivariance conditions was recently introduced in [1]. By choosing appropriate inner products on S and on $\mathfrak{g}$, both the drift and slice truncated normal forms acquire additional symmetry.

For Lie symmetric Hamiltonians, one may transfer the machinery from the case of general vector fields: the dynamics can be split into the drift-slice directions, but it must accommodate the additional Hamiltonian structure. By Noether's theorem, the symmetry group G provides the Hamiltonian system with conserved quantities, called momenta. The symplectic manifold is therefore partitioned into flow-invariant level sets of the *momenta*. The way these level sets intersect the slice can be complicated, especially when G is non-abelian and the relative equilibrium has non-trivial isotropy. This leads to a nontrivial structure on the slice bundle, which in turn induces a nontrivial structure on the slice equations, [4], [5].

A less ambitious task is to compute normal forms near non-isotropic relative equilibria in the case of compact Lie symmetries. Let P be a symplectic manifold with a symplectic action of a compact Lie group G and a G-equivariant momentum map. Consider h a G-invariant Hamiltonian on P such that $z(t) = \exp(t\omega)z_0$ is a non-isotropic relative equilibrium with momentum $\mu_0 \in \mathfrak{g}^*$. By the symplectic slice theorem, there is near $z(t)$ a local model given by $G \times \mathfrak{g}^*_\mu \times Y$, where $\mathfrak{g}^*_\mu$ is the co-Lie algebra of the momentum isotropy subgroup $G_{\mu_{(0)}} := \{g \in G : g\mu_0 = \mu_0\}$ and Y is the symplectic normal space to $G \cdot z_0$. In coordinates (g, μ, y), the relative equilibrium is identified with $(e, 0, 0)$ (e being the identity on G) and we write the hamiltonian $h = h(\mu, y)$. On the slice $\mathfrak{g}^*_\mu \times Y$, the dynamics is given by a Poisson bracket $\{f, g\} = \{f, g\}_{\mathfrak{g}^*_\mu} + \{f, g\}_Y$, where $\{f, g\}_{\mathfrak{g}^*_\mu}$ is the Lie-Poisson (rigid body) bracket on $\mathfrak{g}^*_\mu$ and $\{f, g\}_Y$ is a symplectic bracket on Y. The equations of motion appear as Euler equations on $\mathfrak{g}^*_\mu$ coupled to a canonical Hamiltonian system on Y, i.e. an interaction between "rotational" and "vibrational" motions. The drift equations are $\dot{g} = g\,\frac{\partial h}{\partial \mu}$.

The normal form theory on Poisson manifolds as developed in [2] can be applied to our specific context, with the slice as the Poisson manifold. We obtain, for any $m \geq 2$, coordinate changes taking h into $\hat{h}$ such that $j^m\hat{h}(\mu, y) = \hat{h}^{(1)}(\mu) + \hat{h}^{(2)}(y) + \hat{h}^{(3)}(\mu, y) + \ldots + \hat{h}^{(m)}(\mu, y)$, where $\hat{h}^{(1)}$ is defined on a Cartan subalgebra of $\mathfrak{g}_\mu$ and $X_{\hat{h}^{(2)}}$ is the semi-simple part of the canonical Hamiltonian vector field on Y. The vector field $X_{\hat{h}^{(1)}+\hat{h}^{(2)}}(\mu, y)$ is linear and semi-simple, and the induced normal form symmetry reads $\{\hat{h}^{(1)} + \hat{h}^{(2)},\ \hat{h}^{(k)}\} = 0$ for all k. By the structure of the equations, the normal form of h as calculated in the slice normalizes the drift equations as well. Note that the normal form sketched in the paragraph above applies whenever the symmetry group is compact with a semi-simple Lie algebra. This includes $SO(n)$, $SU(n)$ and $Sp(n)$, i.e., the standard symmetry groups of classical dynamics and quantum theory. In this respect, one may foresee that applications of this method might lead to interesting results.

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How to achieve radial symmetry through simple rearrangements

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We like to think that symmetric problems have symmetric solutions, but symmetry-breaking can be observed everywhere in physics and geometry. Still, shouldn't at least optimal solutions be highly symmetric? Faced with a symmetric optimization problem in several variables, we commonly look for radial solutions, betting that the true optimizer might be among them. After all, balls minimize surface area and electrostatic capacity among bodies of a given volume, and the sharp constants in many integral inequalities, such as the Sobolev inequality, are assumed by particular symmetric decreasing functions. However, it can be difficult to compare a competitor for optimality with a radial solution. This is where symmetrization techniques come in.

