

CMS celebrates longstanding member Dennis Russell

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CMS NOTES de la SMC

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From the Vice-President's Desk

Elena Braverman

CMS Vice-President Western Prov. & Territories

The Broken Link



Imagine a society where toddlers are discouraged from walking. It is strongly advised to keep them in strollers outdoors and in walkers inside the house; children younger than six are rarely seen walking in the street, and even then only when assisted by a parent. Parents who do not follow their kids' every step even in their backyard risk attracting the neighbours' attention. There are several reasons proclaimed for this: the risk of falling for toddlers and hurting themselves while they are too young to be aware of the dangers around; even supervised, young children can easily be involved in car accidents in the streets. There are handicapped kids who cannot walk, run and climb as successfully as the others, which creates a feeling of inequality at an age which can be handled only with great difficulty by psychologists. It is commonly agreed that at the age of six it is allowed to start learning how to walk. Unfortunately, the precious moment is lost, the beginner walkers are too heavy for their undeveloped muscles, and most children cannot really catch up, and can only walk slowly and clumsily for the rest of their lives. There is no public concern about this issue as it is not an obstacle to driving successfully ten years later. The system has many side effects: there are countless tutoring agencies, advertising a "smooth and easy way to walking", as well as private tutors, with many desperate parents of both young and older children relying on them. For rich people, there are elite clubs where walking skills are developed earlier by licensed instructors who do the training at the appropriate age. In addition, at the most advanced level there are free outreach activities

for active runners, especially for promising athletes. However, most students are neither interested nor qualified to participate. The whole idea does not stir society since most parents grew up in a similar environment, walk with difficulty and would definitely feel uncomfortable to assist their children once they move quickly; the same situation is with teachers at school.

The situation described above seems absurd, but is quite appropriate when applied to mathematics, which is sometimes seen as a scarecrow of our school education and even as an obstacle for socially underprivileged students to obtain a college or university education. This is especially surprising since development of mathematical skills requires neither expensive tools, for example when compared to many sports and experimental sciences, nor an "educated" environment to be cultivated, unlike language and humanities skills. The main reason why mathematics is blamed for causing depression in high school and first year undergraduate students is probably that mathematics is not a mirror that is easily distorted. It does not allow anybody to believe what he or she desires but tells the truth. We do not credit students for wrong solutions and answers, though there was an article by a parent in The Calgary Herald once (and in other sources I believe) that a partial score should be granted for totally incorrect mathematical solutions. I do not believe that first year students are more successful in writing essays or analyzing natural science phenomena than in solving mathematical problems, but in the first case evaluation criteria may be not so strict while my colleagues, for example, in geology, do not assume that students have any background. In mathematics, we are very much dependent on the skills our

Gauss, Chaitin and You.



Gauss famously wrote, on the subject of Fermat's 'last theorem': "I could easily lay down a multitude of such propositions, which one could neither prove nor disprove." His assessment of that particular conjecture was of course eventually shown to be wrong. Some have seen it as an arrogant statement, or at least one that only a mathematician of Gauss' stature could make without arrogance. In this editorial I want to show how you, too, can come up with your own "Reader's Conjecture," more likely than Fermat's to stand the test of time.

The idea is simple. The density of prime numbers decreases with $\log(n)$; so the probability that a randomly selected number near 2^n is prime is about $1/n$, and near 2^{2^n} the density is about 2^{-n} . If I throw one dart at the number line near each of $2, 4, 16, 256, \dots, 2^{2^n} \dots$ then the total expected number of primes that I hit is $O(1)$. (You may recall this hardly-new observation from my October 2007 editorial, "Some Polygonal Oddities.")

Now here's where you come into it. Before you start, if you are reading this online, please *print out this editorial* before you continue. Trust me, this is important; you will see why soon. Replace the dart by a function of your choice that is not very predictable. (Pick a few unrelated integer-valued functions that do not grow too fast – one good starting point is $p(n)$, the n^{th} prime, and another is the n^{th} digit of π . Your phone number will add a nice personal touch. Now add them together.) Your personal conjecture will be that $2^{n^2} + f(n)$ takes only finitely many prime values.

And there you have a conjecture in the true Gaussian style; a monument more enduring than brass, if not to your genius, at least to the superabundance of your free time. Write it, as it deserves, in the margin of this page – we have, unfortunately, not left space for the proof.

Is this exercise pointless? Maybe not! You may have heard of Gregory Chaitin's famous theorem that many facts in mathematics are "randomly true" – that is, they are true but for no reason simpler than that they are true. This may seem mysterious, but your already-famous-among-ourselves conjecture is probably a good example! Chances are that it's true: in fact, it's implied by your *Strong Conjecture*

$$(\forall n > N)(2^{n^2} + f(n) \text{ is composite}).$$

(You'll have to decide on a good value for N yourself.) But probably the fastest way to prove any finite fragment of your Strong Conjecture is to find a factor of each $2^{n^2} + f(n)$. Have fun!

Gauss, Chaitin et vous

Au sujet du dernier théorème de Fermat, Gauss a écrit la phrase célèbre suivante : « Je pourrais aisément énoncer une multitude de pareilles propositions, que nul ne pourrait ni prouver ni réfuter. » Il a été démontré par la suite, bien sûr, que son évaluation de cette conjecture était incorrecte. Certains y ont vu une déclaration arrogante, ou du moins une déclaration que seul un mathématicien de la réputation de Gauss pouvait faire sans arrogance. Dans cet éditorial, j'aimerais montrer comment vous pouvez vous aussi proposer votre propre « Conjecture du Lecteur », qui pourrait vraisemblablement résister davantage à l'épreuve du temps que celle de Fermat.

L'idée est simple. La densité des nombres premiers décroît avec $\log(n)$; la probabilité qu'un nombre choisi au hasard près de 2^n soit un nombre premier est donc environ $1/n$, et près de 2^{2^n} , la densité est d'environ 2^{-n} . Si je lance une fléchette sur la droite numérique près de chacun des nombres $2, 4, 16, 256, \dots, 2^{2^n}, \dots$ alors le nombre total prévu de nombres premiers que je touche est $O(1)$. (Vous vous souviendrez peut-être de cette observation pas tellement nouvelle formulée dans mon éditorial d'octobre 2007 intitulé « Quelques bizarreries polygonales ».)

Et voici où vous entrez en jeu : avant de commencer, si vous lisez ce texte en ligne, imprimez-le avant de continuer. Faites-moi confiance, c'est important; vous comprendrez bientôt pourquoi. Remplacez la fléchette par une fonction de votre choix qui n'est pas très prévisible. (Choisissez quelques fonctions à valeurs entières sans liens entre elles qui ne croissent pas trop rapidement – un bon point de départ serait $p(n)$, le $n^{\text{ième}}$ nombre premier, et un autre est le $n^{\text{ième}}$ chiffre de π . Votre numéro de téléphone ajoutera à l'ensemble une touche personnelle. Maintenant additionnez le tout.) Votre conjecture personnelle sera que $2^{n^2} + f(n)$ prend un nombre fini des valeurs premières.

Et voilà, vous avez une conjecture de style tout à fait gaussien, un monument plus durable que l'étain, sinon élevé à votre génie, du moins à la surabondance du temps libre dont vous disposez. Écrivez-la, comme il se doit, dans la marge de cette page – malheureusement, nous n'avons pas laissé d'espace pour la preuve.

Est-ce là un exercice inutile? Peut-être pas! Vous avez peut-être déjà entendu parler du célèbre théorème de Gregory Chaitin selon lequel de nombreux faits mathématiques sont « aléatoirement vrais », autrement dit, qu'ils sont vrais, mais pour aucune autre raison à part qu'ils sont vrais. Cela peut sembler très mystérieux, mais votre conjecture-déjà-célèbre-entre-nous en est probablement un bon exemple! Il est probable qu'elle soit vraie : en fait, cela est rendu implicite par votre *Conjecture Forte*

$$(\forall n > N)(2^{n^2} + f(n) \text{ est un nombre composé}).$$

(Vous devrez trouver vous-même une bonne valeur pour N .) Mais la façon la plus rapide de prouver n'importe quel fragment fini de votre *Conjecture Forte* serait de trouver un facteur pour chaque $2^{n^2} + f(n)$. Amusez-vous bien!

Du bureau du Vice-Président

Elena Braverman

vice-président de la SMC

(provinces de l'Ouest et les territoires)



Le lien brisé

Imaginez une société qui décourage l'apprentissage de la marche en bas âge. Dans cette société, il est fortement recommandé de garder les petits enfants dans une poussette à l'extérieur et dans un trotteur à l'intérieur; rarement voit-on des enfants de moins de six ans marcher

dans les rues, à moins qu'ils ne soient assistés d'un parent. Les parents qui omettent de suivre leurs enfants pas à pas – même dans leur cour arrière – risquent d'éveiller les soupçons de leurs voisins. Il en est ainsi pour de nombreuses raisons : un enfant pourrait se blesser en tombant alors qu'il est trop petit pour prendre conscience des dangers qui le guettent; un jeune enfant, même sous la surveillance d'un adulte, pourrait facilement se faire frapper par une voiture dans la rue. Le fait que les enfants handicapés soient incapables de marcher, de courir et de grimper aussi bien que d'autres crée un sentiment d'inégalité que les psychologues ont beaucoup de mal à gérer à cet âge. Il est communément admis que les enfants peuvent commencer à apprendre à marcher vers l'âge de six ans. Malheureusement, les marcheurs débutants ont perdu un temps précieux, car ils sont devenus trop lourds pour leurs muscles atrophiés. Si bien que la plupart n'arrivent jamais vraiment à rattraper ce retard et sont condamnés à marcher lentement et maladroitement pour le reste de leur vie. Cela n'inquiète personne puisque, dix ans plus tard, les enfants devenus grands peuvent quand même se déplacer facilement en voiture. Corollairement, on ne compte plus les agences de tutorat qui proposent des « méthodes faciles pour apprendre à marcher » ainsi que les tuteurs privés qui sont embauchés par tant de parents désespérés d'enfants jeunes et grands. Les riches, eux, donnent une longueur d'avance à leurs enfants en les envoyant dans des clubs d'élite où, avec le concours d'instructeurs spécialisés, ils apprennent à développer leurs habiletés motrices à l'âge idéal. De plus, au niveau le plus avancé, des programmes gratuits sont offerts aux coureurs actifs et plus particulièrement aux athlètes prometteurs. Rares sont toutefois les enfants qui sont intéressés ou qui ont ce qu'il faut pour en profiter. L'idée ne suscite guère d'engouement dans la société puisque la plupart des parents ont grandi dans des conditions similaires et marchent péniblement. Ces derniers seraient donc bien mal placés pour assister leurs enfants lorsqu'ils commencent à se déplacer rapidement. La situation n'est pas différente pour les enseignants dans les écoles.

Le portrait que je viens de décrire peut paraître absurde, mais il correspond assez bien à la réalité des mathématiques, parfois vues comme la bête noire de notre système d'éducation et

Suite à la page 16

Letters to the Editors Lettres aux Rédacteurs

The Editors of the NOTES welcome letters in English or French on any subject of mathematical interest but reserve the right to condense them. Those accepted for publication will appear in the language of submission. Readers may reach us at **notes-letters@cms.math.ca** or at the Executive Office.

Les rédacteurs des NOTES acceptent les lettres en français ou anglais portant sur un sujet d'intérêt mathématique, mais ils se réservent le droit de les comprimer. Les lettres acceptées paraîtront dans la langue soumise. Les lecteurs peuvent nous joindre au bureau administratif de la SMC ou à l'adresse suivante : **notes-lettres@smc.math.ca**.

NOTES DE LA SMC

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The Broken Link, continued from page 1

students brought from high school. However, we should be careful when comparing between the disciplines. When I complained to my colleague teaching history that our main problem is the poor background of students in math and that this poor background is probably not as much of an issue in the history department, she answered: "How would you teach history to students who have never heard of the first and the second world wars?" (oh happy students! It would be interesting to know where they come from).

