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Canadian Mathematical Society
Société mathématique du Canada

CMS
NOTES
September
2013
de la SMC

Du bureau du Vice-Président

L. Addario-Berry
vice-président de la SMC (Québec)

Contribution Québécoise aux Notes de la SMC

"Le libre accès à la littérature mathématique est un objectif important. Nous pouvons tous contribuer à l'atteinte de cet objectif en diffusant électroniquement autant de nos travaux que possible … Ce faisant, nous élargirons considérablement le volume de documents primaires en libre accès et rendrons un grand service aux scientifiques qui n'ont pas accès à une bonne bibliothèque." – *Comité exécutif de l'Union mathématique internationale, 15 mai 2001.*

J'estime que six mois serait un laps de temps raisonnable pour monopoliser le contenu d'une revue. … Il est peu probable qu'une revue perde beaucoup d'abonnés en donnant libre accès à ses anciens numéros après six mois. Je pense que peu de bonnes revues souffriraient de l'application d'une telle politique. – *Richard J. Roberts, Proc. Natl. Acad. Sci. États-Unis, vol. 98, numéro 2, 381-382, 16 janvier 2001.*

Bien que l'on puisse difficilement s'opposer au « libre accès », les assertions [de Roberts] ne s'appliquent tout simplement pas aux mathématiques, et la réussite de pareille entreprise porterait un coup presque fatal à des éditeurs comme la SMC ou l'AMS. Sans des mesures vigoureuses comme celles prônées par l'UMI/CEIC, nous sommes pris en otage à la fois par les grandes maisons d'édition du domaine scientifique et par la vision embrouillée de nos collègues du monde médical. – *Jonathan Borwein, Notes de la SMC, octobre 2001.*

Qu'est-ce que les membres de la SMC aiment lire dans leur bulletin? Comme tout bon probabiliste, j'ai essayé de répondre à cette question en me basant sur des statistiques ou, si vous préférez, sur l'exploration de données. Dans les cinq dernières années, les 33 articles qui ont fait la couverture des *Notes de la SMC* portaient principalement sur quatre thèmes : les activités de la SMC (12 articles); le financement et l'évaluation des chercheurs en mathématiques (8 articles); l'enseignement des mathématiques (6 articles); les mathématiques et la société (4 articles)1. Tous des sujets importants.

Voici un autre sujet important dont on parle moins dans les *Notes* : le libre accès. Les publications de la SMC, notamment nos revues savantes (le *Journal canadien de mathématiques* et le *Bulletin canadien de mathématiques*), sont d'importantes sources de revenus. Comme la citation reproduite plus haut le montre clairement (*Notes de la SMC,* octobre 2001), la SMC a vite pris conscience de la menace existentielle que l'accès libre représentait (là-dessus, je pense que nous avions plusieurs années d'avance sur Elsevier).

L'accès libre ne va pas disparaître, et la SMC s'y rallie au moins partiellement; les numéros de nos revues deviennent accessibles à tous après cinq ans. Le rapport annuel 2012 de la SMC fait état de débats animés au sein de l'Exécutif sur la question du libre accès. Mon instinct me pousse à vouloir aller plus loin dans cette voie. Parallèlement, il me paraît

1 Date : 16 juillet 2013. Plus précisément, cela éclaire les sujets sur lesquels les membres de l'Exécutif aiment écrire ou au moins choisissent d'écrire.

ENGLISH PAGE 5

Suite à la page 6

CMS Notes
Volume 45 No. 4, September 2013

Letter to the Editor

CMS Notes, June 2013, The Broken Link (Braverman)

To become an educated citizen of Canada requires both literacy and numeracy. For various reasons and through various causes, a major one of which is the negative cultural influence from a neighbouring country, the schools of Canada in at least the English language are progressively failing to produce literate and numerate Canadians to an extent that was achieved in urban areas in decades past. For mathematics in particular, designers of provincial curricula and teachers of their implementation seem to have lost the will and the intention to ensure that pupils in schools acquire not only an understanding of arithmetical and mathematical principles but especially the capability of undertaking arithmetical and mathematical calculations, including mental arithmetic that is the most fundamental stage. Although we have calculators and computers with powerful mathematical software that are invaluable for various pedagogical purposes, the urgent necessity in primary and secondary education is the direct and manual solution of arithmetical and mathematical problems as an intellectual capability. As the author agreed, the shirking of such basic tasks as the addition of fractions and solving a quadratic equation is reprehensible.

I question, however, whether the most efficient and effective development of mathematical skills does not "require .. expensive tools". There is a profound advantage in both teaching and learning to be derived from the use of mathematical software on computers, which process is by no means costless. A significant component of that cost is applicable to the training of teachers to apply these methods efficaciously.

At the level of tertiary or 'professional' education, the emphasis must alter radically. In this environment the cost to achieve mathematical capability increases markedly because of the necessity of providing access to mathematical software, apart from the machines, and the intensive teaching in laboratory style. No longer can one be content with an intellectual knowledge and appreciation of mathematical theory – the practice of mathematics in all aspects of activities must transcend the mere exercise of simple cases, such as to calculate the eigenvalues of a matrix of second order, that was formerly all that could be expected of students. Again, the cost of training instructors to rise to the level of competence to undertake both 'theoretical' and 'practical' instruction, both of which involve mathematical software, is significant. An electronic interactive textbook, *Mathematics for Chemistry with Symbolic Computation*, has been made available gratis at www.cecm.sfu.ca, but its application is severely impeded by the cost of arranging 'laboratory' instruction, in the manner taken for granted of both science subjects and engineering.

The most effective teaching of mathematics can not be done cheaply. Designers and implementers of curriculum in arithmetic and mathematics at all levels must become aware of the practicalities, and respond to the challenge of reversing the decline of mathematical capability up to the secondary level of education and increasing both the understanding and the applicability of mathematics at tertiary level, through the medium of appropriate mathematical software.

Yours sincerely,

J. F. Ogilvie
(B.Sc., M.Sc., Brit. Col., M.A., Ph.D. Cantab.)
(associate of Centre for Experimental and Constructive Mathematics at Simon Fraser University)

Letters to the Editors

The Editors of the NOTES welcome letters in English or French on any subject of mathematical interest but reserve the right to condense them. Those accepted for publication will appear in the language of submission. Readers may reach us at **notes-letters@cms.math.ca** or at the Executive Office.

Lettres aux Rédacteurs

Les rédacteurs des NOTES acceptent les lettres en français ou anglais portant sur un sujet d'intérêt mathématique, mais ils se réservent le droit de les comprimer. Les lettres acceptées paraîtront dans la langue soumise. Les lecteurs peuvent nous joindre au bureau administratif de la SMC ou à l'addresse suivante : **notes-lettres@smc.math.ca**.

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A Statistically Significant Anniversary

Srinivasa Swaminathan,
Dalhousie University

"Statistical thinking will one day be as necessary for efficient citizenship as the ability to read and write." – H. G. Wells

Many statistical associations and institutes have declared 2013 as the International Year of Statistics. It is stated that the main purpose for doing so is threefold: to increase public awareness of the power and impact of statistics on all aspects of society; to nurture statistics as a profession, especially among youngsters, and to promote creativity and development in probability and statistical sciences. Two main reasons are pointed out to explain why this year has been chosen: First, Jacob Bernouilli's *Ars Conjectandi* was published posthumously three hundred years ago in Basel. This work is considered as the foundation of the combinatorial basis of probability theory. Secondly the work of Thomas Bayes, *An essay towards solving a problem in the doctrine of chances* was published posthumously 250 years ago in 1763. This work gave rise to the topic of Bayesian statistics. The year 2013 misses by only 4 years the bicentennial of Gauss's *Theoria motus corporum coelestium in sectionibus conicis solem ambientium* in 1809 that introduced the method of least squares, the normal distribution, and the method of maximum likelihood.

Today, a good knowledge of statistical methods is necessary for anyone to understand clearly many aspects of present day life. This is largely because of the various applications of statistical methods to technology, medicine, biological and industrial processes. Originally statistics used to be taught in colleges and universities as part of courses in mathematics, economics, psychology, etc. Scientist used statistical methods tacitly in their papers, indeed, Einstein did so in his first published paper; Boris Oglewicz wrote in American Statistician 61 (2007) 339-342, "While Einstein is widely quoted as saying "God does not 'play dice' with the universe," the extent to which he used statistical methods and statistical reasoning in his work is not widely known." Einstein's' first publication, 'Conclusions drawn from the Phenomenon of Capillarity' (*Annalen der Physik* 4 (1901) 513-523), written when he was 21-year old graduate student, clearly shows that Einstein was well trained to use statistical arguments in his scientific investigations, largely through self-study. Indeed, two of his 1905 papers, one on the photoelectric effect (for which he won the Nobel prize) and the other dealing with Brownian motion, involve substantial use of statistical reasoning. In his 1901 paper Einstein postulated, "To each atom corresponds a molecular attraction field that is independent of the temperature and of the way the atom is chemically bound to other atoms" and resolved these statements by statistical methods, using linear regression.

A recent example of the interplay of mathematics and statistics is the study of power outages in large cities, with a view to detect signs of excessive electricity use; such a study uses cutting-edge statistical monitoring techniques and wavelet analysis. Motivated by two major black-outs that happened in 2003-2004 in New York City and in Los Angeles, the study involves finding a method that can simultaneously monitor multiple time series, taking account of interrelations between these series, coupled with Bayesian modeling. Mathematically, wavelets are used to turn the information of a signal into coefficients which can be manipulated, stored, transmitted, analyzed or used to reconstruct the original signal; Haar wavelets are used for such analysis and monitoring the very-frequent time series. [*Chance* 23(2) (2010) 28-37]

The International Congress of Mathematicians always includes a section devoted to Probability and Statistics. The International Mathematical Union is supporting the International Year of Statistics by planning some additional activities to be held at the ICM in Seoul (Korea) in 2014. Thus, we take this opportunity to greet our statistics colleagues at this juncture.

FRANCAIS PAGE 4

NOTES DE LA SMC

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Rédacteurs en chef
Robert Dawson, Srinivasa Swaminathan
notes-redacteurs@smc.math.ca

Rédacteur-gérant
Johan Rudnick
jrudnick@smc.math.ca

Rédaction
Éducation : John Grant McLoughlin et Jennifer Hyndman
notes-education@smc.math.ca
Critiques littéraires: Renzo Piccinini
notes-critiques@smc.math.ca
Réunions : Sarah Watson
notes-reunions@smc.math.ca
Recherche : Florin Diacu,
notes-recherche@smc.math.ca
Assistante à la rédaction : Jessica St-James

Note aux auteurs : indiquer la section choisie pour votre article et le faire parvenir au Notes de la SMC à l'adresse postale ou de courriel ci-dessous.

Les Notes de la SMC, les rédacteurs et la SMC ne peuvent être tenus responsables des opinions exprimées par les auteurs.

CMS NOTES

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Editors-in-Chief
Robert Dawson, Srinivasa Swaminathan
notes-editors@cms.math.ca

Managing Editor
Johan Rudnick
jrudnick@cms.math.ca

Contributing Editors
Education: John Grant McLoughlin and Jennifer Hyndman
notes-education@cms.math.ca
Book Reviews: Renzo Piccinini
notes-reviews@cms.math.ca
Meetings: Sarah Watson
notes-meetings@cms.math.ca
Research: Florin Diacu,
notes-research@cms.math.ca
Editorial Assistant: Jessica St-James

The Editors welcome articles, letters and announcements, which can be sent to the CMS Notes at the address below.

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Canadian Mathematical Society - Société mathématique du Canada
209-1725 St. Laurent Blvd., Ottawa, ON, Canada K1G 3V4
tel 613-733-2662 | fax 613-733-8994

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Un anniversaire statistiquement significatif

Srinivasa Swaminathan,
Dalhousie University

« Le jour viendra où le raisonnement statistique sera aussi nécessaire à l'exercice efficace de la citoyenneté que la capacité de lire et d'écrire. » – H. G. Wells

Nombre de sociétés et d'instituts de statistique ont proclamé 2013 « année internationale de la statistique ». L'objectif est triple : sensibiliser le public à la puissance statistique et à l'incidence de cette science sur tous les aspects de la société, promouvoir la statistique comme profession, surtout auprès des jeunes, et stimuler la créativité et le progrès de la science probabiliste et de la statistique. Deux raisons sont avancées pour justifier le choix de cette année : premièrement, elle marque le tricentenaire de la publication d'*Ars Conjectandi* (l'art de la conjecture), ouvrage posthume de Jacob Bernoulli considéré comme le point de départ de la perspective combinatoire de la théorie des probabilités. Deuxièmement, elle correspond également au 250e anniversaire de la publication (en 1763), également posthume, d'un ouvrage de Thomas Bayes intitulé *An essay towards solving a problem in the doctrine of chances* (essai de résolution d'un problème de la doctrine des probabilités), qui est le fondement de la statistique dite bayésienne. Qui plus est, l'année 2013 dépasse de quatre ans seulement le bicentenaire de la *Teoria motus corporum coelestium in sectionibus conicis solem ambientium* (*Théorie du mouvement des corps célestes parcourant des sections coniques autour du soleil*) que publia Gauss en 1809 pour exposer sa méthode des moindres carrés, de la distribution normale et de la probabilité maximale.

Beaucoup d'aspects de notre vie exigent désormais une bonne connaissance des méthodes statistiques. Il suffit de songer à leurs diverses applications à la technologie, à la médecine et aux processus biologiques et industriels. À l'origine, la statistique était au programme des cours de mathématiques, d'économie, de psychologie et d'autres encore des collèges et universités. Les scientifiques en appliquaient implicitement les méthodes dans leurs communications. Pour citer un exemple célèbre, c'est ce que fit Einstein, dans son tout premier article. On rapporte volontiers cette phrase du savant : « Dieu ne joue pas aux dés avec l'Univers », mais on sait moins dans quelle mesure il a utilisé les méthodes et le raisonnement statistiques dans ses travaux. À 21 ans, alors qu'il est étudiant diplômé, il publie un premier article (*Annalen der Physik*, vol. 309, no 3, 1901, 513-523) dans lequel il expose ses « conclusions sur le phénomène de la capillarité ». Selon Boris Iglewicz (*American Statistician*, vol. 61, 2007, 339-342), le texte « [traduction] montre clairement l'habileté d'Einstein à fonder ses investigations scientifiques sur les arguments statistiques, habileté acquise principalement en autodidacte ». En 1905, il publie deux autres communications, dont une sur l'effet photoélectrique qui lui vaudra un prix Nobel et une autre sur le mouvement brownien, où il fait à nouveau largement appel au raisonnement statistique. Dans l'article de 1901, Einstein pose que « [traduction] à chaque atome correspond un champ d'attractions moléculaires indépendant de la température et des liaisons chimiques entre atomes », postulat qu'il résout à l'aide de méthodes statistiques et, plus précisément, de la régression linéaire.

Depuis quelque temps, l'étude des pannes de courant qui affectent les grandes villes met à profit, de même, l'interaction entre mathématique et statistique, afin de détecter les signes d'un usage excessif de l'électricité. L'étude est fondée sur des techniques perfectionnées d'observation statistique et sur l'analyse par ondelettes. Motivée par deux pannes généralisées survenues en 2003-2004 à New York et à Los Angeles, elle vise à trouver une méthode qui permettrait tout à la fois l'observation de multiples séries temporelles en tenant compte de leurs interrelations et l'application du modèle bayésien. Le volet mathématique consiste à utiliser des ondelettes et, plus précisément, les ondelettes de Haar (*Chance*, vol. 23, no 2, 2010, 28-37) pour transformer l'information fournie par un signal en coefficients que l'on peut manier, stocker, transmettre, analyser et utiliser pour reconstruire le signal originel.