In 1838, Steiner gave a "simple proof" of the isoperimetric inequality by comparing a given body not directly with a ball, but rather with a related body of the same volume that has just one hyperplane of symmetry (see Figure 1). For a planar convex body (the case considered by Steiner), the perimeter decreases *strictly* under Steiner symmetrization unless the body is already reflection symmetric. Since the perimeter of a minimizing set cannot be reduced by symmetrization in any direction, the minimizer must be a disk. Steiner's contemporaries objected that this argument does not establish existence, and that the minimizer is identified only within the restricted class of convex bodies. These issues were finally resolved in 1958 by De Giorgi, using a technical notion of perimeter due to Caccioppoli. In the 1970s, symmetrization was recognized by analysts including Talenti, Lieb, and Baernstein as a tool for proving sharp functional inequalities.

A more direct geometric approach is to construct a sequence of symmetrizations that converges to a minimizer. This ultimately leads to the "competing symmetries principle" of Carlen and Loss, which has been used to find optimizers for many conformally invariant problems. The basic idea goes back to 1909, when Carathéodory proved the isoperimetric inequality, using a greedy sequence that reduces the moment of inertia by (almost) as much as possible in each step. How can convergence fail? Of course, the set of directions may simply be too small to generate full rotational symmetry. Yet convergence can be achieved by iterating Steiner symmetrization in finitely many well-chosen directions. Symmetrization along a sequence of directions chosen uniformly at random from the unit sphere converges almost surely to the ball.

Over the last ten years, subtle geometric properties of Steiner symmetrization have become much better understood through the work of Bianchi, Chlebík, Cianchi, Fusco, Gronchi, Klain, Klartag, Lutwak, V. Milman, Van Schaftingen, Volčič, D. Yang, G. Zhang, and very recently Marc Fortier, Greg Chambers, and myself. Steiner symmetrization of a set of finite perimeter in a random direction almost surely decreases perimeter, unless the set is a ball. For any given convex body in $\mathbb{R}^d$ there exists a sequence of $3d$ Steiner symmetrizations that reduce its ratio of outradius over inradius to an absolute constant. Chambers and I found that Steiner symmetrization along d arbitrary linearly independent directions transforms every compact subset of $\mathbb{R}^d$ into a set of finite perimeter. It has been a pleasant surprise to discover new properties of such a classical tool.

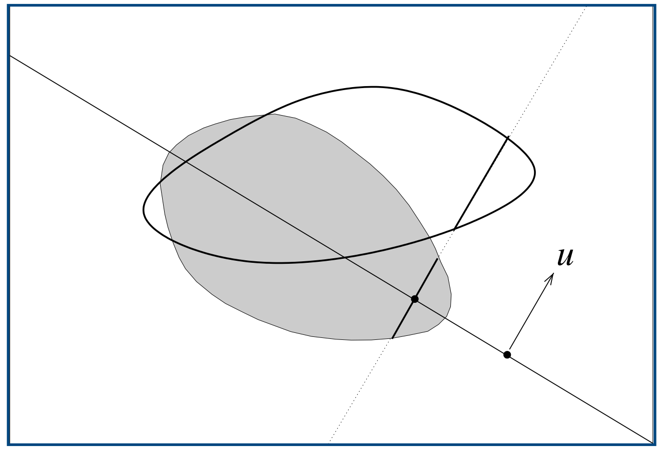

Figure 1 The Steiner symmetrization of a compact set in the direction of a unit vector u. The set is enclosed by a solid curve, and the symmetrized set is shaded. Steiner symmetrization creates a hyperplane of symmetry by replacing the intersection with each line parallel to u with a symmetric interval. Volume is preserved, while perimeter is reduced.

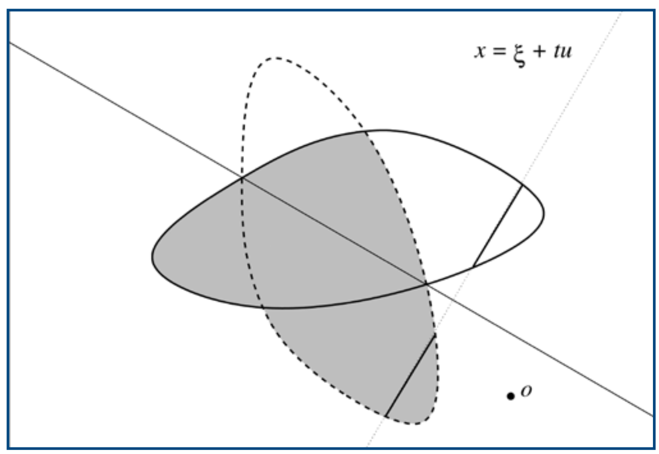

Figure 2 The polarization of a compact set with respect to a reflection. The solid line indicates the hyperplane of reflection, the set is enclosed by a solid curve, its mirror image by a dashed curve, and the polarized set is shaded. Polarization pushes mass towards the origin by replacing the portion of the set that lies in the half-space across from the origin whose reflection is not in the set with its mirror image. Both volume and perimeter are preserved. Note that convexity, smoothness, and non-trivial symmetries can be lost.