One of recommended books for junior high school is the series "Math makes sense". There was a justified critique of these series by Dr. Malgorzata Dubiel, see, for example,

<http://blogs.vancouversun.com/2012/04/13/math-makes-sense-not-with-these-textbooks-expert-says/>

<https://sfu.ca/pamr/issues-experts/2012/assessing-math-teaching.html>

I have just reviewed the textbooks for grades 8 and 9 in this series; I recommend this activity to every colleague involved in teaching first year calculus, as it is quite revealing. Your students have difficulties with adding fractions? There is no reason to be surprised: Unit 3 of the grade 8 book titled "Operations with fractions" deals with multiplication and division but not addition of fractions. Bringing fractions to the common denominator is not outlined; the word "least common multiple" is not even in the index. So, next time, in the topic of "integration with partial fractions", when students choose the product of all the denominators of algebraic fractions as a common denominator, just note that they do the best they can. By the way, concerning algebra, the only element introduced by the end of junior high school is addition of polynomials, multiplication and division by a number and by a monomial (in all the exercises, a polynomial with integer coefficients is a result). The first impression is that the book was inspired by lazy schoolchildren who wanted material that could cause any difficulty removed from the curriculum. It does not matter that you will need these things in the future; if the addition of fractions, squaring a binomial, and solving a quadratic equation can be postponed to senior high school, they probably can be avoided.

However, the insufficient technical skills acquired by students in the first 9 years of their school life, in elementary and junior high schools, is only a symptom. The main problem of the approach accepted in the books is that what is presented is not really mathematics (which is all about why) but some collection of recipes. In the books, I could not find any explanation for why the sum of measures of angles in a triangle is 180° (or for that matter, any other fact in geometry). The best justification that could be found in the book is: draw some triangle, measure, compute and check. Reviewing the books for grades 8 and 9 was very educational for me: now I can understand what my students may mean by "I am good at math" and "I am bad at math" and why their image of their mathematical abilities may be so far from reality. Presumably being good at math is viewed as being able to implement some numerical or algebraical operations according to some prescribed (and even not justified) algorithms quickly and accurately after lengthy

training. It is fortunate I had not been informed earlier; personally, I struggled with long addition and subtraction in grades 2 and 3.

There are several aspects of mathematical education: the first and the most important is developing the mind in general and logical thought in particular. The second one is encouraging independent thinking and judgment: there are minor discoveries even on the way of beginners, and a student can challenge his instructors and parents since only reason but not authority establishes what is true. The third part is acquiring some technical skills and learning certain methods. For some reason, only the third aspect seems to be involved in the present junior high school curriculum, and even then at a rather limited level. For example, the textbook includes exercises outlining problem solving skills; however, practically all the problems require only one step for their solution.

The main cause of this limited representation of mathematics is the one-size-for-all education; sometimes it may mean that the slowest students have no chance to catch up with the material, while the more advanced students have no chance to get exposed to much mathematics (which is all about reasoning and not a set of recipes). I believe while there is still a certain percentage of junior high school students in the former category who would benefit from less intensive or more spread over the years curriculum, an even larger proportion falls into the latter category. There are free outreach activities (math nights at the University of Calgary and similar clubs at practically every Canadian university), but until a certain age it is in fact the parents' initiative: students cannot get to the universities on their own. And when the students grow up and get to high school it is unfortunately too late.

The situation can be resolved in the following way, with some foreign school systems serving as an example. First, in the elementary school when the pressure of school reports is not yet high, students should get exposed to operations with integers (mastering oral counting to 100, for example) and fractions (including addition), as well as some ideas in geometry and basic word problem solving skills. In the beginning of grade seven, as a result of some qualification test, students are divided into several groups (four in the program of reference), run at the same time for several classes at the same level. Starting with the senior high school, there are several streams in mathematics (A,B,C) where A has not only enriched calculus which is at a level between Math 31 and Calculus in IB programs, but also includes high level of algebra, geometry, and trigonometry. Level B roughly corresponds to the well mastered Math 30 program, while C is a weaker level. Students can switch to a higher stream taking exams; it is true that some of them will employ tutors for this. However, this will be incomparable to the level of private assistance that our high school and first year undergraduate students are seeking now. Their present situation can be compared to a joint school race for all grades: first graders have no chance, graduates have no challenge.

In a smooth transition for our high school students to universities, a solid mathematical background plays an important role. However, this link for many of our first year science and engineering students is broken, probably at an earlier stage than senior high school.



JUNE 2013

3-7 PIMS/EQINOCS Automata Theory and Symbolic Dynamics Workshop (UBC, Vancouver) <http://www.pims.math.ca/scientific-event/130603-atsdw>

3-14 Moduli Spaces and their Invariants in Math.Physics (CRM, Montreal, QC) http://www.crm.umontreal.ca/2013/Moduli13/index_e.php

3-28 Noncommutative Geometry & Quantum Groups (Fields Inst. Toronto, ON) www.fields.utoronto.ca/programs/scientific/12-13/quantumgroups

4 18:00-19:00 Erik Demaine, MIT *Algorithms Meet Art, Puzzles, and Magic* (Halifax, NS) <https://cms.math.ca/Events/summer13/abs/pdf/pl-ed.pdf>

4-7 CMS Summer Meeting (Halifax, NS) <http://cms.math.ca/Events/summer13/>

7 11:30-12:30 Eva Knoll, Mount Saint Vincent University *The Hilbert Curve: Algorithms, Processes, Influences, Results* (Halifax, NS) <https://cms.math.ca/Events/summer13/abs/pdf/pl-ek.pdf>

10-13 Canadian Discrete and Algorithmic Mathematics Conference (St. John's, NL)

16-23 51st International Symposium on Functional Equations (Rzeszów, Poland) tabor@univ.rzeszow.pl

17-20 7th Annual International Conference (Athens, Greece) www.atiner.gr/mathematics.htm

17 -28 Algebraic Topology (Berkeley, CA) <http://www.msri.org/web/msri/scientific/workshops/summer-graduate-workshops/show/-/event/Wm9063>

21 Insects, Computers, and Us, Jane Wang (Toronto, ON) <http://mpe2013.org/public-lectures/>

30 - July 20 Geometric Analysis (Berkeley, CA) <http://www.msri.org/web/msri/scientific/workshops/summer-graduate-workshops/show/-/event/Wm9754>

JULY 2013

1-5 Summer School on Geometry, Mechanics & Control (Madrid, Spain) <http://gmcnetwork.org/drupal/?q=activity-detail/867>

1-5 Erdős Centennial (Budapest, Hungary) <http://www.renyi.hu/conferences/erdos100/index.html>

1-12 New Geometric Techniques in Number Theory (Berkeley, CA) <http://www.msri.org/web/msri/scientific/workshops/summer-graduate-workshops/show/-/event/Wm9460>

16-20 HPM 2012 History and Pedagogy of Mathematics The HPM Satellite Meeting of ICME-12 (Daejeon, Korea) <http://www.hpm2012.org>

29-Aug 2 Workshop on Climate Change, etc (Guanajuato, Mexico) <http://www.mca2013.org/en/workshop-on-mathematics-of-climate-change.html>

31-Aug 1 MAA MathFest (Hartford, CT) <http://www.maa.org/mathfest>

AUGUST 2013

3-8 Joint Statistical Meetings (Montreal, QC) www.amstat.org

5-9 Mathematical Congress of the Americas (Guanajuato, Mexico)

28- Sept 2013 Gelfand Centennial Conference (MIT, Cambridge MA) <http://mat.mit.edu/conferences/Gelfand/index.php>

SEPTEMBER 2013

2-5 Royal Statistical Society Conference (Newcastle, UK) www.rssconference.org.uk

9 La prévision des grandes catastrophes, Florin Diacu (CRM, Montreal, QC) <http://mpe2013.org/lecture/la-prevision-des-grandes-catastrophes/>

9-13 Combinatorics, Graph Theory & Applications (Pisa, Italy) <http://www.eurocomb2013.it/>

12-14 Category Theory & Algebraic Topology Summer School (Louvain-la-Neuve, Belgium) Marino.gran@uclouvain.be

22-27 The first Heidelberg Laureate Forum <http://www.heidelberg-laureate-forum.org/>

28-29 Mathematics of Planet Earth 2013 (Banff, Alberta) http://www.crm.umontreal.ca/act/theme/theme_2013/indigenous_population13_e.php

OCTOBER 2013

10 Ocean Waves, Rogue Waves, and Tsunamis, Walter Craig (Fredericton, NB) <http://mpe2013.org/lecture/ocean-waves-rogue-waves-and-tsunamis/>

NOVEMBER 2013

7 Mathématiques de la planète Terre, Christiane Rousseau (Québec, QC)

11-12 IYS Video & Statistical Science Workshop (London, UK) www.statistics2013.org

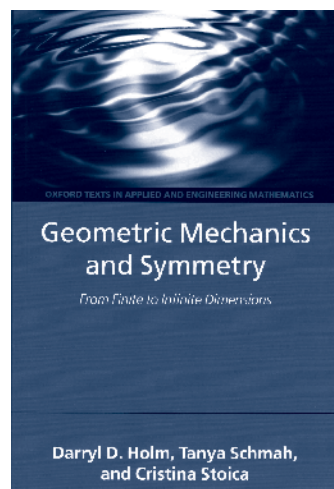


Geometric Mechanics and Symmetry: From Finite to Infinite Dimensions, by D. Holm, T. Schmah and C. Stoica

Oxford University Press 2009

ISBN 978-0-19-921290-3

Reviewed by **Robert Milson**, Dalhousie University, Halifax, NS



The item under review, "Geometric Mechanics and Symmetry" is really two books in one. Part I is a textbook on geometric mechanics. Part II is a research-level monograph on the geometric formulation of PDEs related to fluid flow. The first part consists of standard textbook material that can be found in many other books on mathematical mechanics such as [MR]. The second part reads more like a research survey in integrable systems combining

elements of introductory articles such as [P], more comprehensive books such as [BBT], and more specialized topics found in current research literature.

Despite the apparent disparity between the topics, this amalgamation is far from artificial. In 1966 Arnold discovered that the equations governing the motion of an ideal, incompressible fluid can be formulated as an infinite-dimensional analogue of rigid body motion [A1]. For a rigid body, the configuration space is a Lie group, and the dynamics are determined by an invariant Lagrangian describing the internal energy and external forces acting on the body. Extending configuration space to the infinite-dimensional group of volume preserving diffeomorphisms, and taking kinetic energy as the diffeomorphism-invariant Lagrangian functional, leads naturally to the derivation of the Euler equations for the dynamics of an ideal fluid.

This beautiful connection between classical and fluid mechanics has a profound application to integrable systems. Many non-linear equations related to fluid dynamics, such as the celebrated KdV equation, are integrable. One particular aspect of integrability is the property that the dynamics in question admit an infinite number of conserved quantities and some procedure for dynamical reduction. For the case of rigid-body mechanics, the geometrical setting for integrability is furnished by the moment map, symplectic reduction, and the Liouville-Arnold theorem. For integrable PDEs, the integrals of motions are obtained by means of a bi-Hamiltonian structure, while integrability is related to an

Inverse Scattering Transform. The formulation of the dynamics in terms of an invariant Lagrangian defined on the tangent space to a Lie group, in principle, allows for the derivation of all these features in a uniform and systematic manner. The book in question covers some, but not all, aspects of this conceptualization. For example, inverse scattering is mentioned in the introductory blurb to Chapter 13, but not systematically developed in the text.

