CMS NOTES de la SMC
December 2013

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Société mathématique du Canada

From the Vice-President's Desk

Gregory G. Smith, *CMS Vice-President, Ontario*

Assessment in Mathematics

Assessment in academia refers to the process or means of evaluating work. With the origins of the word `assessment' being intimately connected with taxes, levies, and fines, it naturally acquired a negative connotation. However, I suspect that, within mathematical research and education, the pervasiveness of high-stakes tests also plays a considerable role in establishing this negative association. Nevertheless, assessment is unavoidable in discovery and learning. Can we generate a positive overtone by improving the assessment practises for researchers and students?

For the majority of us, our experience with assessment is dominated by the forms used in teaching mathematics. We can all remember, if only vaguely, what it is like to take a math exam. In contrast, most of us are reacquainted with the joys of grading exams every year at this time. As a postdoc, one of my mentors convinced me to employ a sorting algorithm when marking a large number of exams. The basic idea is to use a variant of Quicksort to partition the exams into classes with equivalent errors. In the first step, I quickly scan each solution to one problem and physically divide the collection of exams into piles. For a typical first pass, these piles might be loosely described as potentially perfect, probably one error, apparently a couple of mistakes, and almost blank. In the second step, I carefully read the solutions in one pile. As soon as I encounter something that differentiates one solution from others in the pile, I stop reading and place it into another pile with similar solutions. All solutions in the remaining pile get the same grade and comments. To complete the algorithm, I recursively apply the two steps. In each iteration, at least one equivalence class is completely graded. Although many solutions are skimmed multiple times, each one is read through carefully only once. Ideally, this procedure reduces decision fatigue. Having used this approach for more than a decade, I think it makes my grading much more consistent and, perhaps counterintuitively, considerably faster.

Beyond consistency and efficiency, high-quality assessment needs to be valid. Are we measuring what we intended to measure? This challenge arises in teaching whenever one creates a quiz, test, or exam. Questions should focus on the essential concepts, evaluate the relevant skills, and not depend on extraneous information or knowledge. For example, a related-rates problem in first-year calculus involving subtle geometry is likely to gauge how much students remember from high school (or perhaps the quality of their high-school math teachers) rather than how well they understand the chain rule. Choosing the right assessment tool is another key component in validity. For an undergraduate, a capstone project almost surely provides much better insights into the student's research potential and communication skills than a grade point average. Graduate students normally receive most of their formative feedback from seminar presentations and regular meetings with their supervisor. Hopefully, this aligns well with their summative evaluations including oral exams and thesis defenses. In my experience, the hardest part in designing a valid assessment strategy lies in clearly identifying what should be measured.

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December Mathematicians

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Which month of the year has the distinction that most mathematicians were born who were famous for their work? Answer: February and December – there were 15 of them in each of these two months!

The book *The Nature of Mathematics,* by Karl J. Smith (8th edition), Brooks/Cole 1998, includes, in order 'to make mathematics come alive,' cartoons, quotations and historical notes. As part of historical notes, a list of mathematicians who were born in each month of the year is provided at the beginning of each chapter, together with some biographical details of one mathematician selected from the list. Those that are listed for the month of December are: Nikolai Lobachevski (non-Euclidean geometry); L. Kronecker (analysis); Karl Jacobi (determinants, etc); Janos Bolyai (non-Euclidean geometry); Sophus Lie (group theory); Nicolo Tartaglia (cubic equations); S. Ramanujan (number theory); Charles Hermite (transcendence of *e*); Isaac Newton (calculus, etc); Charles Babbage (calculating machines); Johannes Kepler (astronomy); John von Neumann (operator theory, etc); Jacob Bernoulli (probability, etc); Thomas Jan Stieltjes (calculus); & Jaime Escalante (teacher).

Most of us will be familiar with everyone in this list (and his speciality) except, perhaps, the last one. What was great about him? His biographical details are as follows: Jaime Escalante Gutierrez was born in Bolivia in 1930, taught there for 12 years and then moved to U.S.A. Earning a degree in mathematics with a teaching credential, he taught at Garfield, a troubled inner-city high school in Los Angeles. He achieved spectacular success in teaching mathematics to gang members and other students who were considered "unteachable"; this attracted national attention. He became a national hero when his story was told in an applauded film called *Stand and Deliver* (1988) and by the release of the book *Escalante: The Best Teacher in America* by Jay Mathews. Then he taught at a high school in Sacramento, CA, and worked with the Foundation for Advancements in Science and Education, and also in Public Broadcasting Service (PBS), where he helped create videos in mathematics. His enthusiasm for teaching was recognized and acclaimed. Regarding the future of mathematics education he is reported to have said: "you do not *enter* the future, rather you *create* it through hard work. This means teachers must teach, students must do their work and parents must take active interest in their children's schoolwork."

Escalante was not a research mathematician; his knowledge of mathematics was confined to the standard college curriculum only! Yet he used this knowledge in a calculus program that continued to grow. However, that was not without its own price. Tensions that surfaced when his career began at Garfield High school escalated.; in his final years at Garfierld, Escalante received threats and hate mail from various individuals. He died on March 30, 2010, aged 79, at his son's home while undergoing treatment for bladder cancer.

It is only a tiny percentage of those who study college math and that happen to teach high school and then go on to do Ph.D work. Most alumni pursue careers elsewhere become artists rather than become mathematicians. We know that kids who study poetry do not become poets. It is all just about a habit of the mind. Our minds do not think abstractly unless they are asked to – and this asking should be done at a relatively young age. The rigor and logic that is needed for understanding and doing mathematics afford a good way for the brain to get trained to deal with problems in later life, no matter when they were born.

Mathematiciens né en Decembre

Quels mois de l'année ont vu naître le plus de mathématiciens devenus célèbres pour leurs travaux? Réponse : février et décembre – 15 d'entre eux sont nés chacun de ces deux mois!

Dans son livre *The Nature of Mathematics* (8e édition, Brooks/Cole 1998), Karl J. Smith ajoute divers éléments – bandes dessinées, citations, notes historiques – pour « rendre les mathématiques plus vivantes », explique-t-il. Il dresse entre autres une liste de mathématiciens nés chaque mois de l'année au début de chaque chapitre, ainsi que des détails biographiques à propos d'un des mathématiciens de cette liste. Les mathématiciens mentionnés en décembre sont : Nikolai Lobachevski (géométrie non euclidienne); L. Kronecker (analyse); Karl Jacobi (déterminants, etc.); Janos Bolyai (géométrie non euclidienne); Sophus Lie (théorie des groupes); Nicolo Tartaglia (équations cubiques); S. Ramanujan (théorie des nombres); Charles Hermite (transcendance de *e*); Isaac Newton (calcul différentiel et intégral, etc.); Charles Babbage (machines à calculer); Johannes Kepler (astronomie); John von Neumann (théorie des opérateurs, etc.); Jacob Bernoulli (probabilité, etc.); Thomas Jan Stieltjes (calcul différentiel et intégral); Jaime Escalante (enseignant).

La plupart d'entre nous connaissent chacun des hommes figurant sur cette liste (et sa spécialité) sauf peut-être le dernier. Qu'est-ce que ce mathématicien a fait de célèbre? Voici quelques détails sur sa vie : Jaime Escalante Gutierrez est né en Bolivie en 1930, où il a enseigné pendant 12 avant d'émigrer aux États-Unis. Après avoir décroché un baccalauréat en mathématiques avec un brevet d'enseignement, il a enseigné dans une école défavorisée du centre-ville de Los Angeles. Il a obtenu des résultats spectaculaires en enseignant les mathématiques à des membres de gangs de rue et à d'autres élèves considérés comme « irrécupérables »; ses résultats ont retenu l'attention sur la scène nationale. Il est devenu un héros national le jour où son histoire a été portée à l'écran dans un film acclamé intitulé *Envers et contre tous (Stand and Deliver* - 1988) et par la parution du livre *Escalante: The Best Teacher in America* de Jay Mathews. Il a par la suite enseigné dans une école secondaire de Sacramento, en Californie, et il a travaillé pour la Foundation for Advancements in Science and Education et la chaîne PBS, où il a contribué à la production de vidéos sur les mathématiques. Sa passion pour l'enseignement a été récompensée et

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Du bureau de la vice-présidente

Gregory G. Smith, *vice-président SMC – Ontario*

L'évaluation en mathématiques

Dans le milieu scolaire, le mot « évaluation » renvoie au processus d'évaluation du travail ou aux moyens employés pour ce faire. En anglais, l'origine du mot équivalent à évaluation, soit assessment, est étroitement liée aux impôts, aux taxes et aux amendes. Ce mot a donc naturellement pris une connotation négative. Cependant, je suppose que dans le milieu de la recherche et de l'enseignement des mathématiques, l'omniprésence de tests qui représentent des enjeux élevés joue aussi un rôle considérable dans l'établissement de cette connotation. Néanmoins, l'évaluation est une composante inévitable de la découverte et de l'apprentissage. Est-il possible de créer une association positive en améliorant les pratiques d'évaluation pour les chercheurs et les étudiants?