The understanding of infinite sequences of Steiner symmetrizations has grown similarly. Every sequence that uses only a *finite* set of directions converges to a body that has at least partial symmetry. On the other hand, convergence can fail even for Steiner symmetrizations of a convex body along a *dense* set of directions (see Figure 3). Symmetrizations along a dense set of directions can be made to converge or diverge by simply reordering the sequence; furthermore, any sequence of directions (convergent or not) can be realized as a subsequence of one that fails to converge. In a forthcoming paper, Bianchi, Gronchi, Volčič, and I show that the known examples of non-convergent sequences of symmetrizations do converge "in shape" (i.e., up to a rotation), to limits that need not be ellipsoids or even convex. The proof relies on the fact that the intersection between any pair of sets can only grow while the perimeter of each set shrinks.

I am interested in the corresponding questions for an even simpler rearrangement, known as "polarization," or two-point symmetrization (see Figure 2). First introduced by Littlewood and Ahlfors in the 1940s for studying conformal invariants, polarization is particularly useful for proving geometric inequalities on spheres. It also lends itself well to quantitative estimates for path integrals. While Steiner symmetrization (when it applies) reduces geometric inequalities to one-dimensional problems, polarization reduces them to combinatorial identities.

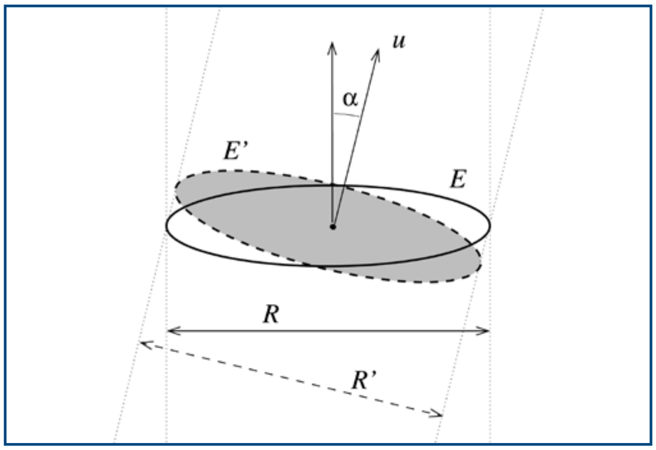

Figure 3 Steiner symmetrization transforms ellipses into ellipses. The unit vector u determines the direction of symmetrization, the original ellipse E is enclosed by a solid curve, and the symmetrized ellipse E' is shaded and enclosed by a dashed curve. Note that u is an axis of symmetry for E'. The diameters of E and E' satisfy $R' > R \cos\alpha$, where α is the angle between u and the minor semiaxis of E. Sequences where the sum of the angles between consecutive directions diverges are dense in S^1. If, moreover, $\prod \cos\alpha_i > 0$, then the corresponding Steiner symmetrizations spin slowly about the origin while their shape converges to an ellipse that is not a ball.

In his 2010 Master's thesis, Fortier presents conditions for sequences of random polarizations to converge to the ball, which sharpen results that Van Schaftingen had proved five years earlier. In subsequent joint work, Fortier and I consider sequences that may be far from uniformly distributed, and may even concentrate on small sets or converge weakly to zero. If the distribution is sufficiently uniform, we show that the symmetric difference to the ball decreases at least proportionally to n^{-1} in the number of steps. Many questions remain open: Are there (non-random) sequences that converge faster? Is a greedy approach the best? We know that random polarizations of a compact set converge in Hausdorff distance, but we have no estimate for the rate. I would like to understand what distinguishes convergent from non-convergent sequences. Is there always convergence in shape?

As corollaries, Fortier and I obtain new convergence results for Steiner symmetrization, including a bound on the rate of convergence that does not require the set to be convex. This bound cannot possibly be sharp, because Steiner symmetrization has much stronger smoothing properties than polarization. For convex bodies, Klartag has shown that the achievable rate of convergence is faster than any power law. Remarkably, his constants grow only polynomially with the dimension. Can this result be extended to non-convex sets? Is the best possible rate of convergence in fact exponential and how does it depend on dimension? Given a configuration of d directions in $\mathbb{R}^d$, what geometric and algebraic conditions determine the ergodic properties of the corresponding Steiner symmetrizations? All these questions await answers.

Canadian Leader's Report, IMO 2012

Jacob Tsimerman

The 53rd International Mathematical Olympiad was held in July 2012 in Argentina. Throughout the year, countries all over the world host their own national mathematical olympiads to select 6 of their brightest young minds to represent their country in this annual competition. Canada's team this year consisted of Matthew Brennan, Calvin Deng, James Rickards, Alex Song, Daniel Spivak, and Kevin Zhou. Accompanying them were myself as team leader, Lindsey Shorser as Deputy Leader, and Ralph Furmaniak as Observer. This year's team has done extremely well, bringing back 3 gold medals, a silver medal and 2 bronze medals, along with a record-setting 5th place world-ranking.

