



2013 CMS Summer Meeting
Réunion d'été SMC 2013

18

IN THIS ISSUE DANS CE NUMÉRO

From the President's Desk

What good is mathematics? 1

À quoi ça sert les mathématiques? 3

Editorial

Telling Our Stories / Nos histoires racontées. . . 2

Calendar of Events 5

NSERC-CMS Math in Moscow Scholarships

Bourse CRSNG-SMC Math à Moscou. 5

Special Article

Current Trends 6

Book Reviews

Heights in Diophantine Geometry 8

Convex Analysis and Monotone Operator

Theory on Hilbert Spaces 10

Education Notes

Understanding studying and

studying understanding. 12

Research Notes

Separation in Convex Polytopes. 14

The Lorentz Gas: Periodic or Not? 16

2013 Doctoral Prize / Prix de doctorat 2013 ... 15

2013 CMS Summer Meeting /

Réunion d' été SMC 2013 18



CMS
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Canadian Mathematical Society
Société mathématique du Canada

CMS NOTES February 2013 de la SMC

From the President's Desk

Keith Taylor

CMS President

What good is mathematics?



How many times have we heard this question in our lives? How many of us have actually asked the question ourselves at some point when we were deciding what we should work on? It is

one of those questions for which the answers are always unsatisfactory because there is so much to say, but soon one says too much. Here are a few ways I answer the question for myself.

In the recent festive season, I talked with a medical doctor who had started her university education as a 'mature' student. She grew up in disadvantaged circumstances and, by her own admission, did not pay any attention to learning mathematics in school because she did not see what it was good for; moreover, her grades told her she was not good at math. When she was finally admitted to university under a special program, she needed remedial work on her writing skills and mathematics. The spark was struck by a great math teacher and little successes led to big ambitions. She grew confident in managing the quantitative and other sciences now made more sense. She still does not know that the uniform limit of a sequence of continuous functions is continuous, but she is skilled with arithmetic, elementary algebra and exponential decay. For her, gaining a command of basic mathematics was life changing. This is not an unusual story.

The Mathematics of Planet Earth movement is drawing attention to the usefulness of

mathematics on a different scale than its usefulness to my friend the doctor. Through the many events organized in Canada and around the world, the usefulness of mathematics as a fundamental component in humanity's efforts to confront the complex interconnected set of problems the planet faces is being highlighted for the general public. Perhaps the best results of MPE2013 will be resources developed for the use of mathematics teachers in grade schools and high schools around the world. Through these, students will have the opportunity to see something that resonates with their own experience and hopefully appreciate the role of mathematics.

Of course, it has been recognized by powerful people in the past that mathematics plays an important role in dealing with many serious issues. One of the best known examples is the code breaking team assembled at Bletchley Park during the Second World War and modern methods of code breaking and encryption are based on number theoretic concepts accessible only to those with an advanced understanding. But this mathematics is not just useful for making war. In a very real sense, much of the commerce of this century depends fundamentally on theory built over several previous centuries.

The ability to make secure financial transactions has enabled an extraordinary transformation in how the world does business and is certainly a factor in the economic transformations underway in what was once called the 'underdeveloped' world. Hundreds of millions have moved out of poverty in the last couple of decades and, although more middle class consumers are adding to the stresses on our environmental and energy

Telling Our Stories

Robert Dawson

Saint Mary's University



In 1958, some years before I was born, Clifton Fadiman published a book called *Fantasia Mathematica*, perhaps the first anthology of specifically mathematical fiction and verse. It included classics such as Aldous Huxley's "Young Archimedes" and the episode from Plato's *Meno* in which Socrates teases

the proof of a simple geometric theorem out of an unlettered slave. It included the episode of Cabell's *Jurgen* in which the hero gives a most unprofessional mathematics lesson to the Queen of Philistia. It included Martin Gardner's hilarious "The No-Sided Professor." It included Heinlein's "And He Built A Crooked House." Some of the stories are science fiction - others are not, by any ordinary standard.

It included - but why must I go on? You can buy it from Amazon for \$15 (\$10 for the Kindle ebook), and it is worth every cent. You can probably read it for free on the Internet if you insist. Not only are the stories mostly treasures, but it is one of the best cures I know for the feeling that nobody out there loves or understands us.

And then, a few years later, Fadiman wrote a sequel. "The Mathematical Magpie" (1962) may have been the second picking, but the harvest was just as rich. Arthur C. Clarke's "The Nine Billion Names of God", W.H. Upson's "Paul Bunyan Versus the Conveyor Belt", and Stephen Leacock's "A, B, and C: the Human Element in Mathematics" are all there, along with many, many more stories and poems. Max Beerbohm, Samuel Beckett, and J. L. Synge add literary street cred; and there is an episode of Lewis Carroll that you may well never have read. (Most of Carroll's *Sylvie and Bruno* is Victorian icky in a way that the *Alice* books never were; but this episode, "Fortunatus' Purse," is not bad.)

Fadiman must have pretty much brought the field up to date; it is not at all easy to find a pre-1962 mathematical story that he did not include. But what since?

I'm pleased to say that there are now two online journals, both currently free, serving this market. (Conflict of interest disclaimer: I've had minor pieces published in both, but don't let that put you off.) The *Journal of Humanistic Mathematics* [2], published by the Claremont Center for the Mathematical Sciences, follows an academic peer-review model and has been printing fiction, poetry, and nonfiction dealing with the human side of mathematics for two years.

More recently, *Imaginaire* [1], privately published, has been doing something similar from the fiction magazine side. *Imaginaire* has only had one issue so far, but seems to be settling down to, perhaps, a twice-yearly publication schedule. Check them out too.

[1] *Imaginaire* <http://www.imaginairemag.com/>

[2] *Journal of Humanistic Mathematics*
<http://scholarship.claremont.edu/jhm/>

Nos histoires racontées

En 1958, quelques années avant ma naissance, Clifton Fadiman publiait un ouvrage intitulé *Fantasia Mathematica*. Il s'agit probablement de la première anthologie de fiction et de poésie sur le thème des mathématiques. On y trouve des classiques comme *Le jeune Archimède* d'Aldous Huxley et l'épisode de *Ménon* de Platon dans lequel Socrate teste un théorème géométrique simple auprès d'un esclave illettré. On y trouve aussi un extrait de *Jurgen* de James Cabell dans lequel le héros donne une leçon tout sauf professionnelle à la reine des Philistins, ainsi que l'hilarant *The No-Sided Professor* de Martin Gardner et *La Maison biscomue* de Robert Heinlein. Certaines de ces histoires tiennent de la science-fiction, d'autres pas du tout.

On y trouve aussi... mais est-il nécessaire de poursuivre cette liste? Vous pouvez vous procurer ce livre sur Amazon pour la modique somme de 15 \$ (10 \$ en format Kindle) – un investissement que vous ne regretterez pas. Et si vous cherchez vraiment, vous pourrez probablement le lire gratuitement sur Internet. Ces histoires sont non seulement de véritables trésors, mais elles constituent aussi l'un des meilleurs remèdes que je connaisse contre l'impression que personne ne nous aime ou ne nous comprend.

Quelques années plus tard, Fadiman publiait une suite à son ouvrage. Malgré son statut de deuxième cuvée, *The Mathematical Magpie* (1962) était tout aussi riche que son prédécesseur. *Les Neuf Milliards de noms de Dieu* d'Arthur C. Clarke; *Paul Bunyan Versus the Conveyor Belt* de W.H. Upson et *A, B, C ou l'élément humain en mathématiques* de Stephen Leacock y sont tous, ainsi que de nombreux autres textes et poèmes. Max Beerbohm, Samuel Beckett et J. L. Synge rehaussent également la valeur littéraire de l'ouvrage, et vous y découvrirez peut-être une œuvre de Lewis Carroll dont vous ne soupçonniez pas l'existence. (La plupart des épisodes des romans *Sylvie et Bruno* de Carroll sont typiques de la littérature populaire victorienne, contrairement aux romans d'*Alice*, mais « La Bourse de Fortunatus » n'est pas mauvais.)

Fadiman a sans doute fait le tour du jardin à l'époque; il est en effet difficile de trouver un texte littéraire à saveur mathématique datant d'avant 1962 dont Fadiman n'aurait pas parlé dans ses ouvrages. Mais que s'est-il écrit depuis?

Je suis heureux de vous apprendre qu'il existe maintenant deux revues en ligne – gratuites pour l'instant – sur le sujet (avis de conflit d'intérêts : j'ai publié de contes dans les deux revues, mais ne laissez pas cette incursion vous distraire). *The Journal of Humanistic Mathematics* [2], publié par le Centre Claremont (Claremont Centre for the Mathematical Sciences), adopte le modèle universitaire de revue avec comité de lecture et publie depuis deux ans des œuvres de fiction, de la poésie et des ouvrages non littéraires traitant de l'élément humain des mathématiques.

Plus récemment, la revue *Imaginaire* [1], publiée à compte privé, donne dans le même sens, mais en fiction seulement. *Imaginaire* n'a publié qu'un numéro à ce jour, mais elle semble viser une publication semestrielle. Allez y faire un petit saut...

[1] *Imaginaire* <http://www.imaginairemag.com/>

[2] *Journal of Humanistic Mathematics*
<http://scholarship.claremont.edu/jhm/>

À quoi ça sert les mathématiques?

Keith Taylor
CMS President



Combien de fois vous a-t-on posé cette question? Combien d'entre nous se la sont déjà posée au moment de décider sur quoi travailler? C'est une de ces questions à laquelle aucune réponse n'est tout à fait satisfaisante parce qu'il y a tant à dire, mais sur laquelle on en dit rapidement trop. Voici quelques-unes des réponses que j'aime donner à cette question.