Part I of the book under discussion consist of 10 chapters on various topics in geometric mechanics. Part II offers an additional 8 chapters on a rather eclectic range of topics. The book begins with a review of mechanics (Chapter 1) and differential geometry (Chapters 2-3). Chapter 4 ties these together by introducing Poisson structure and by recasting the Chapter 1 material in a coordinate-free fashion. Chapters 5 and 6 bring Lie groups into the picture. Then, there is a break for applications, including rigid body mechanics (Chapter 7) and the group theme resumes with a discussion of moment maps and Lie-Poisson brackets in Chapters 8-9. Part I concludes with an application to generalized rigid bodies in Chapter 10. It is a nice example of a more general class of Lie groups used as a configuration space for mechanical systems.

Taken alone, Part I could be used as an advanced undergraduate or beginning graduate text in geometric mechanics. Chapter sections conclude with exercises and solutions — some computational, others more conceptual in their focus. The tone is pedagogical but rigorous, with space devoted to exposition trying to get across the geometrical intuition behind the definitions and theorems.

My chief complaint about Part I is the omission of topics related to integrability. There is no treatment of Liouville's integrability theorem, and with the exception of rigid body mechanics, there is no coverage of the standard integrable mechanical systems that can be found in other books on the subject. Given the structure of the book, such an omission is understandable. The overall focus of the book is on systems whose configuration space is some kind of a group, however, the consequence of this choice is that it will discourage the adoption of the book as a class text. Everything else being equal, one can envision using the book in a mechanics course that focuses mostly on Part I, and covers just the first few chapters from part II as advanced or optional topics. The absence of standard material from Part I means that any such course would require supplemental readings to cover topics such as the Hamilton-Jacobi equation, canonical transformations, and integrability.

Moving onto Part II, the book switches gears in various ways. First, there is the size: 10 chapters spread over 350 pages for Part I, versus 8 chapters and 150 pages in Part II. The presentation is much more terse and the pedagogical tone is largely gone. Chapter 11 introduces the EPDiff equations, which are the analogues of the Euler-Poincare rigid body equations on

an infinite dimensional group (called by other authors the Euler-Arnold equations.) It is in this chapter that the synergy between Parts I and II has the biggest payoff. The variational formulation and the reduction procedure for the PDEs are clearly analogous to their finite-dimensional counterparts. However, very little space is devoted to this synergy. After deriving the equations for an ideal fluid using the group setting, the treatment of the Lagrangian and of the reduction procedure is done in just 2 pages, in Subsections 11.4.2 and 11.4.3.

Next, the book treats a particular example: the Camassa-Holm (CH) equation and its dispersionless limit. Chapter 12 is devoted to soliton-like solutions called peakons and pulsions and Chapter 13 focuses on establishing integrability of the CH equation via the bi-Hamiltonian formalism. The dispersionless limit of the CH equation can be derived as a 1-dimensional instance of the EPDiff equation. Other than that, the geometric machinery of Part I seems to play no role in these chapters. Indeed, strictly speaking, bi-Hamiltonian structures and Magri's Lemma should have been introduced in Part I, since these ideas are perfectly applicable in the finite-dimensional setting. Instead these topics are done en passant before deriving the recursion operator for the CH equation and establishing isospectrality.

Here we find a bit of a disconnect with the Part I material because the general CH equation is not derived using any geometric formalism. It's introduced as a generalization of the 1D EPDiff equations. This editorial choice is very puzzling. In my opinion, a more pedagogical approach would have focused on the KdV equation, a much more standard example which has a clear geometric derivation. All of the concepts in Chapters 12 and 13 have KdV analogues. Doesn't it make more sense to teach the students about solitons before introducing them to peakons?

Chapter 14 is devoted to the EPDiff equations in higher dimensional settings. Here there is no general integrability theory. Rather the authors introduce a particular ansatz that reduces the PDEs to a finite-dimensional Hamiltonian system which can be further reduced using moment maps. The chapter concludes with a brief discussion of numerical methods. There are no details, just some references and some thought-provoking illustrations.

Chapters 15-18 are devoted to a range of applications: computational anatomy and image processing, continuum mechanics, and geophysical fluid dynamics. Here the style is no longer that of a textbook, but rather that of a research-oriented survey. The material should be of interest to applied mathematicians interested in integrability and to advanced students pursuing a topic in this area. Some interesting geometric ideas, like semi-direct products, are introduced along the way, but the treatment is sketchy and the reader is referred to various publications for details.

In summary, this book defies straight-forward categorization. Part textbook, part survey monograph, it will appeal to many

readers, but I dare say, will not satisfy most of them. The infinite-dimensional geometrical setting of integrable PDEs is covered much more thoroughly in texts such as [KW]. By way of illustration, the book under review markedly avoids a discussion of the functional-analytic foundations underlying infinite-dimensional geometry. However a proper formulation of the infinite-dimensional variational theory allows one to treat the fluid equations as an ordinary differential equation on an infinite-dimensional manifold, and has important applications to questions of solution existence and stability [EM]. This is just one way in which the book has an unfinished feel to it.

On the other hand a course in Mathematical Mechanics would be better served by the well known [MR] or even Arnold's classic [A2]. That may be an unfair criticism, because it seems that the ambition of the book under review is to provide a path to integrable systems via classical mechanics. I have reservations about the extent to which this ambition is fulfilled. Part II is too terse, too limited in its choice of core topics at the expense of more specialized developments, to serve as a textbook in integrable systems. The two part organization is a promising idea, but Part I shows far more polish than Part II. In my opinion, the book does not live up to the full potential of such an approach.

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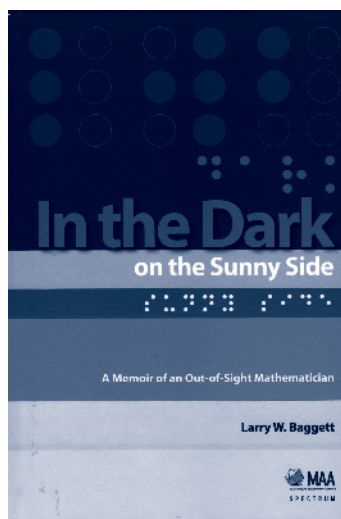
In the Dark on the Sunny Side: A Memoir of an Out-of-Sight Mathematician, by Larry W. Baggett

Mathematical Association of America Spectrum, 2012.

ISBN : 978-0-88385-581-2

Reviewed by **Keith F. Taylor**, Dalhousie University, Halifax

Let me begin with the full disclosure that Larry Baggett is a collaborator and good friend of mine which makes it impossible for me to read his memoir with total objectivity. But allow me to give you my sense of this book, but first I provide some background on the author.



By looking over Larry Baggett's *resumé*, you would judge his academic career as a solid success. He graduated from Davidson College of North Carolina with an honors degree and the University of Washington, where he wrote his thesis under the supervision of J.M.G. Fell, with a doctorate. He started as an Assistant Professor at the University of Colorado in 1966 and enjoyed a 40 year career, rising through the ranks in due course and serving his turn as Chair of their rather strong,

but sometimes prickly, Department of Mathematics. Along the way, he supervised a dozen PhD students, published books on Fourier analysis and functional analysis and established himself as one of the world experts on several themes of abstract harmonic analysis and the foundations of wavelet theory. I imagine his Deans were quite satisfied with his annual reports. What is remarkable is that Baggett has been blind since the age of five.

The book begins with a comical description of the author's last visual memory but it takes a sober turn with the 'quantum moment' where he lost his sight. By page 20, the reader is immersed in this child's life. About half the book is devoted to childhood experiences that are similar to the experiences of many kids, but different in essential ways. We learn that the family spent two years in Boston and young Larry got to start his education at Perkins Institute for the Blind, 'the premier school for the blind anywhere in the world.' It seems that this foundation equipped him with many of the skills to interface with the sighted world. The family then moved to Florida and a less than satisfactory year in the Florida School for the Blind. From grade 4 onward, he was 'mainstreamed'; apparently the first blind student

enrolled in the Orlando Public School System. He was also the first blind student at prestigious Davidson College. He gives a great deal of credit to his mother for insisting that he experience life to the fullest and to many kind teachers who accommodated his disability.

There are various vignettes from the school years in which Baggett's organized thought processes are evident and you can imagine him becoming a mathematician.

However, one gets the impression, probably true, that music was much more important to him in those years than mathematics. He played clarinet in his school band and became an accomplished pianist (this even provided an income source while he was in graduate school). Some of the most interesting, semi-technical, sidebars that he includes provide information on how he thought about music. Perhaps that influenced him in choosing to study harmonic analysis at the University of Washington.

While at the University of Washington, he joined with a group of 60 faculty members and teaching assistants in suing the university over two state laws that required oaths of allegiance to the United States and to swear to not being a subversive person. The case went to the US Supreme Court and the landmark ruling struck down the offending laws. The author's name now is enshrined in case law. Look up *Baggett v. Bullitt*. Although he downplays his social conscience, it is clear that he was always concerned with doing his part for the greater good. He served as his class president for most years in high school and, as a senior, was president of student council.

As is often the case with memoirs and our own memories, time is compressed for adult life. Perhaps the most heartfelt aspects of the second half of the book deal with the birth of his daughters Alice and Molly, who he declares as his best work, and his second wife Christy. In this latter part of the book, we also learn of the early difficulties he faced with writing up his research results and developing teaching strategies. Let me say that he made all this look quite easy to those of us who know him as a productive mathematician. Various technological advances have made reading and writing mathematics much easier for him and he wrote his graduate text on functional analysis with no assistance. Although I found less information about his academic life than I hoped for in the book, he does give the reader a feel for some of the most pleasant parts of his job such as working with graduate students and other collaborators and travel for conferences or sabbatical leaves.

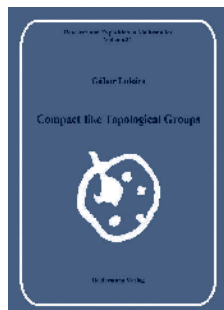
Baggett's writing style is skilled but unpretentious resulting in a book that is a very easy read. The reader becomes comfortable with the author as a regular person who is experiencing life in an unusual manner. He keeps the technical mathematics to an absolute minimum. As a result, the book can be enjoyed by anyone interested in the the process of succeeding at a high level in spite of a major handicap.

Compact-like Topological Groups, by Gábor Lukács

Research and Expositions in Mathematics n. 31
Heldermann Verlag, Lemgo 2009

ISBN 978-3-88538-231-7

Reviewed by **Dikran Dikranjan**, Università degli Studi di Udine, Italy



This book is dedicated to those topological groups that share some of the typical properties of the compact groups.

The first part of the book is mainly oriented to the topological theory of topological groups. Chapter 1 is an introduction to the basic facts of the theory of topological groups, as well as their elementary properties: separations axioms, the

metrization theorem of Birkhoff-Kakutani, complete groups and completions, connectedness and disconnectedness in topological groups.

Chapter 2 treats cardinal invariants of the topological groups with particular emphasis on character, pseudocharacter, weight and Lindelöf number. An entire section is dedicated to a cardinal invariant going back to the notion of \aleph_0 -bounded group, introduced by I. Guran [Dokl. Akad. Nauk SSSR 256 (1981), no. 6, 1305–1307] (a group that can be covered by countably many translates of every non-empty open set); \aleph_0 -bounded groups are precisely the subgroups of products of second countable groups. The author considers a generalization of this concept, using the term τ -precompact group (a group which can be covered by τ many translates of every non-empty open set). This chapter considers also other classes of groups defined by a similar property (namely, topologically isomorphic to (closed) subgroups of products of groups of a certain form). In this sense, it is fair to say that the target of the first part of the book are the compact-like properties coming from general topology.