Pour la majorité d'entre nous, notre expérience de l'évaluation est dominée par les formes utilisées en enseignement des mathématiques. Nous nous souvenons tous, ne serait-ce que vaguement, de ce que signifie passer un examen dans cette matière. En revanche, chaque année à cette période-ci, la plupart d'entre nous redécouvrent les joies de la correction des examens. Lorsque je faisais mon postdoctorat, un de mes mentors m'a convaincu d'employer un algorithme de tri quand je corrigeais un grand nombre d'examens. L'idée de base consiste à utiliser une variante du tri rapide pour répartir les examens dans des catégories comportant des erreurs équivalentes. En premier lieu, je regarde rapidement chaque solution et je constitue des piles de copies. Pour qu'elles soient acceptables au premier abord, ces piles peuvent être vaguement classées ainsi : potentiellement parfaites, comportant probablement une erreur, contenant apparemment quelques erreurs et presque vierges. En deuxième lieu, je lis soigneusement les solutions contenues dans une pile. Dès que je trouve un élément qui différencie une solution des autres, je cesse de lire et je place la copie dans une autre pile qui comporte des solutions similaires. Toutes les solutions de la pile restante reçoivent la même note et les mêmes commentaires. Pour exécuter l'algorithme, j'applique ces deux étapes de façon récursive. Dans chaque itération, au moins une catégorie d'équivalence est entièrement notée. Même si je parcours beaucoup de solutions de nombreuses fois, je lis soigneusement chacune une seule fois. Idéalement, cette procédure diminue la fatigue décisionnelle. J'utilise cette méthode depuis plus de dix ans et je pense que mes corrections sont plus cohérentes et peut-être, contre toute attente, considérablement plus rapides.

Au-delà de la cohérence et de l'efficacité, les évaluations de grande qualité doivent être valides. Évaluons-nous ce que nous avions l'intention d'évaluer? En enseignement, ce défi se pose à chaque fois que l'on conçoit un test, une interrogation ou un examen. Les questions devraient porter sur les concepts essentiels, permettre d'évaluer les

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Letters to the Editors

The Editors of the NOTES welcome letters in English or French on any subject of mathematical interest but reserve the right to condense them. Those accepted for publication will appear in the language of submission. Readers may reach us at **notes-letters@cms.math.ca** or at the Executive Office.

Lettres aux Rédacteurs

Les rédacteurs des NOTES acceptent les lettres en français ou anglais portant sur un sujet d'intérêt mathématique, mais ils se réservent le droit de les comprimer. Les lettres acceptées paraîtront dans la langue soumise. Les lecteurs peuvent nous joindre au bureau administratif de la SMC ou à l'addresse suivante : **notes-lettres@smc.math.ca**.

NOTES DE LA SMC

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Du bureau de la vice-présidente, suite de la page 3

compétences pertinentes et ne pas dépendre d'informations ou de connaissances qui n'ont pas de rapport avec le sujet. Par exemple, en calcul différentiel et intégral en première année, un problème lié aux taux et comprenant de subtils éléments de géométrie permet probablement d'évaluer ce que les étudiants ont retenu de l'école secondaire (ou peut-être la qualité de leurs professeurs de mathématiques à cette époque) plutôt que leur compréhension de la règle de dérivation en chaîne.

Le choix de l'outil d'évaluation adéquat est une autre composante clé de la validité. Le projet de couronnement d'un étudiant de premier cycle va presque assurément être un meilleur indicateur de son potentiel en matière de recherche et de ses habiletés de communication que sa moyenne générale. Aux cycles supérieurs, normalement, un étudiant reçoit la plus grande partie de la rétroaction formative lorsqu'il fait une présentation à un séminaire ou au cours d'une rencontre régulière avec son directeur de thèse. Avec un peu de chance, cette rétroaction correspond au contenu de ses évaluations sommatives, y compris les examens oraux et la soutenance de sa thèse. Selon mon expérience, la partie la plus difficile de la conception d'une stratégie d'évaluation valide est clairement la détermination de ce qui devrait être évalué.

Ces principes s'appliquent aussi à l'évaluation des bourses de recherche. Tout comme les meilleurs enseignants s'efforcent régulièrement d'améliorer leur enseignement, nous devrions continuer à chercher des façons d'augmenter la justesse et la précision des concours de subvention à la découverte du CRSNG. Selon moi, l'instabilité temporelle de ce processus semble être largement reconnue et est bien documentée dans au moins un cas [G]. Devrait-on utiliser un petit échantillon représentatif des demandes de l'année précédente pour définir les classes de valeurs? Quelles autres techniques amélioreraient la qualité du calibrage d'une d'année à l'autre? Pouvons-nous promouvoir la cohérence interne en demandant à tous les membres du groupe d'évaluation de revoir leur classement et leurs recommandations? Je suppose que certains changements structurels relativement simples augmenteraient considérablement la répétabilité et la reproductibilité. Par ailleurs, une meilleure approximation de la valeur vraie sera difficile à faire parce qu'il n'y a tout simplement pas de définition objective de la valeur dans ce contexte. Pour lutter contre le faux raisonnement concernant « la mesure correcte », je pense que nous devons élargir l'étendue des indicateurs de performance. Malheureusement, aucune de ces mesures ne règlera le principal problème : la diminution des ressources allouées à la recherche fondamentale.

Il semble encore plus difficile de garantir la fiabilité et la validité de l'évaluation des champs de recherche. La détermination de ce qui est important entraîne des désaccords réels. Comment prioriser la capacité et la qualité? Comment veiller à la diversité des idées? Comment éviter de tomber dans le piège facile de la surestimation de la capacité de prédiction? Les décisions concernant le financement à ce niveau sont invariablement controversées et délicates sur le plan politique. Ces problèmes empirent lorsqu'un système sporadique ne fait qu'entraîner une perte pour la plupart des parties prenantes. Si l'on s'arrête à un cas d'intérêt particulier, la façon dont le CRSNG devrait répartir les fonds entre les diverses disciplines scientifiques n'est pas claire. Personnellement, j'aimerais que le programme d'évaluation soit plus continuel. Une série de mesures diverses – contenant des indicateurs quantitatifs, des données bibliométriques normalisées et des méthodes délibératives – qui seraient assemblées petit à petit serait peut-être plus efficace. Quelques mesures pourraient être mises à jour chaque année afin que toutes les données soient révisées en suivant un cycle régulier et que la charge de travail d'une année donnée ne soit pas particulièrement élevée.

En reproduisant le Programme des coûts indirects du gouvernement fédéral, il serait possible d'améliorer la répartition des subventions chaque année en utilisant un nouvel agrégat. On pourrait même retarder intentionnellement l'inclusion des mesures les plus récemment révisées au sein de l'agrégat pour obtenir davantage de rétroaction de la part de la communauté et améliorer la transparence. Il serait absurde de déclarer que cette proposition brute serait meilleure que le mécanisme actuel. Je cherche seulement à montrer qu'il existe de nombreuses possibilités d'amélioration.

[G] Nassif Ghoussoub, Grade Inflation in Discovery Grant Competitions by Anthony Quas, Piece of Mind, 23 janvier 2012, accessible à http://nghoussoub.com/2012/01/23/grade-inflation-instability-and-uncertainty-in-discovery-grant-competitions/

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acclamée. À propos de l'avenir de l'enseignement des mathématiques, il aurait dit à peu près ceci : « On n'*entre* pas dans l'avenir, on le *crée* en travaillant fort. Cela veut dire que les enseignants doivent enseigner, que les élèves doivent faire leur travail et que les parents doivent jouer un rôle actif dans les études de leurs enfants. »

Jaime Escalante n'était pas un chercheur en mathématiques; ses connaissances mathématiques se limitaient aux mathématiques standards de niveau collégial! Il a pourtant mis ces connaissances à profit dans un programme de calcul différentiel et intégral qui a continué de grandir. Tout cela ne s'est toutefois pas fait sans un fort prix à payer. Des tensions qui ont fait surface au début de sa carrière à l'école secondaire Garfield se sont amplifiées avec le temps. À ses dernières années dans cette école, le professeur Escalante a reçu des menaces et des messages haineux de plusieurs personnes. Il est mort le 30 mars 2010, à l'âge de 79 ans, installé chez son fils où il recevait des traitements contre le cancer de la vessie.

