IN THIS ISSUE
DANS CE NUMÉRO

From the President's Desk
- The pros and cons of contests. 1
- Le pour et le contre des concours 4

Editorial
- Problem Solving / La résolution de problèmes 2

Calendar of Events 5

Education Notes
- Teaching the Concept of Research Through Games 7
- Using Game-like Scenarios to Reinforce Understanding 9

Book Reviews
- CRM Monograph Series. 10
- Chicago Lectures in Mathematics Series, 2012. 12

Career Opportunity. 13

Research Notes
- Minimal surfaces as extremals of eigenvalue problems 14
- The Halting Problem 16

Mathematics Challenge. 18

2013 CMS Summer Meeting / Réunion d' été SMC 2013 19

Election Notice / Avis d'élection 20

Call for Nominations
- Research 21
- Editorial 22
- Education. 24

Advancing Junior Canadian Number Theorists . . 23

Canadian Mathematical Society
Société mathématique du Canada

CMS NOTES de la SMC
March April 2013

From the Vice-President's Desk

Karl Dilcher
CMS Vice-President Atlantic

The pros and cons of contests

Mathematical contests and competitions have long been among the main components of the Society's educational initiatives. Indeed, the first entry under "Education" on the CMS website is about competitions, and the introduction to those extensive and well-organized pages states that "Competitions are an important part of learning mathematics and a fun activity for students of all ages". It then goes on to describe the CMS's support for competitions. The Society has a Mathematical Competitions Committee, and at least four more of the 14 standing committees are partly and/or indirectly involved with competitions.

Over the years I have also been involved with competitions, both within the CMS and at Dalhousie. As a member and then chair of the Endowment Grants Committee I played a role in the awarding of grants to several excellent proposals involving local, regional, and national competitions. In my own department, before I became chair, I was involved for many years in organizing training sessions for the Putnam and the Science Atlantic (then called APICS) competitions. I supervised the Putnams and organized travel and accommodation to the APICS competitions. I even designed a (now discontinued) problem solving course featuring competition-type problems, aimed at preparing students for various contests.

Before I go on, let me state my unambiguous support for problem solving as a mathematical activity. It was perhaps not such a coincidence that my colleague and office neighbour Swami (S. Swaminathan) independently chose for his editorial in this issue a topic quite similar to what I was going to write about. We both have similar mathematical tastes, and our approaches to mathematics are largely problem based. Along with many other mathematicians, we take delight in beautiful problems, and usually even more delight in our efforts to solve them. For us mathematicians the word "problem" has a positive connotation, which is certainly not the case in everyday non-mathematical usage of the word. In fact, it is difficult to convince a non-mathematician (or, to be fair, a non-scientist), that a problem can actually be beautiful.

It is also true that "Mathematics is not a spectator sport" (the title of a book by George M. Phillips, Springer, 2005). But is it a sport, in the competitive sense? You learn by doing; this is the premise of George Phillips's book, and this is what we tell our students as we give them homework and practice problems. But do we learn better and faster by doing mathematics fast and under time pressure?

This ambiguity was also the topic of a brief article, "Pros and Cons of Math Competitions" (www.artofproblemsolving.com/Resources/articles.php) by Richard Rusczyk, founder of the very interesting and engaging web resource "Art of Problem Solving" which is listed on the CMS website under "Problem Solving Resources". Referring to competitions for middle school and high school students in the

FRANÇAIS PAGE 4

continued on page 6

Problem Solving

Srinivasa Swaminathan,
Dalhousie University

The teaching of undergraduate and graduate courses in mathematics involves routine exposition of standard topics illustrated by solved problems from the texts. Weekly assignments are generally based on exercises from textbooks. Generally mathematics is studied, not for its own sake, but because the ultimate object is merely to pass an examination or to acquire the minimum knowledge necessary for dealing with some other subject of study. In such a situation how much of problem solving ability is acquired by students is doubtful; just propose a problem outside the normal curriculum – one would find that most students are unable to solve it. However, there are gifted students in almost every class. Problem solving sessions are held to train such students so that they can compete in the annual Putnam and similar exams; they learn to apply previously acquired knowledge to new and unfamiliar situations.

Problems can be classified under different headings: mechanical or drill problems, those that require understanding of the concepts; those that require problem-solving skills or original thinking; those that require research or library work; and finally those that are group projects, requiring group participation.

The importance of problem solving in the learning process and also in the growth and development of mathematics has been recognized and emphasized by many prominent authors; for example, George Pólya's *How to Solve It*. New branches of mathematics have arisen from the search for solutions of challenging problems. Noteworthy examples are: the successful attack on the brachistochrone problem by the Bernouilli brothers and the role played by their solution in the evolution of the Calculus of Variations. Mathematical theory of probability arose from the investigations by Pacioli, Cardan, Tartaglia, Pascal and Fermat. Topology and Graph Theory had their origin in Euler's analysis of a problem about crossing bridges. The fact that in some fields (algebraic) the resolution into prime factors is not unique as it is in common arithmetic led Dedekind to restore this highly desirable uniqueness by the invention of *ideals*, an important concept in algebraic geometry.

Many mathematical journals contain problem sections inviting readers to submit solutions. From these solutions the editors select what they consider to be the 'best' solution which they publish along with other interesting solutions, if any. Solution(s) of difficult and challenging problems may lead to interesting further investigations of the devices employed. Selecting proposals pose a more challenging task to the editors than the selection of solutions; the editors seek to have a diversity of high-quality proposals in geometry, analysis, number theory, etc rising above the level of unimaginative text-book exercises. Elegant proposals attract a wide range of would-be solvers. The criteria for elegance can be summarized in the ABCD's of elegance as follows: A for accuracy, B for brevity, C for clarity and D for display of insight, ingenuity, originality and generalization, if possible.

Periodically collections of proposed and solved problems from well known journals are published. Thus the *Otto Dunkel Memorial Problem Book* was published by the Mathematical Association of America in 1957 on the occasion of the 50th anniversary of the *American Mathematical Monthly* which contains a popular section on problems. The most recent such collection is *A Mathematical Orchard* by the Mathematical Association of America which contains 208 challenging, original problems with carefully worked, detailed solutions. One can spend hours browsing through this book, thinking about and trying to solve problems before looking at the solutions. As I was thinking about problem 62 of the book, [which is to find the fifth digit (the ten thousands digit) from the end of the number 5 raised to the power of 5, which is raised to the power of 5,… up to five times!] the idea for writing this editorial occurred to me! [Answer: zero].

La résolution de problèmes

Srinivasa Swaminathan,
Dalhousie University

L'enseignement des mathématiques à l'université, tous cycles confondus, implique une exposition constante aux sujets de base illustrés par des problèmes résolus présentés dans les manuels. Les devoirs hebdomadaires reposent généralement sur des exercices tirés de manuels. De façon générale, on étudie les mathématiques non pas pour elles-mêmes, mais dans le but de réussir un examen ou d'acquérir les connaissances minimales nécessaires pour réussir dans une autre discipline. Dans de tels cas, on peut s'interroger sur la compétence réelle des étudiants en résolution de problèmes; il suffit de proposer un problème qui sort du programme normal pour constater que la plupart des étudiants ne parviendront pas à le résoudre. Toutefois, on trouve des étudiants doués dans presque tous les groupes. Les séances de résolution de problèmes servent à préparer de tels étudiants au concours annuel Putnam ou à d'autres examens du genre; les étudiants y apprennent à appliquer leurs connaissances à de nouvelles situations non familières.

Les problèmes se divisent en plusieurs catégories : les problèmes mécaniques ou d'entraînement, qui nécessitent une bonne compréhension des concepts; les problèmes qui nécessitent une facilité à résoudre des problèmes ou de l'imagination; les problèmes qui nécessitent de la recherche ou du travail à la bibliothèque et enfin les problèmes qui se font en équipe, qui nécessitent la participation d'un groupe.

De nombreux auteurs célèbres reconnaissent et valorisent l'importance de la résolution de problèmes dans le processus d'apprentissage et dans l'évolution des mathématiques, notamment George Pólya dans son ouvrage *How to Solve It*. La recherche de solutions à des problèmes difficiles a donné naissance à de nouveaux domaines mathématiques. Prenons les tentatives réussies de résolution du problème brachistochrone par les frères Bernoulli et le rôle qu'a joué leur résolution sur l'évolution du calcul des variations. La théorie mathématique des probabilités découle des recherches de Pacioli, Cardan, Tartaglia, Pascal et Fermat. La topologie et la théorie des graphes tirent leur origine de l'analyse du problème des sept ponts par Euler. Le fait que dans certains domaines (algébriques) la résolution en facteurs premiers ne soit pas unique comme elle l'est en arithmétique courante a poussé Dedekind à rétablir cette unicité fortement souhaitable en élaborant la théorie des *idéaux*, devenue importante en géométrie algébrique.

De nombreuses revues mathématiques invitent les lecteurs à proposer des solutions à des problèmes qu'elles proposent. Les rédacteurs choisissent, parmi ces solutions, celles qu'ils estiment la « meilleure » solution et la publient avec d'autres solutions intéressantes, s'il y en a. La résolution de problèmes difficiles et complexes peut mener à d'intéressantes études plus approfondies des stratégies employées. Toutefois, le choix des problèmes proposés constitue une tâche plus difficile que le choix des solutions; les rédacteurs souhaitent en effet présenter des problèmes diversifiés de grande qualité en géométrie, en analyse, en théorie des nombres, etc., qui sortent du cadre peu créatif de certains exercices trouvés dans les manuels. Les problèmes élégants attirent un large éventail de « chercheurs de solutions ». Un problème dit « élégant » est un problème exact, bref, clair et démontrant l'intelligence, l'ingénuité, l'originalité et la capacité de généralisation de son auteur, dans la mesure du possible.

Des collections de problèmes accompagnés de leurs solutions parus dans des grandes revues sont parfois publiées. Ainsi, la Mathematical Association of America (MAA) a publié en 1957 le recueil *Otto Dunkel Memorial Problem Book* à l'occasion du 50e anniversaire de la revue *American Mathematical Monthly*, qui contient une section populaire sur les problèmes. La toute dernière collection de la sorte, aussi publiée par la MAA, s'intitule *A Mathematical Orchard* et contient 208 problèmes stimulants et originaux, accompagnés de solutions soigneusement présentées et détaillées. On peut passer des heures à feuilleter ce livre, à réfléchir à des problèmes et à tenter de les résoudre avant d'aller voir les solutions. C'est d'ailleurs en m'attardant au problème nº 62 du livre (qui consiste à trouver le cinquième chiffre [celui des dizaines de milliers] du nombre 5 à la puissance 5, à la puissance 5, et ainsi de suite cinq fois!] que l'idée de cet éditorial m'est venue! [Réponse : zéro]

Letters to the Editors
Lettres aux Rédacteurs

The Editors of the NOTES welcome letters in English or French on any subject of mathematical interest but reserve the right to condense them. Those accepted for publication will appear in the language of submission. Readers may reach us at **notes-letters@cms.math.ca** or at the Executive Office.