Durant la dernière période des Fêtes, je parlais à une femme médecin qui venait de terminer ses études universitaires en tant qu'« étudiante adulte ». Elle a grandi dans un milieu défavorisé et, de ses propres dires, elle ne s'intéressait pas aux mathématiques à l'école parce qu'elle ne voyait pas à quoi elles servaient. De plus, ses résultats scolaires lui montraient qu'elle n'était pas douée pour les mathématiques. Quand elle a finalement été admise à l'université dans le cadre d'un programme spécial, elle a dû suivre des cours d'appoint en rédaction et en mathématiques. C'est un enseignant de mathématiques exceptionnel qui a fait jaillir l'étincelle, tant et si bien que ses petites réussites se sont transformées en grandes ambitions. Elle est devenue de plus en plus habile avec l'aspect quantitatif des choses, ce qui l'a aidée à comprendre les autres sciences. Elle ne sait toujours pas que la limite uniforme d'une séquence de fonctions continues est continue, mais elle s'y connaît en arithmétique, en algèbre de base et en décroissance exponentielle. Dans son cas, la maîtrise des concepts mathématiques élémentaires a transformé sa vie. Et ce n'est pas un cas exceptionnel.

Le mouvement Mathématiques de la planète Terre (MPT) attire l'attention sur l'utilité des mathématiques, mais à une autre échelle que pour mon amie médecin. Grâce aux nombreuses activités organisées au Canada et dans le monde, l'utilité des mathématiques comme élément fondamental des efforts de l'humanité à résoudre les problèmes complexes et interreliés que vit la planète est exposée au grand public. Peut-être que les résultats les plus spectaculaires du mouvement MPT2013 seront la création de ressources destinées à l'enseignement des mathématiques au primaire et au secondaire partout dans le monde. Ces ressources feront voir aux élèves des aspects des mathématiques qu'ils pourront associer à leur propre vie et qui, espérons-le, les amèneront à reconnaître l'utilité des mathématiques.

Bien sûr, des gens importants ont déjà admis que les mathématiques jouaient un rôle important dans la résolution de nombreux problèmes. L'un des exemples les plus connus est

Letters to the Editors Lettres aux Rédacteurs

The Editors of the NOTES welcome letters in English or French on any subject of mathematical interest but reserve the right to condense them. Those accepted for publication will appear in the language of submission. Readers may reach us at **notes-letters@cms.math.ca** or at the Executive Office.

Les rédacteurs des NOTES acceptent les lettres en français ou anglais portant sur un sujet d'intérêt mathématique, mais ils se réservent le droit de les comprimer. Les lettres acceptées paraîtront dans la langue soumise. Les lecteurs peuvent nous joindre au bureau administratif de la SMC ou à l'adresse suivante : **notes-lettres@smc.math.ca**.

NOTES DE LA SMC

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Note aux auteurs : indiquer la section choisie pour votre article et le faire parvenir au Notes de la SMC à l'adresse postale ou de courriel ci-dessous.

Les Notes de la SMC, les rédacteurs et la SMC ne peuvent être tenus responsables des opinions exprimées par les auteurs.

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celui de l'équipe de décodage réunie à Bletchley Park durant la Seconde Guerre mondiale, les méthodes modernes de décodage et d'encodage étant basées sur des concepts théoriques accessibles uniquement aux personnes ayant une compréhension approfondie de ces concepts. Mais ces mathématiques ne sont pas seulement utiles pour faire la guerre. En fait, une bonne partie du commerce qui se fait de nos jours dépend fondamentalement d'une théorie élaborée au cours des siècles précédents.

La possibilité de sécuriser les transactions financières a permis une transformation extraordinaire des pratiques commerciales dans le monde, et joue sans doute un rôle important dans les transformations économiques en cours dans les régions du monde autrefois qualifiées de « sous-développées ». Des centaines de millions de personnes sont sorties de la pauvreté au cours des dernières décennies et, bien que le nombre toujours croissant de consommateurs de la classe moyenne augmente le stress sur l'environnement et sur nos systèmes de production d'énergie, cette victoire de si nombreuses personnes sur la pauvreté est l'un des éléments les plus positifs du développement mondial. Je n'irais pas jusqu'à dire que l'on doit cette transformation aux mathématiques, mais je pense que les mathématiques y ont certainement contribué.

Même si Frédéric le Grand a obligé Euler à faire des calculs pratiques, la grande majorité des découvertes d'Euler n'ont pas été revues par le Bureau des transferts technologiques de l'Académie de Berlin. Ces scientifiques étaient loin de se douter qu'Euler établissait alors les fondements de vastes domaines mathématiques, dont font notamment partie les méthodes modernes de chiffrement. Il a fallu plus de 200 ans pour que les découvertes d'Euler sur les entiers relatifs commencent à produire des retombées économiques. Même avec de très bas taux d'intérêt, la valeur en 1750 de tout revenu en 2012 aurait été pratiquement nulle.

L'économie mondiale serait très différente, je crois, et beaucoup plus instable, si le grand édifice de la théorie des nombres ne reposait pas sur les fondements établis par Euler et d'autres mathématiciens il y a si longtemps. Toutefois, les méthodes standard de calcul de la valeur économique n'accordaient aucune importance à une découverte en théorie des nombres avant l'émergence des méthodes modernes de chiffrement. Il est donc très vraisemblable que certains théorèmes récemment découverts dans des domaines considérés comme des mathématiques totalement pures et sans application seront à la base d'éléments essentiels de la civilisation du 23^e siècle. En l'an 2263, les gens devront beaucoup aux personnes qui contribuent aujourd'hui à l'approfondissement des mathématiques dans des domaines qui semblent inutiles pour l'instant. Il reste que, comme toujours, le plus important pour les mathématiciens doués est de pousser l'avancement des connaissances et d'élargir sa propre sphère de connaissances mathématiques. La meilleure méthode de sélection des mathématiques qui auront de la valeur plus tard est encore de se fier au choix des mathématiciens de bon goût.

What good is mathematics?, continued from cover

generation systems, this movement out of poverty of so many is one of the most positive aspects of global development. Perhaps it is claiming too much for mathematics to take credit for this transformation, but I think mathematics enabled it.

Although Euler was obliged by Frederick the Great to carry out some practical calculations, the vast majority of his discoveries would not get a second look from the Tech Transfer Office of the Berlin Academy. Little did they know that he was building the foundations upon which vast areas of mathematics would be constructed, including modern methods of encryption. However, it took more than 200 years for Euler's discoveries about integers to start to generate economic returns. Even with very low interest rates, the value in 1750 of any amount of income in 2012 would have been virtually zero.

The world's economy would be much different, I believe much weaker, if the grand edifice of number theory had not been built on the foundations laid by Euler and others so long ago. However, the standard methods of computing economic worth would place no value on a discovery in number theory before modern methods of encryption emerged. It is highly likely that some theorems being discovered today in areas of mathematics considered to be totally pure and without application will be the foundation for vital aspects of civilization in the twenty third century. The people of 2263 will owe a great deal to those building mathematical understandings today in areas that appear to be useless. It remains, as it has always been, most important for gifted mathematicians to continue to develop and expand the sphere of mathematical knowledge for its own sake. There is still no better method of selecting mathematics that will be of value than by relying on the taste of mathematicians of good taste.



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Cliquez <http://smc.math.ca/Bourses/Moscou/>



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FEBRUARY 2013

7 The Language of Life: When Mathematics Speaks to Biology, Gerda DeVries (Regina, SK) <http://mpe2013.org/lecture/the-language-of-life-when-mathematics-speaks-to-biology/>

11-14 Workshop on Graph C^* -algebras, etc (Univ. Western Sydney, Australia) <http://sites.google.com/site/amsiuws2012/>

11-15 Sage Days: Multiple Dirichlet Series, Combinatorics, And Representation Theory (Providence, RI) <http://www.icerm.brown.edu/sp-s13-wl>

11-17 Representation Theory, Homological Algebra & Free Resolutions (Berkeley, CA) <http://www.msri.org/web/msri/scientific/workshops/all-workshops/show/-event/Wm8999>

15 The Mathematics of Light and Sound, Nilima Nigam (Montreal, QC) <http://mpe2013.org/lecture/the-mathematics-of-light-and-sound/>

15 PIMS/UBC Distinguished Colloquium by Béla Bollobás (Vancouver, BC) www.pims.math.ca

25-March 1 AIM Workshop: Brauer groups & obstruction problems: Moduli spaces and arithmetic (Palo Alto, CA) <http://www.aimath.org/ARC/workshops/brauermoduli.html>

MARCH 2013

4-8 Whittaker Functions, Schubert Calculus & Crystals (Providence, RI) <http://www.icerm.brown.edu/sp-s13-w2>

4-8 Conference on Combinatorics, Graph Theory & Computing (Boca Raton, FL) <http://www.math.fau.edu>

18 The Mathematics of "Fracking", Anthony Pierce (Montreal, QC) <http://mpe2013.org/lecture/the-mathematics-of-fracking/>

APRIL 2013

4-6 Extension and Interpolation of Functions (Fayetteville, AR) psarrin@uark.edu <http://math.uark.edu/3742.php>

6-7 AMS Spring Eastern Sectional Meeting <http://www.ams.org/meetings/sectional/sectional.html>

8-9 IMA Mathematics in Finance (Edinburgh, UK) www.ima.org.uk

8-12 AIM Workshop: Geometric perspectives in mathematical Quantum field theory (Palo Alto, CA) <http://www.aimath.org/ARC/workshops/geometricqft.html>