Très peu de gens qui étudient les mathématiques au collège et enseignent au secondaire poursuivent ensuite leurs études au doctorat. En fait, on compte plus d'artistes parmi les diplômés que de mathématiciens. Nous savons que les jeunes qui étudient la poésie ne deviennent pas poètes. Ce n'est qu'une habitude de l'esprit. L'esprit ne pense pas de façon abstraite à moins qu'on ne le lui demande, et cette demande devrait se faire tôt dans l'éducation d'un enfant. La rigueur et la logique nécessaires à la compréhension et à la résolution de problèmes mathématiques sont une bonne façon de préparer le cerveau à résoudre d'autres problèmes qui se présenteront dans la vie.

DECEMBER 2013

2–4 Quasilinear PDE's and Game Theory *(Uppsala, Sweden)*
http://www.math.uu.se/quasilinear-pdes-and-game-theory

3 Fields-Origins Institute Seminar I Marcus Feldman (Stanford University) Mathematics and Computation in Phenotypic Switching I Fields Institute http://www.fields.utoronto.ca/programs/scientific/13-14/Fields-Origins/

6-9 CMS Winter Meeting *(University of Ottawa)*
http://cms.math.ca/Events/winter13/ **(SEE PAGE 18)**

7–8 Infinite-dimensional Geometry *(Berkeley, CA)*
www.msri.org/web/msri/scientific/workshops

8–13 Integral Equations Methods: Fast algorithms & applications *(Banff Research Station, Banff, AB)*
http://www.birs.ca/events/2013/5-day-workshops/13w5044

16-17 Workshop on Nonparametric Curve Smoothing I CRM I Organizer : Yogendra P. Chaubey (Concordia University)
http://www.crm.umontreal.ca/2013/NCS13/index_e.php

16–19 deLeónfest 2013, *(Madrid, Spain)* http://www.icmat.es/facilities/howtoarrive.html ; http://www.icmat.es/deLeonfest/

24–29 Understanding Relationships between Aboriginal Knowledge Systems, Wisdom Traditions, and Mathematics: Research Possibilities www.birs.ca/events/2013/5-day-workshops/13w5120

JANUARY 2014

24-26 Combinatorial Algebra meets Algebraic Combinatorics I Organizers: Sara Faridi, Hugh Thomas, Mike Zabrocki *(Location: Dalhousie)*Contact Information: Hugh Thomas http://www.aarms.math.ca/events/index.html

25 –30 From Random Walks to Levy Processes *(ANU, Australia)*
http://maths.anu.edu.au/events/kioloa-conference-kioloa-conference-randomwalk-levy-processes

27–31 AIM Workshop: Arithmetic statistics over finite fields and function fields *(Palo Alto, CA)* http://www.aimath.org/ARCC/workshops/arithstatffield.html

FEBRUARY 2014

17–21 Perfectoid Spaces and their Applications *(Berkeley, CA)*
http://www.msri.org/workshops/731

MARCH 2014

14–28 Representation Theory and Geometry of Reductive Groups, *(Altheim, Germany)* http://www2.math.uni-paderborn.de/konferenzen/conferencespring-school.html

APRIL 2014

4 An afternoon in honor of Cora Sadosky *(Albuquerque, New Mexico)*
http://www.math.unm.edu/conferences/13thAnalysis/

7–11 Tools from Algebraic Geometry *(Los Angeles, CA)*
http://www.ipam.ucla.edu/programs/ccgws2/

MAY 2014

5–9 Kakeya Problem, etc *(Los Angeles, CA)*
http://www.ipam.ucla.edu/programs/ccgws2/

5–9 Projective modules and A1-homotopy theory *(Palo Alto, CA)*
http://www.aimath.org/ARCC/workshops/projectiveA1.html

19 Bers 100 celebration *(CUNY, NY)*
http://fsw01.bcc.cuny.edu/zhe.wang/IB.html

20–25 16th International Conference on Fibonacci Numbers *(Rochester, NY)* http://www.mathstat.dal.ca/Fibonacci/ I http://www.msri.org/web/msri/scientific/workshops/all-workshops/show/-/event/Wm9805

26–30 European Conference on Elliptic & Parabolic Problems *(Gaeta, Italy)* http://www.math.uzh.ch/index.php/conferencedetails

NOVEMBER 2014

17–21 Categorical Structures in Harmonic Analysis Workshop *(Berkeley, CA)* http://www.msri.org/web/msri/scientific/workshops/all-workshops/show/-/event/Wm9805

DECEMBER 2014

1–5 Automorphic Forms, Shimura varieties etc. *(Berkeley, CA)*
http://www.msri.org/workshops/719

Sets of Finite Perimeter and Geometric Variational Problems

by Francesco Maggi

Cambridge Studies in Advanced Mathematics n. 135
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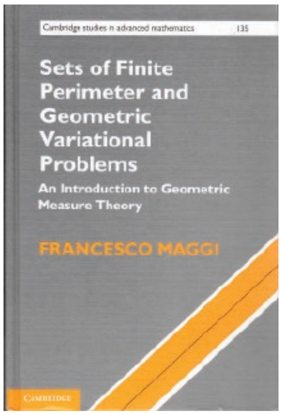
*Reviewed by **Alessio Figalli**, The University of Texas at Austin*



Introduction

The notion of perimeter plays a key role in several geometric problems. A classical example is provided by the:

Isoperimetric problem: Among all sets with fixed volume, find the one which minimizes the perimeter.

The above problem, although very simple to state, presents some non-trivial mathematical difficulties just in its formulation: indeed, while the concept of volume is relatively easy to define (by definition, the volume of a Borel set is its Lebesgue measure), less clear is the definition of perimeter.

While one may be satisfied to consider the isoperimetric problem only among smooth sets, it is particularly difficult to prove that a minimizer exists in such a class. Hence, to overcome this problem, one would like to introduce a notion of perimeter which is defined on a large enough class of sets to ensure that a minimizer exists inside such a class, and then eventually prove that the minimizer is smooth and is actually a ball.

Several notions of perimeter have been introduced during the first half of the 20th century, and the one which has been the most successful is the one introduced by De Giorgi [2]. His starting point is the Gauss-Green formula: a Borel set $E \subset \mathbb{R}^n$ has finite perimeter if there exists a n-tuple of Radon measures $(\mu_E^1, \dots, \mu_E^n)$ such that

$$\int_E \partial_i \varphi(x)\,dx = \int_{\mathbb{R}^n} \varphi(x)\,d\mu_E^i(x) \qquad \forall\,\varphi \in C_c^1(\mathbb{R}^n).$$

Then, by definition, the perimeter of E is given by the total variation of the vector-valued measure $\mu_E := (\mu_E^1, \dots, \mu_E^n)$, that is

$$P(E) := |\mu_E|(\mathbb{R}^n) = \sup\left\{\sum_j |\mu_E(F_j)|, \qquad \bigcup_j F_j = \mathbb{R}^n,\ F_j \cap F_k = \emptyset\right\},$$

where

$$|\mu_E(F_j)| = \sqrt{\sum_{i=1}^n |\mu_E^i(F_j)|^2}.$$

This definition, which a priori may seem very weak, is actually very practical and allows one to prove several nice results. Probably the most basic and important one is De Giorgi's rectifiability theorem [3]: if E is a set of finite perimeter then the measure μ_E defined above is concentrated on a countable union of Lipschitz hypersurfaces, and it can be represented in terms of the $(n-1)$-dimensional Hausdorff measure restricted to these hypersurfaces and of a unit vector field (which plays the role of the outer normal). This shows that there is a "geometric enough" concept of boundary associated to a set of finite perimeter.

In addition to this nice geometric fact, the notion of perimeter of De Giorgi enjoys the following properties: the space of sets with uniformly bounded perimeter is precompact with respect to the L^1_{loc}-convergence, and the perimeter is lower-semicontinuous with respect to the L^1_{loc}-convergence (that is, if $E_k \to E$ in L^1_{loc} and $P(E_k) \leq C$, then $P(E) \leq C$). These two key facts make sets of finite perimeter a particularly suited framework to study geometric variational problems.

A first example is provided by the solution to the isoperimetric problem [4]: among all sets of finite perimeter with fixed volume, balls are the unique minimizers of the perimeter. Another important example is provided by the Plateau problem, where among all surfaces in $\mathbb{R}^n$ with fixed boundary one wishes to minimize the area. At least in codimension one (so, in the context of $(n-1)$-dimensional hypersurfaces), one may look at a surface as the boundary of a set of finite perimeter, and then study the problem in this framework. The setting of sets of finite perimeter permits to prove that a minimizer exists, and to study its regularity properties. One of the most celebrated results in this area states that minimal surfaces are smooth up to dimension 7, and in general they may have a singular set of dimension $n-8$.