Le congrès international des mathématiciens (CIM) consacre toujours un volet de son programme aux probabilités et à la statistique. Pour souligner l'année internationale de la statistique, l'Union mathématique internationale ajoute quelques activités au programme du CIM 2014, qui aura lieu à Séoul (Corée). Nous profiterons de l'occasion pour saluer nos collègues statisticiens.

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CMS Notes from Quebec

L. Addario-Berry
CMS Vice-President – Quebec

"Open access to the mathematical literature is an important goal. Each of us can contribute to that goal by making available electronically as much of our own work as feasible … Our action will have greatly enlarged the reservoir of freely available primary mathematical material, particularly helping scientists working without adequate library access." – *International Mathematical Union Executive Committee, May 15, 2001.*

I would argue that 6 months seems a reasonable time for a journal to monopolize the content … It seems unlikely that a large number of subscriptions would be lost if 6-month-old issues were made freely available. I think rather few worthwhile journals would be adversely affected if they were to institute such a policy. – *Richard J. Roberts, Proc. Natl. Acad. Sci. USA, Vol. 98, Issue 2, 381-382, January 16, 2001.*

While it is hard to oppose 'open access', [Roberts'] assertions are clearly not true of mathematics and any such successful venture would come close to destroying mathematical publishers like the CMS or the AMS. Without vigorous activity such as the IMU/CEIC are promoting, we are hostage to both the large academic publishers and the myopic views of our medical colleagues. – *Jonathan Borwein, CMS Notes, October 2001.*

What do CMS members like to read about in their society bulletin? As a good probabilist should, to help answer this question I turned to statistics; or, if you prefer, data mining. In the past five years, the thirty-three cover stories of the *CMS Notes* have been primarily devoted to four topics: the activities of the CMS itself (12 articles); the funding and evaluation of researchers in mathematics (8 articles); mathematics education (6 articles); mathematics and society (4 articles).[1] Important topics, all.

Here is another important topic, less discussed in these Notes: open access. CMS publications, notably our research journals, the *Canadian Journal of Mathematics* and the *Canadian Mathematical Bulletin*, are an important source of revenue. As the above quote from the October 2001 *CMS Notes* makes clear, the CMS quickly recognized the existential threat posed by open access (in this respect I believe we were several years ahead of Elsevier).

Open access isn't going away, and has been at least partially embraced by the CMS; journal issues become fully open to the public after five years. The 2012 CMS annual report notes that open access is a subject of active discussion among the Executive. My instinct is toward pushing much further in this regard. At the same time, publishing journals through learned societies seems both natural and valuable. Those running such publications are peers, presumably with similar interests to my own; any profits from publication are returned to my community.

It's tempting to monologue about existing and proposed open-access projects in mathematics and the sciences, but I'll save the wood; I can see your eyes glazing over. (Yes, you there; you with the computer and those overdue papers to referee on your desk.) Instead, I'll leave you with a little Laphamesque trivia { not anecdotes so much as *artisanal data* {on the broad theme of open access. One final note: though I've skipped a soliloquy, this is an important issue that our community needs, and will likely be forced, to address. Please feel free to contact me with thoughts or ideas about open access and the CMS. See you in Ottawa for the CMS winter meeting in December!

- Date on which CERN released the first web browser, WorldWideWeb, into the public domain: **April 30, 1993**
- Date of the first accepted submission to the Electronic Journal of Differential Equations, the first peer-reviewed, open access mathematics journal: **May 2, 1993**
- Date of arXiv developer Paul Ginsparg's UNESCO presentation "Winners and losers in the global village", which introduced the idea of peer-reviewed overlay journals based on a publically accessible "raw research archive": **February 21, 1996**.
- Number of submissions to arXiv as of July 16, 2013: **859,781**
- Number of mathematics submissions to arXiv as of December 31, 2012: **157,284**
- Number of millions of downloads from arXiv in 2012: **63.8**
- Number of institutional supporters of arXiv in 2012: **168**
- Number from Canada: **7**
- Total arXiv funding from Canadian institutional supporters in 2012: **$17,000**
- Number of researchers to sign the "Cost of Knowledge" Elsevier boycott, launched on January 22, 2012: **13,753**
- Number of mathematics researchers to sign: **2,304**
- Number of Fields medalists on the editorial board of Forum of Mathematics, which describes its two branches, Pi and Sigma, as the open access alternatives to the leading generalist mathematics journals and specialist mathematics journals, respectively: **4**
- Number of researchers from Canadian institutions: **1**
- Total number of Fields medalists on the editorial boards of Annals of Mathematics, Publications Scientifiques de l'IHÉS, Acta Mathematica, Journal of the AMS, and Inventiones Mathematicae (counted with multiplicity): **5**
- Cost to publish an article in Forum of Mathematics, Pi or Sigma, after October 1, 2015: **$750 USD**
- Cost before October 1, 2015: **$0 USD**
- Canadian Journal of Mathematics cost per page (institutional subscription, 2010): **$1.82 USD**
- Number of papers published by Forum of Mathematics as of July 16, 2013: **2**
- Number of open access journals listed at the Directory of Open Access Journals, doaj.org: **9,914**
- Number of journals on Beall's list of potential, possible, or probable predatory scholarly open-access publishers: **382**
- Number of editors of the Antarctica Journal of Mathematics affiliated with an Antarctic institution of higher education: **0**

1 *Date: July 16, 2013.* More accurately, this illuminates what members of the CMS executive like, or at least choose, to write about.

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Contribution Québécoise aux Notes de la SMC, suite de la couvrir

à la fois naturel et utile de publier des revues par l'entremise des sociétés savantes. Ces publications sont produites par des collègues qui s'intéressent probablement aux mêmes choses que moi, et les profits reviennent à ma communauté.

J'aurais envie de monologuer sur les projets de libre accès en mathématiques et en sciences, mais je vais me retenir; je vois vos yeux se voiler. (Oui, c'est à vous que je m'adresse; vous là, à l'ordinateur, avec des articles en retard à évaluer sur votre bureau.) À la place, je vais vous laisser sur quelques futilités laphamesques – pas tant des anecdotes que des données artisanales – sur le grand thème du libre accès. Un mot en finissant : même si j'ai laissé tomber le soliloque, le libre accès est un dossier important sur lequel nous devons nous pencher, qu'on le veuille ou non. N'hésitez pas à me transmettre vos réflexions et vos idées sur le libre accès et la SMC. Au plaisir de vous rencontrer en décembre à Ottawa, à la réunion d'hiver!

- Date à laquelle le CERN a mis le premier navigateur Web à la disposition du grand public : **30 avril 1993**
- Date du premier article accepté pour publication dans l'*Electronic Journal of Differential Equations* : la première revue de mathématiques à comité de lecture en libre accès : **2 mai 1993**
- Date à laquelle Paul Ginsparg, fondateur d'arXiv, a présenté à l'UNESCO son exposé « Winners and losers in the global village » qui a lancé le mouvement des archives ouvertes et des épi-revues à comité de lecture : **21 février 1996**
- Nombre d'articles soumis à arXiv en date du 16 juillet 2013 : **859 781**
- Nombre d'articles de mathématiques soumis à arXiv en date du 31 décembre 2012 : **157 284**
- Nombre de millions de téléchargements servis par arXiv en 2012 : **63,8**
- Nombre d'établissements membres d'arXiv en 2012 : **168**
- Nombre d'établissements membres au Canada : **7**
- Financement total d'arXiv venant des établissements membres au Canada en 2012 : **17 000 $**
- Nombre de chercheurs associés au mouvement de boycottage d'Elsevier (The Cost of Knowledge), lancé le 22 janvier 2012 : **13 753**
- Nombre de mathématiciens signataires : **2 304**
- Nombre de médaillés Fields siégeant au comité de rédaction du Forum of Mathematics, qui présente ses deux divisions, Pi et Sigma, comme des solutions de rechange libres d'accès aux grandes revues généralistes et spécialisées du domaine mathématique, respectivement : **4**
- Nombre de chercheurs venant d'établissements canadiens : **1**
- Nombre de médaillés Fields siégeant au comité de rédaction d'Annals of Mathematics, des Publications Scientifiques de l'IHE´S, d'Acta Mathematica, du Journal of the AMS et d'Inventiones Mathematicae (comptés avec leur multiplicité) : **5**
- Coût de publication d'un article dans Forum of Mathematics (Pi ou Sigma), après le 1er octobre 2015 : **750 $ US**
- Coût avant le 1er octobre 2015 : **0 $ US**
- Coût par page du *Journal canadien de mathématiques* (abonnement institutionnel, 2010) : **1 82 $ US**
- Nombre d'articles publiés par Forum of Mathematics en date du 16 juillet 2013 : **2**
- Nombre de revues en libre accès inscrites au Directory of Open Access Journals, doaj.org : **9,194**
- Nombre de revues inscrites sur la liste de Beall des éditeurs potentiellement, possiblement ou probablement prédateurs de revues savantes en libre accès : **382**
- Nombre de rédacteurs en chef de l'*Antarctica Journal of Mathematics* rattachés à un établissement d'enseignement supérieur en Antarctique : **0**

2014 CMS MEMBERSHIP RENEWALS

RENOUVELLEMENTS 2014 À LA SMC

REMINDER: Your membership reminder notices have been e-mailed. Please renew your membership as soon as possible. You may also renew on-line by visiting our website at **www.cms.math.ca/forms/member**

RAPPEL : Les avis de renouvellements ont été envoyés électroniquement. Veuillez s-il-vous-plaît renouveler votre adhésion le plus tôt possible. Vous pouvez aussi renouveler au site Web **www.cms.math.ca/forms/member?fr=1**

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SEPTEMBER 2013

9 La prévision des grandes catastrophes, Florin Diacu *(CRM, Montreal, QC)* www.mpe2013.org/lecture/la-prevision-des-grandes-catastrophes/

9 Montreal: La prévision des grandes catastrophes, Florin Diacu *(University of Victoria)* http://cms.math.ca/Events/MPE2013/

16-20 Workshop on Mathematics for an Evolving Biodiversity www.crm.umontreal.ca/2013/Biodiversity13/index_e.php

19 16:00 PIMS Public Lecture - 2013 Year of Statistics: Trevor Hastie *(Earth Sciences Building 1013)* www.pims.math.ca/scientific-event/130919-pplysth

27 Halifax: Harnessing Math to Understand Tipping Points: Mary Lou Zeeman *(Bowdoin College)* http://cms.math.ca/Events/MPE2013/

22–27 The first Heidelberg Laureate Forum www.heidelberg-laureate-forum.org/

30–Oct 3 Arithmetic Dynamics (in honor of Elon Lindenstrauss) *(Fields Inst. Toronto,CA)* www.fields.utoronto.ca

OCTOBER 2013

7–11 Differential Geometry & Global Analysis *(Leipzig, Germany)* www.math.uni-leipzig.de/~rademacher/dgga13.html

10 Fredericton: Ocean Waves, Rogue Waves, and Tsunamis, Walter Craig *(McMaster)* http://cms.math.ca/Events/MPE2013/

10–13 Whittaker Functions: Number Theory, Geometry 7 Physics *(Banff Research Station, Banff, AB)* www.birs.ca

11-12 57e Congrès de l'AMQ « Mathématiques québécoises de la planète Terre » *(Collège militaire royal de Saint-Jean)* www.cmrsj-rmcsj.forces.gc.ca/col-col/amq/amq-eng.asp

11–13 Symposium on Biomathematics & Ecology *(Arlington ,VA)* www.biomath.ilstu.edu/beer

18-20 Science Atlantic Mathematics, Statistics and Computer Science Conference Organizers: Shannon Fitzpatrick, Gordon MacDonald, Chris Vessey, Nasser Saad, Cezar Campeanu (University of Prince Edward Island)

19–20 Route 81 Conference on Commutative Algebra & Algebraic Geometry *(Syracuse, NY)* http://www.comalg.org/Rte81-2013

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CMS Notes
Volume 45 No. 4, September 2013

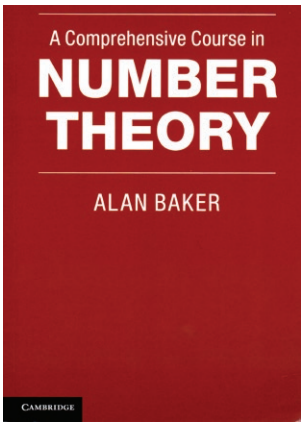
A Comprehensive Course in Number Theory

by Alan Baker

Cambridge University Press, Cambridge 2012

ISBN 978-1-107-01901-1

Reviewed by ***Kenneth Williams****, Carleton University, Ottawa*



This book is a sequel to the author's earlier book *A Concise Introduction to the Theory of Numbers, Cambridge University Press, 1984.* It contains standard introductory material on elementary number theory as well as some advanced material (without proofs) from the author's lecture courses at Cambridge University. The inclusion of this latter material makes the book particularly interesting and valuable. The binding and the printing of the book itself are up to the usual high standards of Cambridge University Press. The book comprises 17 relatively short chapters:

Chapter 1. Divisibility
Chapter 2. Arithmetical Functions
Chapter 3. Congruences
Chapter 4. Quadratic residues
Chapter 5. Quadratic forms
Chapter 6. Diophantine approximation
Chapter 7. Quadratic fields
Chapter 8. Diophantine equations
Chapter 9. Factorization and primality testing
Chapter 10. Number fields
Chapter 11. Ideals
Chapter 12. Units and ideal classes
Chapter 13. Analytic number theory
Chapter 14. On the zeros of the zeta function
Chapter 15. On the distribution of the primes
Chapter 16. The sieve and circle methods
Chapter 17. Elliptic curves

Each chapter is made up of a number of short sections each devoted to a particular topic. The text is easy to read. The proofs are short and clear. Each chapter closes with a valuable section suggesting further reading and a set of exercises. No solutions to the exercises are provided. There is a useful bibliography and an extensive index.

Chapter 1 covers in a concise but readable way standard material on divisibility including the fundamental theorem of arithmetic and the properties of primes. The author also gently introduces the reader to advanced topics such as the representability of primes by polynomials, Bertrand's postulate, Dirichlet's theorem on primes in an arithmetic progression, Goldbach's conjecture, the twin-prime conjecture and Chen's theorem.

Chapter 2 develops the basic properties of Euler's totient function, the Möbius function and the sum of divisors function together with their average orders. Section 2.8 of this chapter introduces the reader to the Riemann zeta-function and its zeros, as well as the Riemann hypothesis. The connection of these topics to both the distribution of primes and to the Möbius function is made. A small comment here — In introducing the functional equation for the Riemann zeta-function the gamma function is used (p. 15), however the gamma function is not defined until p. 177.

Chapter 3 discusses the Chinese remainder theorem, Fermat's little theorem, Euler's theorem, Wilson's theorem, Lagrange's theorem, primitive roots and indices. No advanced material is introduced in this chapter. In connection with the material in this chapter, the author could perhaps have mentioned extensions of the Chinese remainder theorem to moduli which are not co-prime in pairs, and computing problems in connection with the calculation of primitive roots and indices such as the discrete logarithm problem.

Chapter 4 treats quadratic residues, the Legendre symbol and Euler's criterion in a readable and efficient manner. The law of quadratic reciprocity is proved via Gauss' lemma. No advanced material is covered. The author perhaps could have taken the opportunity to mention higher degree reciprocity laws as well as extensions of Gauss' lemma which have appeared in the literature.

Chapter 5 gives an extremely brief introduction to binary quadratic forms, as well as a proof of Lagrange's four squares theorem. In this chapter the author could have included related material on the representability of primes by the forms $x^2 - 2y^2$, $x^2 + 2y^2$, $x^2 + xy + y^2$ and $x^2 + 3y^2$ as well as proving formulae for the number of representations of a positive integer by each of the forms $x^2 + y^2$, $x^2 + xy + y^2$ and $x^2 + 2y^2$. In connection with sums of four squares, Jacobi's formula for the number of representations of a positive integer as a sum of four squares could have been mentioned. Gauss' theory of the genera of binary quadratic forms is not discussed.