The second part of the book is focused on compact-like properties of topological groups related to algebra, analysis and category theory. Chapter 3 introduces the minimal topological groups, namely the Hausdorff groups admitting no strictly weaker Hausdorff group topology and gives a brief outline of the history of this topic. The first section of this chapter describes precompactness, while section 2 discusses various permanence properties of the minimal groups (preservation of minimality under taking closed or dense subgroups, including Banaschewski-Prodanov-Stephenson's criterion for minimality of dense subgroups), as well as totally minimal groups (the minimal groups having all Hausdorff quotients minimal). The behavior of (total) minimality under taking products and extension is studied in Section 3. Two of the most interesting

results in the area of minimal groups are a theorem of I. R. Prodanov and L. N. Stoyanov [C. R. Acad. Bulgare Sci. 37 (1984), no. 1, 23–26] asserting that every abelian minimal topological group is precompact, and a recent theorem of M. Megrelishvili [Topology Appl. 155 (2008) 2105–2127] stating that every topological group G is a retract of a minimal topological group M (so G is simultaneously a closed subgroup of a minimal topological group and a quotient of a minimal topological group). These theorems are given without proof.

Chapter 4 deals with the so-called categorically compact (or, briefly, c-compact) groups. This is the largest chapter of the book, based on the author's Ph.D. thesis ["c-compactness and generalized dualities of topological groups", York Univ., Toronto, ON, 2003]. A well-known theorem due to Kuratowski and Mrówka states that a topological space X is compact if and only if the projection $X \times Y \rightarrow Y$ is a closed map for every Y . E. G. Manes [Gen. Topology Appl. 4 (1974), 341–360] used this property to define compactness in a quite general categories, where binary products as well as an appropriate notion of closed subobject, exist. (The possibility of such a general approach is discussed in detail in a special appendix at the end of the book, containing the necessary background from category theory, including the theory of closure operators.) In the case of the category of topological groups, a topological group G is c-compact if for every topological group H and each closed subgroup F of the product $G \times H$ the image of F under projection is closed in H . C-compact groups were studied by V. V. Uspenskij and the reviewer [J. Pure Appl. Algebra 126 (1998), no. 1-3, 149h168]. Obviously, compact groups are c-compact, while c-compact groups are complete. Following Tonolo and the reviewer [Rivista di Matematica Pura ed Applicata 17 (1995) 95--106], call h -complete a topological group whose continuous homomorphic images are necessarily complete. Since the class of c-compact groups is closed under taking closed subgroups and continuous homomorphism images, one deduces that every c-compact group is h -complete. The author introduces the new notion of hereditary h -complete groups (namely, a h -complete group in which every closed subgroup is still h -complete), observing that c-compact groups are hereditary h -complete. It was shown by Uspenskij and the reviewer [op. cit.], that the c-compact groups are compact under some additional property (e.g., soluble, connected locally compact, etc.); moreover, nilpotent h -complete groups are compact. It still remains unknown whether there exist non-compact c-compact groups. In particular, it is not known whether a discrete c-compact group is always finite. The author raises also the problem to distinguish the notions of c-compact and hereditary h -complete.

There are many other instances of "compact-like behavior" of non-compact topological groups that the reader may expect to see but will not find in the book (e.g., pseudocompactness and countable compactness, to mention just two). But clearly, it is not possible to reasonably cover such a vast area in a short book. The book is very well organized and nicely written. Its first chapters can be used for an introductory course on topological groups.

Jennifer Hyndman, *University of Northern British Columbia*
John Grant McLoughlin, *University of New Brunswick*

Recreational mathematics offers a vehicle for engaging students in mathematical thinking at all levels. This is the second issue dealing with recreational mathematics.

The article in this issue considers examples of mathematical puzzles, games and ideas that can be developed to support mathematical learning from elementary through to undergraduate levels. The ideas here are particularly pertinent to those who may be teaching mathematics in a school context, or offering courses to those who are prospective teachers. The essential ideas are readily adapted to other audiences. Feedback is welcomed, as are your examples to share within future issues of Education Notes. Please contact the editors with your ideas.

Recreational Mathematics: An Avenue to Engaging in Mathematical Development

John Grant McLoughlin

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The article features four examples of mathematical challenges or games that have been effectively drawn into my own teaching. The idea of playing games for fun is healthy, though it is the intentionality of drawing particular forms of challenges or games into play that makes the teaching and learning much more meaningful. That is, the examples here are integrated into situations that are designed to develop number sense, as with the first two examples, or bring forth aspects of mathematical structure and/or proof, as with the latter pair.

Developing Number Sense

Random trial and error is commonly employed to find numbers meeting particular requirements or constraints. Such trials are beneficial to ensure understanding of a problem, though examples here demonstrate how one can go beyond randomness to actually get at the mathematical underpinnings of a challenge.

Alphametics

This is an example of a puzzle that uses letters to represent distinct digits. It is typically assumed that the initial digit of any number represented in the problem is nonzero. Also, any letter that appears more than once must represent the same digit in all instances; further, a digit is to be represented by one letter consistently. For example, if $K = 4$, there is not another letter also representing 4.

Consider the following example that has been used by me in a variety of contexts:

$$ABCD \times 4 = DCBA$$

What letters are represented by each of A, B, C, and D?

If students are given a few minutes to play with the challenge, it is common for some observations to be raised, that when put together get us started. The first points may be:

- A is 1 or 2 since multiplication by 4 results in a product less than 10,000.
- A must be even as it is the last digit of the product and we multiplied by 4.

It follows that $A = 2$, and then the number sense may lead to the following points.

- D must be 8 or 9 since the product is greater than 2000×4 .
- $D \times 4$ ends in 2, and so $D = 8$.

Some students are inclined to revert at this point to trial and error again, though number sense can be applied neatly here. Recall that $4 \times 8 = 32$ and hence, there is a carry of 3 into the tens, thus, guaranteeing that the value of B must be odd. Why? We know that the value of $4C + 3$ is odd. Quickly it follows that $B = 1$ to keep the product to four digits, and finally, $C = 7$.

The above example is one illustration of how number sense can be developed or unearthed, as it may be there beneath the surface already. My tendency is to use the example as a collective problem. Then the following example with a parallel structure and less restrictions can be offered for unsupported work. (It is important to note that the challenges are independent and all digits are available again.)

$$EFGH \times 9 = HGFE$$

Interestingly, 1089 and 2178 are the only four-digit numbers that can be multiplied by a single digit greater than 1 to produce their reversals.

Pick and Find a Number

This problem can serve as an icebreaker. The example below was initially shared at the public lecture opening *Sharing Mathematics: A Tribute to Jim Totten* in May 2009. The names have been maintained, as they were all colleagues of Jim.

Kirk, John (Ciriani), Fae, Sonja and Peter are seated in a circular arrangement. Each person selected a number. The neighbours then added their numbers together and the results are shown. What was John's number?

	63	Kirk	
Peter			71
48		Sonja	
Fae			80
	90	John	

Again there is a tendency for students to use trial and error in many cases. It is helpful to try a possible number and see what happens as a means of understanding the problem. However, number sense or some form of mathematical organization allows for a swift and elegant solution. Observe that twice the sum of the five selected numbers equals $71 + 80 + 90 + 48 + 63$, or 352. Hence, the sum of the five numbers is 176.

One possible approach is to exclude John's number from a sum containing each of the other four numbers. Leaving John's number out, the total of the remaining four numbers is $71 + 48$ (that is, Kirk + Sonja added to Fae + Peter). Hence, John's number must be $176 - (71 + 48) = 57$. Other methods can also be employed. This usually surfaces in the discussion, thus, providing an experience in which multiple approaches are used. The middle school student can do this as a reasoning problem, whereas, the high school student can opt for an algebraic representation. In fact, a way to ensure the problem is understood is to form groups of five to generate a set of sums to be exchanged with other groups for the purpose of solving.

Mathematical Structure and Proof

Two examples of games lending themselves to other aspects of mathematics are provided here.

The first of these, *Fifteen Finesse*, appeared in a Martin Gardner book, *aha! Insight*. The rules of the game are described. Playing the game a few times helps one to understand the core principles. The second example, *Sim*, is spatial in nature. While ties are commonplace in the first of these two games, the proof in the second example depends upon the fact that a tie is impossible in *Sim*.

Fifteen Finesse

The numbers 1, 2, 3, 4, 5, 6, 7, 8, and 9 are used in this game – one of each being available for use. Players alternate turns selecting one of the available numbers with the object being to have three numbers that sum to 15. (Note that 7 and 8, or 2, 3, 4, and 6 would not satisfy the requirement as it must be exactly three numbers.) The game ends in a tie if no player gets three numbers that sum to 15.

What makes this game mathematically intriguing to play? My experience is that students generally play this in a way that that sharpens their thinking. For instance, consider the following sequence of opening number selections with the choices of Player A in bold: **6 8 2 7**. A misconception is that the 6 and 2 are no longer helpful when the choice of 7 blocks the possibility of using them to make 15. However, the 6 and 2 remain with Player A and a subsequent selection of 4 here would actually guarantee a win, as both 5 and 9 remain available for either (6, 4, 5) or (2, 4, 9). Reasonable players will soon find that many games are ending without a winner. Why?

The answer lies in the structure of the game. This game has a structure that is isomorphic to tic-tac-toe with a playing board that consists of a magic square.

4	9	2
3	5	7
8	1	6

That is the object of the game is to claim any of the triples forming a diagonal, row, or column in the magic square. In order to appreciate the structural similarities it is helpful to have the students record the sequencing of number selections in a few games prior to delving into the isomorphism itself. One can readily replay the moves on the magic square board to see how it is that they obviously won or lost a game.

Sim

Six dots are drawn on a piece of paper to form the vertices of a (convex) hexagon. Two players are each assigned a colour. The players take turns joining any two of the dots with a line segment, using their assigned colours. The loser is the player

who completes a triangle with three of the original six dots as its vertices and with all three edges the same colour.

Why is it that the game of Sim is mathematically enriching to play? Practically speaking it offers a curious “equalizer” quality in that spatial perception and logic blend to bring forth strengths/weaknesses not so apparent day to day in a class. My experience is that some struggling math students have gained confidence by matching up with or perhaps bettering students and teachers known to be more successful in mathematics. Indeed the game helps to focus attention on detail – a valuable skill for mathematical development. Further, the game does not take long to play as there are only 15 possible segments that can be drawn.

Mathematically there is a lovely connection to proof. Unlike the preceding game, it is a fact that every game of Sim must have a winner. (In fact, in dire situations with an imminent losing position late in a game it may be worth checking if you won already and did not notice!) Why is a tie impossible?

The essence of the why lies in the idea of a proof by contradiction that is outlined here. Assuming a tie is possible would require that all five possible segments be drawn from each vertex. So we know that at least three segments from any vertex must be the same colour. So suppose that three red segments are drawn from a given vertex to connect with three other vertices. This will create a situation in which two edges of three different triangles, each containing two of the original vertices, are the same colour. Hence, a tie is only possible if the third edge of each of these triangles is the other colour. However, this creates a triangle with three edges all having the other colour. The contradiction is evident.

Conclusion

Select games and other recreational mathematical ideas offer valuable teaching examples that can be intentionally drawn into the development of mathematical work at elementary or advanced levels. The beauty of many examples comes from the invitational space created for the student to engage directly in the mathematical process, thus, enabling deeper appreciation of what is essentially being learned.

Followup note to interested readers of Education Notes

Sim and Fifteen Finesse are two of five ideas shared in Playing Games with Mathematics (Part I) and Playing Games with Mathematics (Part II) in Crux Mathematicorum with Mathematical Mayhem issues 32(5) and 32(6) respectively. These articles are both publicly accessible via the links below:

<https://cms.math.ca/crux/v32/n5> and <https://cms.math.ca/crux/v32/n6>. They appear under the heading of Polya’s Paragon in the Mathematical Mayhem section.

Part I provides the games and challenges, whereas, Part II offers insights into the mathematics underlying them as it is hoped that readers will have played with the ideas beforehand.