Description of the book

The book is divided into four parts.

The first one is very classical and discussed the basic theory of Radon measures in $\mathbb{R}^n$. However, with respect to many other books on this subject, the point of view adopted by the author is very original and geometric: for instance, the concept of tangent space to a submanifold is recast in terms of convergerce of rescaled measures associated to the submanifold, a point of view which allows the author to introduce the concept of rectifiable sets in a very natural way.

This part describes the basic properties of Radon measures (such as representation, compactness, and differentiation), and introduces the basic concept of geometric measure theory (covering theorems, differentiability of Lipschitz functions, Hausdorff measures, area formula, and rectifiable sets).

The second part deals with sets of finite perimeter, proving several classical results for this class of objects. As mentioned in the introduction, these sets are introduced as the family of sets for which a suitable Gauss-Green formula holds.

As shown in this section, sets of finite perimeter can be approximated by smooth sets, they can be sliced (coarea formula), and uniform bounds on the perimeter gives precompactness on family of sets.

Also, as mentioned in the previous section, to each set of finite perimeter it is possible to associate a geometric concept of boundary (supported on countably many Lipschitz hypersurfaces) and of unit outer normal. This allows the author to prove Federer's theorem, which shows that an equivalent natural concept of boundary for sets of finite perimeter is given by the points where the set has density $1/2$. Finally, several useful technical formulas are discussed at the end of the section to obtain some nice basic results on the equilibrium shapes of liquid and sessile drops.

The third part focuses on the regularity theory of (almost) minimal boundaries, that is, the smoothness of the boundary of a set which (almost) minimizes the perimeter inside a domain A among all other sets which coincide with it outside$A$.

The main theorem states that the boundary of such a set is analytic in A, except for a possible closed singular set of codimension 8 (so, in particular, it is empty if $n \leq 7$). The proof of this deep theorem takes the whole chapter, and involves several powerful concept such as the excess, the flatness, the harmonic approximation for minimal surfaces, and the blow-up analysis. Although this part is highly non-elementary, the exposition is extremely clear and the beautiful pictures guide the reader through the proof in a very nice and pedagogical way.

The fourth (and last) part is the most original contribution of the book: for the first time the theory of minimizing clusters, developed by Almgren in the 70's [1], appears in a book form. This beautiful theory, for which several nice results have been obtained, still presents many interesting and important open questions.

A cluster is a finite union of disjoints sets of finite perimeter. The basic idea is that the cluster represents a finite union of chambers which may touch each other, and whenever they touch the common part of the boundary counts only once. A basic example is provided by a union of soap bubbles attached together, and one of the goals of the theory of clusters consists in understanding the possible configurations which minimize the energy, and the kind of singularities which can develop.

Already the existence of minimizing clusters (where one fixes the volume of each chamber, which in the case of soap bubbles corresponds to the air inside the bubble) is a very non-trivial task and requires all the power of the tools previously developed in the book, in addition to several new geometric ideas. Once existence is proved, the natural next step becomes the regularity of them. Two main theorems are proven: a general regularity theory which states that minimal clusters are smooth outside a singular sets of zero $(n-1)$-dimensional measure (the intersection of the boundaries of the different chambers); and a very precise description of the possible singularities in dimension two, which shows that boundaries of minimal clusters (that now are curves) consist of finitely many circular arcs that can only meet by group of three forming angles of 120 degrees. With these two beautiful results, the book ends. Although an analogous description of clusters' singularities is available in dimension 3 [5], the proof of the latter is extremely technical and involved, and the author considers that proving such result in the book would go beyond his scope (and the reviewer completely agrees on this point).

Conclusion

The book under review is written by an expert in the theory who has made important contributions to the subject in the last years. The book is a clear exposition of the theory of sets of finite perimeter, that introduces this topic in a very elegant and original way, and shows some deep and important results and applications.

Let me mention that the prerequisites assumed do not go much beyond a first course in analysis and functional analysis, and the proofs are entirely self-contained. This makes the book accessible to a large audience, including graduate and postgraduate students. Moreover the book is extremely well written and pleasant to read.

Each chapter is opened by rather detailed synopses, and ends with notes that provide interesting comments about the chapter and some additional references.

Although most of the results contained in this book are classical, some of them appear in this volume for the first time in book form, and even the more classical topics which one may find in several other books are presented here with a strong touch of originality which makes this book pretty unique.

I strongly recommend this excellent book to every researcher or graduate student in the field of calculus of variations and geometric measure theory. Naturally, it will also be of interest to many mathematicians in different areas who are simply interested in having an overview of the subject.

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A guide to Groups, Rings and Fields

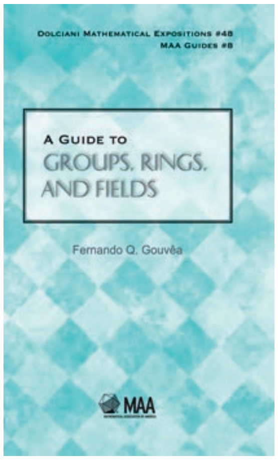
by Fernando A. Gouvêa

Dolciani Math. Expositions n. 48

Mathematical Association of America, 2012

ISBN : 978-0-88385-355-9

Reviewed by ***Keith Johnson****, Dalhousie University, Halifax, NS*



This book is one of the latest in the MAA series of guides to different areas of mathematics intended to summarize the material graduate students need to know in preparing for PhD comprehensive exams. This one in particular, is welcome since the area, abstract algebra is a required exam subject at many universities. The organization of the book is straight forward with 3 brief introductory chapters (totaling 28 pages) giving a bit of history, a very little bit of category theory and some observations on the organization of the subject. These are followed by 3 extended chapters on the 3 topics named in the book's title. For each of these the relevant definitions are stated, the important theorems are given without proofs, and enough connective tissue is added to knit each into a coherent narrative. The writing style is not formal ("cheerful subgroups", "why study modules?", "working without an A") which makes for a pleasant read, although without the proofs it sometimes isn't obvious just how deep some of the stated theorems are. At over 300 pages this guide is over twice as long as some of the others in the series and yet it covers the area with virtually no extranious material. The group theory goes beyond the Sylow Theorems and Jordan-Hölder with sections on permutation and linear groups and a section on finite group representations. The ring theory chapter goes beyond the factorization theorems to include localization, Hom and tensor products and sections on both Noetheriam and Artmian rings. The field theory chapter presents the basics of Galois Theory and includes brief discussions of algorithms for computation and of the inverse Galois problem. It concludes with a section on division algebra. The book covers a lot of ground but it may still not quite cover all that some departments may put on their exam syllabi. In particular algebraic geometers and topologists will wish for a bit more homological algebra (projective and injective modules are covered but not Tor or Ext). By contrast number theorists, operator theorists and students in most other areas will find this book most satisfactory in all respects.

A guide to Functional Analysis

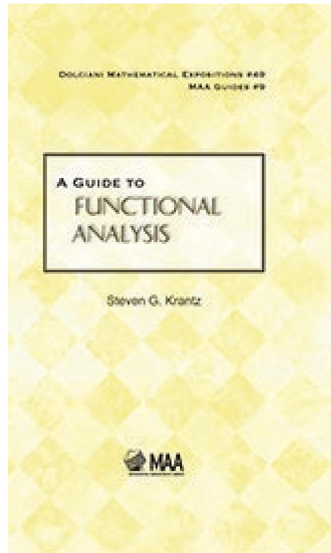
by Steven Krantz

Dolciani Math. Expositions n. 49

Mathematical Association of America, 2013

ISBN : 978-0-88385-357-3

Reviewed by ***Heydar Radjavi****, The University of Waterloo, ON*



This is a brief monograph (135 pages) in the MAA Guides series. The series is, to quote the description given by the MAA, "meant for students, especially graduate students, and faculty who would like an overview of the subject. They will be useful to those preparing for qualifying exams." It takes care of the "fundamentals," the so-called three big results and their applications in the 26 pages of Chapter 1 and then proceeds to cover bounded operators, Banach algebra basics, topological vector spaces, distributions, convexity, fixed point theorems, and spectral theory in the remaining eight chapters. It is a nice review book for somebody who has seen the stuff before or is sufficiently sophisticated in another field of mathematics. Thus the author's claim that the book is "almost completely self-contained" is only technically true. The book is certainly quite useful in preparation for qualifying examinations. I have two minor comments, but they may reflect my personal taste only: (1) I don't understand the apologetic remark in the preface: "Measure theory rears its ugly head in some of the examples... ." Why ugly? (2) and I take exception to the (admittedly informal) claim made in the description of the spectral theorem that "any reasonable bounded operator" on a Hilbert space is normal. This is much stronger than the claim that "any reasonable matrix" is diagonalizable–a circumstance that would make life easier but less interesting for operator-theorists!