Chapter 6 provides a delightful introduction to transcendental number theory including Dirichlet's theorem, Liouville's theorem, continued fractions and quadratic irrationalities. It also describes in a very readable way modern work on transcendental numbers and approximating irrationals by rational numbers. The author himself has made one of the most important contributions to this subject for which he was awarded the Fields Medal in 1970.

Chapters 7, 10, 11 and 12 cover the basics of classical algebraic number theory—quadratic fields, algebraic number fields, algebraic integers, integral bases, discriminants, ideals, norm of an ideal, etc.

Chapter 8 provides an introduction to diophantine equations by developing the theory of the Pell equation, the Thue equation, the

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Mordell equation, the Fermat equation and the Catalan equation as well as the abc-conjecture.

Chapter 9 introduces the reader very briefly to the problem of deciding whether a given integer is prime or not and the problem of factoring a composite integer.

Chapters 13, 14 and 15 cover the basics of classical analytic number theory including the Riemann-von Mangoldt formula and Dirichlet's theorem on primes in arithmetic progression. A highlight of Chapter 15 is a proof of Siegel's theorem.

Chapter 16 develops Selberg's upper bound sieve and uses it to deduce the Brun-Titchmarsh inequality, the twin prime estimate and the Goldbach estimate. I have never found sieve theory to be a subject which particularly appeals to students but the very nice presentation in this book should be well received by them.

Chapter 17 discusses the basic theory of elliptic curves including the Weierstrass $\wp$-function and the Mordell-Weil group.

This is a book which lives up to the word "comprehensive" in its title. It covers the basics of elementary number theory, analytic number theory, algebraic number theory, computational number theory, transcendental number theory, sieve methods and elliptic curves. The first half of the book is ideal for an undergraduate student taking a beginning course in elementary number theory whereas the second half is appropriate for a more advanced student who has successfully completed an elementary number theory course.

The book is a valuable and useful reference for all mathematicians whatever their field of interest and so belongs in every mathematician's library.

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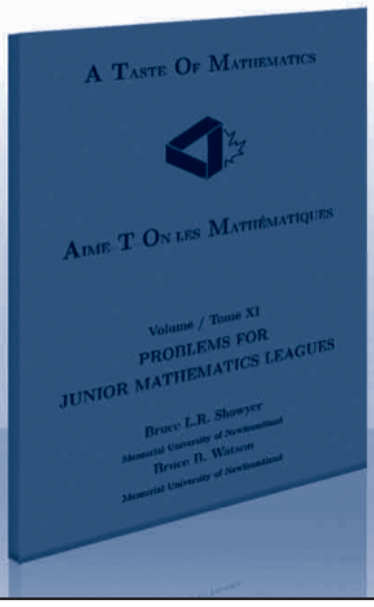

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Volume 45 No. 4, September 2013

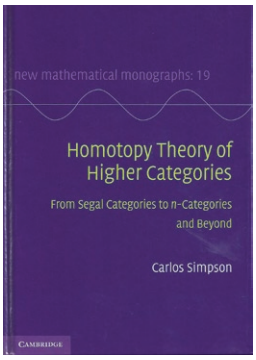
Homotopy Theory of Higher Categories

by Carlos Simpson

Cambridge University Press, Cambridge 2012

ISBN 978-0-521-51695-2

Reviewed by ***Georges Maltsiniotis****, Université de Paris - Jussieu*



Depuis une quinzaine d'années, il y a un intérêt croissant pour la théorie des catégories supérieures, avec des applications en topologie (théorie de l'homotopie, cobordisme), en géométrie algébrique (champs algébriques supérieures, géométrie algébrique dérivée), en physique mathématique (théories de champs quantiques, gravitation quantique) et en informatique théorique (réécriture, théorie des types). La notion de n-catégorie stricte est très simple et date de l'introduction des catégories enrichies. Une n-catégorie stricte est simplement une catégorie enrichie en $(n-1)$-catégories strictes, une 0-catégorie étant un ensemble (et une 1-catégorie une catégorie ordinaire). Dans une n-catégorie stricte, les compositions sont strictement associatives, elles satisfont une « règle d'échange », et les unités sont strictes. Cependant, dans des nombreux exemples ces conditions ne sont satisfaites qu'à isomorphisme, ou équivalence, ou homotopie près. On réservera le terme de n-catégorie pour le concept platonicien de cette version faible de la notion. Pour $n=2$, ce concept a été modélisé par les bicatégories, introduites par Bénabou au milieu des années soixante. Le paradigme est la 2-catégorie dont les objets sont les anneaux, les 1-flèches les bimodules et les 2-flèches les morphismes entre iceux, la composition des 1-flèches étant le produit tensoriel des bimodules (qui n'est associatif et unitaire qu'à isomorphismes cohérents près).

Le cas d'un n général s'avère beaucoup plus délicat. L'exemple principal motivant l'introduction de cette notion est celui du n-groupoïde fondamental d'un espace topologique, généralisant le groupoïde classique de Poincaré. Les objets de ce n-groupoïde sont les points de l'espace, les 1-flèches sont les chemins, les 2-flèches les homotopies de chemins (à extrémités fixes), les 2-flèches les homotopies entre homotopies, ainsi de suite jusqu'à $n-1$, les n-flèches étant les homotopies entre $(n-1)$-flèches à homotopie près. Dans cet exemple, les axiomes des catégories strictes ne sont satisfait qu'à homotopie près (sauf pour les n-flèches pour lesquelles ils le sont strictement), ces homotopies satisfaisant des axiomes de cohérence, mais seulement à homotopie près, etc. Dans « Pursuing Stacks », Grothendieck conjecture en 1983 que les n-groupoïdes (non stricts) modélisent les types d'homotopie n-tronqués (non tronqués si $n=1$), et esquisse une définition de cette notion.

À partir de la fin des années quatre-vingt dix, des nombreux auteurs ont proposé diverses formalisations du concept de n-catégorie (avec parfois $n=\infty$). Il y a des modèles algébriques très proches du point de vue de Grothendieck (Batanin, Leinster, Penon), ou plus lointains (Trimble, May). Dans ces modèles, les opérations et les cohérences font partie de la structure. D'autre part, il y a des modèles non algébriques, oūu l'existence des compositions et des cohérences est simplement exigée, sans qu'un choix explicite soit donné. Certains sont basés sur la notion d'opétope (Baez-Dolan, Hermida-Makkai-Power, Cheng, Leinster), d'autres sur des nerfs généralisés (Joyal, Street, Verity, Tamsamani) et d'autres sur les catégories de modèles de Quillen (Simpson, Joyal, Dwyer-Kan, Bergner, Rezk).

Le livre de Simpson est consacré aux catégories de Segal supérieures appartenant à cette dernière classe de modèles, dont le but est de formaliser le concept de (∞, n)-catégorie (∞-catégorie dont les i-flèches sont faiblement inversibles pour $i>n$). Le point de départ de cette théorie est de prendre la conjecture de Grothendieck comme postulat, et *définir* un ∞-groupoïde comme étant un complexe de Kan, les équivalences de 1-groupoïdes étant les équivalences d'homotopie de complexes de Kan. Ainsi, une $(\infty, 1)$-catégorie n'est rien d'autre qu'une catégorie simpliciale (enrichie en complexes de Kan si on veut). Pour passer aux (∞, n)-catégories, on doit itérer ce procédé. Intuitivement, une (∞, n)-catégorie sera une catégorie enrichie en $(\infty, n-1)$-catégories.Néanmoins, la notion classique de catégorie enrichie est trop rigide et ne conduit pas à la bonne notion de (∞, n)-catégorie. C'est là que les catégories de Segal apparaissent. Une catégorie simpliciale peut être vue, grâce au foncteur nerf, comme un ensemble bisimplicial X tel que $X_{0\bullet}$ soit un ensemble simplicial discret (*i.e.* constant), et satisfaisant à la condition de Grothendieck : pour tout $m>0$, la flèche canonique

$$(*) \qquad X_{m,\bullet} \longrightarrow X_{1,\bullet} \times_{X_{0,\bullet}} \cdots \times_{X_{0,\bullet}} X_{1,\bullet}$$

(le produit fibré comportant m facteurs) est un *isomorphisme* d'ensembles simpliciaux. Pour définir une catégorie de Segal, on remplace la condition de Grothendieck par celle de Segal : la flèche $(*)$ est une *équivalence faible* d'ensembles simpliciaux. On observe que la condition de Segal garde un sens si l'on remplace la catégorie des ensembles simpliciaux par une catégorie de modèles arbitraire M, définissant ainsi la notion de *M-catégorie*. Le but du livre est de dégager des conditions suffisantes sur la catégorie de modèles M permettant de construire une catégorie de modèles ayant comme objets les M-*précatégories*, objets simpliciaux X de M tels que X_0 soit discret (somme de copies de l'objet final), et dont les objets fibrants soient des M-catégories. De plus, on veut que cette nouvelle catégorie de modèles satisfasse à ces mêmes conditions pour pouvoir itérer le processus. Parmi ces conditions, la plus importante est celle de la cartésianité de la catégorie de modèles.

Le livre comporte cinq parties. Dans la première, introductive, l'auteur explique l'insuffisance des n-catégories strictes, en montrant que les 3-groupoïdes stricts ne permettent pas de classifier les 3-types d'homotopie. Il décrit les différentes approches pour définir les catégories supérieures, et en particulier celle conduisant aux catégories de Segal. Il présente le plan du livre. La deuxième partie est consacrée aux rappels sur les catégories de modèles et les catégories localement présentables. Un traitement original d'une classe particulière de localisations de Bousfield à gauche est présenté. Le noyau dur de

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l'ouvrage est formé des parties III et IV, ou la catégorie de modèles des M-précatégories est construite. La dernière partie est dédiée aux applications. Les n-catégories de Segal (resp. de Tamsamani) sont définies par le procédé récursif fondé dans les parties précédentes, en partant des ensembles simpliciaux (resp. des ensembles). Les limites, les colimites, les adjonctions et la localisation sont étudiées dans ce cadre. La $(n+1)$-catégorie de Segal des n-catégories de Segal est introduite; l'existence des limites et colimites est démontrée. Le livre se termine en beauté par une preuve de la conjecture de stabilisation de Baez et Dolan, dans le contexte des n-catégories de Tamsamani. Le livre de Simpson deviendra la référence sur le sujet. La partie I constitue une excellente introduction pédagogique aux n-catégories et une bonne revue historique du développement de la théorie, avec un soucis permanent de l'auteur pour une juste et généreuse attribution des résultats, allant parfois jusqu'à minorer sa propre contribution. Un rare effort est fait pour donner les motivations et expliquer la naissance des notions. Le concept générique (ou « platonicien ») de n-catégorie est présenté de façon limpide. Les seuls reproches pour cette partie : les innombrables « coquilles », l'aspect un peu désordonné de l'exposition et quelques approximations. J'apprécie particulièrement, dans les parties plus techniques du livre, l'effort constant de l'auteur à énoncer et démontrer les résultats intermédiaires dans une forme aussi générale que possible, même si cela rend parfois la lecture difficile. Je conseille vivement la première partie de l'ouvrage à un très large publique intéressé par la problématique des catégories supérieures, et aux personnes désirant étudier de près cette théorie de lire le livre en entier, ligne par ligne, pour apprécier toute sa saveur et sa richesse.

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An Interview with Varadhan

Professor S.R. Srinivasa Varadhan

Professor S.R. Srinivasa Varadhan (**V**) of the Courant Institute, winner of the Abel Prize (2007) and recipient of the National Medal of Science (2010) and many other honours, was in Halifax as the 2012 Distinguished Lecturer at Dalhousie University. On that occasion, he was interviewed by Keith Johnson (**J**) and Chelluri Sastri (**S**). Professor Varadhan was born in Chennai in Southern India.

J : Welcome to Halifax. Can you tell us a little bit about your early life and what led you to a career in mathematics?

V : One doesn't remember all of one's early life, just a few things. I remember the day we got independence, on August 15, 1947. Anyway, in those days I was in grade school. You know, mathematics was always an easy subject for me. I can't explain the reason why, but it came easy. In elementary school, it didn't mean anything - just the standard things: addition, multiplication, subtraction, division and so on. Memorize multiplication tables. I gather that children these days don't do it; those days we did. It was only when I went to high school that I got interested in mathematics - geometry and the logic behind it. There were problems assigned which we used to call riders for some reason. They required some strange constructions which were not so obvious. There were special ways of proving facts about geometric figures using just logic. You couldn't prove it by just drawing pictures, and that I found interesting. There was a high school teacher who encouraged us to visit his home on holidays and weekends and assigned problems to a small group of us. We would spend a couple of hours working on these problems. That is how I developed my interest in mathematics.

J : Your undergraduate years were at Presidency College in Chennai. Was it there that your interest in probability theory developed?

V : I was a student there not of mathematics but of statistics. My major was statistics. It was an interesting program because the mathematics majors and the statistics majors had pure mathematics classes in common. Otherwise, mathematics students took applied mathematics and the statistics students took statistics. Part of the statistics program included probability. And somehow I found probability and statistics easy and natural. I didn't have to struggle to do them. As an undergraduate, I was equally interested in probability and statistics.

S : When you went to the Indian Statistical Institute (ISI) in Calcutta, did you intend working on the National Sample Survey?

V : I wanted to do statistics because I knew it was applicable. I had no idea what research meant. I thought of statistics as a way of getting a job - a job not as a professor at a college but a job in industry. At that time in India, if you were a lecturer at a college, you were just stuck there, whereas a job in industry was more promising. With a Ph.D. in statistics, I thought I could probably work in industry. I was more interested in applied statistics.

S : You tried it for a few months?

V : Yes, I tried for two or three months and found it rather dull. Then Varadarajan[1], Parthasarthy[2], and Ranga Rao[3] were there and they talked me out of it, but I didn't need much talking.

J : This was at the Indian Statistical Institute?

V : Yes.

J : You found that a good environment for a beginning researcher?

V : It was sort of an interesting environment, very different from what one is used to in the US or Canada. You are a research scholar, given a stipend, a desk and a chair and in 3 years you were expected to produce a thesis.

J : Not a lot of direction?

V : If you had a question, you could go to your advisor and ask him, and he might be able to answer, but there was no one who told you what to work on or what was important to study. There was no real hand-holding type of guidance at least in those days. There were three or four of us in a group, and we worked together and formulated our own problems. We tried to solve them, had our own seminars, read material and lectured to each other. There was no formal course work.

J : You learned more from your fellow graduate students than you did form the faculty?

S : To me, it is amazing that from a background of just statistics in Madras, you could reach a stage at which you could look at the literature and say these areas are interesting, these problems are challenging. At that age and at that time in India, it was remarkable achievement.

V : Well, there were other research scholars, and they were all at different levels. Varadarajan had already finished his dissertation. He went through it three years before us.

S : So, all of you were self-starters?

V : We had his experience to rely on. For instance, Ranga Rao interacted with him a lot, and was only a year behind Varadarajan, and Parthasarathy was a year behind him. So there was a ladder-like thing. We were not all at the same level.

S : So the first person to do it was Varadarajan?

V : He interacted with Vaidyanathaswamy[4] a little bit.

S : But he was a topologist.

1 V.S. Varadarajan, Professor of Mathematics, UCLA
2 K.R. Parthasarathy, Professor Emeritus, Indian Statistical Institute, Delhi
3 R. Ranga Rao, Professor Emeritus of Mathematics, University of Illinois, Urbana-Champaign
4 R. Vaidyanathaswamy, Late Professor of Mathematics, University of Madras

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V : But remember that Varadarajan's interests at that time were broad. His thesis was on limit theorems in arbitrary topological spaces. So there was a lot of topology.