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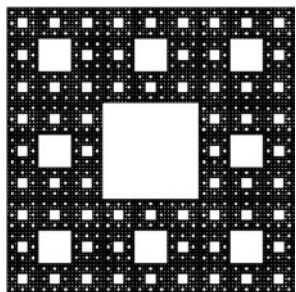
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My Favorite Planar Fractal

Robert L. Devaney

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Unlike most people who adore the Sierpinski triangle, my favorite planar fractal is a Sierpinski curve. By definition, a Sierpinski curve is any planar set that is homeomorphic to the well-known Sierpinski carpet fractal (see the figure below). Here homeomorphic means that there is a one-to-one, onto, continuous map with a continuous inverse that takes this set to the carpet. So a Sierpinski curve is just a continuous deformation of the carpet.



There are three reasons for my fascination with this set. First, a Sierpinski curve is a “universal plane continuum.” Roughly speaking, this means that any compact plane curve, no matter how intricate, can be homeomorphically manipulated to fit inside the carpet, [2]. So the carpet is a “dictionary” of all possible such planar curves. The second reason is that, by a theorem of Whyburn [3], there is a topological characterization of this set: any planar set that is compact, connected, locally connected, nowhere dense, and has the property that any pair of complementary domains are bounded by simple closed curves that are pairwise disjoint is homeomorphic to the carpet.

Then my third reason for loving this set is that Sierpinski curves arise all the time as Julia sets of complex functions. To illustrate this, let's concentrate on the family of rational maps given by

$$F_\lambda(z) = z^n + \frac{\lambda}{z^n}$$

where $n \geq 2$ and $\lambda \in \mathbb{C}$. Let F_λ^j denote the j^{th} iterate of F_λ . Then the set of points $F_\lambda^j(z)$, $j = 0, 1, 2, \dots$, is the orbit of the point z . For these maps, there are $2n$ critical points given by $\lambda^{1/2n}$, but, just as in the case of quadratic polynomials, there really is only one critical orbit because all the orbits of critical points behave symmetrically under iteration of F_λ .

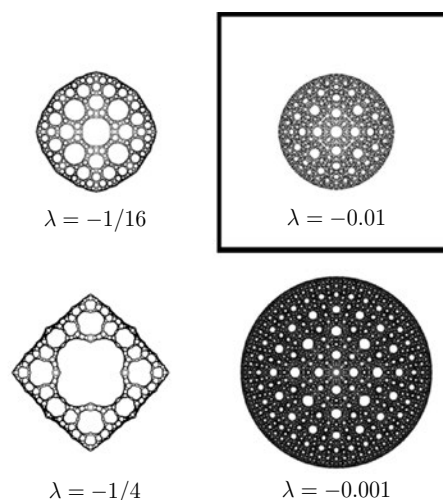
Out near ∞ , F_λ is essentially given by z^n , so all points far from the origin tend to ∞ under iteration. So we have an immediate basin of attraction of ∞ . When $|\lambda|$ is small enough, this is an open disk surrounding ∞ in which all orbits just tend to ∞ . Then, there are infinitely many disjoint preimages of this basin. The Julia set of F_λ , denoted by $J(F_\lambda)$, is, by definition, the boundary of the set of points that escape to ∞ . Equivalently, the Julia set is also the closure of the set of repelling periodic points. So, arbitrarily

close to any point in the Julia set, we have both escaping and periodic points, so the Julia set is the place where chaos occurs for these maps.

So how do Sierpinski curves arise as Julia sets for F_λ ? There are many ways [1]. One way is that, if the orbit of the critical points enter the immediate basin of ∞ at iteration 3 or later, then $J(F_\lambda)$ is a Sierpinski curve. Several such Julia sets are displayed in the figure below. The parameter plane (the λ -plane) for these maps then contains infinitely many open disks called Sierpinski holes, and each parameter in these holes has a Julia set that is a Sierpinski curve.

Another way is the following: it turns out that there are infinitely many small copies of the Mandelbrot set in the parameter plane for these maps. Those Mandelbrot sets that do not extend to the boundary of the parameter plane have main cardioids, and all parameters in these main cardioids have Julia sets that are again Sierpinski curves. And finally, it is known that there are uncountably many closed curves surrounding the origin in the λ -plane on which, again, all parameters have Sierpinski curve Julia sets.

So all of the above Julia sets are the same topologically. However, only parameters drawn from Sierpinski holes or main cardioids that are symmetrically located under complex conjugation or rotation by an $n - 1^{\text{st}}$ root of unity have the same dynamical behavior. The same also holds true on each of the infinitely many curves in the dynamical plane described above. So we find a wealth of Sierpinski curve Julia sets, all with very different dynamics. Understanding all of these different dynamical behaviors is a wide open problem.



Sierpinski curve Julia sets for various values of λ when $n = 2$.

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Optimal Transport: From moving soil to same-sex marriage

By Nassif Ghoussoub (UBC) and
Brendan Pass (U. Alberta)

Optimal mass transport has come a long way since the 1781 “Mémoire sur la théorie des déblais et des remblais” of Gaspard Monge, who was looking for the most economical way of moving soil from one area to another. Mathematically, this amounts to minimizing the total cost $\int_X c(x, Tx) d\mu$ over all possible transport maps T that “push” the initial distribution μ of soil onto a final distribution ν , where $c(x, y)$ is the cost of moving x to y , which (for Monge) was proportional to the Euclidian distance $|x - y|$. Many years later, Kantorovich linearized and compactified the problem by enlarging the constraint set to contain all “transport plans”; that is, he allowed the soil at an initial point x to be divided among two or more destination points, hence multivalued mappings. This relaxed version of Monge’s problem, which earned Kantorovich, together with T. Koopmans, the Nobel Prize in economics in 1975 for their work on optimum allocation of resources, consists of considering the following minimization problem

$$T_c(\mu, \nu) := \inf \left\{ \int_{X \times Y} c(x, y) d\gamma(x, y); \gamma \in \Pi(\mu, \nu) \right\} \quad (1)$$

Here $\Pi(\mu, \nu)$ is the set of measures γ on the product space $X \times Y$ whose marginals are μ and ν . Kantorovich also defined a dual problem, allowing him to relate it to linear programming. It is instructive to think of a manufacturing company shipping resources (say iron) from a distribution $\mu(x)$ of mines on some landscape $X \subseteq \mathbb{R}^n$ to a distribution $\nu(y)$ of factories on a landscape $Y \subseteq \mathbb{R}^n$, where $c(x, y)$ is the cost of shipping one unit of iron from a mine at location x to a factory at location y ; the goal is minimize the total shipping cost. A permissible γ represents a possible transport plan; heuristically $d\gamma(x, y)$ represents the amount of iron that should be shipped from mine x to factory y . For a good outline of the 2-marginal case and its applications, we refer to Villani [7].

Multi-marginal Monge-Kantorovich problems: Suppose now that a manufacturing company is just beginning business and has not yet built their factories. The company is making a certain product, requiring several resources, such as iron, aluminum, nickel, etc. There is a distribution of mines $\mu_i(x_i)$, supported on some set $X_i \subset \mathbb{R}^n$, producing each type of resource, and the cost to ship one unit of the i th resource from x_i to a location y is given by $c_i(x_i, y)$. The company then wants to build its factories in locations that minimize the total shipping costs of all the resources. That is, they want to build a distribution of factories $\nu(y)$ on Y in order to minimize

$$\sum_{i=1}^m T_{c_i}(\mu_i, \nu). \quad (2)$$

Another way to interpret this problem is to consider the function

$$c(x_1, \dots, x_m) = \inf_{y \in Y} \sum_{i=1}^m c_i(x_i, y). \quad (3)$$

Assuming this infimum is always attained at a unique point $y(x_1, x_2, \dots, x_m)$, there is an equivalence between (2) and the problem of minimizing,

$$T_c(\mu_1, \mu_2, \dots, \mu_m) := \inf \left\{ \int_{X_1 \times \dots \times X_m} c(x_1, x_2, \dots, x_m) d\gamma(x_1, x_2, \dots, x_m); \gamma \in \Pi \right\}, \quad (\text{MK})$$

over the set $\Pi := \Pi(\mu_1, \mu_2, \dots, \mu_m)$ of measures γ on $X_1 \times \dots \times X_m$ whose marginals are the μ_i . This is the *multi-marginal optimal transport (or Monge-Kantorovich) problem*. The case $m = 2$ is obviously the above mentioned classical optimal transport problem (1). Intuitively, $d\gamma(x_1, x_2, \dots, x_m)$ represents the amount of resources that are shipped from locations x_1, x_2, \dots, x_m to a certain factory $y(x_1, x_2, \dots, x_m)$. A fundamental problem (largely settled when $m = 2$) is to determine for which *cost functions* c , the infimum in (MK) is attained (uniquely!) by a measure supported by “a graph”, meaning that

$$T_c(\mu_1, \mu_2, \dots, \mu_m) = \int_{X_1} c(x, T_1 x, T_2 x, \dots, T_{m-1} x) d\mu_1(x),$$

for some maps $T_i : X_1 \rightarrow X_{i+1}$ that push the first marginal μ_1 onto μ_{i+1} for $i = 1, \dots, m - 1$. Recently, problems of this general type have begun to attract attention, due to surprisingly diverse applications. But unlike the classical case ($m = 2$), the structure of solutions to multi-marginal problems of form (MK) are not yet well understood. While there has been some progress on the uniqueness and structure of solutions to (MK) (see [6], [5] and the references therein), it has mostly been restricted to cost functions of the form (3), whereas many of these applications involve costs which are *not* of this form. Below, we outline several different applications of this problem.

Multi-agent matching problems in economics: Recent papers link (MK) to a matching problem in economics where agents’ preferences depend on external contracts [1]. For example, consider a collection of consumers, parametrized by the set $X_1 \subseteq \mathbb{R}^n$, looking to buy custom built houses; imagine that the different components x_1^j of a consumer $x_1 = (x_1^1, x_1^2, \dots, x_1^n) \in X_1$ represent characteristics which affect the consumers’ preferences for different types of houses, for example, their income, family size, age, etc. Think of the probability measure $\mu_1(x_1)$ as representing the relative frequency of a consumer of type x_1 . In order to build a house, a consumer must hire several (say $m - 1$) tradespeople: for example, a carpenter, a plumber and an electrician. Imagine, for example, that X_2 parametrizes the set of carpenters available to be hired; the

different components of $x_2 \in X_2$ may represent the age, years of experience and safety record, for example, of the carpenter x_2 , and the measure $\mu_2(x_2)$ the relative frequency of carpenters of type x_2 . The sets X_3, \dots, X_m will have similar interpretations in terms of plumbers, electricians, etc.

Now, suppose the set of houses that can feasibly be built is parameterised by $Y \subseteq \mathbb{R}^n$; the different components of a house $y \in Y$ may represent its size, location, etc. Of course, different consumers prefer different types of houses; let $f_1(x_1, y) \in \mathbb{R}$ represent the utility consumer x_1 would derive from owning a house of type y . Similarly, preferences differ among carpenters, plumbers and electricians as well; let $f_i(x_i, y)$ be the utility worker x_i would derive from building house y . Consumers want to buy houses which they like as much as possible, but also want to pay as little as possible for them. On the other hand, workers want to build houses making their utilities as high as possible, but they also want to be paid as high a wage as possible. Informally, if consumer x_1 hires carpenter x_2 , plumber x_3 , etc, to build some feasible house, then

$$b(x_1, x_2, \dots, x_m) := \sup_{y \in Y} \sum_{i=1}^m f_i(x_i, y)$$

is the maximal total utility that can be obtained by this collection of agents. The link with (MK) is that finding an equilibrium in this market (in other words, an assignment of wages and agents to different types of houses so that no one has an incentive to change jobs) is equivalent to solving (MK), with cost function equal to $-b$.

Symmetric Monge-Kantorovich problems: Consider now problem (MK), but with the additional constraint that the measures γ in $\Pi(\mu_1, \mu_2, \dots, \mu_m)$ should be invariant under the cyclic permutation $\sigma(x_1, x_2, \dots, x_m) = (x_2, x_3, \dots, x_m, x_1)$; note that in this case, the marginals μ_i must all be equal to some common distribution μ . Here, the problem is to determine for which costs c , there exists an optimal measure that is supported on a graph of the form $x \rightarrow (x, Sx, S^2x, \dots, S^{m-1}x)$, where S is a μ -measure preserving m -involution, i.e. $S^m x = x$ a.e.