Training Camp

Our story this year began in beautiful Banff, Alberta where we had our annual two-week long training camp before the contest itself. The Banff International Research Station is kind enough to provide us with food, housing, and a classroom with a blackboard, which is more than enough for us. The very first day that we arrive, we settle into our rooms and have an evening session. Originally we wanted to just take it easy since the students probably wanted to settle in, but when we asked them they overwhelmingly insisted on beginning training. To get things off to a relaxing start Ralph decided to run a session of a Bernoulli Trial, which consists of about 10 yes or no questions that the students have 5 minutes to try and answer. Though this isn't really in the style of the olympiad, it was great fun and we decided to repeat it for a lot of our evening sessions.

This year three of our students were arriving late to the camp. James wanted to attend his high school graduation while Alex and Calvin were already deep in training with the american IMO team at MOSP, which overlapped with our camp by a few days. As such, we invited 3 students to temporarily take their place at the camp: Weilian Chu, Kevin Sun and Leo Lai. A typical day in training included 2 training sessions, one in the morning and one in the afternoon, and a free problem solving session in the evening. That adds up to about 13 lectures, so we had some friends give us a hand: Alex Fink, Hunter Spink, and David Arthur took turns coming in and helping by giving lectures. Someone also had the brilliant idea that James could join us for our evening session by Skyping in. I had my doubts at first about the logistics of such an arrangement but it ended up working really well.

Of course, the hardest thing about having the training in Banff is not giving in to the temptation to drop everything and go hiking every day! This proved to be overwhelming and the team ended up having 2 hikes that I was a part of as well as some hikes afterwards. We also had the privilege of getting a beautiful tour of Banff and its facilities, watched the Canada day fireworks and saw the Amazing Spider-Man in theaters, so there was much fun to be had!

Selection of the Problems

On July 3rd I left for Argentina ahead of the team to join the other countries leaders in selecting the 6 problems that would make up the contest. The only downside to having the IMO in Argentina was that between the first plane trip to Buenos Aires, the bus ride, and the second plane trip to Mar de Plata it took over 24 hours to get there at which point I was absolutely exhausted. Of course, just as I arrive at the hotel I'm handed a list of 28 problems to try and solve over the next 24 hours before we begin the voting process. This is an extremely daunting, infeasible task as the students are given 9 hours over 2 days to do six questions - and they are always the best problem solvers!

As I start looking over the contest I am elated to see a problem that David Arthur showed me a year ago - a very nice game theory problem with an information theory twist. He had mentioned to me during the training camp that he submitted this problem and I was very happy to see that it was selected to appear on the shortlist. The next day we got the official solutions and began the 2 days voting process to select the 6 problems that would appear. During the ensuing discussion David's problem quickly came out as one of the favorites and was selected as problem 3 of the contest.

Once the 6 questions were selected we then spent 2 days translating the paper into over 40 different languages, a remarkable effort by the translation committees! David's problem was by far the longest taking over a paragraph to write down, so this took by far the longest to translate! It was also a pleasure to see how the phrase "liar's game" coined by David for his problem was translated into all the various languages to best fit the cultures of their respective countries.

The contest

The contest itself is split into 2 parts of 3 questions to be written over 4.5 hours each day. At the beginning of each day the leaders are bussed on site to where the students are writing the contest so that for the first hour we can answer clarification questions; it is a tremendous tragedy when a student misunderstands what a problem is asking for and ends up wasting a lot of time! The leaders wait patiently for the questions and hope that no embarrassing questions emerge from their countries (Past questions included such wonderful doozies as "What is a Grasshopper?") This year the questions were all straightforward and uneventful, to everyone's relief. However, as the leaders leave the building the first day we are shocked to discover that the deputy leaders are right outside with us! The leaders are sequestered so that they cannot pass on the questions to their students, and as the deputy leaders stay with the students they cannot be allowed to see the leaders. While the honor code system would surely work for most countries, there have unfortunately been a number of instances of cheating in the past which has led everyone to be extra cautious.

That night, we receive our students script and begin to look over them. I am very pleased to see that everyone on the team got problem 1, 4 people solved problem 2 and we have 3 part solutions to problem 3. As I don't know how other countries did it is hard to judge, but this strikes me as a very solid performance so far. Everyone looks poised for a medal and there are certainly potentials for gold. The leaders tentatively try to gather information over dinner about other countries results, though no one wants to speculate too much. I decide to call it an early night and go to bed.

The next day after the clarification session we are finally unsequestered and check into our new hotel next to the students. I catch up with Lindsey and Ralph and we discuss how the students are doing. When the contest ends we all meet the students and ask how they did. It appears that day 2 also went really well, and we had 6 complete solutions to problem 4. Alex solved all 3 problems so he looks to be a shoo-in for gold. Though one of our strongest competitors, Alex is also the youngest and has 3 more years to compete.

After getting dinner I go to the students hotel and find see the legendary game room: A gigantic space on the first floor filled with everything from video games and table tennis to gymnastic demonstrations and tango lessons. We spend a bit of time playing video games and then its off to bed, to prepare for coordination.