Les rédacteurs des NOTES acceptent les lettres en français ou anglais portant sur un sujet d'intérêt mathématique, mais ils se réservent le droit de les comprimer. Les lettres acceptées paraîtront dans la langue soumise. Les lecteurs peuvent nous joindre au bureau administratif de la SMC ou à l'addresse suivante : **notes-lettres@smc.math.ca**.

NOTES DE LA SMC

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Les Notes de la SMC, les rédacteurs et la SMC ne peuvent être tenus responsables des opinions exprimées par les auteurs.

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Le pour et le contre des concours

Karl Dilcher
vice-président de la SMC (Atlantique)

Les concours mathématiques comptent depuis longtemps parmi les principales activités éducatives de la Société. En effet, l'option « concours » est le premier choix de l'onglet « Éducation » sur le site de la SMC, et l'introduction de ces pages riches et bien organisées commence par la phrase suivante : « Les concours jouent un rôle important dans l'apprentissage des mathématiques et sont une activité amusante pour les élèves de tout âge. » Le reste de l'introduction décrit ensuite l'appui de la SMC aux concours mathématiques. La Société a de plus un Comité des concours mathématiques, et au moins quatre de ses quatorze comités permanents sont liés directement ou indirectement aux concours.

Au fil des ans, j'ai contribué à l'organisation de concours à la SMC et à l'Université Dalhousie. En tant que membre, puis président du Comité des bourses du fonds de dotation, j'ai participé à l'attribution de bourses pour la réalisation de plusieurs excellents projets liés à l'organisation de concours locaux, régionaux et nationaux. Dans mon propre département, avant d'en accepter la direction, j'ai participé de nombreuses années à l'organisation de séances préparatoires aux concours Putnam et du Conseil des provinces atlantiques pour les sciences (CPAS). J'ai surveillé l'examen du Putnam et organisé le transport et l'hébergement aux concours du CPAS. J'ai même conçu un cours de résolution de problèmes (qui n'est plus au programme) proposant des problèmes types de concours et visant à préparer les étudiants à participer à des concours de toutes sortes.

Avant d'aller plus loin, je tiens à déclarer mon appui indéfectible à la résolution de problèmes comme activité mathématique. Ce n'est peut-être pas tant une coïncidence que mon collègue et voisin de bureau Swami (S. Swaminathan) ait choisi pour ce numéro – sans me consulter – un sujet d'éditorial très semblable à ce que j'allais écrire. Nous avons des intérêts mathématiques très similaires, et notre approche mathématique repose tous les deux en grande partie sur la résolution de problèmes. À l'instar de nombreux autres mathématiciens, nous prenons plaisir à aborder de magnifiques problèmes, et généralement encore plus de plaisir à tenter de les résoudre. Pour nous, mathématiciens, le mot « problème » a une connotation positive, ce qui ne correspond certainement pas à l'usage courant non mathématique du terme. En fait, il est difficile de convaincre un non-mathématicien (ou, soyons équitable, un non-scientifique), qu'un problème peut être d'une grande élégance.

Il est aussi vrai que la mathématique n'est pas un sport de spectateurs (clin d'œil au titre d'un livre de George M. Phillips, *Mathematics is not a spectator sport*, Springer, 2005). Mais s'agit-il vraiment d'un sport, au sens compétitif du terme? L'être humain apprend par l'expérience : telle est la prémisse de base de George Phillips, et celle que nous énonçons à nos étudiants en leur donnant des devoirs et des problèmes à résoudre. Mais apprenons-nous mieux et plus rapidement en faisant des mathématiques à toute vitesse et sous pression?

Cette ambiguïté a fait l'objet d'un court article intitulé « Pros and Cons of Math Competitions » (www.artofproblemsolving.com/Resources/articles.php) de Richard Rusczyk, créateur d'un site web très intéressant et stimulant appelé *Art of Problem Solving*, présenté sur le site de la SMC dans la section « Ressources en résolution de problèmes ». À propos de concours à l'intention des élèves de niveau secondaire aux États-Unis, Rusczyk écrit ceci : « La valeur initiale de ces concours est évidente : ils stimulent l'intérêt des élèves pour les mathématiques et les amènent à valoriser le travail intellectuel. Les enfants aiment jouer, et ils sont nombreux à transformer n'importe quelle activité en concours ou, autrement dit, en performance à améliorer. Les concours mathématiques poussent les jeunes à devenir bons en mathématique, tout comme le sport les incite à se mettre en forme. Avec le temps, les élèves mettent le jeu de côté. À ce moment-là, on espère qu'ils ont développé suffisamment d'intérêt pour l'activité sous-jacente. » [traduction libre]

Ce sont là des arguments très solides et convaincants en faveur des concours de mathématiques. Rusczyk poursuit toutefois son argumentaire par une mise en garde à propos de ce qu'il appelle les « concours pédagogiques » et les concours qui exigent beaucoup de vitesse et de mémorisation. Il explique que les concours doivent être bien conçus et qu'ils devraient renforcer chez les élèves leurs compétences en étude et en résolution de problèmes complexes. Rusczyk mentionne deux autres inconvénients, soit le danger de pousser les jeunes au-delà de leurs capacités et de leur faire subir une expérience humiliante et décourageante, qui se voulait au départ une leçon d'humilité et une source d'inspiration, et le danger de l'épuisement, qui risque de détourner les jeunes non seulement des concours, mais aussi des mathématiques en général.

Pourquoi accordais-je autant d'importance à l'article de Rusczyk? Parce qu'il exprime exactement ma propre ambivalence par rapport aux concours mathématiques, à la fois en tant que partie prenante de telles activités comme enseignant et organisateur, et en tant que participant à une autre époque (au début des années 1970) et dans un autre pays. J'étais moi-même attiré vers les mathématiques parce qu'elles étaient – et le sont toujours – une des activités les moins compétitives qui soient. Je pouvais – et je le peux toujours – prendre mon temps, tout mon temps, et parvenir à mes fins. Tout élément compétitif me rebutait et, d'instinct, je me tenais loin des « sujets chauds ».

Suite à la page 6

MARCH 2013

18 The Mathematics of “Fracking”, Anthony Pierce *(Montreal, QC)* http://mpe2013.org/lecture/the-mathematics-of-fracking/

21 The Mathematical Challenges of Earth-System and Weather Prediction, Gilbert Brunet *(Environment Canada)* http://www.mathstat.uottawa.ca/welcome.html

APRIL 2013

4 - 6 Extension and Interpolation of Functions *(Fayetteville, AR)* psharrin@uark.edu, http://math.uark.edu/3742.php

6-7 AMS Spring Eastern Sectional Meeting http://www.ams.org/meetings/sectional/sectional.html

8-9 IMA Mathematics in Finance *(Edinburgh, UK)* www.ima.org.uk

8-12 AIM Workshop: Geometric perspectives in mathematical Quantum field theory *(Palo Alto,CA)* http://www.aimath.org/ARC/workshops/geometricqft.html

8-12 Interactions between Noncommutative Algebra, Representation Theory, and Algebraic Geometry *(Berkeley, CA)* http://www.msri.org/web/msri/scientific/workshops/all-workshops/show/-/event/Wm9063

10 Les mathématiques pour faire parler la Terre (Images, Earthquakes and Plumes), Ingrid Daubechies *(CRM, Montreal, QC)* http://mpe2013.org/lecture/mpe2013-simons-public-lecture-crm/

11 The New Architecture of Our Financial System, Darrell Duffie *(Toronto, ON)* http://mpe2013.org/lecture/the-new-architecture-of-our-financial-system/

15-17 Geomathematics 2013. *(Palatinate, Germany)* http://www.geomathematics2013.de

15-19 Multiple Dirichlet Series, Combinatorics, And Analytic Number Theory *(Providence, RI)* www.icerm.brown.edu/sp-s13-w3

18 Hydrodynamic Quantum Analogs, John Bush *(Quebec, QC)* http://mpe2013.org/lecture/hydrodynamic-quantum-analogs/

29 – May 10 J-holomorphic Curves in Symplectic Geometry, Topology and Dynamics, *(CRM, Montreal, QC)* http://www.crm.umontreal.ca/2013/Curves13/index_e.php

MAY 2013

10 Les ponts de Königsberg, les digues de Hollande et la chûte de Wall Street, Paul Embrechts *(Montreal, QC)* http://www.crm.umontreal.ca/2013/

22-24 Algebra and Topology Conference *(Nantes, France)* http://www.math.sciences.univ-nantes.fr/~LS60/

30 - June 1 Conference: “Geometry and Physics 2013” (GAP 2013) *Location: Centre de Recherches Mathématiques, Montréal, Québec, Canada* http://www.math.uwaterloo.ca/~gap/

JUNE 2013

3-7 PIMS/EQINOCS Automata Theory and Symbolic Dynamics Workshop *(UBC, Vancouver)* http://www.pims.math.ca/scientific-event/130603-atsdw

4-7 CMS Summer Meeting *(Halifax, NS)* http://cms.math.ca/Events/summer13/

3-14 Moduli Spaces and their Invariants in Math.Physics *(CRM, Montreal, QC)* http://www.crm.umontreal.ca/2013/Moduli13/index_e.php

4 Starling Swarms and Fish Schools, Prof. Dr. Charlotte K. Hemelrijk *(Halifax, NS)* http://mpe2013.org/lecture/starling-swarms-and-fish-schools/

10-13 Canadian Discrete and Algorithmic Mathematics Conference *(St. John’s, NL)*

16-23 51st International Symposium on Functional Equations *(Rzeszów, Poland)* tabor@univ.rzeszow.pl

17-20 7th Annual International Conference *(Athens, Greece)* www.atiner.gr/mathematics.htm.

17 -28 Algebraic Topology *(Berkeley, CA)* http://www.msri.org/web/msri/scientific/workshops/summer-graduate-workshops/show/-/event/Wm9063

30 - July 20 Geometric Analysis *(Berkeley, CA)* http://www.msri.org/web/msri/scientific/workshops/summer-graduate-workshops/show/-/event/Wm9754

JULY 2013

1-5 Summer School on Geometry, Mechanics & Control *(Madrid, Spain)* http://gmcnetwork.org/drupal/?q=activity-detaill/867

1-5 Erdös Centennial *(Budapest, Hungary)* http://www.renyi.hu/conferences/erdos100/index.html

1-12 New Geometric Techniques in Number Theory *(Berkeley, CA)* http://www.msri.org/web/msri/scientific/workshops/summer-graduate-workshops/show/-/event/Wm9460

16-20 HPM 2012 History and Pedagogy of Mathematics The HPM Satellite Meeting of ICME-12 *(Daejeon, Korea)* http://www.hpm2012.org

31-Aug 1 MAA MathFest *(Hartford, CT)* http://www.maa.org/mathfest

AUGUST 2013

5-9 Mathematical Congress of the Americas *(Guanajuato, Mexico)*

SEPTEMBER 2013

9 La prévision des grandes catastrophes, Florin Diacu *(CRM, Montreal, QC)* http://mpe2013.org/lecture/la-prevision-des-grandes-catastrophes/

22-27 The first Heidelberg Laureate Forum http://www.heidelberg-laureate-forum.org/

CMS Notes
Volume 45 No. 2, March-April 2013

The pros and cons of contests, continued from cover

U.S., Rusczyk writes, "The most immediate value of these math contests is obvious – they pique students' interest in mathematics and encourage them to value intellectual pursuits. Kids love games, and many will turn just about any activity into a contest, or in other words, something to get good at. Math contests thus inspire them to become good at mathematics just like sports encourage physical fitness. Eventually, students put aside the games. By then, hopefully an interest in the underlying activity has developed."