Monotone maps and polar factorizations of vector fields: When the cost function is given by $c(x_1, x_2, \dots, x_m) = -\sum_{i=2}^m \langle u_i(x_1), x_i \rangle$ for a given family of bounded vector fields (u_2, u_3, \dots, u_m) , the symmetric Monge-Kantorovich problem turns out to be instrumental in the proof of the following representation result for the u_i established by Ghoussoub-Moameni [5]: There exist a cyclically antisymmetric saddle function $H : X^m \rightarrow \mathbb{R}$ (i.e., $\sum_{i=0}^{m-1} H(\sigma^i(\cdot)) \equiv 0$ on X^m) and a measure preserving map $S : X \rightarrow X$ with $S^m = I$ such that

$$(u_2(x), u_3(x), \dots, u_m(x)) = \nabla_{x_2, x_3, \dots, x_m} H(x, Sx, S^2x, \dots, S^m x) \text{ for a.e. } x \in X \quad (4)$$

This extends an earlier decomposition for a single vector field ($m = 2$) by the same authors. Note that in the special case where the u_i 's are *jointly m -monotone*, one can take S to be the identity [4], which extends a well known theorem of Krauss for 2-monotone vector fields. In the other extreme, a classical result of Rockafellar yields that vector fields which are m -cyclically monotone for every m are essentially sub-differentials of convex functions. Taking $u_1 = u$, $u_i = 0$ for $i = 2, 3, \dots, m-1$, the result of Ghoussoub-Moameni then yields that every bounded vector field u is m -monotone up to a measure preserving m -involution.

Matching and the Roommate problem: The economic literature has mostly modeled the marriage market as a bipartite matching game with transferable utility. Yet the bipartite assumption—even in the marriage market—is becoming restrictive in some contexts, where a match does not have to include exactly one individual from each of two prescribed subpopulations, especially now that a growing number of countries have authorized same-sex unions in some form. This leads to problems with symmetry constraints in many types of matching problems [3]. Another example comes from a university housing office trying to assign students to dorm rooms, say three to a room. The problem of finding an assignment which maximizes some measure of overall compatibility between roommates can be formulated as (MK) but again restricted to measures γ which are invariant under any permutation of the arguments. Heuristically, the invariance arises because the population of interest is not a priori partitioned into disjoint subsets; one works with three copies of the original distribution of student types. If it is optimal to match a trio (x_1, x_2, x_3) of student types together, it should also be optimal to match the trio (x_2, x_3, x_1) .

Density functional theory: A fundamental question in chemical physics is to determine the ground state energy of a system of m -interacting electrons (for example, an atom). A partitioning of this search leads to consideration of the Hohenberg-Kohn functional, which in the semi-classical limit, takes the form [2]:

$$F_{HK}[\mu] := \inf_{\gamma \in \Pi(\mu, \dots, \mu)} \int_{\mathbb{R}^{dm}} \sum_{i \neq j} \frac{1}{|x_i - x_j|} d\gamma.$$

This is exactly problem (MK), where the cost function $\sum_{i \neq j} \frac{1}{|x_i - x_j|}$ represents the Coulombic interaction energy between the electrons. Note that in this case, the marginals, which represent the single particle densities of the electrons, are all the same, embodying the indistinguishability of the electrons. The measures γ in $\Pi(\mu, \mu, \dots, \mu)$ represent potential N -particle densities

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Le lien brisé, suite de la page 3

même comme un obstacle à l'éducation collégiale ou universitaire pour les élèves socialement défavorisés. Il y a de quoi s'étonner puisque le développement des aptitudes mathématiques ne requiert pas d'outils dispendieux (contrairement à plusieurs sports et aux sciences expérimentales, par exemple) n'est pas plus facile pour les élèves issus d'un milieu « éduqué », à la différence des langues et des lettres. Si l'on accuse les mathématiques de causer des dépressions chez les élèves du secondaire et les étudiants en première année de baccalauréat, c'est probablement parce que le miroir des mathématiques ne renvoie pas facilement une image déformée. Il ne permet à personne de croire ce qu'il désire; il dit plutôt la vérité. Nous ne donnons pas de points aux élèves pour une solution erronée ou une mauvaise réponse, bien qu'on ait pu lire dans le *Calgary Herald* (et ailleurs, je crois) la lettre d'un parent qui réclamait qu'une partie des points soit donnée pour des solutions mathématiques totalement incorrectes. Je ne pense pas que les étudiants de première année s'en tirent mieux pour rédiger des dissertations ou analyser des phénomènes naturels que pour résoudre des problèmes mathématiques. Dans le premier cas, les critères d'évaluation ne sont pas toujours très stricts; en géologie, par exemple, mes collègues ne présumant pas que les étudiants ont des connaissances préalables. En mathématiques, nous sommes largement tributaires de ce que nos étudiants ont appris à l'école secondaire. Il faut toutefois comparer les disciplines avec circonspection. À ma collègue qui enseigne l'histoire, j'ai fait valoir que notre principal problème, c'était le manque de culture mathématique des étudiants et que ce manque de culture n'était probablement pas aussi problématique pour les professeurs d'histoire. « Comment enseigneriez-vous l'histoire à des étudiants qui n'ont jamais entendu parler des deux grandes guerres mondiales? », m'a-t-elle répondu (chanceux étudiants! J'aimerais bien savoir d'où ils viennent).

Parmi les manuels recommandés pour le premier cycle du secondaire, il y a ceux de la collection « Math makes sense ». La professeure Malgorzata Dubiel critique à juste titre ces manuels; voir par exemple :

<http://blogs.vancouverun.com/2012/04/13/math-makes-sense-not-with-these-textbooks-expert-says/>

<https://sfu.ca/pamr/issues-experts/2012/assessing-math-teaching.html>

Je viens d'examiner les manuels de cette collection pour la 8^e et la 9^e année; c'est un exercice que je recommande à tous mes collègues qui enseignent le calcul différentiel et intégral en première année : vous verrez, c'est très révélateur. Vos étudiants ont du mal à additionner les fractions? Pas étonnant : dans l'unité 3 du manuel de 8^e année, consacrée aux opérations avec des fractions, il est question de multiplication et de division de fractions, mais pas d'addition. Pas un mot sur comment réduire

les fractions au dénominateur commun; le terme « plus petit commun multiple » ne figure même pas dans l'index. Ainsi donc, la prochaine fois que vous aborderez l'intégration par fractions partielles et que vos étudiants choisiront comme dénominateur commun le produit de tous les dénominateurs dans une fonction algébrique, sachez simplement qu'ils font de leur mieux. Parlant d'algèbre, la seule chose que les élèves apprennent au premier cycle du secondaire est l'addition des polynômes, la multiplication et la division par un nombre et par un monôme (dans tous les exercices, un polynôme à coefficients entiers est un résultat). A priori, on dirait que le manuel est inspiré par des écoliers paresseux qui voulaient faire retirer du curriculum tout ce qui pourrait causer des difficultés. Ça ne change rien que ces notions soient nécessaires ultérieurement; si l'addition de fractions, l'élevation d'un binôme au carré et la résolution d'une équation quadratique peuvent être reportées au deuxième cycle du secondaire, c'est qu'on peut probablement en faire abstraction.

Or, l'insuffisance des habiletés techniques acquises par les élèves dans les 9 premières années de leur parcours scolaire, au primaire et au premier cycle du secondaire, n'est qu'un symptôme. Sur l'approche, la grande faiblesse de ces manuels est que les élèves n'apprennent pas vraiment les mathématiques (axées sur le pourquoi), mais plutôt une série de recettes. Dans ces manuels, je n'ai trouvé aucune explication à savoir pourquoi la somme des angles d'un triangle donne 180° (ou, du reste, quelque notion de géométrie). La meilleure explication que j'ai pu y trouver est : dessine un triangle, mesure, calcule et vérifie. L'examen des manuels de 8^e et de 9^e année m'a été fort instructif : désormais je comprends ce que mes étudiants veulent dire par « Je suis bon en maths » ou « Je ne suis pas bon en maths » et pourquoi leur vision de leurs aptitudes mathématiques est parfois si éloignée de la réalité. J'imagine qu'être bon en maths équivaut à savoir réaliser des opérations numériques ou algébriques vite et bien selon des algorithmes prescrits (et même pas justifiés) après une longue formation. Heureusement que je ne l'ai pas su plus tôt; personnellement, j'avais de la difficulté avec les additions et les soustractions complexes en 2^e et en 3^e année.

L'enseignement des mathématiques vise de nombreux objectifs. Le premier et le plus important, c'est de développer l'esprit en général et la pensée logique en particulier. Le deuxième, c'est de favoriser l'indépendance d'esprit et de jugement : même des débutants peuvent faire de petites découvertes, et un élève peut remettre en cause les affirmations de ses professeurs et de ses parents puisque seule la raison et non l'autorité permet d'établir la vérité. Le troisième, c'est d'acquérir quelques compétences techniques et d'apprendre certaines méthodes. Pour une quelconque raison, seul ce troisième objectif semble se retrouver dans le curriculum du premier cycle du secondaire, et même là, il ne prend pas beaucoup de place. Par exemple, le manuel comporte des exercices de résolution de problèmes,

mais presque tous les problèmes peuvent être résolus en une seule étape.

La principale cause de cette sous-représentation des mathématiques, c'est l'uniformisation de l'éducation. Parfois, cela fait en sorte que les élèves les plus lents n'ont pas la possibilité de se rattraper, tandis que les élèves plus avancés n'ont pas la possibilité d'être suffisamment exposés aux mathématiques (qui sont axées sur le raisonnement et non sur une série de recettes). À mon avis, il y a peut-être une partie des élèves du premier cycle du secondaire qui tombe dans la première catégorie et qui bénéficierait d'un curriculum moins intensif ou plus étalé, mais les élèves de la seconde catégorie sont nettement plus nombreux. Des activités éducatives sont offertes gratuitement (l'Université de Calgary organise des « math nights » et l'on trouve des clubs du même genre dans presque toutes les universités canadiennes), mais jusqu'à un certain âge, c'est aux parents d'y emmener leurs enfants parce qu'ils ne peuvent s'y rendre par leurs propres moyens. Et lorsque les élèves arrivent en âge de fréquenter l'école secondaire, il est malheureusement trop tard.

Il y a moyen de corriger la situation en prenant exemple sur certains pays. Premièrement, au primaire, pendant que la pression des résultats n'est pas encore trop forte, il faudrait exposer les élèves aux opérations avec des nombres entiers (savoir compter jusqu'à 100, par exemple) et des fractions (y compris l'addition de fractions) ainsi qu'à certaines notions de géométrie et de résolution de problèmes simples. Au début de la 7^e année, une épreuve de qualification permettrait de répartir les élèves en plusieurs groupes (quatre dans le programme de référence) qui travailleraient simultanément pendant plusieurs séances au même niveau. À compter du deuxième cycle du secondaire, il y aurait plusieurs groupes de mathématiques (A, B, C). Le groupe A recevrait non seulement des notions enrichies de calcul différentiel et intégral à un niveau situé entre le programme Math 31 et les programmes de calcul différentiel et intégral du BI, mais aussi des notions avancées d'algèbre, de géométrie et de trigonométrie. Le niveau B correspondrait grosso modo à une bonne maîtrise du programme Math 30, et le niveau C serait réservé aux élèves plus faibles. Les élèves pourraient passer à un niveau supérieur par voie d'examen; il est vrai que certains recourront à des tuteurs pour ce faire. Cela serait toutefois sans commune mesure avec le niveau d'accompagnement personnalisé dont bénéficient actuellement élèves du secondaire et les étudiants en première année de baccalauréat. Leur situation actuelle pourrait se comparer à une compétition scolaire conjointe à tous les niveaux : les élèves de première année n'ont aucune chance et les diplômés n'ont aucun défi.