8-12 Interactions between Noncommutative Algebra, Representation Theory, and Algebraic Geometry (Berkeley, CA) <http://www.msri.org/web/msri/scientific/workshops/all-workshops/show/-event/Wm9063>

11 The New Architecture of Our Financial System, Darrell Duffie (Toronto, ON) <http://mpe2013.org/lecture/the-new-architecture-of-our-financial-system/>

15-17 Geomathematics 2013. (Palatinate, Germany) <http://www.geomathematics2013.de>

15-19 Multiple Dirichlet Series, Combinatorics, And Analytic Number Theory (Providence, RI) www.icerm.brown.edu/sp-s13-w3

18 Hydrodynamic Quantum Analogs, John Bush (Quebec, QC) <http://mpe2013.org/lecture/hydrodynamic-quantum-analogs/>

MAY 2013

30 - June 1 Conference: "Geometry and Physics 2013" (GAP 2013) Location: Centre de Recherches Mathématiques, Montréal, Québec, Canada <http://www.math.uwaterloo.ca/~gap/>

JUNE 2013

3-7 PIMS/EQINOCs Automata Theory and Symbolic Dynamics Workshop (UBC, Vancouver) <http://www.pims.math.ca/scientific-event/130603-atsdw>

16-23 51st International Symposium on Functional Equations (Rzeszów, Poland) tabor@univ.rzeszow.pl

17 -28 Algebraic Topology (Berkeley, CA) <http://www.msri.org/web/msri/scientific/workshops/summer-graduate-workshops/show/-event/Wm9063>

30 - July 20 Geometric Analysis (Berkeley, CA) <http://www.msri.org/web/msri/scientific/workshops/summer-graduate-workshops/show/-event/Wm9754>

JULY 2013

1-5 Erdős Centennial (Budapest, Hungary) <http://www.renyi.hu/conferences/erdos100/index.html>

1-12 New Geometric Techniques in Number Theory (Berkeley, CA) <http://www.msri.org/web/msri/scientific/workshops/summer-graduate-workshops/show/-event/Wm9460>

16-20 HPM 2012 History and Pedagogy of Mathematics The HPM Satellite Meeting of ICME-12 (Daejeon, Korea) <http://www.hpm2012.org>

NSERC-CMS Math in Moscow Scholarships

The Natural Sciences and Engineering Research Council (NSERC) and the Canadian Mathematical Society (CMS) support scholarships at \$9,000 each. Canadian students registered in a mathematics or computer science program are eligible.

The scholarships are to attend a semester at the small elite Moscow Independent University.

Math in Moscow Program

www.mccme.ru/mathinmoscow/

Application details

www.cms.math.ca/Scholarships/Moscow

Deadline March 30, 2013 to attend the Fall 2013 semester.



Bourse CRSNG-SMC Math à Moscou

Le Conseil de Recherches en Sciences Naturelles et en Génie du Canada (CRSNG) et la Société mathématique du Canada (SMC) offrent des bourses de 9,000 \$ chacune. Les étudiantes ou étudiants du Canada inscrite(s) à un programme de mathématiques ou d'informatique sont éligibles.

Les bourses servent à financer un trimestre d'études à la petite université d'élite Moscow Independent University.

Programme Math à Moscou

www.mccme.ru/mathinmoscow/

Détails de soumission

www.smc.math.ca/Bourses/Moscou

Date limite le 30 mars pour le trimestre d'automne 2013.



Current trends in the NSERC Postdoctoral Fellowship Program 2012

Walter Craig

Chair, Mathematics - NSERC Liaison Committee

Mid-October has already passed, and along with it the autumn leaves and the colder weather have arrived; it is the time at which many of our mathematics graduate students who are planning to finish their dissertation by the end of the academic year are now thinking of their applications for postdoctoral positions. The most talented and most ambitious of our doctoral candidates normally aim for an NSERC Postdoctoral Fellowship (PDFs for short), an award of merit that will allow them to continue their research and to expand their mathematical horizons at an institution of choice. Our NSERC PDFs are intending to work at the best places and with the best mathematicians in Canada. As well, such fellowships offer the holder the opportunity to study abroad, in the US, Japan or Europe, where they may work with the best mathematicians in the world. So it was a bit of a surprise to receive on August 10, 2012, a letter from NSERC regarding recent changes in eligibility requirements for the NSERC Postdoctoral Fellowship Program, informing me that:

“... NSERC has decided to reduce the maximum number of applications an individual may submit in a lifetime to its PDF program from two to one.”

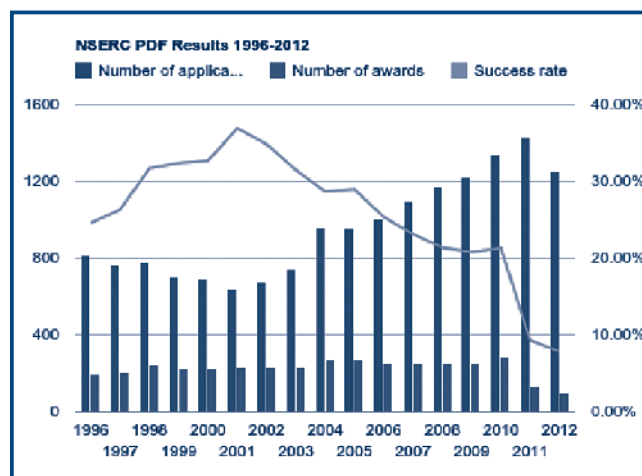
[Serge Villemure, Director Scholarships and Fellowships Division]

So from now on, an individual is only allowed to apply for a NSERC Postdoc one time in their academic career. The rationale for this change has to do with the steep decline in the success rate for such awards. In Villemure's words ‘This change to the eligibility rules will contribute to a better alignment between both the number of applications submitted and the awards available, thereby streamlining the application and review processes’. Since the awards are very competitive, the usual advice to a finishing graduate student on strategy is to apply as you write your dissertation, and if unsuccessful, to plan for a second application the following year. Out of context this seems a strange directive for a very successful and prestigious NSERC program, and so it merits looking into the facts.

Mathematicians have been aware and concerned for quite some time that the NSERC is decreasing the number of its postdoctoral fellowship awards by substantial numbers. The Math NSERC Liaison Committee (MNLCL) has had some discussions as to how to proceed to most effectively work to reverse this trend. Recently we received e-mail from Gabor Kunstetter, President of the Canadian Association of Physicists concerning the severely declining numbers of NSERC PDFs. Data from the NSERC website, when focused on PDFs, is summarized as follows:

PDF AWARD YEAR	# APPLICATIONS	# AWARDS	SUCCESS RATE
2008	1,169	250	21.4%
2009	1,220	254	20.8%
2010	1,341	286	21.3%
2011	1,431	133	9.3%
2012	1,254	98	7.8%

This basic data shows a severe decline in the success rate for the program. While there are some variations in the number of applications, clearly the most important change to the Postdoctoral Fellowships Program is the dramatic decline in the number of awards. The NSERC changes in eligibility for postdoctoral fellowships, that an individual can only apply once to the NSERC PDF program, is not the real problem. It is simply a band-aid on a symptom of the real problem, which is the very specific decline in the actual number of PDFs being awarded annually through the NSERC Postdoctoral Fellowship Program. Brent Pym compiled more data for the MNLCL, including for the PDF program over a 17 year period, which he presents in the following very telling graph. [This was passed to me by Jim Colliander, who has also made it available on his blog (<http://blog.math.toronto.edu/colliand>).]



Nassif Ghoussoub has also written about the trends on his must-read blog (<http://nghoussoub.com/>) and notes that “... the number of opportunities through the traditional PDF program has been dramatically reduced. In the past 7 years, the number of awards has dropped from 255 in 2006 to 98 in 2012, and the success rate from 25.4% to 7.8%.”

The evidence is that this decline is in parallel with the growth of industry research based initiatives, such as the NSERC-IRDF program, and the fellowships made available through the NSERC-CREATE program. In our opinion this is a national science policy issue, for which NSERC is making decisions to direct resources away from

core or basic research and towards 'applications-oriented' and/or 'industrial' research. And this is happening without having a discussion with the sciences community or with the population-at-large, and even without a public announcement of this fact. I note that the CREATE program is not completely dedicated to applied or industrial research, but it has been primarily awarded to applied projects, and currently is becoming increasingly industry focused and its support is going disproportionately to non-basic research areas. Indeed there is not one successful CREATE award in mathematics (there have been a number of proposals submitted, as we have found through informal surveys of our colleagues).

The question is what to do about these trends?

The President of the Canadian Association of Physicists (CAP) has circulated a letter of concern among the science professional societies, including the CMOS, CIC, CASCA, SSC, COMP, and the CSEB. It seems a shame that one of the gemstones of Canadian science, and the source of opportunity for so many young mathematicians and other scientists, including the early careers of many of our faculty colleagues today, is being severely cut back. I think that it is important that we be aware of the trends, and ponder ways to react to them, including discussing them with your colleagues and contacting your local MP. I would be happy to receive your further comments and reactions, as well as those of our colleagues in the other science disciplines, to the direction of these recent changes at NSERC.

Best regards,

Walter
Chair, Math NSERC Liaison Committee

Walter Craig is Professor of Mathematics and Canada Research Chair, Department of Mathematics and Statistics, McMaster University, craig@math.mcmaster.ca



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Applications or nominations should be forwarded to Dr. Roy M. Roshko, Associate Dean, Faculty of Science, University of Manitoba, 247 Machray Hall, Winnipeg, Manitoba, Canada R3T 2N2, email: roy.roshko@ad.umanitoba.ca.