These are indeed very strong and convincing arguments in favour of math competitions. But Rusczyk goes on to caution that there are some pitfalls. In particular, he warns against what he calls "curricular contests" and contests that greatly emphasize speed or memorization. Contests need to be well designed, he argues, and should help students develop the ability to think about and solve complex problems. Rusczyk mentions two further pitfalls, namely extending children beyond their abilities, with the danger of the experience going from humbling and challenging to humiliating and discouraging. Finally he cautions against burnout, with the danger of students not just turning against competitions, but against math in general.

I'm giving so much space to Rusczyk's article because it puts into words my own ambiguous feelings about math competitions, both as someone involved in them as an educator and minor administrator, and as a participant in a different era (the early 70s) and a different country. I myself was always attracted to mathematics because it was, and remains, one of the least competitive endeavours around. I could (and still can) be slow, very slow, and get away with it. Anything competitive has always turned me off, and I instinctively stayed away from "hot topics".

Partly for this reason I must dispute one argument that Rusczyk brings forward in favour of math competitions: "For better or worse, much of life is competition, be it for jobs or resources or whatever." No, it doesn't have to be that way. Collaboration is always better in all spheres of life and society. So, by all means, let's build on children's love of games and competitions. But let's be mindful of the pitfalls and dangers of instilling too much of a sense of competition in children.

What does this mean for the CMS and the wider mathematical community? In spite of my words of caution I believe we are doing alright; many of the competitions are collaborative, and there are Math Camps, Math Circles, Math Leagues, and other less competitive and more collaborative initiatives. So, in most parts of the country there are programs for the slow kids as well as for the fiercely competitive, and everyone in between. In any case, I hope that most will be able to say, as Terence Tao did at the beginning of his "Solving Mathematical Problems: A Personal Perspective" (Oxford U.P., 2006): "But I just like mathematics because it's fun."

Le pour et le contre des concours, suite de la page 4

Pour cette raison, en partie, je dois m'opposer à l'un des arguments de Rusczyk en faveur des concours mathématiques. Il dit en effet que la vie, qu'on le veuille ou non, est faite de compétitions, que ce soit pour un emploi, pour des ressources ou autre chose. À cela je réponds que non, la vie n'est pas forcément une compétition. La collaboration est toujours préférable dans toutes les sphères de la vie et de la société. Exploitons donc par tous les moyens l'intérêt des enfants pour le jeu et la compétition, mais en étant bien conscients du risque de pousser trop loin leur sens de la compétition.

Qu'est-ce que cela signifie pour la SMC et la grande communauté mathématique? Malgré ma mise en garde, je crois que nous faisons bien les choses. Bon nombre de nos concours sont de nature collaborative, et certaines de nos activités (camps, cercles et ligues mathématiques) sont davantage axées sur la collaboration que sur la compétition. Presque partout au pays, on offre donc des programmes pour les jeunes qui aiment prendre tout leur temps et pour ceux qui ne jurent que par la compétition, et aussi pour tous les jeunes qui se situent entre ces deux extrêmes. Mais dans tous les cas, j'espère que la plupart de ces jeunes pourront affirmer, comme Terence Tao l'a fait au début de son livre *Solving Mathematical Problems: A Personal Perspective* (Oxford U.P., 2006) : « J'aime tout simplement les mathématiques parce que c'est amusant. »

In this issue we feature two education research note articles on the use of games for teaching ideas in mathematics. The Hyndman/Casperson article describes ways of using games to introduce the idea of research in non-classroom settings. The MacGillivray article provides an example of the use of game-like practices within the classroom to enhance learning.

Many games can be viewed from the perspective of a light and amusing activity through to an explicit tool for teaching the underlying concepts of research. We challenge you to describe the mathematical thought underlying the integration of games in developing mathematical ideas. Send us along your contributions as it is our wish to share some more examples of the unique place of games in mathematics – whether learning, understanding, or illustrating concepts.

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Teaching the Concept of Research through Games

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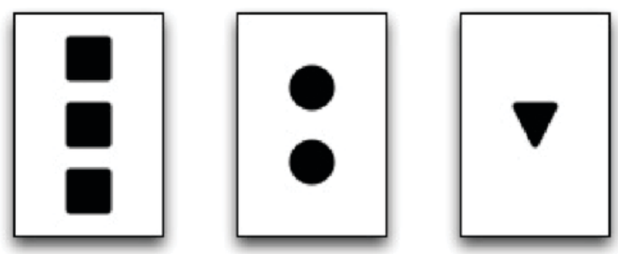
The question "how do you do research in mathematics?" arises fairly often and many mathematicians do not have a good answer suited to a general audience. Mathematicians play games, and games offer an excellent way to answer the question. Here we outline two quite different games and how they can be used to discuss the idea of research. ***Set*** is a card game that can be played by one or many people, as a speed game, competitively, or cooperatively. In contrast, ***Nim*** is a two-player game that has a winning strategy.

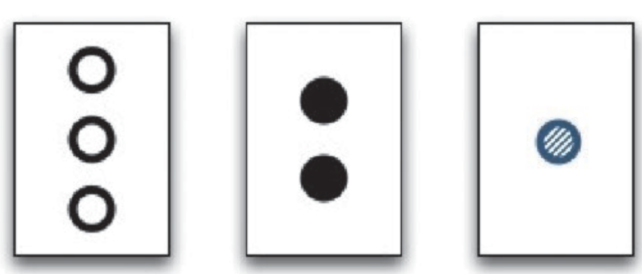
Set

In the game of Set[1] the task is to find three cards that match. Discovering what this means can be used to illustrate research on a small scale. Instead of telling new players what the rules are they watch experienced players pick up matching cards and the new players have to discover what the rules are. A continually replenished tableau of face up cards might include the following three cards.

When the dealer picks up these cards and says they match, the dealer asks the new players why. Usually someone says that they are all circles. The dealer would then choose a matching set such as



and ask why they match. One beginner might say they are all black and another might notice that they all have a solid shading. After several more rounds the beginners see some pattern in the cards so the dealer starts asking the beginners to pick out sets. Invariably something like

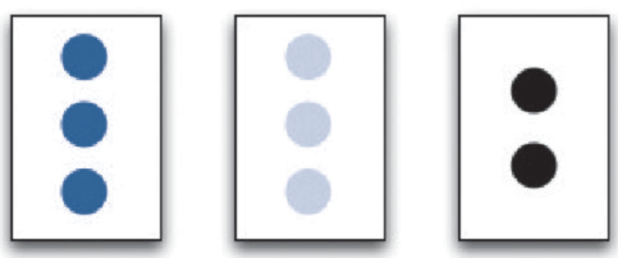


is chosen and then the dealer gets to ask the others why it is not a matching set. Someone notices two black cards and a blue card. This kind of interaction continues until the beginners can reliably pick matching sets.

1 Set is a trademarked game. See www.setgame.com/set/ for history on the development of this game from a tool for tracking traits related to epilepsy in German Shepherds.

The next challenge is to get the new players to articulate what they are doing. The dealer starts this process by asking what the general rules are. Responses tend to be that the cards all have to be black or the cards all have to be blue. The dealer then reviews the sets

and asks leading questions like how does this break your rule. The ensuing discussion usually gets to the idea that the colour all has to be the same or one of each of the three colours must occur. The dealer continues to choose sets and improperly matched sets such as



to check for intuition and ask what other rules there are. When the beginners have noticed a second property, such as number, the dealer asks what the rule is for number. Eventually rules for colour, number, shading and shape are expressed individually. The dealer continues to probe and asks for a general rule that can be stated in one sentence.

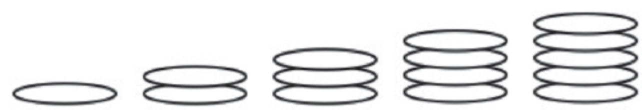
When the new players finally manage to say that there are four properties (colour, number, shading, and shape) and that each property is all the same or all different, then the dealer can get very excited and tell the new players that they have successfully done mathematical research. The research involved collecting information (examples of matching sets), developing descriptive language (colour, number, shading, and shape), making hypotheses (the cards have to be all blue) and testing hypotheses (the dealer answers yes or no to proposed matching sets).

For those who want to do more, the dealer can ask questions such as the following: If there is exactly one of every possible card how many cards are there? If necessary, this can be narrowed down to how many blue, striped cards, etc. Children enjoy creating a deck of Set cards themselves. A much more challenging question is what is the least number of cards that must be visible in order for a matching set to be present.

Nim

Nim is the name of a collection of two-player games involving piles of objects such as coins or poker chips. The players take turns removing objects from the collection of piles until all of the objects are gone. The rule for the version discussed here is that the player who takes the last object(s) is the loser (the so-called misère version).

One particular variant of Nim that is easy to teach to high school students, but complex enough to intrigue undergraduate students, starts from a configuration of five piles containing one, two, three, four, and five objects (written 1-2-3-4-5). In any turn a player may only remove objects from a single pile, but may remove as many objects as she desires, provided that she removes at least one.



Starting Position 1-2-3-4-5

Beginning players quickly discover that certain configurations, for instance 1-1-1 or 2-2, cause them to lose if it is their turn to play. If you pose the question “is the configuration n-n always losing?” players are likely to quickly come to the conclusion that the answer is “yes.” (You may or may not wish to point out that 1-1 is an exception.)

Sample Losing Position 1-2-1-2

For students who have some concept of proof, you may wish to ask “can you prove this?” This often leads to the question “What do you mean by a losing position?” Two natural answers are “you always lose, no matter what” (1-1-1, but not 2-2), or “you always lose against a sufficiently skilled opponent.”

It takes a certain level of mathematical maturity (or at least familiarity with two-player games) to realize that according to this latter definition every position can be described as either *winning* or *losing*. It may be hard for many of us who play games and teach mathematics to remember back to that time where we did not realize that chance-free two-player games are deterministic.

Once players start to become comfortable with the 1-2-3-4-5 game, you can ask about other starting configurations. Indeed, players may feel that they know “the rule” for playing Nim. Challenged to discover what three-pile configurations are losing, they may be surprised to find out that 3-5-6 is such an example.

As players start to formulate rules and counter-examples, they are learning at least implicitly, something about the discovery stage of mathematical research. Because they have concrete instances at hand, they see the utility of concepts like hypotheses, generalizations, and counter-examples.

Many non-mathematicians are surprised to learn that there are open questions in mathematics. Students who are exploring multipile Nim often ask questions in the opposite vein: "is there any simple theory of losing positions?"

For this game the answer is yes (see [2] or [3]). However, for many variants of Nim the existence of a simple-to-state strategy is an open question. Two charming introductions to the theory of two-player games are [2] and [1]. A good introduction to the modern theory of impartial misère games can be found in [3].