2014 CMS Summer Meeting

June 6 - 9, 2014, Delta Winnipeg

Host: University of Manitoba

CALL FOR SESSIONS

The Canadian Mathematical Society (CMS) and the University of Manitoba welcomes and invites proposals for sessions for the 2014 Summer Meeting in Winnipeg from June 6th to 9th, 2014. Proposals should include a brief description of the focus and purpose of the session, the expected number of speakers, as well as the organizer's name, complete address, telephone number, e-mail address, etc. All sessions will be advertised in the CMS Notes, on the web site and in the AMS Notices. Speakers will be requested to submit abstracts, which will be published on the web site and in the meeting program. Those wishing to organize a session should send a proposal to the Scientific Directors by January 30, 2014.

Scientific Directors:

Nina Zorboska: **zorbosk@cc.umanitoba.ca**

Stephen Kirkland: **stephen.kirkland@umanitoba.ca**

Réunion d'été 2014 de la SMC

6-9 juin 2014, Delta Winnipeg

Hôte : Université du Manitoba

PROPOSITION DE SESSIONS

La Société mathématique du Canada (SMC) et l'Université du Manitoba vous invitent à proposer des sessions pour la Réunion d'été 2014 qui se tiendra à Winnipeg du 6 au 9 juin 2014. Ces dernières doivent inclure une brève description de l'orientation et des objectifs de la session, le nombre de conférenciers prévus ainsi que le nom, l'adresse complète, le numéro de téléphone et l'adresse courriel de l'organisateur. Toutes les sessions seront annoncées dans les Notes de la SMC, sur le site web et dans les AMS Notices. Les conférenciers devront présenter un résumé, qui sera publié sur le site web et dans le programme de la Réunion. Toute personne qui souhaiterait organiser une session est priée de faire parvenir une proposition aux directeurs scientifiques au plus tard le 30 janvier 2014.

Directeurs scientifiques :

Nina Zorboska : **zorbosk@cc.umanitoba.ca**

Stephen Kirkland : **stephen.kirkland@umanitoba.ca**

Photo : Dan Harper

CMS Notes
Volume 45 No. 6, December 2013

Jennifer Hyndman, *University of Northern British Columbia*
John Grant McLoughlin, *University of New Brunswick*

Two invitations to do mathematics are presented in this issue of the Notes. The opening piece features some playful mathematics emerging from mistakes that produce correct results, thus, making them misteaks. The second piece offers information to female undergraduate students who may wish to participate in the 2014 Summer School for Women in Math at University of Waterloo.

Mathematical Misteaks: Learning from Errers (yes!)

John Grant McLoughlin,
University of New Brunswick (johngm@unb.ca)

The idea of *mathematical misteaks* was introduced to me by Jack Weiner, as the theme of a talk to the Grand Valley Mathematical Association in the late 1980's. Simply stated a mathematical misteak results from some poor mathematics that produces a correct result. Perhaps the most famous example of this would be the direct cancellation of the 6's in $16/64$ to produce the correct result of $1/4$. Sure this is wrong in a significant manner, yet at the same time thought provoking. Are there other fractions that would be simplified properly by taking similar steps? Rather than focusing on the mathematical missteps, let us turn our attention as mathematicians to the "why" aspect of the misteak. In this example, we could begin by considering when the following is true:

$$\frac{ab}{bc} = \frac{a}{c} \text{ or more precisely } \frac{10a+b}{10b+c} = \frac{a}{c}.$$

"Errers" do indeed err but only those mistakes that produce the correct answer in some observable form are termed *misteaks.* It is these misteaks that are central to this discussion.

Here is another numerical curiosity one may wish to consider:

$$\sqrt{2\frac{2}{3}} = 2\sqrt{\frac{2}{3}}$$

The example is not unique, as there is a family of values that can be found for which

$$\sqrt{a+\frac{b}{c}} = a\sqrt{\frac{b}{c}}$$

I recall reading a *Reader Reflection* (feature of *Mathematics Teacher)* years ago written by a teacher who had asked his students to find fractions that would go in between two other fractions in an ordering from least to greatest. For example, consider the challenge of finding a fraction between $3/5$ and $5/8$. The fractions seem relatively close together and the lowest common denominator of 40 does not seem to provide sufficient space as the equivalent fractions have numerators of 24 and 25 respectively. So imagine the surprise of the teacher when one of his students simply added the numerators and denominators to get a suitable value in each instance. The result of $8/13$ would be obtained with this method in the aforementioned example. Try another and another…and it always seems to work. This appears to be a rich example of a misteak when in fact, it is not. Rather this is the idea of a "mediant." A little algebra shows that the mediant of two inequivalent positive-denominator fractions a/b and c/d, defined as $(a+c)/(b+d)$, always lies between them. It may be helpful to consider baseball: the sum of the hits and the sum of at bats of a player in two successive games will produce an intermediate fractional value. Interested readers ought to examine more about Farey sequences, a concept extending back to Farey in 1816 and Haros in 1802 (see Wikipedia, "Farey Sequence").

My efforts to learn more on this topic took me to an article that was new to me, though hardly recent in that the article by Robert A. Carman [1] appeared over forty years ago in *Mathematics Teacher*. A plethora of examples including the first pair noted above are found in this piece. Many of the examples are credited to sources in recreational mathematics. Among these are so-called 'printer errors' in reference to the dropping of exponents or reversals of digits, thus, producing the actual values. Some examples are shared here:

$$2^5 \cdot 9^2 = 2592$$

$$3^4 \cdot 425 = 34425$$

$$13 \times 62 = 31 \times 26$$

Carman also offers 'fractured fractions' such as $1\ 1/2 \times 3 = 1\ 1/2 + 3$ for which the change of operational symbol does not affect the result. In fact, the article contains a rich array of examples through which much mathematics can be explored. This extends to more advanced topics such as trigonometry including the following identity where the angles are radian measures:

$$\cos(4) + \cos(2) = 2 \cdot \cos(3) \cdot \cos(1)$$

Removing cos from the equation gives $4 + 2 = 2 \cdot 3 \cdot 1$, or $6 = 6$, as required.

The author encourages feedback on this piece including efforts to integrate these ideas into your own classes or more examples of misteaks.

Reference:

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Two Weeks at Waterloo - A Summer School for Women in Math (August 10-23rd, 2014)

Barbara Csima, *University of Waterloo (csima@math.uwaterloo.ca)*

This summer school is an opportunity for up to sixteen outstanding female undergraduate students, from across Canada, to gather together to study topics in mathematics in an intense two-week immersion. The aim is to encourage and inspire these gifted young women to continue on to graduate work in mathematics. The program will provide both enrichment of the undergraduate curriculum and a research component, in a collaborative environment.

This programme ran for the first time in the summer of 2012 at the University of Waterloo welcoming sixteen students, two female instructors and two female teaching assistants. The feedback from the participants was very positive with many speaking of feeling inspired to pursue further education in mathematics and remarking on the benefits of an all-female environment.

The students will participate in two mini courses, taught by Malabika Pramanik (University of British Columbia) and Karen Lange (Wellesley College). Four female guest speakers will talk about their work in mathematically related fields, including a distinguished public lecture by Professor Mary Lou Zeeman, and visits will be made to businesses and institutions employing mathematicians.

The women will be housed at the University of Waterloo. The students' accommodation, meals and travel costs within Canada will be covered, subject to availability of funds.

The summer school is open to female undergraduate students studying mathematics or a related discipline at a Canadian university, with at least one year of studies remaining in their program. Canadians and permanent residents of Canada studying outside Canada are also eligible to apply.

Applications for this very selective program are due January 31, 2014. For more information and an online application form, please go to the website **https://math.uwaterloo.ca/women-in-mathematics/events/summer-school**

For further information please contact the organizers at **wimsum@uwaterloo.ca**.

Another year…and now your turn

This issue of *Education Notes* marks the end of Volume 45, bringing to a close our fourth year as co-editors. Reflecting on the year, it is evident that an emphasis on recreational mathematics has appeared in the pages of Education Notes. Contributions from Gary MacGillivray, Susan Milner, John Grant McLoughlin, David Casperson and Jennifer Hyndman have spoken to different aspects of games in mathematics.

Two other articles merit attention in moving forward. Jennifer Hyndman opened this volume with an article entitled *Studying Understanding and Understanding Studying*. Brian Menounos offered a take on the field of earth science as a lens for teaching mathematics. These two articles may offer channels for readers to consider as possible directions for written submissions. This next year we would like to encourage contributions that address an aspect of mathematical pedagogy, or bring math alive through examining its place in some other field (e.g. earth science). Think in broad terms so this may extend to the discussions of mathematical connections to history, or insight into mathematics in disciplines, or even within a less traditional field, such as mathematics in the lived experiences of artisans.