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Heights in Diophantine Geometry

by Enrico Bombieri and Walter Gubler

Cambridge University Press 2007

ISBN : 97805217112293 (paperback)

ISBN : 9780521846158 (hardback)

Reviewed by **Marc Hindry**, Université de Paris - Jussieu

Le livre de Bombieri et Gubler expose très clairement les multiples notions de hauteurs qui ont été introduites dans le cadre de la géométrie diophantienne, ainsi que les grands résultats de cette théorie.

Les mots *géométrie diophantienne* ont été lancés par Serge Lang il y a un demi-siècle [9, 5]. Ils désignent une approche des équations diophantiennes via la géométrie algébrique, clarifiant la distinction entre notions géométriques (objets définis sur le corps des complexes ou plus généralement sur un corps algébriquement clos) et arithmétiques (objets définis sur \mathbf{Z} ou \mathbf{Q} ou plus généralement sur un anneau ou corps de type fini). Les solutions d'équations diophantiennes sont ainsi vues comme les points entiers ou rationnels de la variété définie par les équations. Un outil fondamental a émergé, la notion de hauteur, qui dans sa forme la plus naïve peut être définie ainsi : un point P de l'espace projectif sur \mathbf{Q} possède des coordonnées homogènes (x_0, \dots, x_n) que l'on peut choisir entières et premières entre elles, on pose alors $H(P) = \max\{|x_0|, \dots, |x_n|\}$.

Un des premiers résultats important du domaine est le théorème de Mordell-Weil qui dit que le groupe des points rationnels d'une courbe elliptique ou plus généralement d'une variété abélienne est de type fini (1922-28). Ce résultat est crucial dans la preuve du théorème de Siegel (1929) : l'ensemble des points entiers d'une courbe affine est fini pourvu que la courbe ait un genre au moins égal à un, ou au moins trois points à l'infini. L'archétype du théorème de

géométrie diophantienne est aujourd'hui le théorème de Faltings [3] (*quondam* conjecture de Mordell) : "*L'ensemble des points rationnels d'une courbe de genre au moins deux est fini*". L'analogie entre les tores algébriques (groupes algébriques isomorphes à une puissance du groupe multiplicatif \mathbf{G}_m – au moins sur la clôture algébrique du corps de définition) et les variétés abéliennes a une longue histoire. Par exemple le groupe des points entiers

sur un corps de nombres K de \mathbf{G}_m n'est autre que le groupe des unités \mathcal{O}_K^\times de l'anneau des entiers algébriques de K ; le théorème des unités de Dirichlet indique que ce groupe est de type fini, comme le groupe de Mordell-Weil. Cette analogie avait suggéré à Serge Lang une généralisation de la conjecture de Mordell, également démontrée en 1991 grâce principalement aux travaux de Vojta et Faltings [4, 13]. Si G est une variété semi-abélienne (extension d'un tore par une variété abélienne) et Γ un sousgroupe de rang fini, l'intersection de Γ avec une sous-variété X fermée dans G n'est pas dense dans X (est contenue dans une hypersurface de X) sauf si X est un translaté de sous-groupe algébrique. La démonstration de ce dernier résultat repose sur la théorie des hauteurs et pas mal de géométrie algébrique mais fait aussi appel à des techniques classiques d'approximation diophantienne qui culminent avec la preuve du théorème de Roth (1955) puis du théorème du sous-espace de Wolfgang Schmidt dans les années 70. Dans une version simplifiée, celui-ci affirme que si les L_i sont des formes linéaires indépendantes à coefficients algébriques, l'ensemble des solutions entières $x = (x_1, \dots, x_n)$ de l'inéquation

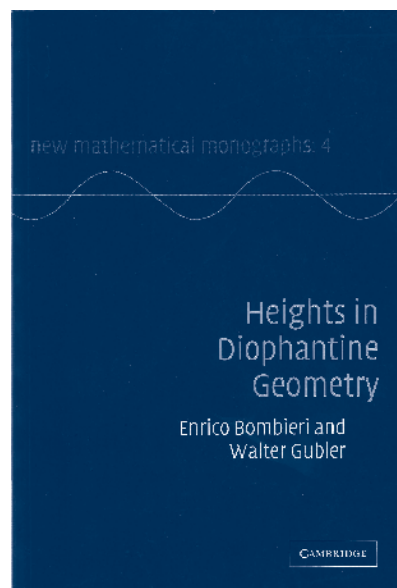
$$\prod_{i=1}^n |L_i(x)| \leq H(x)^{-\varepsilon}$$

est contenu dans une réunion finie de sous-espaces vectoriels stricts. Lorsque α est un nombre algébrique irrationnel, en prenant $n = 2$, $L_1(x, y) = x - \alpha y$, $L_2(x, y) = y$, on retrouve le théorème de Roth : le nombre de rationnels $x/y \in \mathbf{Q}$ tels que $|\alpha - x/y| \leq y^{-2-\varepsilon}$ est fini.

La théorie servant de filigrane à la première démonstration de la conjecture de Mordell est la géométrie d'Arakelov, qui a été brillamment développée depuis. Les idées arakeloviennes sont encore présentes dans le travail novateur de Vojta [12] dont Serge Lang a déclaré que c'était une des plus belles visions mathématiques qu'il ait rencontrée. Il s'agit d'analogies entre la théorie de Nevanlinna (concernant la distribution des valeurs de fonctions de variable complexe) et la théorie de l'approximation diophantienne. Les conjectures de Vojta sont loin d'être démontrées (ou infirmées) aujourd'hui mais ont servi d'inspiration aux travaux précités ; elles entraînent notamment la conjecture *abc* (confer ci-dessous).

Le livre de Bombieri et Gubler présente un exposé systématique et très accessible de l'ensemble de ces thèmes. Serge Lang a publié il y a quelques années un point de vue dans la *Gazette* [7] où il vantait les mérites d'une vision géométrique des équations diophantiennes ; ce point de vue est devenu aujourd'hui une évidence pour beaucoup. Le présent livre concilie les deux points de vue (abstraction géométrique et calculs concrets). On peut regretter que certains aspects soit peu développés comme les développements autour de la conjecture de Zilber-Pink (voir [2]) ou l'apport de la théorie d'Arakelov, mais il sera difficile de boudier son plaisir devant la richesse et la qualité des quelques 600 pages proposées.

Le titre des chapitres décrit assez bien leur contenu : 1 Hauteurs ; 2 Hauteurs de Weil ; 3 Tores linéaires ; 4 Petits points ; 5 Équation aux unités ; 6 Théorème de Roth ; 7 Théorème du sous-espace ;



8 Variétés abéliennes ; 9 Hauteurs de Néron-Tate ; 10 Théorème de Mordell-Weil ; 11 Théorème de Faltings ; 12 Conjecture abc ; 13 Théorie de Nevanlinna ; 14 Conjectures de Vojta. Trois appendices concernent un résumé des outils de géométrie algébrique, de la théorie de la ramification et de la géométrie des nombres. Le chapitre sur la conjecture abc est riche de résultats (Belyi, Elkies, Darmon-Granville) et questions ; la conjecture abc , initialement proposée par Masser et Oesterlé peut se formuler très élémentairement (sur le corps des rationnels). Si $R(n)$ désigne le *radical* de n , i.e. le produit des nombres premiers divisant n , cette conjecture prédit que pour des entiers premiers entre eux vérifiant $a + b + c = 0$ on a l'inégalité

$$\max\{|a|, |b|, |c|\} \leq C_\epsilon R(abc)^{1+\epsilon}.$$

Parmi les textes pré-existant sur la géométrie diophantienne, citons le survey de Serge Lang [10] essentiellement sans démonstration et les ouvrages [6, 9, 11]. Le livre de Bombieri et Gubler est remarquablement bien écrit dans un style précis et détaillé. Il contient des exposés concis, lumineux sur plusieurs sujets non abordés dans d'autres livres comme les hauteurs sur les grassmanniennes, les versions optimales du lemme de Siegel, les estimations explicites des hauteurs, le théorème de Beukers-Schlickewei sur l'équation aux unités, la version de Sprindzhuk du théorème d'irréductibilité de Hilbert, les minoration de hauteurs (problème des petits points de Lehmer ou Bogomolov) et même quelques résultats originaux comme le décompte des points situés sur des droites dans un threefold cubique (chapitre 11). Plusieurs chapitres produisent le meilleur exposé écrit sur leur sujet ; c'est le cas notamment des chapitres sur le théorème de Roth et le théorème du sous-espace, du chapitre donnant la démonstration du théorème de Faltings (suivant la démonstration de Vojta revisitée par le maître [1]) et le dernier appendice sur la géométrie des nombres.

C'est un ouvrage qui mérite de figurer dans la bibliothèque de tout(e) mathématicien(ne).

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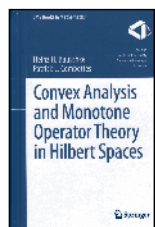
Convex Analysis and Monotone Operator Theory on Hilbert Spaces

by Heinz Bauschke and Patrick Combettes

CMS Books in Mathematics, Springer Verlag

ISBN 978-1-4419-9466-0

Reviewed by **Frank Deutsch**, Pennsylvania State University



This is a wonderful book! The authors's avowed purpose in writing this monograph was to exhibit the close relationship between three general areas of analysis: convex analysis, monotone operator theory, and the theory of nonexpansive operators.

While there are numerous books that have been written about each of the individual areas, the novelty of this book is that it emphasizes the (often surprising and elegant) connection between all three.