Coordination

The way that the contest is marked is that countries look over their own papers and decide on a plausible grade. They then meet with assigned coordinators from the host country and together decide on a final grade. It is usually a very good system. A lot of our problems seem straightforward, and we expect to get either a 7(the maximum score) or a 0. However there is some scavenging for part marks to be done here and there. It also looks like we have 2 solid gold medals from Alex and Calvin, while everyone else is on potential medal cutoffs, so it becomes all the more crucial to argue for each part mark we can get.

Our coordination is overall very smooth, though we quickly start hearing from other countries that problem 4 is a nightmare in coordination. Problem 4 is a relatively straightforward functional equation, with the twist that there are a few strange solutions one might not expect to begin with. The issue is that being the easiest problem on day 2, the marking scheme for the problem demands that each solution be carefully verified with an attention to detail not typically asked for on an olympiad. What's worse, up to 2 points can be lost for failing to do this, so there is a lot of bickering to be done! When the marking scheme was selected me and a few other leaders argued against this system but ultimately lost out.

Sure enough, when our turn comes, James' and Kevin's solutions are targeted for point deductions. We do our best and manage to save a point, but we still come out with a 5 for James and a 6 for Kevin. This is upsetting, but other countries have much more tragic tales (7's that become 2's) so we have no right to complain.

Results

As the results slowly start appearing, it looks like Canada might have a chance for top ten, which is tremendous! In past years it was usually possible to determine fairly quickly the countries rankings (unofficial, as this is an individual contest) and medal cutoffs from the scoreboard, but a few years ago they started blocking out the scores for 1 problem per student making it very hard to make reasonable predictions. We don't spend too much time worrying about it and instead go to the games room for the night.

When the final jury meeting is held, the first order of business is a couple countries that want to argue over points for their students' solutions to problem 4. After much debate the USA manages to get 1 more point for one of its solutions. Then all the scores are revealed and we decide to pick the medal cutoffs. The policy is that the top twelfth of the students get gold medals, the next sixth get Silver medals and the next fourth get Bronze medals, so that half of the students get a medal. This year the cutoffs elegantly end up being multiples of seven: 14,21 and 28. As soon as we vote these are entered into the computer and put online. Within seconds Ralph gets a text message saying "OMG! You guys are fifth!!" and we are all stunned! This is Canada's best result ever, not even accounting for the 'inflation' of the growing number of countries involved. We also have the joyous news of getting 3 gold medals - another record! - and moreover Alex and Calvin placing 5^{th} and 12^{th} for total individual rankings. All in all, this has been a remarkable year for us. As we make our way to the students hotel to tell them how well they did we run into the leader of Thailand's team, with whom we tied for fifth place, and exchange congratulations.

At the closing ceremony, Alex, Calvin and Matthew are awarded gold medals, James gets his silver medal, and Kevin and Daniel claim bronze medals. I would again like to congratulate all the students for doing so well and representing Canada with distinction. It was a joy to work with them at the IMO, and I look forward to hearing of their future accomplishments.

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Benoit Collins (Ottawa, Lyon 1)
Mireille Capitaine (Université Paul Sabatier)

Graham Wright Award for Distinguished Service Prix Graham Wright pour service méritoire

Bernard Hodgson (Université Laval)

Public Lectures | Conférences publiques

Ivar Ekeland (UBC; Paris-Dauphine)

Doyne Farmer (University of Oxford)

Plenary Speakers | Conférences plénières

Graciela Chichilnisky (Columbia)
Ted Hsu (Physicist and Federal MP)
Martin Nowak (Harvard)
D.H. Phong (Columbia)
Catherine Sulem (Toronto)

Scientific Director | Directeur scientifique

Luc Vinet (Montréal)

CENTRE DE RECHERCHES MATHÉMATIQUES

Faculty Position in Graph Theory

Department of Combinatorics & Optimization, University of Waterloo

The Department of Combinatorics and Optimization (**http://math.uwaterloo.ca/co**) at the University of Waterloo invites applications for an open rank tenure-track or tenured faculty position in the area of graph theory.

A Ph.D. degree and evidence of excellence in research and teaching are required. Successful applications are expected to maintain an active program of research, to attract and supervise graduate students, and to participate in undergraduate and graduate teaching. Salary will depend on the candidate's qualifications. The effective date of appointment is July 1 2013.

Interested individuals should apply using the MathJobs site (**http://www.mathjobs.org**). Applications should include a curriculum vitae, research and teaching statements, and up to three reprints/preprints. In addition, at least three reference letters should be submitted.

Inquiries may be addressed to **combopt@math.uwaterloo.ca** or to Alfred Menezes, Chair, Department of Combinatorics and Optimization, University of Waterloo, Waterloo, Ontario, Canada N2L 3G1. Closing date for receipt of applications is December 7, 2012.

All qualified candidates are encouraged to apply; however Canadians and permanent residents will be given priority. The University of Waterloo encourages applications from all qualified individuals, including women, members of visible minorities, native people, and persons with disabilities.