Please do not hesitate to send along a note with ideas as we strive to ensure quality articles of value to the readership of *Education Notes*. In addition, activity-based/informational contributions such as that of Barbara Csima (in this issue) are welcomed as they help us inform the community of efforts in place to promote mathematics and education.

Čebyšev Sets and Monotone Spreads in Hyperspaces

Robert Dawson, *Department of Mathematics and Computing, Saint Mary's University, Halifax, NS*

In linear spaces, the concept of convexity is a very useful one. There are various ways of generalizing it to sets in a metric space; the Čebyšev property is one of these. A subset A of X will be said to have the Čebyšev property (or be a Čebyšev set) if every point of X has a unique nearest neighbor in A.

In Euclidean spaces, this is equivalent to being closed, nonempty, and convex. In general, in a finite-dimensional Banach space, the smoothness of balls implies that every Čebyšev set is convex, and the strict convexity of balls implies the converse for (nonempty) closed sets [6]. So, for instance, in $\mathbf{R}^n$ equipped with the ℓ_1 or ℓ_∞ norm, the Čebyšev property neither implies convexity nor is implied by it.

One interesting class of metric spaces in which we can study Čebyšev sets is the class of *hyperspaces* over Minkowski spaces. A hyperspace is a suitable family of nonempty compact sets, with a metric derived in some way from that of the underlying space. (This idea has also proved fruitful in a non-metric topological setting.) Here, we will be following [2, 5, 4] in studying hyperspaces with the *Hausdorff metric*, defined as

$$\varrho_H(X,Y) = \max\{\max_{x\in X}\min_{y\in Y} d(x,y), \max_{y\in Y}\min_{x\in X} d(y,x)\} .$$

(Informally: restrict a dog to X and a cat to Y, and allow them to reach their natural equilibrium, with the dog as close to the cat as possible and the cat keeping its distance. Now exchange territories and try again; finally, let the cat choose between the two arrangements.)

In 2006, Bogdewicz and Moszyńska [2] studied the hyperspace $\mathcal{K}^n$ of compact convex subsets of $\mathbf{R}^n$ with the Hausdorff metric, and the related hyperspace $\mathcal{K}^n_0$ of convex bodies. They showed that the family $\mathcal{B}^n$ containing all balls is Čebyšev in $\mathcal{K}^n_0$, while the family containing all balls and singletons is Čebyšev in $\mathcal{K}^n$. Closed affine segments of the form $(A+\lambda B^n|\lambda=[0,1])$, nonempty convex sets of singletons, and strictly affine convex sets were also shown to be Čebyšev in $\mathcal{K}^n$. In 2009, Moszyńska and I [5] showed that any closed strongly nested family (that is, with $A_i \subset \mathrm{int} A_j$ for $i<j$) is Čebyšev in $\mathcal{K}^n$ or $\mathcal{K}^n_0$. We also introduced the hyperspace $\mathcal{O}^n$ of strictly convex compact sets, and showed that, in this hyperspace, families of translates are Čebyšev.

An arc of convex bodies $A(t)$ is called *monotone* if for each $\mathbf{u}\in S^{d-1}$ the height function $f_u(A_t)$ is monotone increasing, monotone decreasing, or constant as a function of t; and if it is constant, the points at which maximum height is achieved on different bodies in the arc are distinct. This concept generalizes both the concept of strongly nested arc and the concept of translational arc; and any such arc, if compact, has [4] the Čebyšev property in $\mathcal{O}^n$. That is, every strictly convex body has a unique Hausdorff-closest approximation in the arc.

An affine segment $[A_0, A_1]$ is monotone unless the terminal bodies A_0 and A_1 are supported in the same sense, at the same point, by the same hyperplane. There are also other monotone arcs, such as the set of all ellipses inscribed in a quadrilateral (including the diagonals as degenerate ellipses). It should be noted that except in degenerate cases a monotone arc is never determined (among monotone arcs) by its endpoints or indeed by any finite subset; in this regard, strongly nested arcs are more typical examples than the "rigid" affine segments.

Let $A(t)$ be bodies in $\mathcal{O}^d$, and let $N_t : \partial A(t) \to S^{d-1}$ be the Gauss map that takes a boundary point to its normal vector. As the bodies are strictly convex, the inverses $N_t^{-1} : S^{d-1} \to \partial A(t)$ are well-defined. Let

$$D_{s,t} : \mathbf{u} \mapsto N_s^{-1}(\mathbf{u}) - N_t^{-1}(\mathbf{u}) .$$

If $A(s)$ and $A(t)$ are in a monotone arc, $D_{s,t}(\mathbf{u})$ is never zero, and hence can be retracted canonically onto S^{d-1}. Thus, to each pair of bodies corresponds to an element of $H_{d-1}(S^{d-1}) = \mathbf{Z}$, the *index*; and as the arc is continuous, the index is invariant over all pairs of bodies in the arc. Informally, every two bodies in the arc have the same configuration relative to each other. The possible values are the integers less than or equal to 1. A nested arc has index 1; a translation-like arc, 0; and the arc of ellipses inscribed in a quadrilateral, -1.

In recent work (still unpublished), I define a *monotone spread* to be a collection of bodies such that any two of them are joined, within the collection, by a monotone arc. (Such an arc is always unique within the spread.) A compact monotone spread in $\mathcal{O}^n$ is easily shown to have the Čebyšev property: for if two elements A_0 and A_1 of the spread are equidistant from B, the monotone arc joining them is Čebyšev and must thus contain a closer body. It may be shown that the index is invariant over all arcs in a monotone spread, and is hence an invariant of the spread.

Any monotone spread with index 1 (that is, with nested bodies) is just an arc. Spreads with index 0 can have up to d parameters (for instance, all translates of a given body). It is not so easy to find spreads with negative index that are not just arcs; one example in the plane is given by the set of all ellipses that can be inscribed in a unit square, with any orientation, centered at 0 (Figure 1).

What happens in higher dimensions? Amazingly, at least for centered ellipsoids, the answer depends on a theorem of Adams, Lax, and Phillips [1] on the the maximum dimension $R_H(n)$ of a real linear space of symmetric real invertible $d\times d$ matrices. In $1, 2, 3\ldots$ dimensions, this is given by the sequence $(1, 2, 1, 3, 1, 2, 1, 5, \ldots)$ closely related to the Radon sequence. In particular, $R_H(3) = 1$; so in $\mathcal{O}^3$, there are no two parameter monotone spreads of centered ellipsoids!

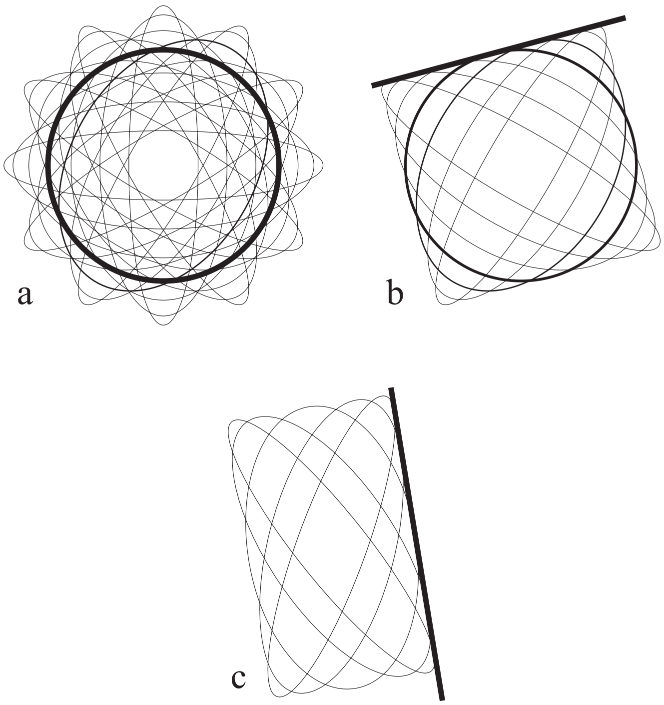

Figure 1 : A spread of dimension 2 and index -1 in $\mathcal{O}^2$, and two of its arcs.

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Assessment in Mathematics, continued from cover

These principles also apply to the evaluation of individual research grants. Just as the finest educators work steadily to enhance their teaching, we should continue to look for ways to raise both the accuracy and precision in the NSERC Discovery Grant competitions. From my perspective, the temporal instability in this process seems to be widely recognized and is well-documented in at least one case [G]. Should a small representative sample of applications from the previous year be used to calibrate the bins? What other techniques would improve the quality of the calibration between years? Can we promote internal consistency by having the entire evaluation group review their rankings and recommendations? I suspect that some relatively simple structural changes would significantly increase both the repeatability and the reproducibility. On the other hand, a better approximation of the true value will be difficult to achieve, because there is simply no objective definition for value in this context. To combat the ``one correct metric'' fallacy, I believe that we need to broaden the range of performance indicators. Unfortunately, none of these developments will address the greatest problem: the decrease in resources dedicated to fundamental research.