One of the many excellent features was the way the book was decomposed into bite-size, easily digestible chapters. There are 29 chapters of approximately 15 pages each. It is rigorously written yet detailed enough to be suitable for self-study. It is also an excellent reference and textbook. As a text, however, the sheer size of the book (468 pages) will force any instructor to make some judicious choices for a one-semester course. Each chapter ends with a collection of exercises that test the material of that chapter, and there are 400 exercises included in the book. In practice, the basic first year graduate course in analysis would be more than enough background for an intelligent reading of this book.

The authors wisely chose to restrict themselves to working only in a Hilbert space H rather than more general Banach spaces, where incorporating the additional background material would have necessitated a much larger book and probably fewer readers. But even this restriction is still more general than in most other books on convex analysis that I am aware of in which the authors worked only in finite-dimensional Euclidean spaces.

Briefly, the first two chapters contain essential background material and the relevant Hilbert space theory needed for the full understanding of the material in the book. Chapters 3-19 are concerned with convex analysis, chapters 20-25 with monotone operator theory, chapters 19 and 26 with convex optimization, chapter 27 with proximal minimization, and chapters 28-29 with best approximation in Hilbert space via projections.

There is a very useful section at the end of the book in which all the symbols and notation that are used in the book is collected with the relevant page where it is first defined and used. (However, I do have one minor quibble about one of the terms used. The authors call a real number x "positive" (respectively, "negative") if $x \geq 0$ (respectively, $x \leq 0$). Using this terminology, zero is both positive and negative! What most people call positive, namely $x > 0$, the authors call "strictly positive".)

In chapter 2, there is given a special case of a useful identity that doesn't seem to be as widely known as it should be. Namely, for each $x, y \in H$ and each real number α , it follows that

$$\|\alpha x + (1 - \alpha)y\|^2 = \alpha\|x\|^2 + (1 - \alpha)\|y\|^2 - \alpha(1 - \alpha)\|x - y\|^2.$$

One immediate consequence of this identity is that the function $f = \|\cdot\|^2$ is strictly convex.

Chapter 3 on Convex Sets contains basic convex set theory as well as basic approximation theory in Hilbert space. The main result here is that all closed convex sets are *Chebyshev* (i.e., admit unique nearest points or "best approximations") to each point in the space.

Chapter 4 on Convexity and Nonexpansiveness includes a nice introduction to nonexpansive mappings as well as some stronger properties. The classic example of a nonexpansive map is the metric projection (i.e., nearest point mapping) P_C onto a closed convex set C . Conversely, Phelps proved in 1957 that if the metric projection P_S onto a Chebyshev set S is nonexpansive, then S must be convex. The most interesting examples of nonexpansive maps are not linear in general. Under reasonable conditions on a mapping T , it can be shown that the fixed point set of T , $\text{Fix } T$, is nonempty. One useful result in this direction is the theorem of Browder-Göhde-Kirk which states that if D is a nonempty closed bounded convex set in H and $T : D \rightarrow D$ is nonexpansive, then $\text{Fix } T \neq \emptyset$.

Chapter 5 studies Fejér Monotonicity and Fixed Point Iterations. A sequence (x_n) that is Fejér monotone relative to a convex subset in H is central to the study of various iterative methods. In particular, it is useful for the construction of fixed points of nonexpansive operators. The chapter develops all the machinery necessary to conclude with a short proof of the von Neumann-Halperin theorem.

Convex cones lie somewhere between a general convex set and a linear subspace. Chapter 6 on Convex Cones and Generalized Interiors develops this concept. For example, there is the result of Moreau that every Hilbert space can be decomposed into the sum of any given closed convex cone and its polar cone. Also, tangent and normal cones, as well as recession and barrier cones, are described. Several variants of the interior of a set are discussed.

Chapter 7 on Support Functions and Polar Sets discusses support points, support functions, and dual cones. Two of the main results here are the Bishop-Phelps theorem that the set of support points of a closed convex set C are dense in the boundary of C , and the fact that every closed convex set is the intersection of the collection of all the closed half-spaces that contain it.

The three chapters 8-10 are on Convex Functions, Lower Semicontinuous Convex Functions, and Convex Functions: Variants, respectively. Here convex functions and other variants are studied (e.g., strictly convex, uniformly convex, strongly convex, quasi-convex, and uniformly quasi-convex). These are fundamental in modern optimization. Basic facts about such functions, both algebraic and topological, are developed. One of the most important properties any real-valued function may possess is lower semicontinuity, since such functions always attain their minimum on compact sets. Moreover, proper lower semicontinuous convex functions always possess a continuous affine minorant.

Chapter 11 on Convex Variational Problems discusses the existence and uniqueness of minimizers of convex functions, and some properties of minimizing sequences.

Chapter 12 on Infimal Convolution discusses the infimal convolution of two extended real-valued functions on H . The infimal convolution of two proper convex functions is again convex. Moreover, various “envelopes” of functions can be defined in terms of the infimal convolution (e.g., the Pasch-Hausdorff envelope and the Moreau envelope). Also defined and studied is the *proximal* or *proximity* operator Prox_f of a function f . Its name seems to stem from its connection to the metric projection. Namely, when f is the indicator function of a closed convex set C , then $\text{Prox}_f = P_C$. Moreover, for any proper lower semicontinuous function f on H , the set of points which minimize f is precisely the set of fixed points of Prox_f .

Chapter 13 on Conjugation includes a rather detailed study of the (Fenchel) conjugate f^* of an extended real-valued function f on H , and exhibits numerous examples. Among other things it is shown that $f = f^*$ if and only if $f = 1/2 \|\cdot\|^2$, and the Fenchel-Moreau result that if f is a proper function, then $f = f^{**}$ if and only if f is a lower semicontinuous convex function.

Chapter 14 on Further Conjugation Results contains several deeper results on conjugation. Included is the Moreau decomposition theorem in which, if f is a proper lower semicontinuous convex function on H , then $\text{Prox}_f + \text{Prox}_{f^*} = I$, the identity operator. Also included are the Moreau-Rockafellar and Toland-Singer theorems.

Chapter 15 is on Fenchel-Rockafellar duality. One of the main results here is the Attouch-Brézis theorem which gives a condition guaranteeing when the conjugate of sum is the infimal convolution of the conjugates. Also, several applications are given, one of which is von Neumann's minimax theorem.

Chapters 16 is on Subdifferentiability. The subdifferential ∂f of an extended real-valued function f on H is a useful tool for analyzing nondifferentiable convex functions. Conditions that guarantee the *sum rule*, i.e., when $\partial(f + g) = \partial f + \partial g$ holds, are given. Among other things, this yields the Brønsted-Rockafellar theorem.

Chapters 17 and 18 are on Differentiability of Convex Functions, and Further Differentiability Results, respectively. The various notions of differentiability (Fréchet, Gâteaux, directional) and subdifferentiability are described, and fundamental results are established for the relationships between them. For example, the “max formula” relates the directional derivative to the support function of the subdifferential at a given point. The further results include the Ekeland-Lebourg theorem (about the denseness of the set of points at which a convex function is Fréchet differentiable), the Subdifferential of a Maximum, and the Baillon-Haddad theorem.

In chapter 19, on Duality in Convex Optimization, it is shown that a convex optimization problem can be paired with a dual problem involving the conjugates of the functions appearing in its (primal) formulation. This interplay is studied here. In particular, special attention is given to minimization under equality and inequality constraints.

Chapter 20 is on Monotone Operators. The set-valued monotone operators play a central role in many areas of nonlinear analysis. The notion of a monotone operator is a far-reaching generalization of the classic notion of an increasing or decreasing function on the real line. Basic properties of monotone (set-valued or single-valued) operators are developed. The single-valued monotone operators are closely related to the nonexpansive operators. A single-valued *linear* monotone operator A must be a “positive operator”, i.e., $\langle Ax, x \rangle \geq 0$ for all x . The subdifferential of a proper extended real-valued function is monotone; if the function is also lower semicontinuous and convex, then its subdifferential is *maximally monotone*, i.e., its graph is not properly contained in the graph of a monotone operator. Every monotone operator has a maximal monotone extension. Using the “Fitzpatrick” function, it is even possible to explicitly describe such an extension.

Chapter 21 on Finer Properties of Monotone Operators explores some of the deeper results concerning monotone operators. For example, Minty's theorem shows that a monotone operator $A : H \rightarrow 2^H$ is maximally monotone if and only if the range of the sum $A + I$ is the whole space H . For a maximally monotone operator, the closure and interior of its domain and range are all convex. This fact is used in an elegant proof of a beautiful theorem of Bunt: If C is a Chebyshev subset of a *finite*-dimensional Hilbert space, then C must be convex. (Whether this result is true in an *infinite*-dimensional Hilbert space is one of the major unsolved problems in abstract approximation theory today.) However, I believe that the authors were overly generous in attributing credit to another author by calling this result the “Bunt-Motzkin” theorem. Indeed, Bunt proved this result in a Dutch doctoral thesis in 1934. Motzkin, a year later in 1935, only proved that this fact holds in a *two*-dimensional Euclidean space! Moreover, Motzkin's proof does not seem to extend to any Hilbert space of dimension greater than two in any obvious way. Indeed, if it did, he surely would have proved it himself. Further, Kritikos in 1938, apparently unaware of the earlier results, proved the result in any finite-dimensional Hilbert space. Thus, if anyone else's name is to be recognized next to that of Bunt, one could argue that it should be Kritikos.

Other important theorems established in Chapter 21 include the Rockafellar-Vesely theorem and Kenderov's theorem.