Related Activities | Activités liés

Canadian Launch of Mathematics of Planet Earth (MPE2013)

Lancement canadien de Mathématiques de la planète Terre 2013 (MPT2013)

Friday, December 7, time and location TBD
Vendredi, 7 décembre, détails à venir

Student Lecture | Conférence pour les étudiants

Yvan Saint-Aubin (Montréal)
Friday, December 7, 9:00-12:00
Vendredi, 7 décembre, 9h à 12h

Student workshop | Atélier pour les étudiants

Friday, December 7, details TBD
Vendredi, 7 décembre, détails à venir

Mathematical Science Investigation (MSI): The Anatomy of Integers and Permutations

Friday, December 7, 16:30 - 18:00 and Saturday, December 8, 20:30 - 22:00
Vendredi, 7 décembre, 16h30 à 18h et samedi, 8 décembre, 20h30 à 22h

This is an experimental work that blurs the boundaries between pure mathematics, live performance and graphic novel. Andrew Granville, mathematician and writer of popular articles; Jennifer Granville, actor and screenwriter; Michael Spencer, performance designer; and Robert Schneider, musician and composer, have collaborated to present this rehearsed reading.

Thrill to mysterious murders, marvel at detectives' deductions, and groan at the mathematical puns! Don't miss this opportunity to be present at an unusual theatrical and mathematical event.

CMS-CRM-IMS Cegep MPE2013 Activities

Activité MPT2013 cégeps SMC-CRM-ISM

Friday, December 7
Vendredi, 7 décembre

Julien Arino (Manitoba) - Modélisation des épidémies, leur contrôle et leur propagation spatio-temporelle

Florin Diacu (Victoria) - Les systèmes planétaires en mécanique

Aurélie Labbe (McGill University) - De la statistique à la génétique, en passant par les mathématiques : identifier les gènes responsables de maladies complexes

Marc Laforest (École Polytechnique) - Les ouragans: un engin de destruction

Justin Leroux (Institut d'économique appliquée, HEC Montréal) - Un défi pour les mathématiciens : tarifer l'eau de manière équitable

Louis-Paul Rivest (Laval) - L'emploi de méthodes de capture et recapture pour l'estimation de la taille de populations humaines et animales

Launch of the LRP Report

Saturday, December 8, 12:30-14:00
Samedi, 8 décembre, 12h30 à 14h

Mathematics for the Life Sciences, Panel and Discussion

Sunday, December 9, 8:30 - 10:00
Dimanche, 9 décembre, 8h30 à 10h

A number of Canadian universities have taken steps toward creating (or have created) mathematics and/or statistics courses appropriate for future biologists (and life scientists in general). In this session, we plan to discuss issues, challenges and successes surrounding the creation, development and teaching of math and stats courses for life sciences students (MSLS for short).

The objectives of this session are:

(1) To provide an opportunity for the faculty in charge of thinking about *designing* teaching math and stats for life sciences courses to come together and exchange ideas and experiences.

(2) To create a network of faculty interested in teaching math for life sciences. This could be a good opportunity for faculty from departments across Canada to collaborate, in order to develop good math courses.

(3) Decide on ways to continue this dialogue and further the collaboration between faculties at different universities.

CMS Townhall Meeting | Séance de discussion ouverte SMC

Sunday, December 9, 12:30 - 13:30
Dimanche 9 décembre, 12h30 à 13h30

The CMS Executive is inviting all CMS members and meeting participants to join them at an informal luncheon to learn what CMS has planned for 2013 and to discuss any interests or concerns that members of our community may have. Unlike the AGM that focuses on what was achieved last year, this meeting focuses on what lies ahead. There will be a short presentation followed by questions and answers. This is an opportunity for participants to get together with the CMS Executive and discuss emerging issues as well as directly voice their opinions, concerns and interests.

Please visit the website for up to date information

Veuillez visiter le site web pour des informations à jour

MATHEMATICS EDUCATION

York University's Faculty of Education and Faculty of Science and Engineering invite applications for a joint full-time tenure stream appointment in **Mathematics Education** at the rank of Assistant Professor.

The Faculty of Education offers innovative pre-service, graduate and professional development programs. The Faculty values collaboration and an interdisciplinary orientation to education in a multicultural and social justice framework. Its staffing approach encourages faculty in Education to work with educators seconded from their schools and with colleagues from other academic departments across the University.

The Department of Mathematics and Statistics in the Faculty of Science and Engineering, offers an innovative mathematics major in Mathematics for Education which features a common core with all mathematics majors, in the first three semesters, along with a breadth of upper year mathematics courses and three courses designed for future teachers, but open to all mathematics majors (in Geometry, History of Math, and Research in Mathematics Education). A number of these majors are in concurrent programs with the Faculty of Education while others are preparing for Consecutive Education. In addition, the Graduate Program in Mathematics and Statistics offers a course-based MA in Mathematics for Teachers, with courses targeted at current teachers at high school or college. The Graduate Programs in Education and in Mathematics and Statistics jointly offer a group of courses focused on mathematics education as a Graduate Diploma in Mathematics Education.