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Using Game-like Scenarios to Reinforce Understanding

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Puzzles and games can be an excellent way of introducing topics, or ways of thinking, or reinforcing them. The "Pirates Puzzle" [3,4] is a logic problem that introduces induction, analyzing problems by beginning with small cases, and taking care over the precise wording of conditions. The graph game Cops and Robber [1,2] also introduces induction, as well as sets, relations, quantifiers, and generalization. Game-like scenarios seem to generate more student involvement and interest than standard examples.

All that said, puzzles and games appear in my classes only once or twice per term, and only when the analysis of the puzzle or game leads to, or uses, main topics in the course. There are always time concerns when teaching a prescribed curriculum, but that is not the real reason. I want students to see mathematics as a collection of related methods rather than as a bunch of hard problems that are solved by a disconnected collection of clever tricks. I will only introduce puzzles and games when they help achieve that goal.

On the other hand, game-like scenarios are a part of most lectures, typically in the context of what strategy would be used to convince a friend that a given object has, or lacks, some property. For example, rather than stating the definition of a reflexive relation, doing some standard examples, and then moving on, there seems to be some benefit in revisiting the examples after a discussion that starts with "Suppose you and your friend are having an argument over whether a relation is reflexive. He says it is; you say it isn't. How are you going to convince him you're right?". Then, later "Ok, now suppose you are the one who says it is reflexive. What are you going to do to win this argument?"

Countable and uncountable sets, and in particular the method of Cantor Diagonalization, is a topic in our first year class *Logic and Foundations* that students have, historically, found hard. The following approach has been successful for me, that is, it works with my personality and style, and most students now seem to develop an understanding of the method and why it works. At the time Cantor Diagonalization is introduced, the students are comfortable with the idea that a set being countable is synonymous with there being a list that is guaranteed to contain each of its elements (at least once). The next step is to have a discussion of what it would mean for a set to be uncountable. A goal is to bring the discussion around to the conclusion that one way to demonstrate a set is uncountable is to have a strategy whereby *whatever list of elements someone shows you, it is possible to find, or construct, an element not in the list*. And then the conclusion that *if you have a strategy that is guaranteed to always find such an element, then it must be that no list containing all of the elements of the set can possibly exist.*

The first example is usually showing the uncountability set of infinite binary sequences. To get started, I write down a list of actual binary sequences and, together, we construct a sequence that is not in the list by changing the diagonal elements. And then we do it again, except that they construct the sequence (the last example is still on the board). The main idea added here is that as soon as the list can be seen, it is possible to find an element that is not in it. And then we do it in the abstract. Making the jump from a concrete list of sequences to an abstract one is a places where students can have trouble. It helps to have convinced the students that the method always works, and then present this step as describing the strategy, and why it works, in general.

The next example is the demonstration that the interval (0, 1) of real numbers is uncountable. This proceeds the same way. I make sure that, on the second concrete example, a poor choice of changes along the diagonal leads to constructing a different representation of a number that is in the list. Discussing this subtlety leads back to the idea that the strategy must be *guaranteed* to find an element not in the list, and what needs to be added in the argument to assure that the guarantee is there. The last step is as before: write out a detailed abstract description of the strategy and explanation of why it works.

continued on page 11

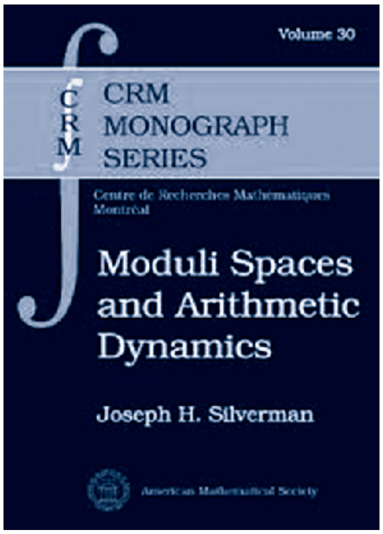
Moduli Spaces and Arithmetic Dynamics

by Joseph H. Silverman

American Mathematical Society 2012 CRM Monograph Series n. 30

ISBN 978-0-8218-7582-7

*Reviewed by **Laura De Marco**, University of Illinois at Chicago*



As a student of Curt McMullen, who was himself a student of Dennis Sullivan, I was raised under the guiding principle of "Sullivan's dictionary" between the theory of holomorphic dynamical systems on the Riemann sphere and the theory of Kleinian groups. Students today have another dictionary to develop, "Silverman's dictionary" between the arithmetic of elliptic curves (or more general varieties) and the arithmetic of algebraic dynamical systems. This book offers an interesting collection of results and questions that fit well within this dictionary.

The monograph is an expanded version of Silverman's lecture notes from a workshop held in May 2010, on the topic of moduli spaces of dynamical systems. It is a pleasure to read, as much as it was a pleasure to attend the lectures. Silverman writes well: he explains the mathematics clearly and includes many explicit examples. Best of all, he has peppered the text with leading questions and side remarks, opening doors to hundreds of potential research projects. It will be a useful reference for students and senior researchers alike.

The intended audience is one already familiar with basic notions in arithmetic dynamics, at the level of Silverman's recently-published textbook *The Arithmetic of Dynamical Systems*. Many of the results presented are due to Silverman; many are due to others. Most of the questions posed are motivated by results about elliptic curves or abelian varieties, but Silverman does not assume fluency in the arithmetic of elliptic curves. On the contrary, he makes an extra effort to remind the reader about relevant facts from arithmetic geometry (however elementary) as motivation for his comments about dynamical systems. He also makes reference to many results from the theory of holomorphic dynamics that have arithmetic analogs.

The content: The basic object of study is a moduli space of morphisms

$$\phi : \mathbb{P}^n \to \mathbb{P}^n.$$

Choosing homogeneous coordinates on $\mathbb{P}^n$ and expressing ϕ in terms of its coefficients, the space of morphisms of degree d on $\mathbb{P}^n$ is identified with a Zariski open subset

$$\mathrm{Hom}_d^n \subset \mathbb{P}^N$$

for some appropriate $N = N(d,n)$. In fact, Hom_d^n is the complement of a divisor in $\mathbb{P}^N$ (defined as the zero locus of the resultant of the $n+1$ polynomials defining ϕ), making Hom_d^n a smooth affine variety. Equivalence of morphisms is defined by conjugation,

$$\phi \sim \psi \iff \phi = A^{-1} \circ \psi \circ A \text{ for some } A \in \mathrm{PGL}_{n+1}.$$

In this way the group PGL_{n+1} acts on Hom_d^n, and we may define a quotient space

$$M_d^n = \mathrm{Hom}_d^n / \mathrm{PGL}_{n+1}$$

as the moduli space of dynamical systems on $\mathbb{P}^n$.

Silverman discusses fundamental facts about these moduli spaces M_d^n in Chapter 2. For example, the moduli space exists as a geometric quotient, in the sense of Mumford's Geometric Invariant Theory. It fails to be a fine moduli space, because of the existence of nontrivial symmetries of certain morphisms. In the case of $n = 1$, it is known that M_d^1 is a rational variety, and M_2^1 is isomorphic to the affine plane A^2, but almost nothing is known about the structure of M_d^n in general.

Silverman introduces the arithmetic notions of good and bad reduction and the minimal resultant in Chapter 3, explaining analogies with the setting of elliptic curves or abelian varieties. He also makes several diversions from the context of morphisms of $\mathbb{P}^n$, for example to discuss results about polynomial automorphisms of affine space (that do not extend to morphisms of $\mathbb{P}^n$) and automorphisms of K3 surfaces. In Chapter 4, Silverman concentrates on the case $n = 1$, where he defines dynamical modular curves. He discusses a few cases where their structure is understood.

Chapter 5 is devoted to the theory of dynamical height functions. For a global field K, the canonical height function of a morphism $\phi \in \mathrm{Hom}_d^n(\overline{K})$ is defined, via a construction attributed to Tate, by

$$\hat{h}_\phi(P) = \lim_{m\to\infty} \frac{1}{d^m} h(\phi^m(P))$$

for $P \in \mathbb{P}^n(\overline{K})$, where h is a standard Weil logarithmic-height function. The difference $\hat{h}_\phi - h$ is bounded on $\mathbb{P}^n(\overline{K})$ for any morphism ϕ. When K is a number field, we have $\hat{h}_\phi(P) = 0$ if and only if p has finite orbit under ϕ. The existence of the height function allows for a more careful analysis of the arithmetic dynamics of ϕ: for example, the set of preperiodic points is a set of bounded Weil height, and we might hope for a Uniform Boundedness result, as conjectured by Morton and Silverman in a paper published in 1994:

Conjecture : For all n, d, D, there is a constant $C = C(n, d, D)$ such that

$$\#\mathrm{PrePer}(\phi, \mathbb{P}^n(K)) \leq C$$

for all number fields K of degree at most D over $\mathbb{Q}$ and all $\phi \in \mathrm{Hom}_d^n(K)$.

The set $\mathrm{PrePer}(\phi, \mathbb{P}^n(K))$ consists of all preperiodic K-rational points for ϕ. The far-reaching consequences of this conjecture are described in Chapter 1.

The emphasis of Chapter 6 is on the postcritically-finite maps of $\mathbb{P}^1$. Silverman discusses Thurston's rigidity result about these maps and some arithmetic proofs in special cases. The postcritically-finite maps form a Zariski dense subset of M_d^1, and they might play an analogous role within M_d^1 to that of abelian varieties with complex multiplication within a moduli space of abelian varieties. Silverman provides partial results and conjectural ideas in this direction.

Finally, in Chapter 7, Silverman introduces the question of when the field of moduli for $\phi \in \mathrm{Hom}_d^n$ can also be a field of definition. For example, with $n = 1$ and d even, it is always possible; Silverman offers an example in degree 3 where it fails.

Every chapter of this monograph offers multiple entries, many conjectural, in the "Silverman dictionary." For anyone already familiar with both the arithmetic theory of elliptic curves and the traditionally analytic results about these dynamical systems, recent results in arithmetic dynamics make the analogies beautifully apparent; with this book, Silverman helps the rest of us to appreciate the similarities.

Using Game-like ... Understanding, continued from page 9

To conclude, there is an in-class exercise that takes about five minutes. Each of the seventy students is given a sheet of paper with a question similar to the problem below. The previous examples are still on the board. They are encouraged to talk to each other, and to the instructor (and TA if she's in the room) in the process of working through it. Asking the answer is acceptable, and I would tell anyone who asked, but in practice the students really just want hints and reassurance; they want to be able to answer the question. Aside from the reinforcement that comes from working through the problem, there is enormous value in having the students explain their solutions to each other.

Problem

Gary claims he has a list that contains every infinite sequence whose elements s_{ij} come from {A, B, C, D}:

1) $s_{11}, s_{12}, s_{13}, s_{14}, \ldots$

2) $s_{21}, s_{22}, s_{23}, s_{24}, \ldots$

3) $s_{11}, s_{12}, s_{13}, s_{14}, \ldots$

$\quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \ddots$

Show that Gary is wrong by:

a) describing how to construct a sequence that is not in his list,

b) explaining how you know your sequence is not in his list, and

c) explaining why this proves that he is wrong.