Chapter 22 is on Stronger Notions of Monotonicity, where such things as para, strict, uniform, strong, and cyclic monotonicity are studied. A major result here is the theorem of Rockafellar which characterizes those cyclic monotone operators that are maximal as being the subdifferentials of proper lower semicontinuous convex functions.

Chapter 23 is on Resolvents of Monotone Operators. The resolvent and the Yosida approximation are useful single-valued Lipschitz continuous operators that can be associated with a monotone operator. There is a strong interplay between firmly nonexpansive mappings and monotone operators. This chapter is a study of these operators and the interplay.

While the sum of two monotone operators is monotone, the sum of two maximal monotone operators is not maximal monotone in general. Chapter 24, on Sums of Monotone Operators, investigates useful sufficient conditions for maximality. One seemingly unrelated

continued on page 13

Jennifer Hyndman, *University of Northern British Columbia*
John Grant McLoughlin, *University of New Brunswick*

Understanding Studying and Studying Understanding

Jennifer Hyndman
UNBC

Every mathematician recognizes that adding fractions of polynomials is the same as adding integer fractions — one first factors each fraction as much as possible; finds a least common denominator; rewrites the fractions with that denominator; and then adds the numerators. What is actually happening in our understanding of this? We are able to see the pattern for adding and are able to move this pattern from a simple situation to a more complex situation.

My first-year calculus students make mistakes in adding fractions of polynomials. When I show them a pattern of adding fractions of integers that mimics their incorrect polynomial addition rule, their body language response is one of total understanding of the integer situation.

What is it that they actually understand about addition of fractions of integers? They can find a prime factorization of an integer. They can find common factors of two integers. They can find a least common multiple of two integers. They can multiply integers. They can add integers. However, I doubt that the students could articulate this list of actions as part of their understanding of how to add fractions. What I am not sure of is whether this negatively influences their ability to add fractions of integers or whether this affects their ability to transfer their understanding to adding fractions of polynomials.

I continually observe how people collect knowledge and compare it to how my students learn mathematics. Gary, a member of my family, has over 1000 CD's and listens to music 18 hours a day, yet, he cannot always identify the time signature of a song. He will happily ask me what the time signature is. With a little thought I can tell him. He knows the history of every artist on every CD while I might not be able to name the band. Which one of us has a better understanding of music? Which student has a better understanding of mathematics? The student who can describe the process of adding fractions or the student who can do the process? What about the student who can both add fractions and describe the process? Most of us would agree that this latter student has the best understanding of the three students. What about the music listener who can identify the artist and the musical structure? Does that person have a better understanding of music than Gary or I? Or just a different understanding? Does this person enjoy music more or less because they instinctively hear (and cannot ignore) the underlying structure of every piece of music they hear?

As I believe (at least I think I do) that more knowledge increases understanding and enjoyment, I have been giving one-on-one sessions and running study skills workshops on how to learn mathematics for several years. The workshops were developed and initially run with my colleague, Vivian Fayowski, the Co-ordinator of UNBC's Academic Success Centre. For one year they were also part of a UNBC Early Alert research project with Dan Ryan, Kerry Reimer, and Peter MacMillan. The focus of these

workshops has been, in essence, to explore how to organize mathematical information and how to internalize and articulate mathematics.

The most common response to the question of how a student studies is that they "do" problems. Initially they are unable to articulate what they mean by "doing" a problem. Several minutes of prompting eventually yields words like write, read, copy, draw, type, speak, hear, listen and rewrite. Is their inability to describe their own actions relevant to their difficulties in learning mathematics? I think so. However, I also believe one needs to be able to articulate what one is doing and then internalize it so it is non-verbal.

I am part of my own observations of learning. As a student in dance classes I am continually being challenged by learning new styles of dance and new choreography. Not long ago I suffered the misfortune of not being able to figure out exactly where I was supposed to be while on stage in a group number. This was unusual for me and, to prevent it from happening again, I thought long and hard about what had happened. I certainly knew the choreography thoroughly as I had been talking the group through the steps to help us practice. This talking turned out to be the problem. I had "learned" the choreography as if it included speaking. When I walked on stage and had to smile instead of talking, I was literally lost. On the verbal, visual, and kinesthetic scales of learning I am (respectively) highly kinesthetic, very visual, and almost non-verbal. Speaking the steps had interfered with my own ability to reproduce the steps without speaking. What are our students actually learning when they study mathematics or when they study any subject? What do we actually test for in a midterm or exam? Are our students self-aware enough to realize which study methods work for them and which don't? Do they even know more than one study method?

While thinking about his article I asked my family how they studied in university and how they learned to study. The actions they described all fit under my umbrella of things to do to study. What was more interesting were their comments on how they learned to study. David C's first reaction was that he had no memory of ever receiving specific instruction, and then he said he might have had some in high school English. David H's comment was that it was like learning to be a parent; you just do it. David H's son thinks his college course on time management is a waste of time as he is learning nothing new. When I work with students who are failing courses they frequently and proudly admit they spent very little time on the courses and think that they will fix their grades by "spending more time studying". However, when asked what they will do in this additional time they say "do problems" which brings us back to the earlier mentioned inability of students to describe what this means.

As instructors, what should we or can we do to assist students to be more self-aware in their studying (without giving time management courses that are a waste of time)? The lucky students, like the Davids and I, figured out effective study techniques that fit in the time we had available. Other

students do less well than they are capable of. I think we should be teaching study skills as part of the ongoing education of our students.

Here are some of the things I would do if I was to teach the perfect course as part of the perfect university degree. The course objectives provided to the students would have components of both mathematical content and study skills. The lectures would have study techniques embedded in the content development. The content to be examined would include techniques for studying the material. Here is an illustration of the second idea: discuss the definition of continuity and then discuss techniques for memorizing a definition, such as, writing it out several times, reading it out loud, reading it silently, reciting it from memory with your eyes closed versus with your eyes open. An exam question could be as simple as “list three techniques for memorizing a definition” or as self-reflective as “what study technique works best for you when you try to memorize the definition of continuity?”

Of course, at least in my opinion, learning is an activity that spirals. One initially learns a very rudimentary approximation of a concept and then rethinks and refines the approximation until, with focused attention, one understands the same thing as others do. Where does studying study skills fit in this spiral? It cannot be too early but it must be early enough to be useful. When I work with students I often come to the conclusion that they have to be *ready* to hear what I have to say about study skills (or any subject) before they can actually take in the knowledge.

Returning to learning how to add fractions of polynomials, the spiral of knowledge for this starts with the spiral for adding fractions of integers, layers on the language of polynomials, and then repeats the original spiral another time. How could we help our students learn to add fractions? Test questions like “explain the steps in adding the following fractions of integers (polynomials)”. This would be preceded by homework questions like “Build mind maps for integers and for polynomials that illustrate the concepts of adding two fractions. Discuss the similarities and differences.” The intrinsic patterns that mathematicians see can be brought into the light for our students.

Announcement



Canadian Mathematics Education Study Group (CMESG) / Groupe Canadien d'étude en didactique des mathématiques (GCEDM) is holding its **2013 Meeting** at Brock University, St. Catharines from May 24 to 28. The organization is made up of a collection of mathematicians and educators who meet from Friday evening through Tuesday morning. The core of the program is the four or five working groups that meet over three full mornings around themes related to mathematics education. This is supplemented by plenary talks, invited presentations including some by recent Ph.D. graduates in mathematics education, and a blend of social and academic activities. The registration costs of approximately \$350 include an annual membership in CMESG (which comes with a subscription to *for the learning of mathematics*) and all meeting costs (including meals) aside from accommodation. Typically the vast majority of the 100 or so participants stay on site at the host institution. Further information is available on the Brock University website: **www.brocku.ca**

consequence of this is an elegant result of Anderson and Duffin: If H is finite-dimensional and C and D are linear subspaces of H , then $P_{C \cap D} = 2P_C(P_C + P_D)^\dagger P_D$. (Here A^\dagger denotes the generalized inverse of A .)

Chapter 25, on Zeros of Sums of Monotone Operators, begins by characterizing the zeros of sums of monotone operators. Then basic algorithms are presented to construct such zeros iteratively. They are called *splitting algorithms* in the sense that they involve the operators individually. The main algorithms presented are the Douglas-Rachford algorithm, Forward-backward algorithm, and Tseng's algorithm. Variational inequalities are also considered. Let f be a proper lower semicontinuous convex mapping on H and suppose that $B : H \rightarrow 2^H$ is maximal monotone. The associated variational inequality problem is: to find $x \in H$ for which there exists $u \in Bx$ such that

$$\langle x - y, u \rangle + f(x) \leq f(y) \text{ for all } y \in H.$$

With special choices of f and B , one can recover solutions to (1) $\text{Prox}_f z$, (2) $P_C z$ and (3) The complementarity problem. Splitting algorithms can also be used to solve variational inequalities.

Chapter 26 is on Fermat's Rule in Convex Optimization. Various consequences of the basic Fermat theorem (that characterizes the minimizers of a function f as the zeros of its subdifferential) are obtained by specializing the function. In particular, minima are characterized for functions subject to constraints, and Karush-Kuhn-Tucker type theorems are highlighted.

Proximity operators are firmly nonexpansive so they can be used to devise efficient algorithms to solve minimization problems. Such algorithms are called proximal algorithms and are investigated in chapter 27 on Proximal Minimization. They include the *proximal-point* algorithm the *Douglas-Rachford* algorithm, the *forward-backward* (or *proximal*) algorithm, the *projection-gradient* algorithm, *Tseng's splitting* algorithm, and a *primal-dual* algorithm.