J : Moving on to your PhD thesis, you defended it at age 23, or was it 22?

V : Let's say 23.

J : And Kolmogorov was the external examiner?

V : In those days, for an Indian to receive a PhD, you didn't really trust the faculty. So the rules required that there should be two external examiners outside the country. And for me, they were Kolmogorov and Doob. Doob sent his response immediately and said it was a good thesis. He said that we had such good probabilists in India that we didn't need an external examiner. But Kolmogorov never sent his report–for almost 10 or 11 months.

J : You must have despaired!

V : It is normal. A thesis in India takes a year or so. What happened was that Parthasarathy graduated and went to work as a postdoc in Moscow. He got hold of Kolmogorov and said that he had better send the report.

J : And it was very flattering once it arrived.

V : Yes, it was quite positive.

J : What were your thesis results that he was so pleased with?

V : The limit theorems in probability at that point were mostly for finite-dimensional Euclidean spaces. And "limit theorems" means convolution properties are studied. You use the abelian nature and Fourier transforms. To analyze distributions, you analyze their characteristic functions, which are basically Fourier transforms. You have continuity theorems going back and forth. My thesis involved two things: one was trying to extend these results to abelian groups, locally compact abelian groups (LCAG), because they have the Fourier transforms and you have the continuity theorems to some extent; so you can extend some of these results to this context, but there is a little bit of a technical issue that needs to be resolved. When you do characteristic functions and limit theorems and so on, sometimes the distribution has to be centered. So you compute its mean and subtract it in Euclidean space. In a locally compact abelian group it is complicated. If the distribution is fairly degenerate, the centering is by a very little amount. A distribution concentrated near the identity will have a small mean. Locally compact abelian groups can have a discrete component. Then you can't shift by a little. You have to handle the discrete part separately. What it means in practical terms is that you have to be able to take the log for a character. In Euclidean space, it is just itx. There is no analog of that in LCAG. That difficulty has to be overcome somehow. And it is important that the log you take be linear in the group character variable. And the question is how you construct such a thing. Then what we did, with Ranga Rao and Parthasarathy, in joint work, was this: you can't quite take a log, but you can define an object which is linear in the group character variable, which, when exponentiated, agrees with the character in a neighborhood of the identity. And that is sufficient for proving limit theorems. And so the way you can center is by integrating this function. It turns out that for a discrete group this construction gives you zero. The neighborhood can be just one point.

J : Are you working in the connected component of the identity?

V: No, the focus is on the connected component and somehow combines the two in some reasonable fashion. That was part of the work. The other part of the work was to extend these limit theorems to distributions with values in a Hilbert space. There you lose local compactness, and the continuity theorem doesn't hold so you need some compactness criterion that Prohorov had worked out. You somehow combine it. And the surprising thing was that the limit theorems had statements identical to those for Euclidean spaces. But that is only for a Hilbert space. If you go from a Hilbert space to a Banach space, the situation changes dramatically.

S : Your work dealt with Hilbert spaces. I have heard that Ranga Rao did something with Banach spaces.

V : He proved the law of large numbers (LLN) for random variables with values in a Banach space. And there, it is almost sure convergence. He proved various theorems of the French school. His method was very original. He used weak convergence methods and proved the LLN very quickly without much trouble.

S : So you could look at it and get some ideas?

V : It is very different. It is just the LLN, so the limit distributions are all degenerate. It is like the difference between the central limit theorem (CLT) and the LLN.

J : In 1963, you moved to New York to the Courant Institute as a postdoctoral fellow. What did you work on when you got there? Was it a continuation of your thesis work?

V : Not really. I finished my thesis work in April 1962. And in 1963 Varadarajan had come back, and we started working on Lie groups, representations of Lie groups. I studied a little bit of Harish-Chandra's work. So when I came to New York University (NYU), I thought I would do a little bit of group representations. I had some interest in mathematical physics, and I was interested also in Markov processes and diffusion theory. That required a lot of PDEs, and Courant was strong in that. So I was a little undecided about what to do.

S : This is quite a wide range.

V : These are the things I was exposed to as a graduate student. If I had been exposed to algebraic geometry, I would have wanted to read Grothendieck.

J : You have stayed at the Courant Institute even since you arrived. What about it makes it so conducive to doing research?

V : I think part of it was I liked New York City (NYC). I grew up in Chennai, lived in Calcutta for four years. Somehow in NYC, even if you are a foreigner, you don't feel like a foreigner.

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J : Because everybody is a foreigner?

V : Sort of. So I am very comfortable in the city. In terms of working conditions, I found the place very supportive. In terms of scientific work, I was interested in probability and its connections to analysis, and they had a very strong group. In probability we had Donsker[5], and we had a constant stream of visitors. At that point, we had 35 to 40 postdocs each year. In addition, there was Rockefeller University uptown and Mark Kac was there; Henry McKean was there - he moved later to Courant - and Feller[6] was visiting from Princeton. Because of McKean's connections with Japan, there were always two or three Japanese probabilists. So there was this mixture of visitors and regulars. A strong group of 10 to 12 probabilists that was very attractive. In addition to the probabilists, there were the analysts. I never formally learnt PDEs as a graduate student. There was no course, and it is not something you learned as an undergraduate. And I learnt PDEs not even attending courses but attending seminars and oral exams, where I was an examiner. You are in a group of two or three people, and somebody else is asking a question, and the student is answering. It is a marvelous way to learn a subject. After three or four exams you are an expert! And you can ask questions!

J : You have mentioned Donsker, one of your research colleagues at Courant. When you first arrived, were you his postdoctoral fellow? Did he supervise you?

V : We don't really have supervisors for postdoctoral fellows (pdfs). The institute gives you a fellowship, and it is up to you to interact with anyone you like. It was natural for me to interact with him because he was the closest one to my subject.

J : Can you say a little bit about what his research was at the time?

V : He had a student who had just finished his thesis on large deviations. I had read a little bit about large deviations, and about Cramer's theorem because Ranga Rao and Bahadur[7] had worked on it in India when I was s graduate student. So I was familiar with it– the idea that one could do such things in infinite dimensional spaces. The student of Donsker, Michael Schilder, had proved this theorem on large deviations for Brownian motion paths. The idea was also independently discovered by Strassen, who used it to prove the law of the iterated logarithm. However, I didn't know anything about Strassen's work at that time. Schilder's work involved finite dimensional approximations, doing large deviations basically in finite dimensions and then getting some uniform estimates, so you can pass to infinite dimensions. I looked at it and thought that one should be able to do it directly. After all, there are tools in functional analysis which should allow you to do it. The problem is usually one of compactness which is not so easy to prove in infinite dimensions. So the proof may require a couple of special lemmas to take care of that. Once you have that, you have a theory in function spaces directly. So that is how I got interested in large deviations.

J : For a while during your early years you did some work with Stroock[8] on martingale theory. Can you say a little bit about that and why you didn't continue in that area?

V : Stroock finished his PhD and came to Courant in 1966, which was the year I became a faculty member. We had joint seminars with Rockefeller. We would go from downtown and share a taxi on the way back, four of us. Somebody mentioned a result of Ciesielski. It is an interesting result. Take Brownian motion: it has a wonderful resolution, very explicit. You can take a region in space and look at the fundamental solution for the region with Dirichlet boundary conditions. That is always dominated by the full fundamental solution. That is because the Dirichlet condition cuts it down. And then you look at the ratio and you ask: as $t \to 0$, does the ratio go to 1? This was important for Mark Kac in analyzing part of his paper "Can you hear the shape of a drum?" The question was, does the ratio go to 1? And the answer is essentially that it does if the region is convex. If you think about it, the ratio compares the probability of going from x to y in a very short time … the ratio of the probability of going without reaching the boundary because the Dirichlet boundary conditions don't contribute if you go to the boundary. So it is the ratio of the two, and ratios are usually conditional probabilities. Given that in a small time you went from x to y, did you hit the boundary before you went out? And if you say that in a short time you are going to go in a straight line, then you are really saying that all the straight lines remain in the domain. It is a natural question to ask, and it occurred to me during the taxi ride that this is the way to formulate the problem. And the natural question is, if you do it more generally, in a Riemannian space, then the shortest way to get there is a geodesic, and "convex" should be replaced by "geodesically closed". What this required was an expression for the Green's function for the fundamental solution. For the heat equation it looks like $exp\left[-\frac{(x-y)^2}{2t}\right]$; so in a Riemannian space, $(x-y)^2$ should be replaced by the square of the geodesic distance. So you want a theorem that states that for small t, the fundamental solution behaves like the exponential of minus geodesic distance square divided by $2t$. So, I looked for a theorem of the type and couldn't find it. There are things that sort of hinted at it, but those were for values of x and y close, but you want it when x and y are a fixed distance apart. You want to fix x and y and let $t \to 0$. And I did not find a result in that context. So I proved it. That was my last year as a postdoc. In the end I gave a proof that uses just PDE. Basically you want to get some estimates of exit times, and so on. You can either use martingales and Doob's inequality, which gives you exit times, or you can do it by analyzing the Laplace transform of the exit time. It satisfies a PDE. So in some sense the two are not that far apart. It is just the language you express it in. And I realized that the only property I used about this diffusion process, Brownian motion on

5 Monroe D. Donsker
6 William Feller
7 R.R. Bahadur, Late Professor of Statistics, University of Chicago
8 Daniel W. Stroock, Professor of Mathematics, MIT

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a Riemannian manifold, was these martingales. That was all that was used. So the question was, what about characterizing this process in terms of these martingales? Dan Stroock was finishing up and liked this problem and said, let's work on it. And so we worked on it, and that is how we got interested in the martingale formulation. It has the advantage that you want to define a Markov process, which means that you are given some data and you want to construct something. The data given to you are the diffusion coefficients, which are some positive definite matrix-valued functions defined on some $\mathbb{R}^d$, and they may depend on time or may not. The object you want to construct is a stochastic process, which is a measure on the space of continuous paths. And the link between the two is, you look at the parabolic PDE that corresponds to the coefficients, which has a fundamental solution. And you use the fundamental solution as the transition probability and construct the diffusion process that way. I thought this was a rather circuitous way of doing it and looked for a more direct link between the measure you want to construct and the given coefficients. The martingale problem is a direct link. So it formulates the question of the diffusion process, which is a measure on a function space, purely in terms of the coefficients. So you bypass this PDE altogether. And it just becomes, for given coefficients, looking at a process with certain properties. If this process exists, is it unique? Existence is very easy, and the difficulty is in proving uniqueness. If the PDE that you would have solved can really be solved, that implies uniqueness. Because, basically the PDE tells you how to compute the expectations, and if you compute enough expectations, the process is determined. Or, Ito has his own method for solving stochastic differential equations in order to construct these processes. You can also prove that if Ito's method works, that also gives uniqueness. And it has an added advantage in the sense that uniqueness is a local property. You can prove that. So if the coefficients are s.t. in one region the PDE method applies, and in another region Ito's method applies, you are still in business. So it means that the martingale method gives you a lot of flexibility. In terms of handling limit theorems, it is easier to prove here because you have integrated equations that are very stable, rather than DEs which are not so stable. So we worked on this with Stroock for six or seven years. There were several papers, but three major ones.

J : Is this distinct from your work on large deviations?

V : It is very distinct.

J : Your work on large deviations started in 1966?

V : I worked on large deviations from 1963 to 1966 and then shifted to martingale problems from 1966 to 1972 and then went back to large deviations in 1973.

J : Could you say a little bit about what the Large Deviation Principle (LDP) is, maybe with an example?

V : This is sort of the end of the story. It took me a long time to come to this conclusion. In models, usually in probability theory, they are stochastic models that describe some phenomena. The model has some parameters in it and also some size involved in it. Sometimes the parameters are implicit or the model can be nonparametric, which means that the parameter space is just too large. When the size of the system becomes very large, certain events, such as the LLN for example, occur with probability nearly 1. Certain events, such as the complements of these events, naturally occur with very small probability. So there is a whole class of events whose probability goes to zero exponentially fast in the size.

S : What size are you talking about?

V : It doesn't matter. For example, in equilibrium statistical mechanics, it should be the volume that is the size. In normal statistics, when you make observations, it is the sample size. So we mean some size that is natural to the context. If you just start with a finite state, all the properties are fixed, and there is no limit to be taken.

So these probabilities decay exponentially, and the problem is, what is the constant of the exponential decay? And what does the constant depend on? How do you compute this constant? Well, it turns out that the way you compute it is to change the model so that the event which has probability going to zero now has the probability going to 1. Cramer did that by changing the density by an exponential factor. It is called the Cramer tilt. This is not unique; you can do it in many ways by changing the model so the event occurs with probability nearly 1. There is no reason to do an exponential tilting. If you want to change the mean, there are many ways to change the mean of a distribution. So when you change the model, there is a cost involved. The cost, it turns out, is the relative entropy. So when we change from one model to another, we have two different probability distributions. You can compute the relative entropy of one w.r.t. the other. Basically, in statistical mechanics it is computing the log of the partition function. Usually, if you take the entropy, this already involves taking the log. And this depends only on the model. And you look at the constant and think of it as the cost to produce the effect if you change the model in this manner because different changes in the model have different costs. And you minimize the cost. And the least cost is the cheapest way of achieving your goal. It turns out to be the answer.

There is a little bit of a problem here. It is easy to say, change the model. How do you change the model? The class of models over which you want to optimize should be reasonable in the sense that you should be able to calculate a few things, it should be large enough so that the lower bound you get matches with the upper bound you get by some other means so you have a limit theorem. This requires some thought. Usually natural classes appear. In terms of Cramer's work, for example, if you have independent observations it is natural to change to independent observations again but change their common distribution. You compute the relative entropy; you minimize it, fixing the new mean. The minimizer is the exponential tilt. Things like that were discovered over a period of time providing a unified theory. All

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these probabilities of large deviations of different contexts have a common unified theme, which is really relative entropy or the Kullback-Liebler information, which basically controls everything.

S : The power of the principle is that it covers independent as well as dependent random variables, discrete, finite-dimensional and infinite-dimensional distributions. That is quite a wide sweep!

V : It is basically Jensen's inequality. The proofs are all, at some level, easy. But what is often difficult is identifying the class of perturbations that is natural for the problem. For example, if you have independent random variables, your LLN is for functions of the type $\sum f(X_i, X_{i+1})$, two adjacent points. You can have an LLN for this. It turns out that the perturbations have to be in the class of Markov processes, not independent ones because the fact that the function you are tying to optimize involves X_i, X_{i+1} introduces a coupling, but the coupling is just one step back. So it is Markov. The way to think of it is, if you want to do large deviations, then you are introducing an exponential factor. It is very much like statistical mechanics. You are introducing an exponential energy term. Then you want to compute the new partition function. And then large deviations are like the variational formula for free energy in terms of specific energy and entropy. So they are related. There is a nice expository article by Oscar Lanford (1972) on it.

S : Can you explain the notion of large deviations to a non-probabilist, perhaps with a simple example?

V : The simplest example is, take a coin. Toss it, assume it's a fair coin, and if you toss it a large number of times, then the number of heads is supposed to be approximately half of the number of trials. But it doesn't have to be. Maybe you had 75% of heads, although it is not very likely. But the probability is not zero. You can compute the probabilities; if you toss it 10,000 times, it's $2^{-10000}\binom{10000}{7500}$, and you can write down the formula and use Stirling's formula to evaluate it asymptotically. That will turn out to be the relative entropy of a binomial with probability $\frac{3}{4}, \frac{1}{4}$ to a binomial with probability $\frac{1}{2}, \frac{1}{2}$. Stirling's formula for $n!$ has the leading term $n \log n$. The terms $p \log p$ and $q \log q$ come from that. So when you compute $(np)!$, you are going to get $p \log p$. In a sense, this is to be expected because in statistical mechanics, entropy is supposed to be a measure of the volume. Combinatorially, it is the count and that is why it is natural that the entropy comes from size. The way to think about it is, when you compute the probability of an event in a very large system, you are adding a lot of very small probabilities, a very large number of them. So you are adding an exponentially large number of exponentially small things. The advantage in adding exponentials is that the sum of two exponentials is the same as the maximum, approximately. You are only interested in the exponential rate of things, so you can replace the summation by the maximum. That is what large deviation theory is.