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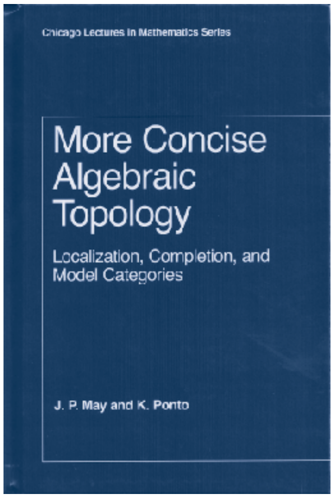
More Concise Algebraic Topology

by J.P.May and K. Ponto

The University of Chicago Press 2012, Chicago Lectures in Mathematics Series

ISBN 978-0-2265-1178-8

Reviewed by ***Guido Mislin****, Department of Mathematics, ETH Zürich*



The basic idea of localization and completion techniques in algebraic topology mimics constructions in algebra, involving the integers $\mathbb{Z}$ and studying $\mathbb{Z}$ by passing to its localizations $\mathbb{Z}_{(p)}$, p a prime number, resp. the p-completions $\hat{\mathbb{Z}}_p$ (the p-adic integers), the rationals $\mathbb{Q}$ and the ring $A = \hat{\mathbb{Z}} \otimes \mathbb{Q}$ of finite adeles for $\mathbb{Q}$, with $\hat{\mathbb{Z}}$ (=$\prod \hat{\mathbb{Z}}_p$) the finite completion of $\mathbb{Z}$. Viewing $\mathbb{Q}$ and $\hat{\mathbb{Z}}$ as subrings of A, one can reconstruct $\mathbb{Z}$ by taking their intersection. In an analogous way one constructs for a space X localizations $X_{(p)}$, p-completions $\hat{X}_p$, the rationalization $X_{(0)}$ and a "formal completion" X_A, such that for good enough spaces, X can be reconstructed up to homotopy from these pieces and the natural maps between them (fracture theorems). Many application arise from these constructions. In particular, the relationship between etale homotopy theory and completion of spaces links algebraic geometry and algebraic topology tightly together (cf. E.M. Friedlander 1982, Annals of Mathematics Studies, Vol. 104).

The first systematic treatments of localization and completion techniques go back to the early 1970s (A.K. Bousfield \& D.M. Kan 1972, Springer Lecture Notes in Mathematics Vol. 304, and D. Sullivan 1974, Annals of Mathematics). However, these early accounts are technically difficult and have over time gone through many transformations. May and Ponto have done a great job in their book in simplifying the exposition and arguments, to make the whole topic more accessible. They begin with an extensive section on nilpotent spaces (spaces of the homotopy type of CW-complexes such that the Postnikov decomposition of each connected component admits a principal refinement). The reminder of the first part of the book contains an elementary treatment of localization and completion for nilpotent spaces, which goes parallel with a description of localization and completion in the setting of nilpotent groups. Indeed, for a connected nilpotent space X, the localization map $X \to X_{(p)}$ induces p-localization on the level of homotopy groups. The situation is slightly more complicated in the case of p-completion, because the p-completion $K(A,n)^\wedge_p$ of an Eilenberg-Mac Lane space $K(A,n)$, A an abelian group, has in general two non-zero homotopy groups. But for nilpotent spaces X of finite type, this anomaly cannot occur and the natural map $\pi_*(X) \to \pi_*(\hat{X}_p)$ is just p-completion. In particular, for A finitely generated abelian, $K(A,n)^\wedge_p$=$K(\mathrm{proj}\lim A/p^kA, n)$. The authors pay a lot of attention to a careful analysis of several variations of fracture theorems, with many new proofs, explaining the relationship between local and global information. They also consider the localization of a space at a set of primes T (corresponding to passing from $\mathbb{Z}$ to the subring of $\mathbb{Q}$ consisting of fractions expressible with denominators not involving any prime from T). Applications include the mixing of homotopy types, as initiated by A. Zabrodsky in the 1970s.

The second half of the book has two parts. In the first one, model category theory in the sense of D. Quillen (Springer Lecture Notes in Mathematics, Vol 43) is introduced. It is then used to prove that localization and completion exist in full generality. The techniques are applied to construct the E-localization $X \to X_E$ of a space X in the sense of Bousfield, where E_* denotes a homology theory satisfying the usual axioms, but not necessarily the dimension axiom. This E-localization generalizes the classical localization of nilpotent spaces, if one takes E to be ordinary homology with coefficients in $\mathbb{Z}_{(p)}$. The general E-localization has also proven to be very useful in the context of stable homotopy theory (E-localization of spectra).

The last part of the book deals with bialgebras and Hopf algebras. The authors follow the classical paper by J. Milnor and J.C. Moore (Annals of Mathematics, 1965), with divers simplifications and extensions. The classical results on connected, graded-commutative Hopf algebras in all characteristics are discussed and related to structure theorems on Lie algebras. Many of the results apply to ungraded bialgebras, which possess a complete augmentation ideal filtration. Several applications are presented. For instance, rational H-spaces are described using structure theorems for rational Hopf algebras. A proof of complex Bott periodicity is given, using the $\mathbb{Z}$-Hopf algebra structure of the cohomology of the space BU.

The May-Ponto book fills a gap in the existing literature on modern algebraic topology. It is written in a lucid style, making it a pleasure to read and to browse through. I can recommend it in highest terms.

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Minimal surfaces as extremals of eigenvalue problems

Ailana Fraser
Department of Mathematics, University of British Columbia

Given a smooth compact surface M, the choice of a Riemannian metric g on M gives a Laplace operator Δ_g. In local coordinates on M,

$$\Delta_g = \frac{1}{\sqrt{g}} \sum_{i,\, j=1}^{2} \frac{\partial}{\partial x^i} \left(\sqrt{g} g^{ij} \frac{\partial}{\partial x^j} \right).$$

Δ_g is a nonnegative self-adjoint operator and has a discrete set of eigenvalues

$$\lambda_0 = 0 < \lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_k \leq \ldots \to \infty.$$

We may then view the eigenvalues as functionals of the metric $g \mapsto \lambda_i(g)$. If we restrict to metrics of fixed area, there is an upper bound on the first nonzero eigenvalue λ_1, which depends only on the topology of the surface ([9]). This suggests the following extremal problem: Assuming we fix the area, what is the metric that maximizes the first eigenvalue? Does such a metric exist? If so what can we say about its geometry?

Assuming that there exists a smooth metric g that realizes the maximum, then it turns out that the multiplicity of the eigenvalue is always at least 3, and the maximizing condition implies that there are independent eigenfunctions $u_1, \ldots, u_{n+1}$ such that the map $u = (u_1, \ldots, u_{n+1})$ defines an isometric minimal immersion from M to S^n with $n \geq 2$ ([7]). That is, $\Sigma = u(M)$ is a minimal surface in S^n, i.e. the mean curvature of Σ is zero. Furthermore, the optimal metric g is a positive constant times the induced metric on Σ from S^n.

There are only a few surfaces for which maximizing metrics are known to exist. For S^2 the standard constant curvature metric uniquely maximizes the first eigenvalue, [3]. For $\mathbb{R}P^2$ the standard constant curvature metric is the unique maximum, [6]. The fact that $\mathbb{R}P^2$ can be isometrically minimally embedded into S^4 plays a key role in the proof. For the torus T^2 the flat metric on the 60^0 rhombic torus is the unique maximum, [7]. It can be minimally embedded into S^5 by first eigenfunctions. For the Klein bottle the extremal metric is smooth and unique, but not flat, [5], [7], [1]. The metric arises from an explicit minimal immersion of the Klein bottle into S^4, [5].

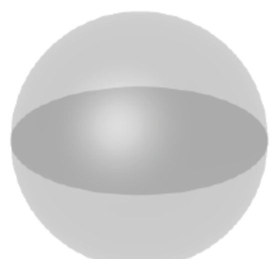
The case of the torus and the Klein bottle are much harder than the case of S^2 and $\mathbb{R}P^2$; in particular, they rely on the existence of a smooth maximizing metric, with an outlined proof in [7]. For higher genus surfaces, the question of existence of maximizing metrics remains open, and it would likely be difficult to characterize them explicitly. For the surface of genus 2, there is a conjectured maximizing metric, [4].

It is natural to ask similar questions and look for optimal metrics on surfaces with boundary. The most standard eigenvalue problems on surfaces with boundary are the Dirichlet and Neumann problems. But another important problem in applications, which also has an interesting connection to a minimal surface problem, is the Steklov problem. Steklov eigenvalues are eigenvalues of the Dirichlet-to-Neumann map, which sends a given smooth function on the boundary to the normal derivative of its harmonic extension to the interior. The Dirichlet-to-Neumann map is a nonnegative, self-adjoint operator with discrete spectrum

$$\sigma_0 = 0 < \sigma_1 \leq \sigma_2 \leq \ldots \leq \sigma_k \leq \ldots \to \infty.$$

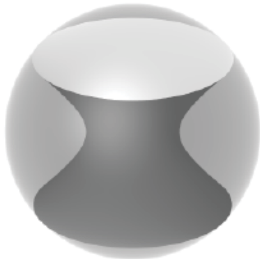
A basic question is: what is the metric on M that maximizes the first eigenvalue σ_1, if we fix the boundary length. Does such a metric exist? If so, what is its geometry?

A classical result of Weinstock shows that the Euclidean disk maximizes σ_1 over all metrics of fixed boundary length on a disk, [8]. Note that this metric is induced from a minimal embedding of the disk in B^3, meeting the sphere orthogonally, as an equatorial plane disk:



In general, on a surface M with boundary, assuming that there exists a smooth metric g that realizes the maximum, it can be shown that there are independent eigenfunctions $u_1, \ldots, u_n$ such that the map $u = (u_1, \ldots, u_n)$ defines a proper conformal map to unit ball $B^n, n \geq 3$, [2]. This implies that the image $\Sigma = u(M)$ is a minimal surface in B^n that is orthogonal to the sphere at the boundary. We call such surfaces *free boundary minimal surfaces* in the ball. Furthermore, the optimal metric g is equivalent to the metric induced on Σ from $\mathbb{R}^n$.

In joint work with R. Schoen, [2], we show that for any compact surface M with boundary, there exists a smooth metric g on M that maximizes σ_1, provided the conformal structure is controlled for any metric near the maximum. For surfaces of genus zero with arbitrarily many boundary components, we prove boundedness of the conformal structure for nearly maximizing metrics. Thus, there exists a maximizing metric on any surface of genus zero with $k \geq 1$ boundary components. In the case of the annulus we are able to explicitly characterize the maximizing metric as arising from the induced metric on the *critical catenoid*. This is the unique portion of a suitably scaled catenoid which defines a free boundary minimal surface in B^3:



In the case of the Möbius band we are also able to explicitly characterize the maximizing metric as arising from the induced metric on a free boundary minimal Möbius band in B^4, the *critical Möbius band*.