Ensuring reliability and validity in the assessment of research fields appears to be even more challenging. There are genuine disagreements about what is important. How should we prioritize capacity and quality? How should we ensure diversity of ideas? How do we avoid the easy trap of overestimating our predictive powers? Funding decisions at this level are invariably contentious and politically charged. These problems are further exacerbated when a sporadic system all but guarantees a loss for most stakeholders. Focusing on a particular case of interest, it is not clear how NSERC should allocate funds between the various scientific disciplines. Personally, I would like to see a much more continual assessment program. A diverse suite of metrics—containing quantitative indicators, norm-referenced bibliometrics, and deliberative methods—that is assembled piecemeal might work best. Each year a few metrics would be updated so that all of the data is revised on a regular cycle and the workload in any given year is not especially high. Mimicking the Federal Indirect Costs Program, refinements to the funding allocations could be made each year using a new aggregate. One could even intentionally delay the inclusion of the most recently revised metric within the aggregate to allow for greater community feedback and transparency. Claiming that this rough proposal would be better than the current mechanism seems to be a vacuous statement. My intent is just to illustrate that there is ample room for improvement.

[G] Nassif Ghoussoub, Grade Inflation in Discovery Grant Competitions by Anthony Quas, *Piece of Mind*, 23 January 2012, available at http://nghoussoub.com/2012/01/23/grade-inflation-instability-and-uncertainty-in-discovery-grant-competitions

Bragg Diffraction of Meyer Sets

Nicolae Strungaru, *Department of Mathematics and Statistics, Grant MacEwan University, Edmonton, AB*

Over the past 100 years, physical diffraction has been a powerful tool in the study of crystals. Physical diffraction is modeled as follows: given a sample of the material, if $F \subset \mathbb{R}^3$ is set of positions of atoms in this sample, then the intensity of the diffraction pattern of the sample is

$$I(z) = \frac{1}{\sharp F} \left| \sum_{x \in F} e^{2\pi i < x,z>} \right|^2 ,$$

where $< x, z >$ denotes the usual dot product in the Euclidian space, and $\sharp F$ denotes the number of elements of F.

The mathematical model we consider is an infinite set $\Lambda \subset \mathbb{R}^3$ (or more generally $\Lambda \subset \mathbb{R}^d$) which represents the set of positions of atoms in an ideal solid. We will assume that the solid is distributed in a regular way, with the atoms not being too close or too far from each other. More exactly we will assume that Λ has two properties:

- There exists a minimal distance $r > 0$ between the points in Λ (we say that Λ is *uniformly discrete*)
- There exists some $R > 0$, such that any point of $\mathbb{R}^d$ is within distance R of some element of Λ (we say that Λ is *relatively dense*)

The diffraction I_Λ of Λ is the positive measure defined as the limit of the diffraction of the finite sets $\Lambda \cap [-n, n]^d$:

$$I_\lambda = \lim_n I_{\Lambda \cap [-n,n]^d} .$$

Here the limit is taken in the vague topology for measures, and the issue of its existence can easily be avoided by replacing the sequence of measures by some subsequence. We omit the technical details around these issues, and refer instead the reader to [1].

It was proved by Hof [1], that there exists a measure γ, called the autocorrelation measure of Λ, whose Fourier transform in the sense of tempered distributions is exactly the measure $\widehat{\gamma} = I_\Lambda$. Like any measure, $\widehat{\gamma}$ has a Lebesgue decomposition

$$\widehat{\gamma} = \widehat{\gamma}_{pp} + \widehat{\gamma}_{ac} + \widehat{\gamma}_{sc} ,$$

into pure point, absolutely continuous and singularly continuous components. The measure $\widehat{\gamma}_{pp}$ corresponds to the bright spots in the diffraction pattern, and the existence of a large pure point component is sign of long range order in Λ. In contrast, the continuous component $\widehat{\gamma}_{ac} + \widehat{\gamma}_{sc}$ corresponds to the diffuse background in the diffraction, and is a measure of disorder.

The pure point (or discrete) part $\widehat{\gamma}_{pp}$ of the diffraction is thus of special interest to us, as it encapsulates (some of) the long range properties of Λ. As any pure point measure, it can be written in the form

$$\widehat{\gamma}_{pp} = \sum_{\chi \in \mathcal{B}} \widehat{\gamma}(\{\chi\}) \delta_\chi ,$$

where $\mathcal{B} = \{\chi \in \mathbb{R}^d \mid \widehat{\gamma}(\{\chi\}) \neq 0\}$ is the set of Bragg peaks. We always have $0 \in \mathcal{B}$ and moreover, 0 has the highest possible intensity among all the Bragg peaks.

A periodic crystal is a solid which repeats periodically in all directions. Mathematically it is described as

$$\Lambda = L + F = \{l + f | l \in L, f \in F\} ,$$

where L is a lattice in $\mathbb{R}^d$ and F is a finite set. Its diffraction is supported inside the dual lattice L^0 and thus is a pure point measure. If the diffraction of a periodic crystal is invariant under a rotation by an angle $\theta \in (0, 2\pi]$, it can easily be proven that $\theta = \dfrac{2\pi}{n}$ and $n \in \{1, 2, 3, 4, 6\}$.

For a long time physicists believed that only periodic crystals can produce diffraction diagrams consisting only of Bragg peaks. In 1984, Dan Shechtman announced the discovery of a solid with a pure point diffraction measure, but whose diffraction was invariant under rotations by $\frac{2\pi}{10}$. The new solids were named quasicrystals, and Shechtman was awarded the 2011 Nobel Prize in Chemistry for this discovery.

Because of Shechtman's discovery, the International Union of Crystallography redefined the term of "crystal" to mean, "any solid with essentially discrete diffraction diagram." Thus, a crystal is any solid for which the set $\mathcal{B}$ of Bragg peaks is relatively dense in $\mathbb{R}^d$.

A Meyer set is a subset $\Lambda \subset \mathbb{R}^d$ which is relatively dense, and for which the Minkowski difference

$$\Delta := \{x - y \mid x, y \in \Lambda\} ,$$

is uniformly discrete. While this condition seems weak, the complete characterization of Meyer sets in [2] shows that this is actually a very strong requirement.

We show in [3] that the set of Bragg peaks of any Meyer set $\Lambda \subset \mathbb{R}^d$ is relatively dense, thus Meyer sets are good models for (aperiodic) crystals. We also prove that for each $\epsilon > 0$, we can find a relatively dense set T of translations, such that each translation changes the intensity of every Bragg peak by at most ϵ.

In a real diffraction experiment, we can only see the Bragg peaks whose intensity exceed a threshold $a > 0$. We call these Bragg peaks the a-visible Bragg peaks.

The second result we prove in [3], is that, given a Meyer set $\Lambda \subset \mathbb{R}^d$, for all $0 < a < \widehat{\gamma}(\{0\})$ the set of a-visible Bragg peaks of Λ is also a Meyer set. This shows not only that the Meyer sets exhibit an essentially discrete diffraction diagram, but also that the set of Bragg peaks is highly ordered.

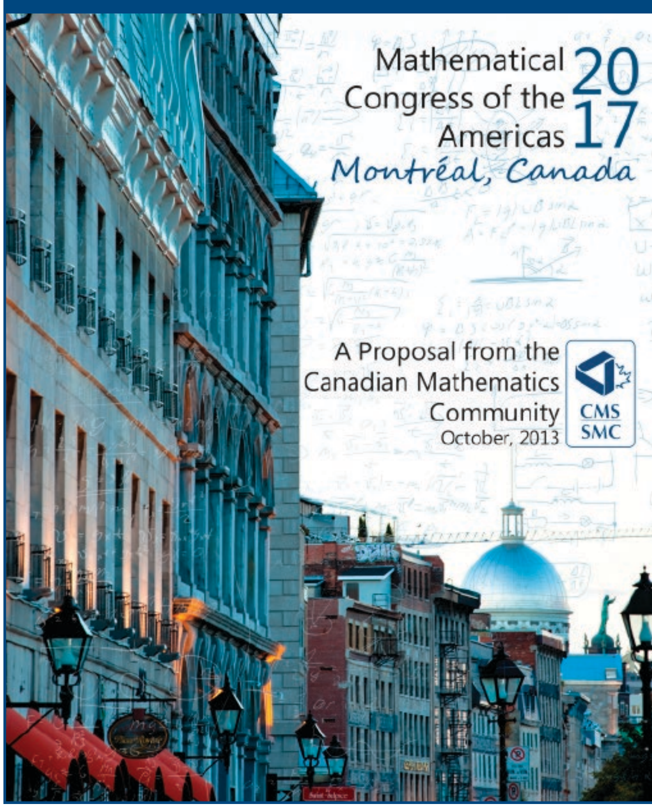
References

[1] A. Hof, *Diffraction by aperiodic structures, in The mathematics of long-range aperiodic order*, (R. V. Moody, ed.), Kluwer, 239-268, 1997.