J : One of the applications made of this in 1983 with Donsker was the proof of Pekar's conjecture. Could you say a little bit about what it was and how you resolved it?

V : I think it is some problem that comes from statistical physics. What it involves is evaluating, for Brownian motion, large time asymptotics for a function which depends on the entire past history of the Brownian motion. So it is a problem with long range dependence, which makes it basically noncomputable. Of course, if it is not computable, you can say that's it and leave it, but it is a problem with a parameter in it, and as the parameter goes to a certain limit, then asymptotically, the answer is supposed to be computable. That was Pekar's conjecture, the formula for it. It is a formula for some integral, and you are interested in the log of the integral; there is a parameter and as it goes to infinity the log behaves in some fashion: it is a power of the parameter, and there is a constant in front which has a variational formula, and that is Pekar's conjecture. At that point, we had done large deviation theory, basically for Markov processes. We looked at the occupation times, the number of times a Markov process visits various sites. In principle, there is no reason you can't do large deviations for the entire history of the Markov process. As the process keeps marching on, the entire history gets a bit longer and the thing gets shifted. If you have a function that depends very weakly on the tail, it can be very well approximated by a finite number of terms, and the theory can be pushed forward. When you push it forward, you have a variational formula for the limit. The variational formula is a mess. You can't do anything with it. But there is a parameter in it, which is the same one as before. As it goes to infinity, the variational formula simplifies: it collapses and becomes Pekar's variational formula. So, basically it involves generalizing the theory to things that have infinite history but weak dependence on what happens.

S : Why was it considered a hard problem–because Lieb said it took him twelve years to solve it by a different method?

V : You have a messy integral. And the integral involves the time t, you take the log of the integral and divide it by t and take the limit. It is a mess. You can prove it exists but you have no idea what it is. There is a parameter in that problem and you want to study what happens to the limit as the parameter goes to another limit. If at one stage, you have a variational formula, then you have something concrete to work with. So what the LDP allows you to do is, for the intermediate step, it gives you a concrete formula. So you can work with it and go to the next step.

S : Are there other problems like that you have solved using large deviations? Is that the first example?

V : No, there was something before that - the Wiener sausage. What you look at is the Schrodinger operator with a random potential, and you compute the density of states, and look at its behaviour: at one end small energies: if you have a potential you think of it as hard spheres, then basically what you want to calculate asymptotically is the probability that a Brownian motion avoids these traps for a long time. So the question is - of course the traps are randomly located - where does the major contribution come from? It is a conjecture of a physicist - physicists are very good at making conjectures and they are mostly correct - that the best way to achieve this is for the Brownian particle to find a hole without

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traps and just stay there. So the question you have to look at is, what is the probability of finding no traps in some large domain? They are randomly distributed in terms of the volume, and you can compute it in terms of the volume. Then you can ask: what is the probability that a Brownian path doesn't leave this region up to time t? That also is something you can compute. And you can balance one against the other because the radius of this sphere (which is the best shape for the region) is up to you to choose. For each t, if you choose it too small, the cost of staying there is prohibitive. If you choose it too big, finding it empty of traps is very unlikely. So you have to optimize at various levels, and the optimal answer must be the right answer. It is not clear why the answer has to be just a circle; you could have more complicated regions, and the region doesn't have to be nonrandom, could be a random region of some kind. You need to find a formal way of justifying this. It turns out that what is needed here is: instead of having hard spheres of some radius, you can just say that with a Brownian path, I can build a sausage around it of that radius and simply say the sausage has no traps. That is the Wiener sausage. So what you need is the volume of this, because this is the region where you don't want to have traps. In a Poisson process the probability of not having any traps in the region is just $e^{-\lambda(\text{volume})}$. So what you need is the behaviour of the volume of the sausage. The LDP that we had considered with Donsker involves mainly the weak topology for probability distributions, because in a Brownian path, it is a very low-dimensional thing, whereas the limits of occupation times would have nice densities in some huge higher-dimensional space, and this transition cannot take place. The strong topology, the L_1 topology, has to replace the weak topology because singular objects are converging to absolutely continuous objects. So volume then is very unstable in the weak topology, it is stable in the L_1 topology. What helps you is your Wiener sausage. The sausage is like a mollification. Essentially, you are looking at the range when you convolute it by the indicator function of a ball. So the question really is, is this sufficient? Is there sufficient smoothing here to converge to something that is valid in the weak topology? Does it compactify to give you a theorem valid in the strong topology, which generally implies the property of the sausage? That is the hard part. This is analysis because when you rescale, the radius of the sausage shrinks, because there is time involved; the time is getting large. As time gets large, the radius gets large and relative to that radius, your sausage is the same size; the traps are the same size. The amount of convolution you have, when you rescale things, is going to zero. So there is a fight between these two, and to resolve these fights you have to use ideas like ε-entropy and so on.

J : Maybe we can finish up with a more general question: the examples you have shown us suggest a very large role for probability theory in mathematics. How far do you think that extends? Are there parts of mathematics that are not touched by probability theory, or is it everywhere?

S : Mumford, in a famous lecture around 2000, said that we were entering the age of stochasticity. He even suggested that Euclid's books be stochasticized. What do you think?

V : Let me put it this way: probability plays a role in many things. There is a lot of uncertainty, and as long as you have to deal with uncertainty, you have to understand it. Probability is one way of understanding and dealing with it in everyday life. Some of it is very trivial, like what is the probability it is going to rain today, and should I take an umbrella or not? On the other hand, in physics and engineering and so on, if you are dealing with objects at a macro level, probability plays a very minor role, because averaging takes place and you don't really see the randomness. But if you are operating at a microlevel, probability is very important. I think what is happening these days is more at the microlevel and so probability is beginning to play more and more of a role.

S : I remember you saying that in PDEs, instead of giving purely analytical proofs of existence and uniqueness, one can give probabilistic proofs.

V : For me, probability *is* analysis. Probability is just a type of analysis. So I would like to say that somehow the intuition that comes from probability provides you with certain analysis tools, which you probably do not see if you are not a probabilist. So in some sense, it enlarges the tools at your disposal. And that is true in PDEs.

S : So, probability helps in dealing with questions in analysis. What about topology, algebra, and geometry? Is probability useful in those areas?

V : Well, there are many counting problems, for example. And probabilistic intuition can help you count certain things. The prime example of that is the connection between random matrices and the number of representations of permutation groups, all the work done by Tracy, Widom and Deift and so on. It is a sort of connection between probability on one side and group theory on the other. It is a question of enumeration, in which probability can provide you with a point of view.

J : Thank you for your visit and for this interesting and insightful interview.

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Jennifer Hyndman, *University of Northern British Columbia*
John Grant McLoughlin, *University of New Brunswick*

As part of the MPE2013 initiative, Brian Menounos gave a talk on glaciers in Jennifer's Calculus class. The students were very engaged by the talk and this led to Brian and Jennifer talking about what motivates students and what would have motivated Brian to do more mathematics earlier in his career. Many mathematicians see the beauty in mathematics without needing a context. However, many students need the context in order to engage in the mathematics enough to actually see the beauty and value the mathematics. This article illustrates some basic first-year calculus that has meaning for glaciers. Brian has also provided questions that could be used for a class assignment.

Teaching Mathematics through the Lens of Earth Science

Brian Menounos
University of Northern British Columbia
Brian.Menounos@unbc.ca

1. Introduction

Let me begin by stating that I am not a mathematician. Although I did well in mathematics in high school, I never saw any real reason why I should continue to study it at university. My ideal profession at that time was to be a free-lance writer. Required coursework as an undergraduate exposed me to Physical Geography where I learned, and became fascinated with, how glaciers and rivers shaped Earth's surface. Soon, I realized that my real calling was to be an earth scientist. In order to move beyond simple description of Earth's landscape, however, I needed to broaden my understanding of mathematics and physics.

I often wonder if I would have found my career path sooner if my high school teachers had used examples of how mathematics can be used to answer interesting questions about Earth's dynamic landscape? In this note I provide an example that instructors could use in their courses to demonstrate the importance of mathematics as a tool to understand Earth's surface and how it works. At the end of this note, I provide ways that these examples could be altered to provide problem sets which could be used in the classroom.

2. Glaciers

Glaciers provide a visually appealing example through which applied mathematics can be taught to students. These frozen bodies form in areas where the net mass of winter snowfall exceeds mass lost during summer. Because glaciers deform under their own weight, they can grow and shrink in response to changes in climate. Glaciers tremendously vary in size from permanent snowfields which are only hundreds of meters wide to the Antarctic Ice Sheet, which has the potential to raise current sea level by 60 m if the entire ice sheet disappeared. Luckily for us, collapse of the Antarctic Ice Sheet will not happen in our lifetime, but sea-level rise of 1-2 m from melting glaciers by the end of this century is plausible.

Given their direct coupling to the atmosphere and their ability to flow, glaciers are complex earth surface systems. Their behaviour, however, can be appreciated in introductory courses in calculus and physics.

2.1. Ice flow

Ice and snow begin to deform under its own weight once the overlying ice and snow exceed its internal strength. For a given depth in the ice and snow below the surface, a unit volume of ice will have a given density (mass/volume). The product of these two terms and gravity yield the weight of ice which is a force (mass ·acceleration). Like a stationary object on an inclined plane, the weight of the ice can be divided into a two components, one component of the weight acts normal (perpendicular) to the slope whereas the second component acts parallel to the slope. For any depth in the ice, this force acting parallel to the surface slope of the ice can be expressed as a stress (force/unit area of ice) as:

$$\tau = -\rho g h \sin(\alpha) \tag{1}$$

where ρ is the density, g is the gravitational acceleration, h is the depth, and α is the surface slope of the ice. The surface slope is simply the change in elevation (dh) over some distance (dx). The formula contains a negative sign since stress acts downslope, that is, a positive stress corresponds to decreasing elevation.

Mountain glaciers where their ice is near or at 0ºC flow downhill both by internal deformation of the ice itself, and by slippage of the basal ice when water exists at the sole of the glacier. For this article, we'll focus just on internal deformation of the ice and ignore basal slippage.

In one dimension, ice deformation can be represented as linear strain (ϵ), which is the change in an object's length (L) from its initial size (L_o):

$$\epsilon = \frac{L - L_o}{L_o} \tag{2}$$

Ice is a three-dimensional body, and a full derivation of the the relationship between stress and strain requires tensor calculus. Interested readers can consult Cuffey and Patterson (2010) for a full derivation in three dimensions. For one dimension, however, strain can be expressed as a rate ($\frac{d\epsilon}{dt}$), and the flow of ice is governed by the strain rate of the ice:

$$\frac{d\epsilon}{dt} = A\tau^n \tag{3}$$

where A is the flow-rate parameter which depends on the temperature and impurities of the ice, and n is an exponent which based on theoretical and empirical evidence is taken to be 3. For ice at 0ºC, A can be treated as a constant. Equation 3 is known as Glenn's Flow Law, named after the glaciologist who first described it.

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Figure 1 Heavily fractured (crevassed) surface of Perito Moreno Glacier, Argentina. This crevassed surface arises from tensional forces applied to the brittle portion of the glacier's surface. Although these fractures appear to be bottomless pits, they are not; crevasse depths relate to the depth at which the ice changes from becoming a brittle material to one which can flow under it own weight. For glaciers with ice temperatures close to 0ºC, this depth is around 30-40 m.

2.2. Ice thickness and ice-sheet profiles

For most glaciers with ice near or at 0ºC, ice begins to flow at a shear stress of about 100,000 Pa (Pa stands for Pascal, a unit of measurement for stress and pressure). Let's use this value to define a critical yield stress (τ_{crit}) or the value at which the glacier begins to flow. Insertion of this value for τ in Equation 1 and neglecting other terms in Equation 1 which do not significantly vary for ice which is at its yield stress (e.g. ρ and g) allows us to make some general observations about a glacier and its surface slope:

$$h \propto \frac{1}{\sin \alpha} \tag{4}$$

Equation 4 suggests that there is an inverse relation between ice thickness and its surface slope. Steep ice is thus expected to be thinner than gently-sloped ice, which is confirmed by many radar surveys of ice thickness.

To a first approximation, Equation 1 can also be used to construct the surface profile of an ice sheet [3]. Consider the idealized cross section of a circular ice sheet (Figure 2). The ice sheet in planform (bird's eye) view has radius (R) so that the ice sheet attains its maximum thickness (H) at its centre. A simplifying assumption is to treat ice as a perfect plastic such that when the ice reaches a critical depth, it begins to flow. Perfect plasticity implies that the ice sheet adjusts its thickness everywhere where the basal shear stress (τ_b) equals the yield stress of the ice (τ_o). Thickness (h) for any point on the ice sheet can be determined as a function of $R - x$ (Figure 2). This ice sheet also has surface slope ($\frac{dh}{dx}$) and $\sin \alpha$ can be approximated by α for low slope angles which are common for ice sheets. This approximation of Equation 1 provides a separable differential equation which can be solved. Using limits of integration from an arbitrary point to the outside edge of the ice sheet yield an equation of ice thickness at that point:

$$\tau_o = -\rho g h \alpha \tag{5}$$

$$\tau_o = -\rho g h \frac{dh}{dx} \tag{6}$$

$$\tau_o \int dx = -\rho g \int h dh \tag{7}$$

$$\tau_o (R - x) = \rho g \frac{h^2}{2} \tag{8}$$

$$h \approx \sqrt{\frac{2\tau_o (R - x)}{\rho g}}. \tag{9}$$

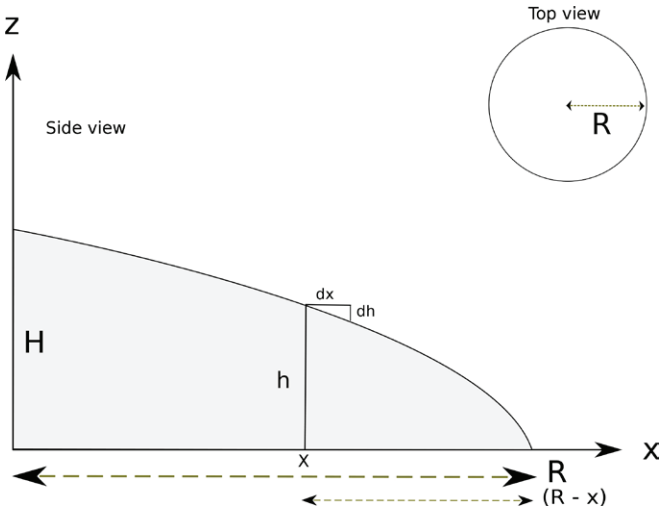

Figure 2 Profile of an idealized, circular ice sheet and coordinate system. R is the radius of the ice sheet and x is the distance from the edge of the ice sheet. After Cuffey and Patterson (2010).

At the centre of an ice sheet ($x = 0$) the total thickness is $H = (\frac{2\tau_o R}{\rho g})^{\frac{1}{2}}$. Using τ_o=100,000 Pa we can estimate the maximum thickness of the Greenland Ice Sheet [3]. A first-order estimate yields a thickness of 3150 m compared to its true depth of 3200 m.

2.3. Rate of sea-level rise

Another interesting topic related to glaciers is sea level. Over time scales of a year or more, changes in global sea level is driven by thermal change of sea water (heat causes water to expand) and changes in the volume of freshwater stored on the continents. Over the past 130 years, sea level rose by about 200 mm (Figure 3).