For surfaces of genus 0 with $k \geq 3$ boundary components, while we are not able to explicitly characterize the maximizing metrics, we show in [2] that the metrics arise from free boundary surfaces in B^3 which are embedded and star-shaped with respect to the origin, and we analyze the limit as k goes to infinity. For large k, the surface is approximately a pair of nearby plane disks joined by k boundary bridges:

In particular, these results provide the existence of a free boundary minimal surface in the ball B^3 of genus zero with any number of boundary components.

Acknowlegement. The author would like to thank Robert Dawson for rendering the computer figures after her sketches.

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The Halting Problem

Cristian S. Calude
The University of Auckland, New Zealand

The *Entscheidungsproblem*, posed in 1928 by D. Hilbert, asks for an algorithm that takes as input a statement of a first-order logic and outputs 1 or 0 according to whether the statement is universally valid (i.e. in every structure satisfying the axioms of the logic) or not. By the completeness theorem of first-order logic, a statement is universally valid if and only if it can be deduced from the axioms of the logic, so solving the Entscheidungsproblem means finding an algorithm to decide whether a given statement is provable in the first-order logic.

Church [7] and Turing [9] published, independently, different negative solutions to the Entscheidungsproblem. In his proof, Turing introduced a computing machine, now called "Turing machine" (shortly, machine), as a mathematical model for the informal notion of algorithm, and showed how it can be programmed. He then constructed a *universal machine* which can simulate the execution of any machine; in a sense, the universal machine is the blueprint of the modern computer.

Page 247 of Turing's paper is a proof that a certain problem—now called (after M. Davis) *the halting problem* (for Turing machines)—cannot be solved by any machine. The halting problem asks for a machine $Halt$ that takes as input an arbitrary machine M and an input x, and outputs 1 or 0 according to whether M will come to a halt or not on x. We assume that a machine incorporates its input and a halting machine outputs a bit-string. As machines can be systematically enumerated (say, lexicographically) and $Halt$ itself always halts, Turing's proof follows by diagonalisation. The proof below gives also a "reason" for unsolvability. For convenience we assume from now on that all programs include their input data. Suppose, by absurdity, the existence of a machine $Halt$. Construct the following machine $T(N)$ (T from "trouble"):

> read a positive integer N in binary; list all machines up to N bits in size; use $Halt$ to remove from the list all machines which do not halt; simulate the running of all machines on the list; output the largest result computed by these machines plus 1.

For every N, the machine $T(N)$ halts. Its size is less than $\log_2 N + \mathrm{O}(1)$ bits, which is smaller than N, for large N. Accordingly, $T(N)$ generates itself at some stage of its computation, and a simple analysis of the output of $T(N)$ leads to a contradiction. Unsolvability is a consequence of coding scarcity.

In modern terms, machines are programs written in a programming language. In this framework, a universal machine is a universal programming language L, i.e. a language such that if any other language L' can program a machine in K bits, then L can do it in less than $K + c_{L,L'}$ bits, where $c_{L,L'}$ is a fixed constant. A universal programming language L can be constructed with the following five instructions: $=\mathtt{r1},\mathtt{r2},\mathtt{r3}$ (branching instruction), $\&\mathtt{r1},\mathtt{r2}$ (assigning instruction), $+\mathtt{r1},\mathtt{r2}$ (sum), $!\mathtt{r1}$ (read one bit), $\%$ (halt). Registers $\mathtt{r1},\mathtt{r2},\ldots$ can contain arbitrary large non-negative integers. A program consists of a finite list of labeled instructions from the above list, with the restriction that the halt instruction appears only once, as the last instruction. The input data is a bit-string which follows immediately after the halt instruction. The result of the computation, if any, is a non-negative integer stored in a fixed output register. A program not reading the whole data or attempting to read past the last input bit results in an error.

Systematically enumerate all programs (say, lexicographically) and define the real number H whose Nth bit h_N tells us whether or not the Nth program halts. Clearly, H is incomputable, i.e. the function $N \mapsto h_N$ is not computable by any program. This "coding" of the halting problem is rather wasteful, because N instances of the halting problem have only $\log_2 N$ bits of mathematical information: one only needs to know *how many* of these N programs halt to be able to determine which ones halt. One can obtain a more compact coding using the halting probability (or Chaitin's Ω) which is defined by the formula:

$$\Omega = \sum_{p \text{ halts}} 2^{-(\text{size of } p \text{ in bits})};$$

Ω is a probability because, due to the syntax of L, if p halts, then no prefix or extension of p halts. There are infinitely many halting probabilities, one for each universal programming language.

All Ω numbers share a few remarkable properties. Like H, Ω "codes" the halting problem, but in a more efficient way because if one "knows" the first N bits of Ω, then one can solve the halting problem for all programs of size smaller than $N+1$: $2^{N+1}-1$ instances of the halting problem are coded into N bits. While H is incomputable, there are programs computing infinitely many values h_N. No program can do a similar computation for Ω; in fact, Ω is strongly incomputable, that is, there is a positive integer Ω such that any program can compute at most ω values of digits of Ω (ω, which depends on the programming language, can be any non-negative integer). Any Ω has two apparently contradictory properties: computable enumerability, i.e. Ω is the limit of a computable sequence of rationals in the unit interval, and incompressibility (also called algorithmic randomness), i.e. the smallest program for computing the first N bits of Ω has at least $N - \mathrm{O}(1)$ bits. The converse result is also true: every real satisfying the above two conditions is an Ω number for some universal programming language. Finally, Ω is transcendental and normal. Two of the above properties can be expressed in terms of mathematical provability. Assume ZFC is sound. Strong incomputability implies unprovability: ZFC cannot prove more than Ω values of the bits of Ω (a quantum random generator can behave in a similar manner, [1]). Incompressibility implies logical irreducibility: Any sound extension of ZFC that can prove the values of the first N bits of Ω must have at least $N - \mathrm{O}(1)$ bits of axioms. See more in [2].

An extensive simulation, whose correctness was mathematically proved, solved the halting problem for all programs in L of up to 80 bits in size and calculated the first 40 bits of $\Omega = \Omega_L$ [3]: 0001000000010000101001110111000011111010.

Because the Riemann hypothesis can be written in the form $\forall n P(n)$, where P is a computable predicated, programs in L can systematically search for a counter-example; such a program RH stops if and only if the conjecture is false. Hence, if we knew the first 2,745 bits—the length of such an RH—of Ω_L we would know whether the Riemann hypothesis is true [6]. Searching for a counter-example for the conjecture $P \neq NP$—which is of the form $\forall n \exists m R(n, m)$, where R is a computable predicate—is more difficult; the solution is to use the *same* programs in L, but with a different semantics. A classical computation produces a result only in case the computation stops; the result is then recorded in a special output register. An *inductive* program P produces the same results as the classical program P, but sometimes a result is obtained when P runs an infinite computation (of course, not all inductive computations produce results). In [4] an inductive program PNP of 6,495 bits was constructed to return 0 if and only if $P \neq NP$; PNP is more complex than RH not only because it is longer, but also because it's solution needs a programming language with higher computational power. Indeed, there is an inductive program in L which solves the halting problem for all classical programs in L. However, no inductive program can solve the halting problem for all inductive programs.

While deterministically unsolvable, the halting problem can be probabilistically solved [5, 8]. Fix a rational ε in (0,1). One can effectively construct a program which stops on every program p (as input) and outputs either: a) p *halts*, and in this case the result is correct, or b) p *does not halt*, and in this case the probability that the result is wrong is less than ε.

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The Sun Life Financial Canadian Open Mathematics Challenge

Robert Woodrow
(Calgary), Chair of the COMC Committee

In November 2012 thousands of high school students across Canada wrote the 2012 COMC competition and hundreds took the challenge at a non-Canadian site where some were Canadian students studying abroad and others were just attracted by the COMC. As Canada's national mathematics competition for students with an interest in mathematics, the COMC encourages them to explore, discover, and learn more about mathematics and problem solving.

This year the competition was staged with and supported by a cross-Canada network of university partners: Memorial University, Dalhousie University, University of Prince Edward Island, University of New Brunswick, Laval University, University of Toronto, University of Manitoba, University of Saskatchewan, University of Calgary, University of British Columbia, Carlton University, and the University of Ottawa.

The competition consisted of 12 problems in 3 sets of increasing difficulty. From a decrease in quartile scores, over 2011, it would appear that some of the problems this year proved more challenging than perhaps expected. The problem that was thought to be the most challenging is the following.

Problem C4: For any positive integer *n*, an *n*-tuple of positive integers $(x_1,x_2,\ldots,x_n)$ is said to be *super-squared* if it satisfies both of the following properties:

1. $x_1 > x_2 > x_3 > \ldots > x_n$
2. The sum $x_1^2 + x_2^2 + \ldots + x_k^2$ is a perfect square for each $1 \le k \le n$.

For example, (12, 9, 8) is super-squared, since $12 > 9 > 8$ and each of 12^2, $12^2 + 9^2$, and $12^2 + 9^2 + 8^2$ are perfect squares.

a. (2 marks) Determine all values of *t* such that (32, *t*, 9) is super-squared.

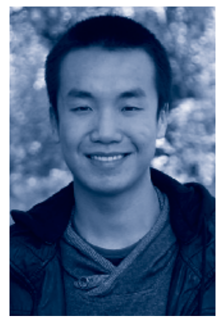
b. (2 marks) Determine a super-squared 4-tuple (x_1,x_2,x_3,x_4) with $x_1 < 200$.

c. (6 marks) Determine whether there exists a super-squared 2012-tuple.

Students performed very well this year – there were 65 national award and honour roll citations as well as almost 400 at the provincial and regional levels. Internationally, 4 participants earned awards.

Based on their COMC performance, 52 students are being invited to write the Sun Life Financial Canadian Mathematical Olympiad (CMO) and a further 69 were invited to write the Sun Life Financial Repêchage, the results of which will earn some of them an invitation to write the Canadian Mathematical Olympiad. Student performance in CMS competitions is the major factor in being selected to be a member of Math Team Canada and compete in the 54th International Mathematics Olympiad (IMO 2013) being held this year in Columbia.

Alex Song
National Gold Medal



Zhiyao (John) Ma,
National Silver Medal

Daniel Spivak
National Bronze Medal

Information on the COMC can be found at: **http://cms.math.ca/Competitions/COMC/2012/#**. *Enquiries regarding interest in or development of the COMC can be directed to Robert at woodrow@ucalgary.ca*.