Applicants are invited to visit the Faculty of Education's website at www.edu.yorku.ca and the Mathematics and Statistics Department's website at http://www1.math.yorku.ca/new/ Candidates should have completed a Master's degree in Mathematics or Statistics and have completed a doctorate in Mathematics Education with evidence of an established or ability to establish a strong program of scholarly research in Mathematics Education. The candidate will also have knowledge of mathematics content and pedagogy as reflected in the Ontario curriculum. The successful candidate should show excellence or the promise of excellence in teaching and will teach courses and develop curriculum in the undergraduate and graduate programs in both the Faculty of Education and the Department of Mathematics and Statistics, and be able to contribute to professional development programming. The successful candidate must be suitable for prompt appointment to the Faculty of Graduate Studies. A commitment to offer leadership in the Mathematics for Education program will be essential. Contributions that build upon ongoing collaborations in the area of mathematics education within the University and beyond will be an asset.

All positions at York University are subject to budgetary approval. The appointment will commence July 1, 2013. Applicants should submit a detailed ***letter of application*** describing their qualifications and research in Mathematics Education in relation to the advertised position and to the context described above, an ***up-to-date curriculum vitae***, and one or two samples of ***recent scholarly writing***. Applicants are asked toarrange for ***three scholarly references*** to be submitted by the deadline to the address below or via e-mailto edudean@edu.yorku.ca clearly marked as "support for position in Math Education".

Dr. Alice Pitt, Dean of Education
York University
c/o 239 Winters College
4700 Keele Street
Toronto, Ont. M3J-1P3

The closing date for complete application packages is January 15, 2013.
Please note: Other than references, electronic applications/materials will NOT be accepted.

York University is an Affirmative Action Employer. The Affirmative Action Program can be found on York's website at www.yorku.ca/acadjobs/ or a copy can be obtained by calling the affirmation action office at 416-736-5713. All qualified candidates are encouraged to apply; however, Canadian citizens and Permanent Residents will be given priority.

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CMS MPE2013 Lecture Series

Mathematics has many things to tell us about the world around us; not only about where we are, but whence we came and where we are going. The range of topics is immense, going from the long term evolution of climate to the split-second behaviour of financial markets. As part of an international celebration of the Mathematics of Planet Earth during the year 2013, the Canadian Mathematical Society, in collaboration with the Atlantic Association for the Advancement of Research in the Mathematical Sciences (AARMS), the Centre de Recherches Mathématiques (CRM), the Fields Institute and the Pacific Institute for the Mathematical Sciences (PIMS), is proud to sponsor a cross-Canada series of lectures during the year, given by some of the most eminent experts in the various fields touched upon by this wide theme.

Scientific audience

Calgary: *A Computational Mathematician Combusts: Simulation of in-situ Combustion for Heavy Oil Recovery*, Margot Gerritsen (Stanford), date to be determined

Edmonton: Andrea Bertozzi (UCLA), April 5th, 2013

Montreal: *The Mathematics of "Fracking"*, Anthony Peirce (UBC), March 18, 2013

Québec: *Hydrodynamic Quantum Analogs*, John Bush (MIT), April 18, 2013

Toronto: *The New Architecture of Our Financial System*, Darrell Duffie (Stanford), April 11, 2013

General audience

Fredericton: *Ocean Waves, Rogue Waves, and Tsunamis*, Walter Craig (McMaster), October 10, 2013

Halifax: Mary Lou Zeeman (Bowdoin College), September 27, 2013

Montreal: *The Mathematics of Light and Sound*, Nilima Nigam (SFU), February 15, 2013

Ottawa: *The Mathematical Challenges of Earth-System and Weather Prediction*, Gilbert Brunet (Environment Canada), March 2013

Québec: *Mathématiques de la planète Terre*, Christiane Rousseau (Montréal), Fall 2013

Regina: *The Language of Life: When Mathematics Speaks to Biology*, Gerda de Vries (Alberta), February 7, 2013

St. John's: *The Mathematical Challenges of Earth-System and Weather Prediction*, Gilbert Brunet (Environment Canada), March 2013

Toronto: *Insects, Computers, and Us*, Jane Wang (Cornell University), Fall 2013

Winnipeg: *The Mathematics of Light and Sound*, Nilima Nigam (SFU), date to be determined

Vancouver: *How Does Google Google? The Math Behind the Internet*, Margot Gerritsen (Stanford), January 14, 2013

Vancouver: *An Applied Mathematician's Journey Into the Complex World of Climate Science*, Andrew Weaver (Victoria), March 10-16, 2013

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Conférences MPT2013 de la SMC