Decreased volume of the planet's glaciers and ice sheets and thermal expansion of sea water substantially contributed to twentieth and early twenty-first century sea-level rise [4]. Both of these processes are linked to Earth's surface air temperature which increases with increasing concentration of CO_2 in our atmosphere. Higher CO_2 concentrations in the future will thus lead to an increase in global sea level. Recent research [6] proposes that the rate of sea level rise ($\frac{dH}{dt}$) can be approximated by surface air temperature (T) and the rate of temperature change:

$$\frac{dH}{dt} = a \cdot (T - T_o) + b \cdot \frac{dT}{dt} \tag{10}$$

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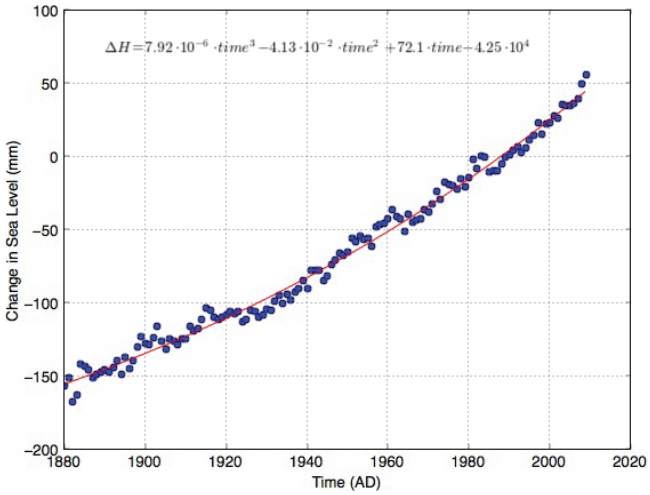

Figure 3 Observed sea-level rise over the period 1880-2009. A third-order polynomial fit to the data explains over 99% of the variance in observed sea-level change. Data for this graph can be obtained from www.cmar.csiro.au/sealevelsl_data_cmar.html

where T is the global temperature above a mean reference state (e.g. 1900-1950), T_o is a baseline temperature in which sea level is in equilibrium with climate (e.g. $\frac{dH}{dt} = \frac{dT}{dt} = 0$), and a and b are empirically-derived coefficients which can be obtained by fitting Equation 10 against the instrumental record of sea level and temperature change. Solution of Equation 10 yields values of -0.41ºC, 56 mm yr^{-1} ºC^{-1} and 49 yr^{-1} ºC^{-1} for T_o, a, and b respectively.

Changes in future sea level displayed in Figure 4 are based on empirical (statistical) relations between observational data. Empirical models do not account for changes in the physical behaviour of a system. Changes in ice dynamics of the Antarctic and Greenland ice sheets provides one example where prediction from such models breaks down. Changes in movement of ice at its bed due to the presence of deformable sediments or water can accelerate discharge of ice directly into the ocean from these ice sheets. Modeling the behaviour of Earth's great ice sheets is an area of active research [1]; it requires students who understand and enjoy mathematics.

3. Conclusion and further reading

A basic understanding of elementary mathematics is essential to understand the movement of energy and matter on Earth's dynamic surface. Earth science requires students with solid foundations in mathematics and physics; applied earth-science problems provide one way for mathematics instructors to peak the interest of students about the power and beauty of math. Glaciers are and will continue to be in the public spotlight due to their continued retreat and contribution to sea level rise, and they provide one just one case to illustrate the importance of mathematics in earth science. Middleton and Wilcock (1994), and Anderson and Anderson (2010) provide mathematicians with many other wonderful examples of the way to teach mathematics through the lens of earth science in the classroom.

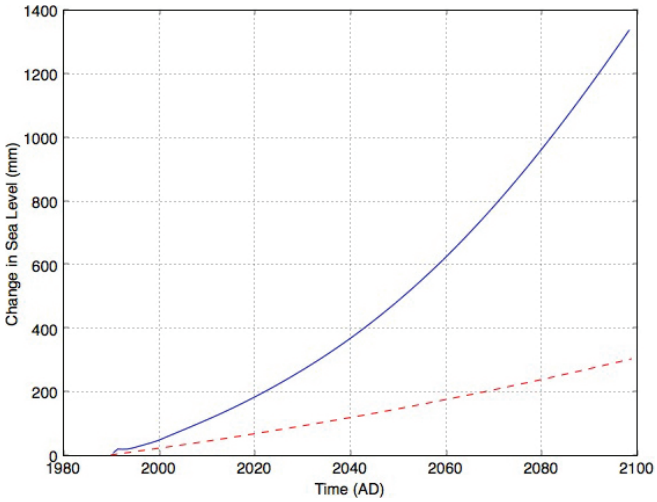

Figure 4 Predicted sea-level rise to the year 2100 based on polynomial fit to data (dashed line) and Equation 10 from Vermeer (2009). The temperature series used in Equation 10 is the global surface temperature estimates averaged over 18 different CO_2 emission scenarios and 19 different general circulation models.

4. Questions for students

- Equation 3 is dimensionally correct (the units on the left side of the equation have to equal those on the right hand side). What are the units of A?
- Use 1 and a value of 100,000 Pa for τ_{crit} to determine the thickness of a glacier with an average slope of $\frac{dh}{dx} = 0.1$ and ice density of 920 $kg\,m^{-3}$.
- Ice is not unique to Earth. On Mars, for example, the planet's two ice caps are primarily comprised of carbon dioxide. Construct ice sheet profiles for water ice on earth and compare to a profile on Mars where τ_{crit} for carbon dioxide = 20,000 Pa, gravity is 3.8 $m\,s^{-2}$, and the density of the ice is 1718 $kg\,m^{-3}$.
- Polynomial functions are useful to interpolation but are problematic for extrapolation. Explain the difference between the terms 'interpolation' versus 'extrapolation'. Why are polynomial functions problematic for extrapolating beyond the range of observational data?

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PIMS Postdoctoral Fellowship Competition

The Pacific Institute for the Mathematical Sciences (PIMS) invites nominations of outstanding young researchers in the mathematical sciences for Postdoctoral Fellowships for the year 2014-2015. Candidates must be nominated by at least one scientist or by a Department (or Departments) affiliated with PIMS. The fellowships are intended to supplement support provided by the sponsor, and are tenable at any of the PIMS Canadian member universities: the University of Alberta, the University of British Columbia, the University of Calgary, the University of Lethbridge, the University of Regina, the University of Saskatchewan, Simon Fraser University and the University of Victoria, as well as at the PIMS affiliate, the University of Northern British Columbia.

Program Features

Nominees must have a Ph.D. or equivalent (or expect to receive a Ph.D. by December 31, 2014) and must be within three years of their Ph.D. at the time of the nomination (i.e., they should have received their Ph.D. on or after January 1, 2011). The fellowship may be taken up at any time between September 1, 2014 and January 1, 2015. The fellowship is for one year and is renewable, contingent on satisfactory progress, for at most one additional year. The amount of the PIMS award for 2014-15 will be $20,000 and the sponsor(s) is (are) required to provide additional funds to finance a minimum total stipend of $40,000. PIMS Postdoctoral Fellows are expected to participate in all PIMS activities related to the fellow's area of expertise and will be encouraged to spend time at more than one site. To ensure that PIMS Postdoctoral Fellows are able to participate fully in Institute activities, they may not teach more than two single-term courses per year.

Application Process

The PIMS PDF nomination/application process takes place entirely online, utilizing the MathJobs service provided by the American Mathematical Society. Having selected their nominees, sponsors direct them to apply online at mathjobs.org/jobs/PIMS. (Detailed instructions regarding all aspects of the MathJobs application procedure may be found in the online MathJobs user guides.) Please note that **application is by nomination only**; unsolicited applications will not be considered. Please note that all nominees, **including those associated with PIMS Collaborative Research Groups** should apply through MathJobs.

- Nominees should upload a list of publications, a curriculum vitae and a statement of research interests. Special justification statements should be included if the applicant plans to either (i) continue to work with his/her PhD advisor, or (ii) remain at their current institution.
- Nominees should arrange for two reference letters to be uploaded to MathJobs. Letters should be preferably from outside referees who are at arm's length from the candidate and/or his/her PhD advisor.
- Sponsors must upload both their own reference letter and a **separate** statement of financial support that identifies the source of matching funds and the level of teaching required by the candidate in as much detail as possible. Vague or incomplete statements may influence the panel decision. Sponsors will receive instructions as to how to proceed via an email from MathJobs.

Selection Criteria

Rankings of candidates are made by the PIMS PDF Review Panel based on the following criteria:

- The scientific qualifications of the candidate;
- The fit between the research interests of the candidate and those of the sponsor;
- Adequacy of matching funds
- A maximum of teaching of 2 courses per year, (no extra consideration will be given for lower teaching loads.)

Deadlines

Complete applications must be uploaded to MathJobs by **December 1, 2013**. For further information, visit: **www.pims.math.ca/scientific/postdoctoral** or contact: **assistant.director@pims.math.ca**.

Johan Rudnick, CMS Executive Director, reviewing materials with one of the MCA2013 participants.

The CMS exhibit at the Mathematical Congress of the Americas 2013, August 5-9 in Guanajuato, Mexico was very well received in part, as a result of having the most popular exhibitor promo items – CMS stylus pens and Canadian flag pins! The promo items came along with a new Canadian Mathematical Sciences Community overview booklet that the CMS developed specifically for the event. The guide not only introduced participants to the various institutes, societies, research and education groups in Canada, the listing of Canadian universities offering math programs was an excellent supplement to Government of Canada materials about studying in Canada.

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Where Ignorance is Bliss

Christian Genest
Department of Mathematics and Statistics, McGill University

Sometimes you're better off not knowing. This can happen even in science, as I will now show using recent results from multivariate analysis, a branch of statistics concerned with the study of more than one variable at a time.

For simplicity, consider a pair (X, Y) of random variables, say the ages at death of husband and wife measured on a continuum. Its distribution is characterized by the map defined, at all $x, y \in \mathbb{R}$, by $H(x, y) = \Pr(X \le x, Y \le y)$.

Clearly, knowledge of H implies knowledge of its margins, defined for all $x, y \in \mathbb{R}$, by $F(x) = \Pr(X \le x)$ and $G(y) = \Pr(Y \le y)$. The latter characterize the individual behaviour of the random variables X and Y, respectively. But H cannot be reconstructed from F and G except in special cases, e.g., independence between X and Y, which means $H(x, y) = F(x)G(y)$ for all $x, y \in \mathbb{R}$.

To emphasize the role of F and G in the joint behaviour of X and Y, write

$$\forall_{x,y\in\mathbb{R}} \quad H(x, y) = C\{F(x), G(y)\}. \tag{1}$$

The map $C : [0, 1]^2 \to [0, 1]$ involved in this representation is called a *copula* [4]. Assuming that H is continuous, C is simply the restriction to $[0, 1]^2$ of the joint distribution of the pair $(U, V) = (F(X), G(Y))$ having uniform margins on $[0, 1]$. Thus for all $u, v \in [0, 1]$, $C(u, v) = \Pr(U \le u, V \le v)$ with $\Pr(U \le u) = u$ and $\Pr(V \le v) = v$.

For example, H is said to have a Farlie–Gumbel–Morgenstern (FGM) distribution [3] if there exists $\rho \in [-1/3, 1/3]$ such that $C = C_\rho$, where

$$\forall_{u,v\in[0,1]} \quad C_\rho(u, v) = uv + 3\rho uv(1 - u)(1 - v).$$

Furthermore, X and Y are independent if and only if $C = C_0$.

In practice, H is rarely known. Typically, it must be estimated from $n \ge 2$ independent copies $(X_1, Y_1), \ldots, (X_n, Y_n)$ of (X, Y). Alternatively, one could estimate F, G, and C separately, and then combine them via (1). In particular, an estimate of C is often of interest, because it describes the association between X and Y, irrespective of their marginal behaviour.

For example, life expectancy is much longer in Canada than in Mali but it could still be that married life affects the partners' survival in the same way, i.e., the copula is identical for both countries. The broken heart syndrome is one manifestation of this association.

When F and G are not of immediate interest, they are called "nuisance parameters" in statistical parlance. Yet if they were known, this information might conceivably be useful. For, one could then transform the random sample $(X_1, Y_1), \ldots, (X_n, Y_n)$ from H into a random sample $(U_1, V_1), \ldots, (U_n, V_n)$ from C by setting, for all $i \in \{1, \ldots, n\}$, $U_i = F(X_i)$ and $V_i = G(Y_i)$.

Is this information worthwhile? Surprisingly, the answer is "not always." To see why, first take F and G to be known. Given $u, v \in [0, 1]$, the probability $p = C(u, v)$ can then be estimated by the sample proportion, viz.

$$C_n(u, v) = \frac{1}{n}\sum_{i=1}^{n} \mathbf{1}(U_i \le u, V_i \le v),$$

where $\mathbf{1}(A)$ is the indicator of the set A.

The random variable $n\hat{p}_n = nC_n(u, v)$ is Binomial (n, p). The Central Limit Theorem then states that, for all $t \in \mathbb{R}$, $\Pr\{\sqrt{n}\,(\hat{p}_n - p) \le t\} \to \Phi\{t/\sqrt{p(1-p)}\}$ as $n \to \infty$, where Φ denotes the distribution function of a standard Normal variate. To emphasize the role of u, v, this weak convergence result is written

$$\mathbb{C}_n(u, v) = \sqrt{n}\,\{C_n(u, v) - C(u, v)\} \rightsquigarrow \mathbb{C}(u, v),$$

where $\mathbb{C}(u, v)$ is Gaussian with zero mean and variance $C(u, v)\{1 - C(u, v)\}$.

Using a bivariate version of the Central Limit Theorem, it can also be shown that for any $u, v, s, t \in [0, 1]$ the pair $(\mathbb{C}(u, v), \mathbb{C}(s, t))$ is jointly Normal and

$$\operatorname{cov}\{\mathbb{C}(u, v), \mathbb{C}(s, t)\} = C(u \wedge s, v \wedge t) - C(u, v)C(s, t), \tag{2}$$

where for arbitrary $a, b \in \mathbb{R}$, $a \wedge b = \min(a, b)$.

Better yet, the result can be extended as follows by viewing $\mathbb{C}_n$ as a *process*, i.e., a random element in $\ell^\infty[0, 1]^2$, the space of all bounded functions from $[0, 1]^2$ to $[0, 1]$ endowed with the sup norm $\|\cdot\|$; see, e.g., [6].

Proposition 1 *As $n \to \infty$, $\mathbb{C}_n = \sqrt{n}\,(C_n - C)$ converges weakly in $\ell^\infty[0, 1]^2$ to a pinned C-Brownian sheet $\mathbb{C}$, i.e., a centered Gaussian process with continuous trajectories and covariance function given, for all $u, v, s, t \in [0, 1]$, by* (2).

When F and G are unknown, they must be estimated. Natural candidates are their empirical distribution functions defined, for all $t \in \mathbb{R}$, by

$$F_n(t) = \frac{1}{n}\sum_{i=1}^{n} \mathbf{1}(X_i \le t), G_n(t) = \frac{1}{n}\sum_{i=1}^{n} \mathbf{1}(Y_i \le t).$$

From the Glivenko–Cantelli Theorem, we know that, as $n \to \infty$, $\|F_n - F\| \to 0$ and $\|G_n - G\| \to 0$ almost surely. These consistent estimators can thus be used to construct pseudo-observations $(\hat{U}_1, \hat{V}_1), \ldots, (\hat{U}_n, \hat{V}_n)$ from C by setting, for all $i \in \{1, \ldots, n\}$, $\hat{U}_i = F_n(X_i)$ and $\hat{V}_i = G_n(Y_i)$. An

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analogue of C_n for the unknown-margin case is then defined, for all $u, v \in [0,1]$, by

$$\hat{C}_n(u,v) = \frac{1}{n}\sum_{i=1}^{n} \mathbf{1}(\hat{U}_i \le u, \hat{V}_i \le v).$$

This so-called *empirical copula* is often used in practice, e.g., to check whether C belongs to a specific family of copulas such as the FGM. For a review of inferential procedures for copula models, see [1].