Basic properties of the metric projection onto a closed convex set are investigated in chapter 28 on Projection Operators. Moreover, specific formulas for the metric projections onto special polyhedral sets are established.

In chapter 29, on Best Approximation Algorithms, two general and useful algorithms are given for computing best approximations from convex sets that are intersections of a finite number of closed convex sets. Dykstra's algorithm and Haugazeau's algorithm both reduce the problem of finding best approximations from $C = \bigcap_1^m C_i$ to that of finding the limit of a sequence of best approximation problems from the individual closed convex sets C_i . Thus these algorithms are valuable in practice when it is easy to compute best approximations from the C_i (e.g., when the C_i are half-spaces, hyperplanes, or certain linear subspaces or cones).

In my view, any mathematical monograph that one always keeps conveniently on his desk (as opposed to in his bookcase) since it is frequently opened, studied, or referred to, is a classic. According to this definition, I expect the Bauschke-Combettes book to become a classic.

Separation in Convex Polytopes

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One of the more famous conjectures in Discrete or Combinatorial Geometry is attributed to I. Gohberg, H. Hadwiger, and A. Markus. It is that the smallest number, $h(K)$, of smaller positive homothetic copies of a compact convex set K (with non-empty interior) in real d -space \mathbb{R}^d with which it is possible to cover K , is at most 2^d , with equality only if K is an affine d -cube. The G-M-H Conjecture is confirmed for $d = 2$, open for $d \geq 3$, and has a rich history with various partial results and equivalent formulations, cf. [2]. For example, M. Lassak has shown that $h(K) \leq 20$ for any K in \mathbb{R}^3 , C.A. Rogers has shown that $h(K) \leq 2^d (d \log d + d \log \log d + 5d)$ for any centrally symmetric K in \mathbb{R}^d , V. Boltyanski has verified that $h(K)$ is the smallest number of unit vectors in \mathbb{R}^d whose directions illuminate the boundary of K , and the equivalent formulation of interest here that is due to K. Bezdek: if the origin is in the interior of K then $h(K)$ is the smallest number of hyperplanes in \mathbb{R}^d required to strictly separate the origin from any face of the polar of K . Strict separation means that the origin and a face are in distinct open half-spaces determined by the hyperplane. Since the polar of a polytope is also a polytope, this formulation is of particular interest in the case of polytopes and leads to the following problem.

SEPARATION PROBLEM. Let P denote a convex d -polytope in \mathbb{R}^d , $d \geq 3$. Determine the smallest number $s(P)$ such that any facet of P is strictly separated from an arbitrary fixed interior point of P by one of $s(P)$ hyperplanes in \mathbb{R}^d .

It is natural to first consider the Separation Problem for d -polytopes that have the maximum number of facets for a given fixed number of vertices. Such polytopes are the neighbourly polytopes, and the cyclic polytopes are the best known examples of neighbourly polytopes. For cyclic d -polytopes P , we have shown with K. Bezdek that there is a number $s(d) < 2^d$ such that $s(P) \leq s(d)$; cf [1], and I. Talata has verified that $s(d) \leq (d+1)^2/2$. For non-cyclic neighbourly d -polytopes (even d is the difficult case), there has been progress in the past decade in the case that $d = 4$ and the d -polytopes are totally-sewn.

Let $P = [x_1, \dots, x_n]$ denote a neighbourly 4-polytope in \mathbb{R}^4 with n vertices x_1, \dots, x_n . The facets of such a P are simplices, and any two vertices x and y of P determine an edge $[x, y]$ of P . Such a P is not intuitive (if $n > 5$), but it has the very useful property that the combinatorial structure (face lattice) of P determines the combinatorial structure of every subpolytope of P ; cf. [3]. One method of generating neighbourly 4-polytopes with $n+1$ vertices from P is by a construction called *sewing*. A sewing onto P is possible if P has an universal edge. An edge E^* of P is *universal* if E^* and any vertex of P , not in E , determine a 2-face of P . A way to picture this is that there is a projection of P from E^* into a plane that is a convex polygon C with $n-2$ vertices: the vertices of C correspond to the 2-faces of P that contain E^* and the edges of C correspond to facets of P that contain E^* . If such an universal edge

E^* of P exists, then there is a method to choose a point $x^* \in \mathbb{R}^4$ with the property that $P^* = \text{conv}(P \cup \{x^*\})$ is a neighbourly 4-polytope with $n+1$ vertices. We say that P^* is constructed by sewing x^* onto P with the *sewing edge* E^* .

Let $n \geq 8$ and $P = [x_1, \dots, x_n]$ be *totally-sewn*; that is, there is a sequence $\{P_m\}$, $7 \leq m \leq n-1$, of subpolytopes of $P = P_n$ such that $P_m = [x_1, \dots, x_m]$ and $P_{m+1} = [x_1, \dots, x_m, x_{m+1}]$, is obtained by sewing x_{m+1} onto P_m . We note that a cyclic d -polytope $P = [x_1, \dots, x_n]$ that satisfies Gale's Evenness Condition with respect to the ordering $x_1 \leq \dots \leq x_n$ is totally-sewn. It is known that $s(P) \leq 9$ for certain classes of P (for example, the cyclic 4-polytopes) and in the case that $n \leq 10$ (there are 431 combinatorial types of neighbourly 4-polytopes with at most 10 vertices, of which 287 are totally-sewn). It thus appears that " $s(P_n) \leq 9$ for any n " break down if n is sufficiently large.

There is an upper bound for $s(P)$, independent of n , for arbitrary totally-sewn 4-polytopes P . Specifically, F. Fodor and I have verified that $s(P) \leq 16$ although $s(P) < 16$ is the likely answer. Success here is due to the fact that we can distinguish the vertices of totally-sewn 4-polytopes P in a way that is distinct from the *sewing order* (P_{m+1} being obtained from P_m for $m \leq 7$ yields the sewing order of $x_1, x_2, \dots, x_6, \dots, x_m, x_{m+1}, \dots, x_n$). In addition to the sewing order, we distinguish vertices in the following manner: for $6 \leq m < s \leq n$, each vertex x_s of P stems from a *unique* universal edge E of P_m via a sequence of vertices that are sewn after x_m and before x_s , and all vertices in $\{x_{m+1}, \dots, x_n\}$ that stem from the same E are said to be of the *same type with respect to P_m* . If we think of sewing a vertex as sewing a button then the different types of vertices would correspond to different coloured buttons. In the finished garment, it is not the order in which the buttons were sewn that is important, but rather the groupings of buttons of the same colour. The verification of $s(P) \leq 16$ is based upon the following observations:

If X is a set of vertices of a given fixed type with respect to P_m , then the convex hull of X is disjoint from any hyperplane spanned by vertices from $\{x_1, \dots, x_n\} \setminus X$.

The number of vertex types with respect to P_m of P is very small (less than m) compared to the number $n(n-3)/2$ of facets of $P = P_n$. For example, a cyclic P contains only one type of vertices.

The next step is to determine upper bounds for $s(P)$ (independent of the number of vertices of P) for any neighbourly 4-polytope P , or for any totally-sewn d -polytope P , $d \geq 6$ and even. We do not yet know if the preceding approach will be of use in case of the former, but it should be extendable in case of the latter.

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CALL FOR NOMINATIONS 2013 Doctoral Prize



The CMS Doctoral Prize recognizes outstanding performance by a doctoral student. The prize is awarded to a candidate who received a Ph.D. from a Canadian university in the preceding year (January 1st to December 31st) and whose overall performance in graduate school is judged to be the most outstanding. Although the dissertation is the most important criterion (the impact of the results, the creativity of the work, the quality of exposition, etc.) other publications, activities in support of students and other accomplishments will also be considered.

Nominations that were not successful in the first competition will be kept active for a further year (with no possibility of updating the file) and will be considered by the Doctoral Prize Selection Committee in the following year's competition.

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Candidates must be nominated by their university and the nominator is responsible for preparing the documentation described below, and submitting the nomination to the Canadian Mathematical Society. No university may nominate more than one candidate.

The documentation shall consist of:

- A curriculum vitae prepared by the student.
- A résumé of the student's work written by the student and which must not exceed ten pages. The résumé should include a brief description of the thesis and why it is important, as well as of any other contributions made by the student while a doctoral student.
- Three letters of recommendation of which one should be from the thesis advisor and one from an external reviewer. A copy of the external examiner's report may be substituted for the latter. More than three letters of recommendation are not accepted.

The deadline for the receipt of nominations is **February 15, 2013**. All documentation, including letters of recommendation, must be submitted electronically to docprize@cms.math.ca

APPEL DE MISES EN CANDIDATURE Prix de doctorat 2013



La SMC a créé ce Prix de doctorat pour récompenser le travail exceptionnel d'un étudiant au doctorat. Le prix sera décerné à une personne qui aura reçu son diplôme de troisième cycle d'une université canadienne l'année précédente (entre le 1er janvier et le 31 décembre) et dont les résultats pour l'ensemble des études supérieures seront jugés les meilleurs. La dissertation constituera le principal critère de sélection (impact des résultats, créativité, qualité de l'exposition, etc.), mais ne sera pas le seul aspect évalué. On tiendra également compte des publications de l'étudiant, de son engagement dans la vie étudiante et de ses autres réalisations.

Les mises en candidature qui ne seront pas choisies dans leur première compétition seront considérées pour une année additionnelle (sans possibilité de mise à jour du dossier), et seront révisées par le comité de sélection du Prix de doctorat l'an prochain.