Les mathématiques nous informent sur le monde qui nous entoure : non seulement sur où nous sommes, mais aussi sur d'où nous venons et sur où nous allons. L'étendue des sujets possibles est immense, allant de l'évolution géologique du climat aux décisions presque instantanées du comportement des marchés financiers. Dans le cadre d'une célébration internationale des Mathématiques de la Planète Terre, la Société Mathématique du Canada, en collaboration avec l'Atlantic Association for the Advancement of Research in the Mathematical Sciences (AARMS), le Centre de Recherches Mathématiques (CRM), le Fields Institute et le Pacific Institute for the Mathematical Sciences (PIMS), ainsi qu'un groupe d'universités hôtes canadiennes, est fière de présenter une série pan-canadienne de conférences durant cette année, données par quelques uns des experts les plus éminents des domaines touchés par cette vaste thématique.

Auditoire scientifique

Calgary: *A Computational Mathematician Combusts: Simulation of in-situ Combustion for Heavy Oil Recovery*, Margot Gerritsen (Stanford), date à déterminer

Edmonton: Andrea Bertozzi (UCLA), le 5 avril 2013

Montréal: *The Mathematics of "Fracking"*, Anthony Peirce (UBC), le 18 mars 2013

Québec: *Hydrodynamic Quantum Analogs*, John Bush (MIT), le 18 avril 2013

Toronto: *The New Architecture of Our Financial System*, Darrell Duffie (Stanford), le 11 avril 2013

Grand public

Fredericton: *Ocean Waves, Rogue Waves, and Tsunamis*, Walter Craig (McMaster), le 10 octobre 2013

Halifax: *Mary Lou Zeeman (Bowdoin College)*, le 27 septembre 2013

Montréal: *The Mathematics of Light and Sound*, Nilima Nigam (SFU), le 15 février 2013

Ottawa: *The Mathematical Challenges of Earth-System and Weather Prediction*, Gilbert Brunet (Environment Canada), mars 2013

Québec: *Mathématiques de la planète Terre*, Christiane Rousseau (Montréal), automne 2013

Regina: *The Language of Life: When Mathematics Speaks to Biology*, Gerda de Vries (Alberta), le 7 février 2013

St. John's: *The Mathematical Challenges of Earth-System and Weather Prediction*, Gilbert Brunet (Environment Canada), mars 2013

Toronto: *Insects, Computers, and Us*, Jane Wang (Cornell University), automne 2013

Winnipeg: *The Mathematics of Light and Sound*, Nilima Nigam (SFU), date à déterminer

Vancouver: *How Does Google Google? The Math Behind the Internet*, Margot Gerritsen (Stanford), le 14 janvier 2013

Vancouver: *An Applied Mathematician's Journey Into the Complex World of Climate Science*, Andrew Weaver (Victoria), du 10 au 16 mars 2013

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2013 CMS Summer Meeting

June 4 – 7, 2013

Dalhousie University and St. Mary's University
Halifax, Nova Scotia

www.cms.math.ca

Call for sessions

The Canadian Mathematical Society (CMS) welcomes and invites proposals for sessions for this Meeting; in particular, we encourage submissions from all universities in the Atlantic Provinces. Proposals should include a brief description of the focus and purpose of the session, the expected number of speakers, as well as the organizer's name, complete address, telephone number, e-mail address, etc. All sessions will be advertised in the CMS Notes, on the web site and in the AMS Notices. Speakers will be requested to submit abstracts, which will be published on the web site and in the meeting program. Those wishing to organize a session should send a proposal to the Scientific Directors by December 31, 2012.

Scientific Directors:

Robert Milson, **rmilson@dal.ca**
Robert J. MacG. Dawson, **rdawson@cs.stmarys.ca**

Réunion d'été SMC 2013

De 4 au 7 juin 2013

Universités Dalhousie et St. Mary's
Halifax (Nova Scotia)

www.smc.math.ca

Appel de sessions

Nous vous invitons à proposer des sessions pour ca Réunion, nous incitons particulièrement les universités des provinces de l'atlantique à faire des propositions. Votre proposition doit inclure une brève description de l'orientation et des objectifs de la session, le nombre de conférenciers prévues, ainsi que le nom, l'adresse complète, le numéro de téléphone et l'adresse courriel. Toutes les sessions seront annoncées dans les Notes SMC, sur le site web et dans le AMS Notices. Les conférenciers devront présenter un résumé qui sera publié sur le site web et dans le programme de la réunion. Toute personne qui souhaiterait organiser une session est priée de faire parvenir une proposition aux directeurs scintifiques avant 31 décembre 2012.

Directeurs scientifiques

Robert Milson, **rmilson@dal.ca**
Robert J. MacG. Dawson, **rdawson@cs.stmarys.ca**

CMS
SMC

Summer Meeting
Réunion d'été

2013

June 4 - 7 • Halifax, Nova Scotia • Dalhousie University, Saint Mary's University
4 - 7 juin • Halifax, Nouvelle-Écosse • Université Dalhousie , Université Saint Mary's

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