Une bonne culture mathématique facilitera beaucoup le passage du secondaire à l'université. Or, pour de nombreux étudiants de première année en sciences et en génie, ce lien est brisé, et cette rupture survient probablement bien avant le deuxième cycle du secondaire.

Optimal Transport: From moving..., continued from page 15

of the system, each with single particle density μ . Heuristically, we can think of $F_{HK}[\mu]$ as representing the minimal (semi-classical) energy of all configurations of electrons, with single particle density μ . The problem in density functional theory is then to minimize $F_{HK}[\mu]$ (or $F_{HK}[\mu]$ plus an external potential) over all possible single particle densities μ . We should note that it is physically natural to impose that the measures γ are invariant under any permutation of the arguments. This does not effect the value of $F_{HK}[\mu]$, as symmetrizing the measure γ does not change the common marginal μ or the value $\int_{\mathbb{R}^{dm}} \sum_{i \neq j} \frac{1}{|x_i - x_j|} d\gamma$, due to the symmetry of the cost function. It is relevant, however, to questions about the structure and uniqueness of the optimal γ . In the case of two electrons (i.e., $m = 2$), it can be shown that the infimum $F_{HK}(\mu)$ is then attained at a measure $\tilde{\gamma}$ which determines the co-motion function $x \rightarrow (x, Sx)$, with $S^2 = I$. The case when $m \geq 3$ is wide open.

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CMS celebrates longstanding member Dennis Russell

The Canadian Mathematical Society (CMS) is pleased to recognize Dennis Russell's longstanding commitment to the Canadian mathematical community, as a member of both the Canadian Mathematical Congress (CMC) and the CMS.

Dennis Russell first joined the CMS as part of the CMC in the spring of 1961, and attended his first Summer Meeting of the CMC in June of the same year. While at this pivotal meeting in Montreal, Russell made the acquaintance of Irvine Pounder, who at the time was working to form a mathematics department at the then brand-new York University.

"In the fall of 1961 I received a letter from Murray Ross, the York University President, inviting me to Toronto to 'discuss the possibility of heading up our mathematics department,'" said Russell. "So it was that I arrived in Toronto in the summer of 1962 and set about the task of recruiting the best people we could find."

"I remember taking part in the debate as to whether to change from the CMC to CMS – there were traditionalists who preferred

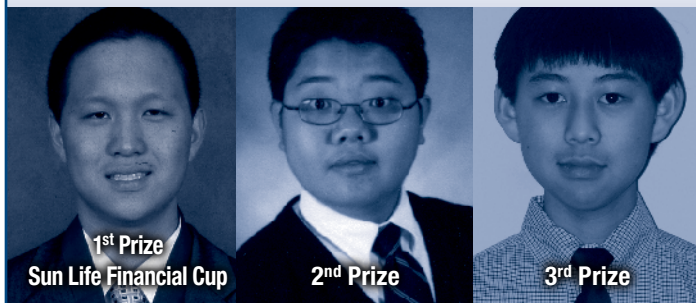
the status quo, but the argument against CMC was that it could present us ambiguously as just a meetings organizer rather than as a representative of Canadian mathematics on a par with other national societies—and of course CMS won out," said Russell.

For over 50 years, Russell has generously dedicated his time to serving the mathematical community as a member of both the CMC and CMS. Russell served as the unofficial co-ordinator for the Ontario Mathematics Meetings – a series of meetings that occurred four times per year, and circulated amongst the Ontario universities. Russell coordinated some 80 meetings over the 19 years he held this position. Additionally, he served as Chairman for the Arrangements Committee for the CMC and CMS for six years, and served on the Society's Research Committee for an additional seven years.

"For 50 years it never occurred to me to not renew my membership – it was a very small price to pay to be counted in helping to maintain a strong voice for Canadian Mathematics," said Russell.



Sun Life Financial

**Calvin Deng**NC School of Science
& Math, Cary, NC**ZhuoQun (Alex) Song**Phillips Exeter Academy,
Exeter, NH**Kevin Sun**Naperville Central High School,
Naperville, IL

Sixty-eight students, invited mainly on the basis of high performance in the Sun Life Financial Canadian Open Mathematics Challenge, or the Repêchage, wrote the 2013 Canadian Mathematical Olympiad. This year, information about high performers in the Concours de l'Association mathématique du Québec came early enough for invitations to be issued.

The CMS Staff, including Sarah Watson and Jessica St. James, ensured the success of the 2013 CMO. In addition, special thanks to CMO Committee member Adrian Tang, who was omnipresent: problem setting, Repêchage, and grading.

The first prize winner is (again) Calvin Deng of the North Carolina School of Science and Mathematics, with (again) a perfect score of 35.

Question 1 was relatively accessible. "Determine all polynomials $P(x)$ with real coefficients such that $(x + 1)P(x + 1) - (x + 1)P(x)$ is a constant polynomial." Of the 68 students writing, 24 got a perfect 7. Question 2 was of a number-theoretic character, basically a parity argument. It was very well done, with 28 students getting full marks. This year, there were 2 geometric questions. Question 3 was not particularly difficult, but there were only 5 full marks. Many more students made partial progress. Question 5 was more difficult and there were only 2 perfect scores, again with substantial progress in several other papers. Question 4 was a delicate inequality. Part marks were relatively common, but Calvin Deng had the only full solution.

Andrew Adler, Chair, CMO Committee Department of Mathematics, UBC

Call for proposals:

2013 Endowment Grants Competition

The Canadian Mathematical Society is pleased to announce the 2013 Endowment Grants Competition. The CMS Endowment Grants fund projects that contribute to the broader good of the mathematical community. Projects funded by the Endowment Grants must be consistent with the interests of the CMS: to promote the advancement, discovery, learning and application of mathematics. An applicant may be involved in only one proposal per competition as a principal applicant. Proposals must come from CMS members, or, if joint, at least one principal applicant must be a CMS member. the deadline for applications is September 30, 2013. Successful applicants will be informed in December 2013 and grants will be awarded in January 2014. Further details about the endowment grants and the application process are available on the CMS website: www.cms.math.ca/Grants/EGC The Endowment Grants Committee (EGC) administers the distribution of the grants and adjudicates proposals for projects. The EGC welcomes questions or suggestions you may have on the program. Please contact the Committee by e-mail at chair-egc@cms.math.ca.

Appel de projets :

Concours de bourses du fonds de donation 2013

La Société mathématique du Canada (SMC) est heureuse d'annoncer la tenue du Concours de bourses du fonds de dotation 2013. Les bourses du fonds de dotation de la SMC financent des activités contribuant à l'essor global de la communauté mathématique. Les projets financés à partir des bourses du fonds de dotation doivent correspondre aux intérêts de la SMC : soit promouvoir et favoriser la découverte et l'apprentissage des mathématiques, et les applications qui en découlent. Un demandeur ne peut présenter qu'un projet par concours en tant que demandeur principal. Les projets doivent venir de membres de la SMC. S'il s'agit d'un projet conjoint, au moins un des demandeurs principaux doit être membre de la SMC. La date limite pour présenter sa demande est le 30 septembre 2013. Les projets retenus seront annoncés en décembre 2013, et les bourses distribuées en janvier 2014. Pour vous procurer un formulaire ou pour de plus amples renseignements sur l'appel de projets, passez sur le site de la SMC au : www.smc.math.ca/Grants/EGC/f Le Comité d'attribution des bourses du fonds de dotation (CABFD) gère la répartition des bourses et évalue les projets. Pour toute question ou tout commentaire sur les bourses du fonds de dotation, veuillez communiquer par courriel avec le comité à pres-egc@smc.math.ca.

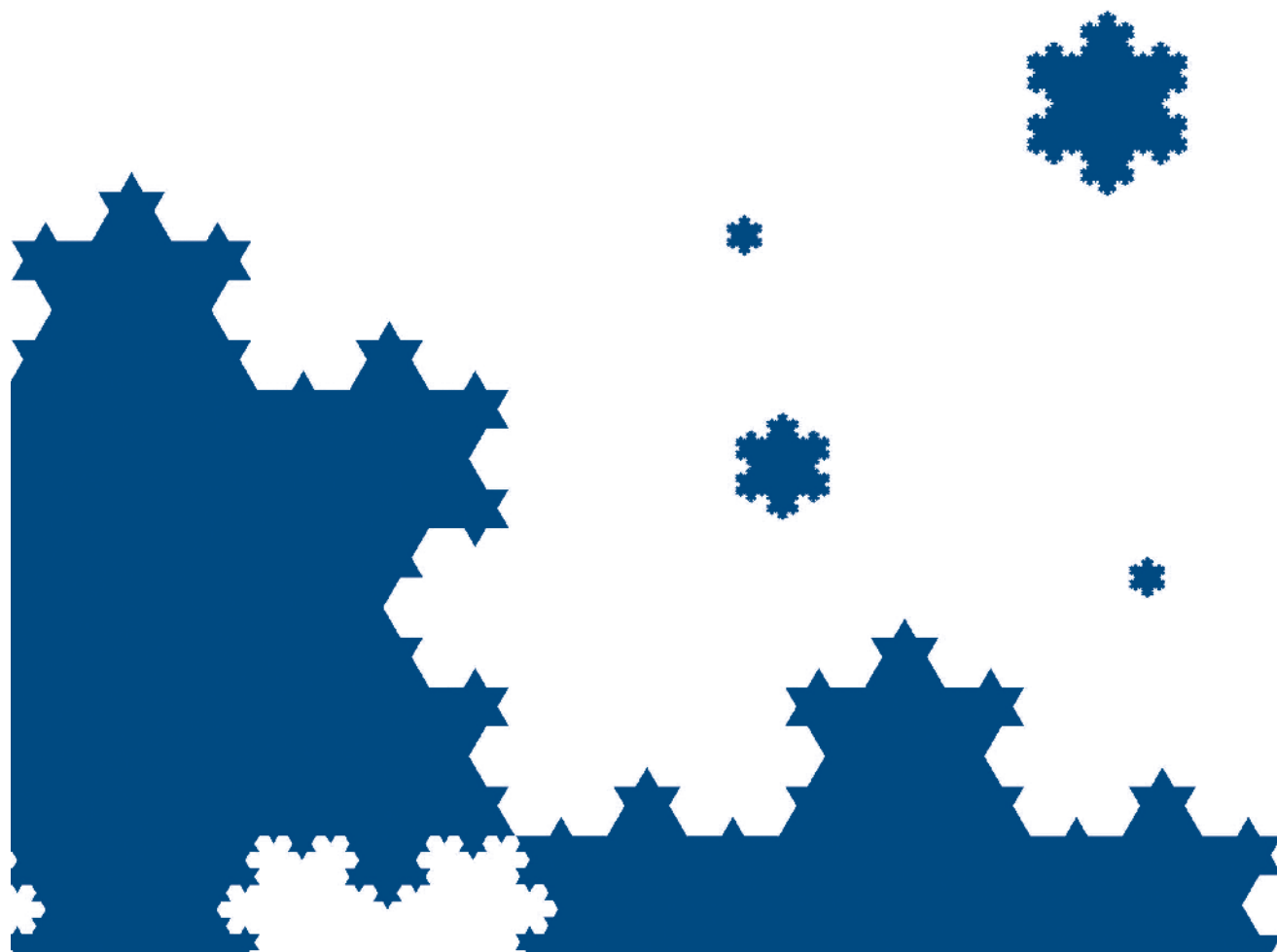


Winter Meeting

December 6-9, 2013
Ottawa Marriot, Ottawa

Réunion d'hiver

6 - 9, décembre 2013
Ottawa Marriott, Ottawa



Adrien-Pouliot Award

Nominations of individuals or teams of individuals who have made significant and sustained contributions to mathematics education in Canada are solicited. Such contributions are to be interpreted in the broadest possible sense and might include: community outreach programs, the development of a new program in either an academic or industrial setting, publicizing mathematics so as to make mathematics accessible to the general public, developing mathematics displays, establishing and supporting mathematics conferences and competitions for students, etc.