Le lauréat du Prix de doctorat de la SMC aura droit à une bourse de 500 \$. De plus, la SMC lui offrira l'adhésion gratuite à la Société pendant deux ans et lui remettra un certificat encadré et une subvention pour frais de déplacements lui permettant d'assister à la réunion de la SMC où il recevra son prix et présentera une conférence.

Candidatures

Les candidats doivent être nommés par leur université; la personne qui propose un candidat doit se charger de regrouper les documents décrits aux paragraphes suivants et de faire parvenir la candidature à la Société Mathématique du Canada. Aucune université ne peut nommer plus d'un candidat.

Le dossier sera constitué des documents suivants :

- Un curriculum vitae rédigé par l'étudiant.
- Un résumé du travail du candidat d'au plus dix pages, rédigé par l'étudiant, où celui-ci décrira brièvement sa thèse et en expliquera l'importance, et énumérera toutes ses autres réalisations pendant ses études de doctorat.
- Trois lettres de recommandation, dont une du directeur de thèse et une d'un examinateur de l'extérieur (une copie de son rapport serait aussi acceptable). Le comité n'acceptera pas plus de trois lettres de recommandation.

Les candidatures doivent parvenir à la SMC au plus tard le **15 février 2013**. Veuillez faire parvenir tous les documents par voie électronique avant la date limite à prixdoc@smc.math.ca

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The Lorentz Gas: Periodic or Not?

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The Lorentz gas. The reconciliation between classical and statistical mechanics remains a long-term goal of mathematical physics. This is a big challenge, or even a paradox, since the former is time-reversible, whereas the latter is not. Ludwig Boltzmann's work in the kinetic theory of gases was often attacked by "positivist" skeptics, who sought to discredit it. But Boltzmann's work survived because it proved useful. Oscar Lanford's 1974 rigorous mathematical derivation of the Boltzmann equation is perhaps its best endorsement so far.

In this note, we describe surprising results in a related area: the so-called Lorentz gas problem, which models the dynamics of noninteracting point particles moving in an array of immobile obstacles. The problem takes its name from Hendrik Lorentz's 1905 proposal to describe the motion of electrons in a metal by a kinetic equation (more precisely, a *linear Boltzmann equation*). Lorentz's idea was this: the metallic atoms are rigid balls that do not recoil when struck by the much lighter electrons, while collisions between electrons are rare events that can be neglected. By letting $f \equiv f(t, x, v)$ be the electron distribution function (i.e., the number density at time t of electrons located at position x with velocity v), then mimicking Boltzmann's arguments, Lorentz derived the following equation satisfied by f :

$$\left(\frac{\partial f}{\partial t} + v \cdot \nabla_x f \right) = Nr^2 |v| \int_{|n|=1} (f(t, x, v - 2(v \cdot n)n) - f(t, x, v))(v \cdot n)_+ dn.$$

Here r is the atomic radius and N the number density of atoms. Lorentz's original equation included an external electric field, but here we disregard the electric field and focus on the equation's right-hand side (the so-called collision integral), where the problem of irreversibility resides.

By assuming the atoms' centers to be spatially distributed via a Poisson law, Giovanni Gallavotti rigorously derived Lorentz's equation in an appropriate scaling limit, [1]. But since metals commonly occur as crystals, it's also natural to investigate a periodic distribution of atoms. To see where this leads, we look first at a few fundamental notions in kinetic theory.

The distribution of free path lengths. In kinetic theory, one obtains the mean free path by appropriately averaging the so-called free path length. Given a position $x \in \mathbf{R}^3$ and a direction $\omega \in \mathbf{S}^2$, the free path length of a particle starting from x in the direction ω is the largest $\ell > 0$ such that the particle moves freely, i.e. without

collision, on the segment $[x, x + \ell\omega]$. We denote this free path length by $\ell(x, \omega)$.

Assuming, as Gallavotti did, that the atoms' centers are distributed according to a Poisson law means that the probability of an (infinite) configuration of these centers meeting a given Borel set A in exactly k points is $(\lambda|A|)^k e^{-\lambda|A|}/k!$, where $|A|$ is the volume of A and $\lambda > 0$ is called the intensity of the Poisson process. By choosing A to be a cylinder of length $s > 0$ and diameter $2r$, one finds that the probability $p_r(s)$ that $\ell(x, \omega) > s$ is $e^{-\lambda s \cdot 4\pi r^2}$ (in space dimension $d = 3$). Integrating in s , one finds that the expected value of $\ell(x, \omega)$ (i.e., the mean free path) is $\langle \ell \rangle = 1/4\pi r^2 \lambda$. Thus $p_r(s) = e^{-s/\langle \ell \rangle}$, and this exponential decay also holds in any space dimension.

However, in the periodic setting, the distribution of free path lengths $p_r(s)$ cannot be exponential. The story of how this came to light begins with a seemingly unrelated estimate, [2], and continues to complete proofs: [3], [4]. Here we give some of the ideas involved.

First, the averaging procedures used to find the mean free path in the random versus the periodic case are quite different (in the random case, one averages over configurations; in the periodic case, the obstacle system is fixed but one averages jointly over initial positions running through one period and initial directions running over the unit sphere, with uniformly distributed initial positions and directions). But the distinctive feature of the periodic case that gives it a fundamentally different distribution of free path lengths is the presence of infinitely long open corridors in the domain through which electrons move. In space dimension $d = 2$, these corridors take the form of open strips oriented in rational directions $(p, q) \in \mathbf{Q}^2 \setminus \{(0, 0)\}$, satisfying

$$p^2 + q^2 < \frac{1}{4r^2},$$

where each strip has width $w = (p^2 + q^2)^{-1/2} - 2r$. This inequality gives an estimate of the number of open strips passing through a period of the obstacle system. On the other hand, if x belongs to an open strip of width w and direction Ω , and if $\omega \in \mathbf{S}^2$, then

$$\ell(x, \omega) \geq b/|\sin(\Omega, \omega)|,$$

where b is the distance from x to the boundary of the strip. These inequalities lead to a less-than-exponential lower bound for the distribution of free path lengths $p_r(s)$. With more work, in any dimension d and for some $C' > C > 0$, one gets both upper and lower bounds on $p_r(s)$ of the form

$$\frac{C}{sr^{d-1}} \leq p_r(s) \leq \frac{C'}{sr^{d-1}} \quad \text{for all } s > 1/r^{d-1}.$$

These apply to the simplest periodic configuration of atoms, i.e., balls of radius r centered at points of the cubic lattice \mathbf{Z}^d .

The Boltzmann-Grad limit ($r \rightarrow 0$) of the periodic Lorentz gas. We now show how all this bears on the Lorentz gas problem. Start from an initial distribution function of electrons of the form $f_r(0, x, v) := f^{in}(r^{d-1}x, v)$, and set $f_r(t, x, v) := f^{in}(\Phi_r(t/r^{d-1}, r^{d-1}x, -v))$, where $\Phi_r(t, x, v) := (x_t, v_t)$,

with x_t (resp. v_t) designating the position (resp. direction) at time t of an electron moving in the periodic system of atoms of radius r centered at integer lattice points and starting from (x, v) at time $t = 0$. The Banach-Alaoglu theorem shows that the family f_r is weak-* relatively compact in $L^\infty(\mathbf{R}_+ \times \mathbf{R}^d \times \mathbf{S}^{d-1})$ as $r \rightarrow 0$. But because of the slow decay of $p_r(s)$ discussed above, one can show that no limit point of f_r can satisfy Lorentz's equation (see [5] for a proof). This is quite surprising, and shows that the validity of the linear Boltzmann equation is more delicately dependent on spatial configurations than previously thought.

But, if Lorentz's gas does not obey Lorentz's equation in the periodic case, what sort of kinetic equation does it obey? The key to this is to seek an equation posed in an "enlarged" phase space; i.e., we consider

$$f_r(t, x, v) \rightarrow f(t, x, v) := \int_0^\infty \int_{-1}^1 F(t, x, v, s, h) ds dh \quad \text{as } r \rightarrow 0,$$

where F is a number density defined on a larger phase space involving the extra s and h , in addition to the usual time, position, and velocity variables t, x, v . For each gas particle, s measures the time until its next collision with an obstacle, while h is its impact parameter at the next collision (i.e., the sine of the angle between the particle velocity and the outward normal to the obstacle at the collision point). The number density F is more complex than the usual distribution function, but there is good reason for using it: F is governed by an equation "in closed form" (i.e., one that involves no unknown quantities other than F itself). This equation takes the form

$$(\partial_t + v \cdot \nabla_x - \partial_s) F(t, x, v, s, h) = \int_{-1}^1 P(s, h, h') F(t, x, R[h']v, 0, h') dh'$$

where $R[h']$ rotates the velocity appropriately upon impact, and $P(s, h, h')$ is the probability density that a particle leaving an obstacle with impact parameter h' will hit the next obstacle after time s and with impact parameter h . Remarkably, P does not depend on the particle's direction. It's not hard to check that, if $P(s, h, h') = \sigma e^{-\sigma s}$, then $F(t, x, v, s, h) = f(t, x, v) \sigma e^{-\sigma s}$ satisfies the equation above if and only if f satisfies Lorentz's equation with $\sigma = N\pi r^2$. But as explained earlier, P does not decay exponentially fast in the periodic case. It turns out to be more complex, mixing s, h , and h' in a nontrivial way. For $d = 2$, the equation above for F was found in [6], and an expression for P was found in [6] and [7]. Complete proofs of the equation's validity in all space dimensions and for all lattices were recently obtained by J. Marklof and A. Strömbergsson in [8].

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