June 4-7, Dalhousie University (Halifax, NS)
4-7 juin, Université Dalhousie (Halifax, NS)

www.cms.math.ca/Events/summer13

Prizes | Prix

Krieger-Nelson Prize | Prix Krieger-Nelson
Chantal David (Concordia)

Coxeter-James Prize | Prix Coxeter-James
to be announced | à être annoncé

Excellence in Teaching Award
Prix d'excellence en enseignement
Ján Minác (Western)

Public Lecture | Conférence publique

Prof. Dr. Charlotte K. Hemelrijk *(University of Groningen, Netherlands)*
Erik Demaine *(MIT)*

Plenary Lectures | Conférences plénières

Jose Carrillo *(Imperial College, UK)*
Irena Peeva *(Cornell, USA)*
Pavel Winternitz *(Université de Montréal, Canada)*

Education Sessions

Mathematics Camps, Outreach in the Schools, Popularization in Mathematics
John McLoughlin *(UNB)*, Andrew Hare *(Saint Mary's)*, Danielle Cox *(Dalhousie)*, and Ryan Jones *(UNB)*

Sessions

Discrete and Combinational Geometry
Géométrie discrète et combinatoire
Wendy Finbow-Singh *(SMU)*, Ted Bisztriczky *(Calgary)*

Commutative Algebra and Combinatorics
Combinatoire et algèbre commutative
Jennifer Biermann *(Lakehead)*, Sara Faridi *(Dalhousie)*
Andrew Hoefel *(Queen's)*, Adam Van Tuyl *(Lakehead)*

Pseudogroups and their Applicatoin
Pseudogroupes et leurs applications
Francis Valiquette *(Dalhousie)*, Abraham Smith *(Fordham University)*

Hopf algebras and Tensor Categories | Algèbres de Hopf et catégories tensorielles
Margaret Beattie *(Mount Allison)*, Mitja Mastnak *(Saint Mary's)*, Bob Pare *(Dalhousie)*, Yuri Bahturin *(Memorial)*

Analysis, Geometry and Topology on Fractals, Wavelets and Self-similar Tilings | Analysis, géométrie et topologie sur les fractales, ondelettes et pavages autosimilaires
Tara Taylor *(StFX)*, Franklin Mendivil *(Acadia)*, Eva Curry *(Acadia)*

Progress in Higher Categories
Progrès sur les catégories supérieures
Peter LeFanu Lumsdaine *(IAS)*, Michael Warren *(IAS)*

Nonlinear Partial Differential Euations and their Applications
Équations différentielles partielles non linéaires et leurs applications
George Chen *(Cape Breton)*, Rodney Scott *(Cape Breton)*

Nonlocal Interactions in Social, Physical, and Biological Sciences
Interactions non locales en sciences sociales, physiques et biologiques
Theodore Kolokolnikov *(Dalhousie)*, Michael Ward *(UBC)*

Contributed Papers | Communications libres

Combinatorial Game Theory | Théorie combinatoire des jeux
Kristine Bauer *(Calgary)*, Marcy Robertson *(Western)*

The Functional Analytic and Representation Theoretic Foundations of Wavelet Theory | Analyse fonctionelle et théorie des représentations: fondements de la théorie des ondelettes.
Jean-Pierre Gabardo *(McMaster)*, Vignon Oussa *(Bridgewater State University)*, Keith Taylor *(Dalhousie)*

Experimental Methods in Number Theory
Méthodes expérimentales en théorie des nombres
Karl Dilcher *(Dalhousie)* Michael John Jacobson Jr. *(Calgary)*
Renate Scheidler *(Calgary)*

AARMS-CMS Student Poster Session
Présentations par affiches pour étudiants
Amanda Malloch, *(Victoria)*

Nonlinear PDE's, control and reaction-diffusion systems
Équations différentielles partielles non linéaires, contrôle et systèmes à réaction-diffusion
Lia Bronsard *(McMaster)*, Holger Teismann *(Acadia)*

Ergodic Theory, Dynamical Systems and Applications | Théorie ergodique, systèmes dynamiques et applications
Shafiqul Islam *(UPEI)*, Pawel Gora *(Concordia)*

Selected Topics in Biomathematics: Applications to Ecology and Aging | Sujets choisis en biomathématiques : applications relatives à l'écologie et au vieillissement
Arnold Mitnitski *(Dalhousie)* Joe Apaloo *(StFX)*

Women Mathematicians' Lunch and Panel Discussion
Wednesday June 5, 12:30 - 2:00 pm

Workshop

AAC International Workshop in Combinatorial Algebra
Atelier international en algèbre combinatoire
Yuri Bahturin, Margaret Beattie, Sara Faridi, Mikhail Kotchetov, Mitja Mastnak and Hamid Usefi

This year the CMS will be electing seventeen (17) officers and directors and the Nominating Committee wishes to announce its initial list of candidates for the 2013 elections. Each person on the list has agreed to be a candidate and to furnish the committee with biographical information.

You are invited to nominate other members to be candidates and their nominations will be accepted by the Nominating Committee **prior to March 29, 2013** (deadline extended) provided that each person nominated: (i) is supported in writing by at least five (5) other members of the CMS; and (ii) has given written acceptance to stand for office and to supply biographical information.

Additional nominations together with supporting materials should be e-mailed to **nominations-2013@cms.math.ca** or sent to:

Nominating Committee Chair
Canadian Mathematical Society
209 - 1725 St. Laurent Blvd.
Ottawa, ON K1G 3V4 Canada

The CMS Nominating Committee proposes the following initial slate of candidates for the Executive Committee:

- **President-Elect**: Lia Bronsard (McMaster);
- **Vice-President** – Atlantic Provinces: Robert van Den Hoogen (St. Francis Xavier);
- **Vice-President** – Quebec: Louigi Addario-Berry (McGill);
- **Vice-President** – Ontario: Gregory G. Smith (Queen's); and
- **Vice-President** – Western Provinces and Territories: Mark Lewis (Alberta).

Nominations are also being solicited for Board of Directors members (see link below for the most current list of nominees):

- **Atlantic** – 2 members to be elected;
- **Quebec** – 2 members to be elected;
- **Ontario** – 4 members to be elected;
- **Western Provinces and Territories** – 2 members to be elected; and
- **At Large** – 2 members to be elected.

For 2013, the CMS will hold the election electronically. Information and voting instructions will be communicated to all individual CMS members. Updated information will be periodically e-mailed to members and posted on the CMS website at **http://cms.math.ca/Elections/2013/.**

Edwin Perkins
Chair, CMS Nominating Committee

Cette année, la SMC élira dix-sept (17) dirigeants et administrateurs et le Comité des mises en candidature souhaite annoncer sa première liste des candidats pour les élections de 2013. Chaque personne sur la liste a accepté d'être candidat(e) et de fournir au comité leurs renseignements biographiques.

Vous êtes invités à nommer d'autres candidats et leurs nominations seront acceptées par la Comité des mises en candidature **avant le 29 mars 2013** (date prolongée), à condition que chaque personne nommée : (i) ait reçu l'appui par écrit d'au moins cinq (5) autres membres de la SMC; et (ii) ait accepté par écrit d'être candidat(e) et de fournir leurs renseignements biographiques.

D'autres nominations accompagnées des documents doivent être envoyer par courier électronique à **nominations-2013@smc.math.ca** ou envoyés à :

Président du Comité des mises en candidature
Société mathématique du Canada
209 - 1725 boul. St. Laurent
Ottawa, ON K1G 3V4 Canada

Le Comité des mises en candidature propose la première liste des candidats pour le Comité exécutif :

- **Président élu** : Lia Bronsard (McMaster);
- **Vice-président** – provinces de l'Atlantique : Robert van Den Hoogen (St. Francis Xavier);
- **Vice-président** – Québec : Louigi Addario-Berry (McGill);
- **Vice-président** – Ontario : Gregory G. Smith (Queen's); et
- **Vice-président** – provinces de l'Ouest et les Territoires : Mark Lewis (Alberta).

Le comité invite les nominations pour le poste de : (voir le lien ci-dessous pour le liste la plus récente des candidats) :

- **Atlantique** – 2 membres doivent être élus;
- **Québec** – 2 membres doivent être élus;
- **Ontario** – 4 membres doivent être élus;
- **Provinces de l'Ouest et les Territoires** – 2 membres doivent être élus; et
- **En général** – 2 membres qui seront élus.

Pour 2013, la SMC tiendra l'élection par voie électronique. Les renseignements et les instructions de vote seront communiqués à tous les membres individuels de la SMC. Mises à jour des renseignements seront communiqués régulièrement par courrier électronique aux membres et affichés sur le site Web de la SMC au **http://cms.math.ca/Elections/2013/.f**.

Edwin Perkins
Président du Comité des mises en candidature

Research Nominations

The CMS Research Committee is inviting nominations for three prize lectureships. These prize lectureships are intended to recognize members of the Canadian mathematical community.

The Coxeter-James Prize Lectureship recognizes young mathematicians who have made outstanding contributions to mathematical research. The recipient shall be a member of the Canadian mathematical community. Nominations may be made up to ten years from the candidate's Ph.D: researchers having their PhD degrees conferred in 2003 or later will be eligible for nomination in 2013 for the 2014 prize. A nomination can be updated and will remain active for a second year unless the original nomination is made in the tenth year from the candidate's Ph.D. The prize lecture will be given at the 2014 CMS Summer Meeting.

The Jeffery-Williams Prize Lectureship recognizes mathematicians who have made outstanding contributions to mathematical research. The recipient shall be a member of the Canadian mathematical community. A nomination can be updated and will remain active for three years. The prize lecture will be given at the 2014 CMS Winter Meeting.

The Krieger-Nelson Prize Lectureship recognizes outstanding research by a female mathematician. The recipient shall be a member of the Canadian mathematical community. A nomination can be updated and will remain active for two years. The prize lecture will be given at the 2014 CMS Summer Meeting.

The deadline for nominations is June 30, 2013.

Nominators should ask at least three referees to submit letters directly to the CMS by September 30, 2013. Some arms-length referees are strongly encouraged. Nomination letters should list the chosen referees, and should include a recent curriculum vitae for the nominee, if available. Nominations and reference letters should be submitted electronically, preferably in PDF format, by the appropriate deadline to the corresponding email address:

Coxeter-James: **cjprize@cms.math.ca**

Jeffery-Williams: **jwprize@cms.math.ca**

Krieger-Nelson: **knprize@cms.math.ca**

Appel de mises en candidature de recherché

Le Comité de recherche de la SMC lance un appel de mises en candidatures pour trois de ses prix de conférence. Ces prix ont tous pour objectif de souligner l'excellence de membres de la communauté mathématique canadienne.

Le prix Coxeter-James rend hommage aux jeunes mathématiciens qui se sont distingués par l'excellence de leur contribution à la recherche mathématique. Cette personne doit être membre de la communauté mathématique canadienne.

Les candidats sont admissibles jusqu'à dix ans après l'obtention de leur doctorat : ceux qui ont obtenu leur doctorat en 2003 ou après seront admissibles en 2013 pour le prix 2014. Toute mise en candidature est modifiable et demeurera active l'année suivante, à moins que la mise en candidature originale ait été faite la 10e année suivant l'obtention du doctorat. La personne choisie prononcera sa conférence à la Réunion d' été SMC 2014.

Le prix Jeffery-Williams rend hommage aux mathématiciens ayant fait une contribution exceptionnelle à la recherche mathématique. Cette personne doit être membre de la communauté mathématique canadienne. Toute mise en candidature est modifiable et demeurera active pendant trois ans. La personne choisie prononcera sa conférence à la Réunion d'hiver SMC 2014.