The asymptotic behaviour of the process $\hat{\mathbb{C}}_n = \sqrt{n}\,(\hat{C}_n - C)$ is difficult to establish because the pseudo-observations are not mutually independent. This was the object of much work since the 1970s; for a review, see [5], where the following result may be found. For arbitrary $u, v \in (0,1)$, let

$$\dot{C}_1(u,v) = \frac{\partial}{\partial u} C(u,v), \quad \dot{C}_2(u,v) = \frac{\partial}{\partial v} C(u,v).$$

The latter are known to exist almost everywhere; see, e.g., [4].

Proposition 2 *Suppose that $\dot{C}_1$ is continuous on $(0,1)\times[0,1]$ and $\dot{C}_2$ is continuous on $[0,1]\times(0,1)$. Then, as $n \to \infty$, $\hat{\mathbb{C}}_n = \sqrt{n}\,(\hat{C}_n - C)$ converges weakly in $\ell^\infty[0,1]^2$ to a centered Gaussian process defined, for all $u, v \in [0,1]$ by*

$$\hat{\mathbb{C}}(u,v) = \mathbb{C}(u,v) - \dot{C}_1(u,v)\,\mathbb{C}(u,1) - \dot{C}_2(u,v)\,\mathbb{C}(1,v).$$

For many years, the terms which distinguish $\hat{\mathbb{C}}$ from $\mathbb{C}$ were interpreted as "the price to pay for not knowing the margins." It was generally thought that if F and G are known, it is then preferable to base the inference on C_n rather than on $\hat{C}_n$. But somewhat surprisingly, the covariance of the process $\hat{\mathbb{C}}$ is *uniformly smaller* than the covariance of the process $\mathbb{C}$ under broad conditions [2].

Proposition 3 *Suppose that $\dot{C}_1(u,v) \le C(u,v)/u$ and that $\dot{C}_2(u,v) \le C(u,v)/v$ for all $u, v \in (0,1)$. Then for all $u, v, s, t \in (0,1)$,*

$$\mathrm{cov}\{\hat{\mathbb{C}}(u,v), \hat{\mathbb{C}}(s,t)\} \le \mathrm{cov}\{\mathbb{C}(u,v), \mathbb{C}(s,t)\}.$$

The conditions on the derivatives of C, loosely referred to as LTD for "left-tail decreasingness," imply a form of positive association in the pair (X,Y) which also occurs in the limiting case of independence. The FGM copula C_ρ with $\rho \ge 0$ is LTD; many other examples are given, e.g., in [4].

Proposition 3 has several interesting consequences for inference, as outlined in [2]. For example, the correlation coefficient $\rho = \mathrm{corr}\{F(X), G(Y)\}$ can *always* be estimated by the empirical correlation $\hat{\rho}_n$ computed from the pseudo-random sample $(\hat{U}_1, \hat{V}_1), \ldots, (\hat{U}_n, \hat{V}_n)$. This estimator is in fact nothing but Spearman's *rank correlation coefficient* because for arbitrary $i \in \{1, \ldots, n\}$, $n\hat{U}_i$ is the rank of X_i among $X_1, \ldots, X_n$ while $n\hat{V}_i$ is the rank of Y_i among $Y_1, \ldots, Y_n$.

Now in the few instances where F and G are known, ρ could also be estimated by the ordinary empirical correlation ρ_n derived from the random sample $(U_1, V_1), \ldots, (U_n, V_n)$. The limit $\xi_C = \lim_{n\to\infty} \mathrm{var}(\rho_n)/\mathrm{var}(\hat{\rho}_n)$ can then be used to assess the asymptotic relative efficiency of ρ_n with respect to $\hat{\rho}_n$. If C is an LTD copula, it follows from Proposition 3 (after some work) that $\xi_C > 1$, and often considerably so; for instance, $\xi_{C_\rho} \to 7$ as $\rho \to 0$ in the FGM model.

In fact if the underlying copula is LTD, $\hat{\rho}_n$ is not only a better estimator than ρ_n asymptotically but generally in small samples too. To paraphrase Thomas Gray, "Where ignorance [of F and G] is bliss, 'tis folly to be wise [by using ρ_n]."

For additional examples and discussion, see [2]. As a final teaser, note that while Propositions 1 and 2 extend readily to random vectors of dimension $d > 2$, it is not yet clear how Proposition 3 could be generalized to that broader context. There is empirical evidence that it does, however. So the search continues!

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CMS Notes
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Bound-Preserving High Order Accurate Schemes

Chi-Wang Shu[1]
Division of Applied Mathematics, Brown University

Many time-dependent partial differential equations (PDEs) from applications have bound-preserving properties for their solutions. For example, the entropy solution of a scalar hyperbolic conservation law

$$u_t + \bigtriangledown_x \cdot f(u) = 0, \qquad u(x,0) = u^0(x), \tag{1}$$

which may be discontinuous, satisfies a maximum-principle. Namely, for all $t > 0$:

$$\min_\xi u^0(\xi) \le u(x,t) \le \max_\xi u^0(\xi).$$

The same property holds for nonlinear convection-diffusion equations as well. For certain hyperbolic systems, similar bound-preserving properties may hold as well. For example, entropy solutions to the Euler equations of ideal gas (in one-dimension for simplicity), namely equation (1) with

$$u = (\rho, \rho v, E)^T, \qquad f(u) = (\rho v, \rho v^2 + p, v(E+p))^T$$

where ρ is density, v is velocity, E is total energy, and $p = (\gamma - 1)(E - \frac{1}{2}\rho v^2)$ is the pressure, with $\gamma = 1.4$ for air, satisfies a positivity-preserving property for density and pressure: if the density and pressure are both non-negative at $t = 0$, then they are non-negative for all $t > 0$.

It is desired that a numerical scheme approximating these solutions obeys the same bound-preserving properties. This is often important physically, for example if the solution u represents a percentage of a certain component in a mixture, then $u < 0$ or $u > 1$ does not make any sense. Similarly, if we are computing a probability density function u, then $u < 0$ is nonsensical. Sometimes it is also important mathematically, for example, in the Euler equations mentioned above, when density or pressure becomes negative, the PDE is no longer hyperbolic and is ill-posed, consequently the numerical scheme typically blows up quickly after negative density or pressure appears, due to the nonlinear instability corresponding to the ill-posedness of the PDE.

Let us consider finite volume schemes [3] and discontinuous Galerkin schemes [1] for solving equation (1) in scalar one-dimension as examples. The computational domain (taken as [0, 1] without loss of generality) is discretized into cells

$$I_j = (x_{j-\frac{1}{2}}, x_{j+\frac{1}{2}}), \quad h_j = x_{j+\frac{1}{2}} - x_{j-\frac{1}{2}}, \quad j = 1, 2, \cdots, N$$

with $x_{\frac{1}{2}} = 0$ and $x_{N+\frac{1}{2}} = 1$.

1 Research supported by NSF grant DMS-1112700.

A finite volume scheme evolves the cell averages

$$\bar{u}_j = \frac{1}{h_j} \int_{x_{j-\frac{1}{2}}}^{x_{j+\frac{1}{2}}} u(x,t)dx$$

in time, while a discontinuous Galerkin scheme evolves a piecewise polynomial (the numerical solution $u_j(x)$ is a polynomial of a given degree k in cell I_j). For both schemes, if we use forward Euler time discretization for simplicity, the evolution of the cell averages has a very simple formula

$$\begin{aligned} \bar{u}_j^{n+1} &= \bar{u}_j^n - \lambda\left(\hat{f}(u^-_{j+\frac{1}{2}}, u^+_{j+\frac{1}{2}}) - \hat{f}(u^-_{j-\frac{1}{2}}, u^+_{j-\frac{1}{2}})\right) \\ &\equiv G_\lambda\left(\bar{u}_j^n, u^-_{j-\frac{1}{2}}, u^+_{j-\frac{1}{2}}, u^-_{j+\frac{1}{2}} u^+_{j+\frac{1}{2}}\right), \end{aligned} \tag{2}$$

where $\lambda = \tau/h_j$ with τ being the time step size, the superscript n refers to the time step, $u^\pm_{j+\frac{1}{2}} = u^n(x^\pm_{j+\frac{1}{2}})$, and $u^n(x)$ is the piecewise polynomial numerical solution either reconstructed from the cell averages for a finite volume scheme, or evolved for a discontinuous Galerkin scheme. $\hat{f}(a,b)$ is a monotone numerical flux, namely it is Lipschitz continuous in both arguments, non-decreasing in the first argument and non-increasing in the second argument, and consistent $\hat{f}(a,a) = f(a)$. If higher than first order accuracy is needed in time, strong stability preserving (SSP) Runge-Kutta or multi-step time discretizations [2] can be used, which is a convex combination of several Euler forward stages. Hence it is enough to discuss (2) for bound-preserving properties.

It is rather easy to obtain a maximum-principle for a first order scheme, namely the polynomial degree $k = 0$ for the piecewise polynomial numerical solution $u^n(x)$. In this case the scheme (2) becomes

$$\begin{aligned} \bar{u}_j^{n+1} &= \bar{u}_j^n - \lambda\left(\hat{f}(\bar{u}_{j-1}^n, \bar{u}_j^n) - \hat{f}(\bar{u}_j^n, \bar{u}_{j+1}^n)\right) \\ &\equiv H_\lambda\left(\bar{u}_{j-1}^n, \bar{u}_j^n, \bar{u}_{j+1}^n\right), \end{aligned}$$

and it is easy to verify that H_λ is a monotonically increasing (non-decreasing) function of all three arguments under a suitable CFL condition

$$\lambda \le \frac{1}{L_1 + L_2} \equiv \lambda_0$$

where L_1 and L_2 are the Lipschitz constants of the numerical flux $\hat{f}$ with respect to its two arguments. We also clearly have consistency $H_\lambda(u,u,u) = u$. If we have $m \le \bar{u}_j^n \le M$ for all j, then it is easy to obtain

$$\bar{u}_j^{n+1} = H_\lambda\left(\bar{u}_{j-1}^n, \bar{u}_j^n, \bar{u}_{j+1}^n\right) \le H_\lambda(M, M, M) = M.$$

Similarly we can prove $m \le \bar{u}_j^{n+1}$. The maximum-principle-preserving property of first order monotone schemes is thus established.

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An attempt to mimic this process for a high order scheme (2) would immediately encounter difficulties, as the function G_λ in (2) is a decreasing rather than increasing function of its third and fourth arguments. Therefore, it is easy to build a counter example, such that all five arguments of G_λ are within the bounds $[m, M]$, but $\bar{u}_j^{n+1}$ is less than m or bigger than M no matter how small the CFL number $\lambda > 0$ is. Thus it is not enough to require the cell averages $\bar{u}_j^n$ and the cell boundary values $u_{j+\frac{1}{2}}^{\pm}$ to be within the desired bounds $[m, M]$ in order to guarantee a maximum principle for the next time step.

In [5], the following simple procedure is designed to guarantee a maximum principle. Notice that the numerical solution $u^n(x)$ is a piecewise polynomial of degree k, hence its cell average can be computed exactly by a suitable Legendre Gauss-Lobatto quadrature rule:

$$\bar{u}_j^n = \sum_{\ell=0}^{m} \omega_\ell u_j^n(x_j^{(\ell)})$$

where $x_j^{(\ell)}$ are the Legendre Gauss-Lobatto quadrature points of the cell I_j, with $x_j^{(0)} = x_{j-\frac{1}{2}}$ and $x_j^{(m)} = x_{j+\frac{1}{2}}$. $\omega_\ell > 0$ are the quadrature weights satisfying $\sum_{\ell=0}^{m} \omega_\ell = 1$. The scheme (2) can then be rewritten as

$$\begin{aligned}
\bar{u}_j^{n+1} &= \omega_m \left[u^-_{j+\frac{1}{2}} - \frac{\lambda}{\omega_m} \left(h(u^-_{j+\frac{1}{2}}, u^+_{j+\frac{1}{2}}) - h(u^+_{j-\frac{1}{2}}, u^-_{j+\frac{1}{2}}) \right) \right] \\
&+ \omega_0 \left[u^+_{j-\frac{1}{2}} - \frac{\lambda}{\omega_0} \left(h(u^+_{j-\frac{1}{2}}, u^-_{j+\frac{1}{2}}) - h(u^-_{j-\frac{1}{2}}, u^+_{j-\frac{1}{2}}) \right) \right] + \sum_{\ell=1}^{m-1} \omega_\ell u_j^n(x_j^{(\ell)}) \\
&= H_{\lambda/\omega_m}(u^+_{j-\frac{1}{2}}, u^-_{j+\frac{1}{2}}, u^+_{j+\frac{1}{2}}) + H_{\lambda/\omega_0}(u^-_{j-\frac{1}{2}}, u^+_{j-\frac{1}{2}}, u^-_{j+\frac{1}{2}}) + \sum_{\ell=1}^{m-1} \omega_\ell u_j^n(x_j^{(\ell)}).
\end{aligned}$$

Clearly, $\bar{u}_j^{n+1}$ is now written as a convex combination of monotonically increasing functions of $u_j^n(x_j^{(\ell)})$ for $0 \le \ell \le m$, provided a more restrictive CFL condition

$$\lambda \le \frac{\lambda_0}{\omega_0} \tag{3}$$

is satisfied (notice that $\omega_0 = \omega_m$). We therefore obtain $m \le \bar{u}_j^{n+1} \le M$ provided $m \le u_j^n(x_j^{(\ell)}) \le M$, under the CFL condition (3).

This step is to ensure that the cell averages $\bar{u}_j^{n+1}$ are within the desired bounds. It is crucial and is the most difficult step for previous numerical schemes. Once the cell averages are under control, a simple scaling limiter described in [5] can be applied to guarantee that $m \le u_j^{n+1}(x_j^{(\ell)}) \le M$, without affecting the original high order accuracy of the scheme. We have therefore obtained a high order accurate finite volume or discontinuous Galerkin scheme, which satisfies the maximum principle in the following sense. If the numerical solution u_j^n, which is a polynomial of degree k in each cell I_j, satisfies $m \le u_j^n(x_j^{(\ell)}) \le M$ for all the Legendre Gauss-Lobatto quadrature points $x_j^{(\ell)}$, then the numerical solution at time level $n+1$ also satisfies the same property $m \le u_j^{n+1}(x_j^{(\ell)}) \le M$.

This class of maximum-principle-satisfying finite volume and discontinuous Galerkin schemes is very simple to implement, since it involves only a simple scaling limiter beyond the original schemes. The technique can be easily extended to multi-dimensions and on unstructured meshes. Similar techniques can be designed for passive convection in a multi-dimensional divergence-free velocity field, and for two-dimensional incompressible Euler equations in the vorticity-streamfunction formulation. More importantly, the technique can be generalized to bound-preserving high order schemes for certain hyperbolic systems, for example to Euler equations of compressible gas dynamics for preserving the positivity of density and pressure, and to shallow water equations for preserving the positivity of water heights. Other generalizations include the preservation of positivity of density functions for Vlasov-Boltzmann transport equations, and the preservation of positivity of population density in a hierarchical size-structured model. We refer to the survey paper [6] for more details and further references. Interesting applications include simulations for gaseous detonations containing very strong shock waves [4]. More recently the technique has been generalized to convection-diffusion equations [7].