Nominations must be received by the CMS Office no later than July 15, 2013.

Please submit your nomination electronically, preferably in PDF format, to apaward@cms.math.ca.

Nomination requirements:

- Include contact information for both nominee and nominator.
- Describe the nominated individual's or team's sustained contributions to mathematics education. This description should provide some indication of the time period over which these activities have been undertaken and some evidence of the success of these contributions. This information must not exceed four pages.
- Two letters of support from individuals other than the nominator should be included with the nomination.
- Curricula vitae should not be submitted since the information from them relevant to contributions to mathematics education should be included in the nomination form and the other documents mentioned above.
- If nomination was made in the previous year, please indicate this.
- Members of the CMS Education Committee will not be considered for the award during their tenure on the committee.

Renewals

Individuals who made a nomination last year can renew this nomination by simply indicating their wish to do so by the deadline date. In this case, only updating materials need be provided as the original has been retained.

Prix Adrien-Pouliot

Nous sollicitons la candidature de personne ou de groupe de personnes ayant contribué d'une façon importante et soutenue à des activités mathématiques éducatives au Canada. Le terme « contributions » s'emploie ici au sens large; les candidats pourront être associés à une activité de sensibilisation, un nouveau programme adapté au milieu scolaire ou à l'industrie, des activités promotionnelles de vulgarisation des mathématiques, des initiatives, spéciales, des conférences ou des concours à l'intention des étudiants, etc.

Les mises en candidature doivent parvenir au bureau de la SMC avant le 15 juillet 2013.

Veuillez faire parvenir votre mise en candidature par voie électronique, de préférence en format PDF, à prixap@smc.math.ca.

Conditions de candidature

- Inclure les coordonnées du/des candidats ainsi que le(s) présentateur(s).
- Décrire en quoi la personne ou le groupe mise en candidature a contribué de façon soutenue à des activités mathématiques. Donner un aperçu de la période couverte par les activités visées et du succès obtenu. La description ne doit pas être supérieure à quatre pages.
- Le dossier de candidature comportera deux lettres d'appui signées par des personnes autres que le présentateur.
- Il est inutile d'inclure des curriculums vitae, car les renseignements qui s'y trouvent et qui se rapportent aux activités éducatives visées devraient figurer sur le formulaire de mise en candidature et dans les autres documents énumérés ci-dessus.
- Si la mise en candidature a été soumise en l'année précédente, s'il vous plaît indiquez-le.
- Les membres du Comité d'éducation de la SMC ne pourront être mise en candidature pour l'obtention d'un prix pendant la durée de leur mandat au Comité.

Renouvellements

Renouveler une mise en candidature Il est possible de renouveler une mise en candidature présentée l'an dernier, pourvu que l'on en manifeste le désir avant la date limite. Dans ce cas, le présentateur n'a qu'à soumettre des documents de mise à jour puisque le dossier original a été conservé.



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Editorial Nominations

The Publications Committee of the CMS solicits nominations for five Associate Editors for the Canadian Journal of Mathematics (CJM) and the Canadian Mathematical Bulletin (CMB). The appointment will be for five years beginning January 1, 2014. The continuing members (with their end of term) are below.

The deadline for the submission of nominations is November 15, 2013.

Nominations, containing a curriculum vitae and the candidate's agreement to serve, should be sent to the address below ;

Nantel Bergeron, Chair

CMS Publications Committee
Department of Mathematics & Statistics
York University
N520 Ross Bldg, 4700 Keele Street
Toronto, ON M3J 1P3
bergeron@yorku.ca

CURRENT MEMBERS:

CJM Editors-in-Chief

Henry Kim (Toronto)	12/2016;
Robert McCann (Toronto)	12/2016.

CMB Editors-in-Chief

Terry Gannon (Alberta)	12/2015;
Volker Runde (Alberta)	12/2015.

Associate Editors

Florin Diacu (Victoria)	12/2016;
Ilijas Farah (York)	12/2015;
Skip Garibaldi (Emory University)	12/2016;
Robert Leon Jerrard (Toronto)	12/2016;
Izabella Laba (UBC Vancouver)	12/2015;
Anthony To-Ming Lau (Alberta)	12/2016;
Alexander Litvak (Alberta)	12/2016;
Alexander Nabutovsky (Toronto)	12/2015;
Erhard Neher (Ottawa)	12/2016;
Vladimir Pestov (Ottawa)	12/2013;
Gordon Slade (UBC Vancouver)	12/2013;
Frank Sottile (Texas A&M)	12/2015;
Roland Speicher (Universität des Saarlandes)	12/2013;
Vinayak Vatsal (UBC Vancouver)	12/2013;
McKenzie Wang (McMaster)	12/2016;
Michael Ward (UBC Vancouver)	12/2015;
Jie Xiao (Memorial)	12/2013;
Efim Zelmanov (UCSD)	12/2016.

Appel de mises en candidature de rédaction

Le Comité des publications de la SMC sollicite des mises en candidatures pour cinq postes de rédacteurs associés pour le Journal canadien de mathématiques (JCM) et pour le Bulletin Canadien de mathématiques (BCM). Le mandat sera de cinq ans à compter du 1er janvier 2014. Les membres qui continuent (avec la fin de leur terme) sont ci-dessous.

La date limite pour les soumissions est le 15 novembre 2013.

Les mises en candidature, incluant un curriculum vitae et l'accord du candidat à servir, doit être envoyé à l'adresse ci-dessous :

Nantel Bergeron, Président

Comité de publication de la SMC
Département de mathématiques et statistiques
Université York
N520 Ross Bldg, 4700 rue Keele
Toronto (Ontario) M3J 1P3
bergeron@yorku.ca

MEMBRES ACTUELS:

Rédacteurs-en-chef JCM

Henry Kim (Toronto)	12/2016;
Robert McCann (Toronto)	12/2016.

Rédacteurs-en-chef BCM

Terry Gannon (Alberta)	12/2015;
Volker Runde (Alberta)	12/2015.

Rédacteurs associés

Florin Diacu (Victoria)	12/2016;
Ilijas Farah (York)	12/2015;
Skip Garibaldi (Emory University)	12/2016;
Robert Leon Jerrard (Toronto)	12/2016;
Izabella Laba (UBC Vancouver)	12/2015;
Anthony To-Ming Lau (Alberta)	12/2016;
Alexander Litvak (Alberta)	12/2016;
Alexander Nabutovsky (Toronto)	12/2015;
Erhard Neher (Ottawa)	12/2016;
Vladimir Pestov (Ottawa)	12/2013;
Gordon Slade (UBC Vancouver)	12/2013;
Frank Sottile (Texas A&M)	12/2015;
Roland Speicher (Universität des Saarlandes)	12/2013;
Vinayak Vatsal (UBC Vancouver)	12/2013;
McKenzie Wang (McMaster)	12/2016;
Michael Ward (UBC Vancouver)	12/2015;
Jie Xiao (Memorial)	12/2013;
Efim Zelmanov (UCSD)	12/2016.

Research Nominations

The CMS Research Committee is inviting nominations for three prize lectureships. These prize lectureships are intended to recognize members of the Canadian mathematical community.

The Coxeter-James Prize Lectureship recognizes young mathematicians who have made outstanding contributions to mathematical research. The recipient shall be a member of the Canadian mathematical community. Nominations may be made up to ten years from the candidate's Ph.D: researchers having their PhD degrees conferred in 2003 or later will be eligible for nomination in 2013 for the 2014 prize. A nomination can be updated and will remain active for a second year unless the original nomination is made in the tenth year from the candidate's Ph.D. The prize lecture will be given at the 2014 CMS Summer Meeting.

The Jeffery-Williams Prize Lectureship recognizes mathematicians who have made outstanding contributions to mathematical research. The recipient shall be a member of the Canadian mathematical community. A nomination can be updated and will remain active for three years. The prize lecture will be given at the 2014 CMS Winter Meeting.

The Krieger-Nelson Prize Lectureship recognizes outstanding research by a female mathematician. The recipient shall be a member of the Canadian mathematical community. A nomination can be updated and will remain active for two years. The prize lecture will be given at the 2014 CMS Summer Meeting.

The deadline for nominations is June 30, 2013.

Nominators should ask at least three referees to submit letters directly to the CMS by September 30, 2013. Some arms-length referees are strongly encouraged. Nomination letters should list the chosen referees, and should include a recent curriculum vitae for the nominee, if available. Nominations and reference letters should be submitted electronically, preferably in PDF format, by the appropriate deadline to the corresponding email address:

Coxeter-James: cjprize@cms.math.ca

Jeffery-Williams: jwprize@cms.math.ca

Krieger-Nelson: knprize@cms.math.ca



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Appel de mises en candidature de recherché

Le Comité de recherche de la SMC lance un appel de mises en candidatures pour trois de ses prix de conférence. Ces prix ont tous pour objectif de souligner l'excellence de membres de la communauté mathématique canadienne.

Le prix Coxeter-James rend hommage aux jeunes mathématiciens qui se sont distingués par l'excellence de leur contribution à la recherche mathématique. Cette personne doit être membre de la communauté mathématique canadienne.

Les candidats sont admissibles jusqu'à dix ans après l'obtention de leur doctorat : ceux qui ont obtenu leur doctorat en 2003 ou après seront admissibles en 2013 pour le prix 2014. Toute mise en candidature est modifiable et demeurera active l'année suivante, à moins que la mise en candidature originale ait été faite la 10^e année suivant l'obtention du doctorat. La personne choisie prononcera sa conférence à la Réunion d'été SMC 2014.

Le prix Jeffery-Williams rend hommage aux mathématiciens ayant fait une contribution exceptionnelle à la recherche mathématique. Cette personne doit être membre de la communauté mathématique canadienne. Toute mise en candidature est modifiable et demeurera active pendant trois ans. La personne choisie prononcera sa conférence à la Réunion d'hiver SMC 2014.

Le prix Krieger-Nelson rend hommage aux mathématiciennes qui se sont distinguées par l'excellence de leur contribution à la recherche mathématique. La lauréate doit être membre de la communauté mathématique canadienne. Toute mise en candidature est modifiable et demeurera active pendant deux ans. La lauréate prononcera sa conférence à la Réunion d'été SMC 2014.

La date limite de mises en candidature est le 30 juin 2013.

Les proposants doivent faire parvenir trois lettres de référence à la SMC au plus tard le 30 septembre 2012. Nous vous incitons fortement à fournir des références indépendantes. Le dossier de candidature doit comprendre le nom des personnes données à titre de référence ainsi qu'un curriculum vitae récent du candidat ou de la candidate, dans la mesure du possible. Veuillez faire parvenir les mises en candidature et lettres de référence par voie électronique, de préférence en format PDF, avant la date limite, à l'adresse électronique correspondante:

Coxeter-James: prixcj@smc.math.ca

Jeffery-Williams: prixjw@smc.math.ca

Krieger-Nelson: prixkn@smc.math.ca

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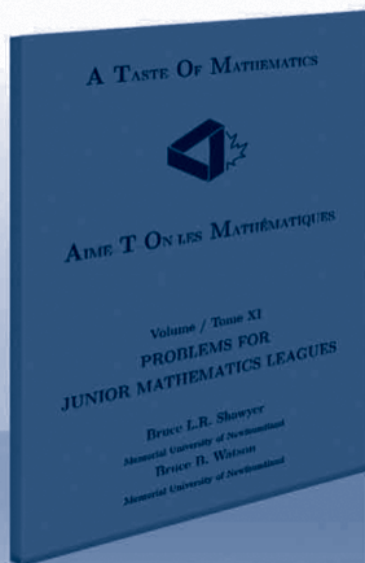
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RENOUVELLEMENTS 2013 À LA SMC

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