Le prix Krieger-Nelson rend hommage aux mathématiciennes qui se sont distinguées par l'excellence de leur contribution à la recherche mathématique. La lauréate doit être membre de la communauté mathématique canadienne. Toute mise en candidature est modifiable et demeurera active pendant deux ans. La lauréate prononcera sa conférence à la Réunion d'été SMC 2014.

La date limite de mises en candidature est le 30 juin 2013.

Les proposants doivent faire parvenir trois lettres de référence à la SMC au plus tard le 30 septembre 2012. Nous vous incitons fortement à fournir des références indépendantes. Le dossier de candidature doit comprendre le nom des personnes données à titre de référence ainsi qu'un curriculum vitae récent du candidat ou de la candidate, dans la mesure du possible. Veuillez faire parvenir les mises en candidature et lettres de référence par voie électronique, de préférence en format PDF, avant la date limite, à l'adresse électronique correspondante:

Coxeter-James: **prixcj@smc.math.ca**

Jeffery-Williams: **prixjw@smc.math.ca**

Krieger-Nelson: **prixkn@smc.math.ca**

CMS Notes
Volume 45 No. 2, March-April 2013

Editorial Nominations

The Publications Committee of the CMS solicits nominations for five Associate Editors for the Canadian Journal of Mathematics (CJM) and the Canadian Mathematical Bulletin (CMB).
The appointment will be for five years beginning January 1, 2014. The continuing members (with their end of term) are below.

The deadline for the submission of nominations is November 15, 2013.
Nominations, containing a curriculum vitae and the candidate's agreement to serve, should be sent to the address below ;

Nantel Bergeron, Chair
CMS Publications Committee
Department of Mathematics & Statistics
York University
N520 Ross Bldg, 4700 Keele Street
Toronto, ON M3J 1P3
bergeron@yorku.ca

CURRENT MEMBERS:

CJM Editors-in-Chief

Henry Kim (Toronto)	12/2016;
Robert McCann (Toronto)	12/2016.

CMB Editors-in-Chief

Terry Gannon (Alberta)	12/2015;
Volker Runde (Alberta)	12/2015.

Associate Editors

Florin Diacu (Victoria)	12/2016;
Ilijas Farah (York)	12/2015;
Skip Garibaldi (Emory University)	12/2016;
Robert Leon Jerrard (Toronto)	12/2016;
Izabella Laba (UBC Vancouver)	12/2015;
Anthony To-Ming Lau (Alberta)	12/2016;
Alexander Litvak (Alberta)	12/2016;
Alexander Nabutovsky (Toronto)	12/2015;
Erhard Neher (Ottawa)	12/2016;
Vladimir Pestov (Ottawa)	12/2013;
Gordon Slade (UBC Vancouver)	12/2013;
Frank Sottile (Texas A&M)	12/2015;
Roland Speicher (Universität des Saarlandes)	12/2013;
Vinayak Vatsal (UBC Vancouver)	12/2013;
McKenzie Wang (McMaster)	12/2016;
Michael Ward (UBC Vancouver)	12/2015;
Jie Xiao (Memorial)	12/2013;
Efim Zelmanov (UCSD)	12/2016.

Appel de mises en candidature de rédaction

Le Comité des publications de la SMC sollicite des mises en candidatures pour cinq postes de rédacteurs associés pour le Journal canadien de mathématiques (JCM) et pour le Bulletin Canadien de mathématiques (BCM). Le mandat sera de cinq ans à compter du 1er janvier 2014. Les membres qui continuent (avec la fin de leur terme) sont ci-dessous.

La date limite pour les soumissions est le 15 novembre 2013.
Les mises en candidature, incluant un curriculum vitae et l'accord du candidat à servir, doit être envoyé à l'adresse ci-dessous :

Nantel Bergeron, Président
Comité de publication de la SMC
Département de mathématiques et statistiques
Université York
N520 Ross Bldg, 4700 rue Keele
Toronto (Ontario) M3J 1P3
bergeron@yorku.ca

MEMBRES ACTUELS:

Rédacteurs-en-chef JCM

Henry Kim (Toronto)	12/2016;
Robert McCann (Toronto)	12/2016.

Rédacteurs-en-chef BCM

Terry Gannon (Alberta)	12/2015;
Volker Runde (Alberta)	12/2015.

Rédacteurs associés

Florin Diacu (Victoria)	12/2016;
Ilijas Farah (York)	12/2015;
Skip Garibaldi (Emory University)	12/2016;
Robert Leon Jerrard (Toronto)	12/2016;
Izabella Laba (UBC Vancouver)	12/2015;
Anthony To-Ming Lau (Alberta)	12/2016;
Alexander Litvak (Alberta)	12/2016;
Alexander Nabutovsky (Toronto)	12/2015;
Erhard Neher (Ottawa)	12/2016;
Vladimir Pestov (Ottawa)	12/2013;
Gordon Slade (UBC Vancouver)	12/2013;
Frank Sottile (Texas A&M)	12/2015;
Roland Speicher (Universität des Saarlandes)	12/2013;
Vinayak Vatsal (UBC Vancouver)	12/2013;
McKenzie Wang (McMaster)	12/2016;
Michael Ward (UBC Vancouver)	12/2015;
Jie Xiao (Memorial)	12/2013;
Efim Zelmanov (UCSD)	12/2016.

Advancing Junior Canadian Number Theorists

The Canadian Mathematical Society, in partnership with the Tutte Institute for Mathematics and Computing (TIMC) staged a special additional Number Theory session as part of the 2012 CMS Winter Meeting on December 6 and 7.

TIMC is Canada's world-class mathematical and computational institute that conducts classified research in the areas of cryptology and knowledge discovery. The TIMC mission is to support the Canadian Cryptologic Program and its international partners by providing leading-edge solutions to emerging complex problems. The unique and robust solutions developed by TIMC staff have very real-life implications for the security of all Canadians and that of our Allies. Because of the importance and impact of the TIMC work, there is a very high level of personal and professional satisfaction as well as pride in TIMC achievements, albeit understandably cloaked within a secret rubric.

As TIMC is contemplating the inauguration of a Post Doctoral Program in the future, it was considered important to witness potential fellows in action. To this end, an announcement of this satellite meeting was sent to Canadian university mathematics departments, soliciting applications to speak at this event.

A committee made up of Chantal David, Eyal Goren and Andrew Granville selected the speakers. In the end, there were 42 talks presented in two parallel sessions. TIMC was able to provide full funding for the 18 attendees who are Canadians; the remaining speakers were partially supported by the Number Theory Foundation.

The session proved to be a great success - not only were there many interesting papers presented, but the quality and richness of the contributions was very high; abstracts of the presentations are posted at **http://cms.math.ca/Events/winter12/abs/nss**. Furthermore, the session provided a unique opportunity for this group of very talented young people to talk about their work, share ideas and get to know each other.

The sessions were all very well attended and, judging by the number of questions put to the speakers, the talks were of great interest to the audience. Many of the people who came to this session stayed throughout the CMS meeting to attend several Special Sessions, plenary talks and the play, "Math Science Investigation (MSI)." There was also a great deal of socializing among the attendees outside of the CMS events. From the perspective of TIMC, it was very useful to see the range and breadth of knowledge of these enthusiastic young people. From the perspective of the talented junior number theorists, the engagement with TIMC provided them a unique insight into the potential opportunities as a TIMC recruit through both long and short term contracts.

Anyone with a potential interest in working with TIMC on some incredibly intriguing problems across a remarkably wide spectrum of mathematics and who at the same time is a Canadian citizen who has or would be able to garner the requisite security clearances should check out the TIMC website [**http://www.cse-cst.gc.ca/tutte/**] and fell free to e-mail me directly at: Hugh.Williams@cse-cst.gc.ca.

Hugh Williams is the Director of TIMC and is internationally recognized as an expert in computational number theory and its application to cryptography; in particular, he is a world authority on computing in number fields.

CMS Notes
Volume 45 No. 2, March-April 2013

Education Nominations

The award recognizes individuals or teams of individuals who have made significant and sustained contributions to mathematics education in Canada. Such contributions are to be interpreted in the broadest possible sense and might include: community outreach programs, the development of a new program in either an academic or industrial setting, publicizing mathematics so as to make mathematics accessible to the general public, developing mathematics displays, establishing and supporting mathematics conferences and competitions for students, etc.

The deadline for nominations is April 30, 2013. Please submit your nomination electronically, preferably in PDF format, to **apaward@cms.math.ca**.

Nomination requirements:

- Include contact information for both nominee and nominator.
- Describe the nominated individual's or team's sustained contributions to mathematics education. This description should provide some indication of the time period over which these activities have been undertaken and some evidence of the success of these contributions. This information must not exceed four pages.
- Two letters of support from individuals other than the nominator should be included with the nomination.
- Curricula vitae should not be submitted since the information from them relevant to contributions to mathematics education should be included in the nomination form and the other documents mentioned above.
- If a nomination was made in the previous year, please indicate this.
- Members of the CMS Education Committee will not be considered for the award during their tenure on the committee.

Renewals:

Individuals who made a nomination last year can renew this nomination by simply indicating their wish to do so by the deadline date. In this case, only updating materials need be provided as the original has been retained.

Appel de mises en candidature d'éducation

Le prix récompenser aux personnes ou aux groupes qui ont fait une contribution importante et soutenue à l'enseignement des mathématiques au Canada. Le terme « contribution » s'emploie ici au sens large; les candidats pourront être associés à une activitée de sensibilisation, un nouveau programme adapté au milieu scolaire ou à l'industrie, des activités promotionnelles de vulgarisation des mathématiques, des initiatives, spéciales, des conférences ou des concours à l'intention des étudiants, etc.

La date limite pour des mises en candidature est le 30 avril 2013. Veuillez faire parvenir votre mise en candidature par voie électronique, de préférence en format PDF, à **prixap@smc.math.ca**.

Conditions de candidature :

- Inclure les coordonnées du/des candidats ainsi que le(s) présentateur(s).
- Décrire en quoi la personne ou le groupe mise en candidature a contribué de façon soutenue à des activités mathématiques. Donner un aperçu de la période couverte par les activités visées et du succès obtenu. La description ne doit pas être supérieur à quatre pages.
- Le dossier de candidature comportera deux letters d'appui signées par des personnes autres que le présentateur.
- Il est inutile d'inclure des curriculums vitae, car les renseignements qui s'y trouvent et qui se rapportent aux activités éducatives visées devraient figurer sur le formulaire de mise en candidature et dans les autres documents énumérés ci- dessus.
- Si la mise en candidature a été soumise en l'année précédente, s'il vous plaît indiquez-le.
- Les membres du Comité d'éducation de la SMC ne pourront être mise en candidature pour l'obtention d'un prix pendant la durée de leur mandat au Comité.

Renouveler une mise en candidature :

Il est possible de renouveler une mise en candidature présentée l'an dernier, pourvu que l'on en manifeste le désir avant la date limite. Dans ce cas, le présentateur n'a qu'à soumettre des documents de mise à jour puisque le dossier original a été conservé.

If undelivered, please return to:

Si NON-LIVRÉ, prière de retourner à :

CMS Notes / Notes de la SMC
209 - 1725 St. Laurent Blvd
Ottawa, ON K1G 3V4 Canada