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CUMC Report

Jean Lagacé, *President CUMC Organization Committee*

La conférence en première vue.

Cette année, le Congrès Canadien des Étudiants en Mathématiques (CCÉM) fêtait ses 20 ans. De McGill, à Montréal en 1994 jusqu'à l'Université de Montréal en 2013, c'était en quelques sortes un retour aux sources pour cet anniversaire. En tout et pour tout, plus de 225 personnes ont pu profiter d'une partie des 120 conférences étudiantes, et assister à nos 7 conférences pléniaires, cinq en anglais et deux en français. Nous avons reçu pour nos conférences pléniaires, dans cet ordre, Pr. Dror Bar-Natan (University of Toronto), Pr. Christiane Rousseau (Université de Montréal), Pr. Mike Roth (Queen's University), Pr. Paul Charbonneau (Université de Montréal), Pr. Mylène Bédard (Université de Montréal), Pr. Franco Saliola (Université du Québec à Montréal) et Dr. Steven Sivek (Harvard), qui nous ont parlé respectivement de la topologie des noeuds sphériques, de dynamique des vols spatiaux, de géométrie algébrique, de modélisation du Soleil, de méthodes MCMC en statistiques, de combinatoire ainsi que de théorie des noeuds. Je crois qu'on peut dire sans se tromper que le congrès cette année a été un succès sur toute la ligne.

Il n'y a pas que les maths!

Nous avons aussi profité de la présence des étudiants en mathématiques de partout au Canada à Montréal pour leur faire découvrir la vie étudiante éclectique et les soirées festives que la ville a à leur offrir. Bien que nous encouragions les gens à visiter la métropole par eux-même, nous en avons profité pour les inviter à venir chanter au karaoké avec nous, histoire de tisser aujourd'hui des liens entre les mathématiciens des années à venir. Nous en avons aussi profité pour les inviter à aller voir les feux d'artifices dans le vieux port de Montréal du concours annuel se déroulant durant l'été.

Partenariats

Il est évident qu'un congrès de cette envergure ne peut s'organiser sans aide financière extérieure. À ce niveau, nous aimerions remercier l'Université de Montréal, et en particulier le département de mathématiques et statistique, ainsi que le Centre de Recherche en Mathématiques pour leur aide, non seulement financière, mais aussi au niveau du temps et des ressources qu'ils nous ont accordés. Nous étions aussi heureux de pouvoir compter sur le Centre de Sécurité des Télécommunications du Canada, qui a été partenaire du CCÉM pour la première fois cette année. Plusieurs autres organismes nous ont évidemment aidé au niveau du financement, ils sont trop nombreux pour tous les nommer ici.

Accomplissements

Nous avons réussi cette année à organiser un CCÉM dont les gens vont se rappeler longtemps. Les étudiants de partout au Canada ont pu voir Montréal comme une ville particulièrement active en mathématiques, à travers n'importe laquelle des 4 grandes universités s'y trouvant. Nous ne pouvons que souhaiter bonne chance pour l'an prochain aux gens de Carleton University à qui nous avons transmi le flambeau.

CUMC website : http://cumc.math.ca/2013/index.php?lang=EN&content=home

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Editorial Nominations

The Publications Committee of the CMS solicits nominations for five Associate Editors for the Canadian Journal of Mathematics (CJM) and the Canadian Mathematical Bulletin (CMB).
The appointment will be for five years beginning January 1, 2014.
The continuing members (with their end of term) are below.

The deadline for the submission of nominations is November 15, 2013.
Nominations, containing a curriculum vitae and the candidate's agreement to serve, should be sent to the address below ;

Nantel Bergeron, Chair
CMS Publications Committee
Department of Mathematics & Statistics
York University
N520 Ross Bldg, 4700 Keele Street
Toronto, ON M3J 1P3
bergeron@yorku.ca

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CMB Editors-in-Chief

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Jie Xiao (Memorial)	12/2013;
Efim Zelmanov (UCSD)	12/2016.

Appel de mises en candidature de rédaction

Le Comité des publications de la SMC sollicite des mises en candidatures pour cinq postes de rédacteurs associés pour le Journal canadien de mathématiques (JCM) et pour le Bulletin Canadien de mathématiques (BCM). Le mandat sera de cinq ans à compter du 1er janvier 2014. Les membres qui continuent (avec la fin de leur terme) sont ci-dessous.

La date limite pour les soumissions est le 15 novembre 2013.
Les mises en candidature, incluant un curriculum vitae et l'accord du candidat à servir, doit être envoyé à l'adresse ci-dessous :

Nantel Bergeron, Président
Comité de publication de la SMC
Département de mathématiques et statistiques
Université York
N520 Ross Bldg, 4700 rue Keele
Toronto (Ontario) M3J 1P3
bergeron@yorku.ca

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Colombia: Six students, six problems, six medals!

Robert Morewood,
Chair, CMS IMO Committee and Deputy Leader, Math Team Canada

After an intense 12-day training camp at the beautiful Banff International Research Station and following a lengthly intercontinental journey, the Canadian team found itself in the tropical clime of Santa Marta, Colombia. There, they ably represented Canada, connecting with students from all around the world and demonstrating world-class mathematical talent. As a team, they tied with Japan for 11th in the world, just behind Iran and the United Kingdom and ahead of Isreal and Australia.

But the International Mathematical Olympiad is a competition between individuals, and leading the Canadian team (sometimes with such determination that his passport got left far behind) was Calvin Deng. Born in Saskachewan and graduating from the North Carolina School of Science and Mathematics, he was the top finisher from all of the Americas, tying for 8th in the world with Alexander Gunning of Australia. Calvin was the only Canadian to solve problem 3:

Problem 3 (Day 1). *For triangle ABC, let A_1 be the point of tangency on side BC of the excircle opposite to the vertex A, with B_1 and C_1 on CA and AB defined analogously. Suppose that the circumcentre of triangle $A_1B_1C_1$ lies on the circumcircle of triangle ABC. Prove that triangle ABC is right-angled.*

Setting the circle through the three excenters as the unit circle in the complex plane, Calvin reduced the problem to factoring:

$$\left(2pqr(p+q+r)+\sum_{\text{sym}} p^3q\right)$$
$$\left(2pqr(pq+qr+rp)+\sum_{\text{sym}} p^2r^3\right)$$
$$-pqr(p+q)^2(q+r)^2(r+p)^2$$

and found a clever way to complete the factorization with time enough left to partially solve problem 2.

Canada's other (repeat) Gold medalist was Alex Song, born in China, raised in Waterloo, and is currently attending the prestigious Phillips Exeter Academy. Along with Calvin, Alex was one of only seven students to solve problem 6:

Problem 6 (Day 2). *Let $n \geq 3$ be an integer, and consider a circle with $n+1$ equally spaced points marked on it. Consider all labellings of these points with the numbers $0, 1, \ldots, n$ such that each label is used exactly once; two such labellings are considered to be the same if one can be obtained from the other by a rotation of the circle. A labelling is called beautiful if, for any four labels $a < b < c < d$ with $a + d = b + c$, the chord joining the points labelled a and d does not intersect the chord joining the points labelled b and c.*

Let M be the number of beautiful labellings, and let N be the number of ordered pairs (x, y) of positive integers such that $x + y \leq n$ and $\gcd(x, y) = 1$. Prove that

$$M = N + 1$$

The key result in Alex's proof is the lemma that, if three chords have equal endpoint sums, then one has endpoints on each of the two arc joining the endpoints of the other two.

Canada's first Silver medalist was John Ma, raised in Toronto and graduating from Interlake High School in Bellevue, Washington. John provided an elegant proof for problem 2:

Problem 2 (Day 1). A configuration of 4027 points in the plane is called Colombian if it consists of 2013 red points and 2014 blue points, and no three of the points of the configuration are collinear. By drawing some lines, the plane is divided into several regions. An arrangement of lines is good for a Colombian configuration if the following two conditions are satisfied:

- no line passes through any point of the configuration;
- no region contains points of both colours.

Find the least value of k such that for any Colombian configuration of 4027 points, there is a good arrangement of k lines.

He considered the more general problem of $2k + 1$ points with any colouration of at most two colours and showing that there is always a "good" arrangement with k lines by induction on k, looking for consecutive points of the same colour on the convex hull, or using the interior points when the convex hull has only alternating colours. Daniel Spival, born in Israel and graduating from Toronto's Bayview Secondary School, also won Silver for Canada, earning a rare point on problem 6 and providing one of Canada's solutions to problem 5:

Problem 5 (Day 2). Let $\mathbb{Q}_{>0}$ be the set of positive rational numbers. Let $f : \mathbb{Q}_{>0} \to \mathbb{R}$ be a function satisfying the following three conditions:

i for all $x, y \in \mathbb{Q}_{>0}$, we have $f(x)f(y) \geq f(xy)$;
ii for all $x, y \in \mathbb{Q}_{>0}$, we have $f(x + y) \geq f(x) + f(y)$;
iii there exists a rational number $a > 1$ such that $f(a) = a$.

Prove that $f(x) = x$ for all $x \in \mathbb{Q}_{>0}$.

He used a combination of inductions, showing that all values of $f(x)$ must be positive and that $f(a^n) \leq a^n$, along with contradictions, contructing from an element y with $y < f(y)$ and element z (involving a large power of a) for which $f(z) < 0$ and then contructing from an element y with $y > f(y)$ an element z (again using a large power of a) for which $f(z) < z$. Canada's first Bronze medal was earned by Alexander

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Whatley, born in Vancouver and currently being home schooled in Houston, Texas. In addition to giving another solution to problem 5, Alexander joined the entire Canadian team in solving problems 4 and 1:

Problem 1 (Day 1). Prove that for any pair of positive integers k and n, there exist k positive integers $m_1, m_2, \ldots, m_k$ (not necessarily different) such that:

$$1 + \frac{2^k - 1}{n} = \left(1 + \frac{1}{m_1}\right)\left(1 + \frac{1}{m_2}\right)\cdots\left(1 + \frac{1}{m_k}\right)$$

Alexander's solution used induction on k. Even Canada's youngest participant, Kevin Sun, with family ties to the Waterloo area and plans to join Alex Phillips Exeter Academy, earned a medal with points on four of the six problems.

Problem 4 (Day 2). *Let ABC be an acute-angled triangle with orthocentre H, and let W be a point on the side BC, lying strictly between B and C. The points M and N are the feet of the altitudes from B and C, respectively. Denote by ω_1 the circumcircle of BWN, and let X be the point on ω_1 such that WX is a diameter of ω_1. Analogously, denote by ω_2 the circumcircle of CWM, and let Y be the point on ω_2 such that WY is a diameter of ω_2. Prove that X, Y and H are collinear.*

Kevin gave a very short proof to problem 4 using cyclic quadrilaterals and the radical axis theorem along with the power of a point. However, as much as this event is explicitly about the intellectual challenge these students need to fully develop their remarkable skills, the international connections which these students make seem just as important as I see students, and even leaders and organizers, making new friends and greeting old friends from around the world and exchanging ideas in sessions long into the night.

David Borwein Distinguised Career Award

The David Borwein Distinguished Career award recognizes mathematicians who have made exceptional, broad, and continued contribution to Canadian mathematics. The deadline for nominations is November 30, 2013.

A complete nomination dossier consists of:

- a signed nomination statement from a present or past colleague, or collaborator (no more than three pages) having direct knowledge of the nominee's contribution to mathematical sciences and the community;
- a short curriculum vitae, no than five pages;
- two to four letters of support in addition to the nomination;
- other supporting material may be submitted, no more than 10 pages.

A nomination can be updated and will remain active for three years. All documentation must be submitted electronically, preferably in PDF format, by the appropriate deadline to **dbaward@cms.math.ca**

Prix David-Borwein de mathématicien émérite

Le prix David-Borwein de mathématicien émérite pour l'ensemble d'une carrière rend hommage à un mathématicien qui a fait une contribution exceptionnelle et soutenue aux mathématiques canadiennes. La date d'échéance pour les candidatures est le 30 novembre 2013.

Le dossier de candidature comprendra les éléments suivants :

- une lettre de mise en candidature signée par un collègue ou un collaborateur actuel ou des années passées (trois pages maximum) qui connaît très bien les réalisations de la personne proposée aux sciences mathématiques et à la communauté;
- un bref curriculum vitae, maximum de cinq pages;
- de deux à quatre lettres d'appui, en plus de la mise en candidature;
- tout autre document pertinent, maximum de 10 pages.

Toute mise en candidature est modifiable et demeurera active pendant trois ans. Veuillez faire parvenir tous les documents par voie électronique, de préférence en format PDF, avant la date limite à **prixdb@smc.math.ca**

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December 6 – 9, 2013, The Ottawa Marriott
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Adrien Pouliot Award | Prix Adrien-Pouliot
recipient to be announced | lauréat à confirmer

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Math Art Exhibit featuring Manuel Baez who investigates the fundamental integrative principles of form, structure and generative processes.

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Regular Sessions | Sessions générales

Complex Analysis and Complex Geometry
Analyse complexe et géométrie complexe
Damir Kinzebulatov(Fields), Rasul Shafikov (Western)

Connections Between Noncommutative Algebra and Geometry | Liens entre l'algèbre non commutative et la géométrie
Jason Bell (Waterloo), Colin Ingalls (New Brunswick)

Functional Differential Equations and Applications
Équations différentielles fonctionnelles et applications
Victor Leblanc (Ottawa)

Geometric Group Theory
Théorie des groupes géométrique
Inna Bumagin (Carleton), Benjamin Steinberg (CUNY)

Geometric Group Theory and Low Dimensional Topology
Théorie des groupes géométrique et topologie en basse dimension
Adam Clay (Manitoba), Mikael Pichot (McGill), Eduardo Martinez-Pedroza (Memorial)

Harmonic Analysis on Groups over Local Fields
Analyse harmonique sur des groupes définis sur des corps locaux
Monica Nevins (Ottawa), Hadi Salmasian (Ottawa)

History and Philosophy of Mathematics
Histoire et philosophie des mathématiques
Tom Archibald (SFU)

Infectious Disease Modelling
Modélisation de maladies infectieuses
Robert Smith? (Ottawa)

Lie Algebras, Representations and Cohomological Invariants Algèbres de Lie, représentations et invariants cohomologiques
Yuli Billig (Carleton), Alistair Savage (Ottawa), Kirill Zaynullin (Ottawa)

Mathematical Genomics | Génomique mathématique
David Sankoff (Ottawa)

Matrix Theory in Quantum Information
Théorie des matrices en information quantique
David Kribs (Guelph), Rajesh Pereira (Guelph), Sarah Plosker (Brandon)

Modular Forms and Physics
Physique et formes modulaires
Abdellah Sebbar (Ottawa)

Number Theory | Théorie des nombres
Damien Roy (Ottawa)

Operator Algebras | Algèbres d'opérateurs
Benoit Collins (Ottawa), Thierry Giordano (Ottawa), Mehrdad Kalantar (Carleton), Matthew Kennedy (Carleton)

Random Walks and Geometry
Marches aléatoires et géométrie
Giulio Tiozzo (Harvard)

Representations of Algebra | Représentations d'algèbres
Vlastimil Dlab (Carleton), Ragnar-Olaf Buchweitz (Toronto), Shiping Liu (Sherbrooke)

Contributed Papers | Communications libres
to be confirmed | à venir

AARMS-CMS Student Poster Session
Présentations par affiches pour étudiants
Leo Belzile (McGill)

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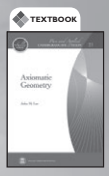

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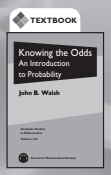

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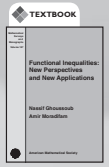

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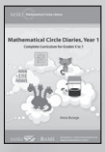

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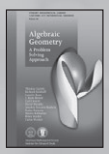

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