[2] R. V. Moody, *Meyer sets and their duals, in: The mathematics of long-range aperiodic order*, (R. V. Moody, ed.), NATO ASI Series, Vol C489 , Kluwer, Dordrecht, 403-441, 1997.

[3] N. Strungaru, *On the Bragg Diffraction Spectra of a Meyer Set*, Canadian Journal of Mathematics 65, no. 3, 675-701, 2013.

MATHEMATICAL CONGRESS COULD BE COMING TO CANADA!



Following the successful launch this year of the Mathematical Congress of the Americas (MCA) in **Guanajuato, Mexico**, the interim MCA council solicited bids for hosting another congress in 2017. Two countries submitted bids: Argentina, and Canada.

The Canadian bid was orchestrated by Jacques Hurtubise (McGill) and prepared by the CMS, who worked with PIMS, Fields, CRM, AARMS, and others, to develop a dynamic program proposal. The proposal envisioned using both commercial and academic venues as well as opportunities to significantly expand session offerings to the over 1,000 expected students and researchers from throughout the Americas. A decision on the 2017 venue is expected by February 2014. If the Canada bid is successful, Montreal will host the event in late July 2017.

In the interim, the CMS is expected to represent Canada on the first official council when it is established in 2014.

CALL FOR NOMINATIONS

2014 Doctoral Prize

The CMS Doctoral Prize recognizes outstanding performance by a doctoral student. The prize is awarded to a candidate who received a Ph.D. from a Canadian university in the preceding year (January 1st to December 31st) and whose overall performance in graduate school is judged to be the most outstanding. Although the dissertation is the most important criterion (the impact of the results, the creativity of the work, the quality of exposition, etc.) other publications, activities in support of students and other accomplishments will also be considered.

Nominations that were not successful in the first competition will be kept active for a further year (with no possibility of updating the file) and will be considered by the Doctoral Prize Selection Committee in the following year's competition.

The CMS Doctoral Prize consists of a $500 award, a two-year complimentary membership in the CMS, a framed Doctoral Prize certificate and a stipend for travel expenses to attend the CMS meeting to receive the award and present a plenary lecture.

Nominations

Candidates must be nominated by their university and the nominator is responsible for preparing the documentation described below, and submitting the nomination to the Canadian Mathematical Society. Universities may nominate more than one candidate.

The documentation shall consist of:

- A curriculum vitae prepared by the student.
- A resumé of the student's work written by the student and which must not exceed ten pages. The resumé should include a brief description of the thesis and why it is important, as well as of any other contributions made by the student while a doctoral student.
- Three letters of recommendation of which one should be from the thesis advisor and one from an external reviewer. A copy of the external examiner's report may be substituted for the latter. More than three letters of recommendation are not accepted.

The deadline for the receipt of nominations is **January 31, 2014**. All documentation, including letters of recommendation, must be submitted electronically to **docprize@cms.math.ca**

APPEL DE MISES EN CANDIDATURE

Prix de doctorat 2014

La SMC a créé ce Prix de doctorat pour récompenser le travail exceptionnel d'un étudiant au doctorat. Le prix sera décerné à une personne qui aura reçu son dipôme de troisième cycle d'une université canadienne l'année précédente (entre le 1er janvier et le 31 décembre) et dont les résultats pour l'ensemble des études supérieures seront jugés les meilleurs.

La dissertation constituera le principal critère de selection (impact des résultats, créativité, qualité de l'exposition, etc.), mais ne sera pas le seul aspect évalué. On tiendra également compte des publications de l'étudiant, de son engagement dans la vie étudiante et de ses autres réalisations.

Les mises en candidature qui ne seront pas choisies dans leur première compétition seront considérées pour une année additionelle (sans possibilité de mise à jour du dossier), et seront révisées par le comité de sélection du Prix de doctorat l'an prochain.

Le lauréat du Prix de doctorat de la SMC aura droit à une bourse de 500 $. De plus, la SMC lui offrira l'adhésion gratuite à la Société pendant deux ans et lui remettra un certificate encadré et une subvention pour frais de déplacements lui permettant d'assister à la réunion de la SMC où il recevra son prix et présentera une conférence.

Candidatures

Les candidats doivent être nommés par leur université; la personne qui propose un candidat doit se charger de regrouper les documents décrits aux paragraphes suivants et de faire parvenir la candidature à la Société Mathématique du Canada. Les universités peut nommer plus d'un candidat.

Le dossier sera constitué des documents suivants :

- Un curriculum vitae rédigé par l'étudiant.
- Un résumé du travail du candidat d'au plus dix pages, rédigé par l'étudiant, où celui-ci décrira brièvement sa thèse et en expliquera l'importance, et énumérera toutes ses autres réalisations pendant ses études de doctorat.
- Trois lettres de recommandation, dont une du directeur de thèse et une d'un examinateur de l'extérieur (une copie de son rapport serait aussi acceptable). Le comité n'acceptera pas plus de trois lettres de recommandation.

Les candidatures doivent parvenir à la SMC au plus tard le **31 janvier 2014**. Veuillez faire parvenir tous les documents par voie électronique avant la date limite à **prixdoc@smc.math.ca**

2014 CMS Winter Meeting

December 5 - 8, 2014, Hamilton Sheraton

Host: McMaster University

CALL FOR SESSIONS

The Canadian Mathematical Society (CMS) and McMaster University welcomes and invites proposals for sessions for the 2014 Winter Meeting in Hamilton from December 5th to 8th, 2014. Proposals should include a brief description of the focus and purpose of the session, the expected number of speakers, as well as the organizer's name, complete address, telephone number, e-mail address, etc. All sessions will be advertised in the *CMS Notes*, on the web site and in the *AMS Notices*. Speakers will be requested to submit abstracts, which will be published on the web site and in the meeting program. Those wishing to organize a session should send a proposal to the Scientific Directors by March 30, 2014.

Scientific Directors:

Nicholas Kevlahan: **kevlahan@mcmaster.ca**

Deirdre Haskell: **haskell@math.mcmaster.ca**

Réunion d'hiver 2014 de la SMC

5-8 décembre 2014, Hamilton Sheraton

Hôte : Université McMaster

PROPOSITION DE SESSIONS

La Société mathématique du Canada (SMC) et l'Université McMaster vous invitent à proposer des sessions pour la Réunion d'hiver 2014 qui se tiendra à Hamilton du 5 au 8 décembre 2014. Ces dernières doivent inclure une brève description de l'orientation et des objectifs de la session, le nombre de conférenciers prévus ainsi que le nom, l'adresse complète, le numéro de téléphone et l'adresse courriel de l'organisateur. Toutes les sessions seront annoncées dans les *Notes de la SMC*, sur le site web et dans les *AMS Notices*. Les conférenciers devront présenter un résumé, qui sera publié sur le site web et dans le programme de la Réunion. Toute personne qui souhaiterait organiser une session est priée de faire parvenir une proposition aux directeurs scientifiques au plus tard le 30 mars 2014.

Directeurs scientifiques :

Nicholas Kevlahan : **kevlahan@mcmaster.ca**

Deirdre Haskell : **haskell@math.mcmaster.ca**

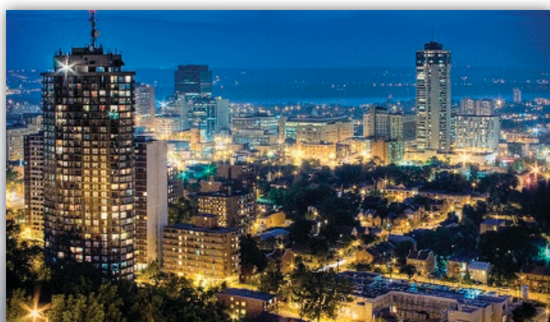


Photo : Dan Harper

December 6 – 9, 2013, The Ottawa Marriott
Host: University of Ottawa

www.cms.math.ca

6-9 décembre 2013, Hôtel Marriott Ottawa
Hôte : l'Université d'Ottawa

www.cms.smc.ca

Prizes | Prix

Jeffery-Williams Prize | Prix Jeffery-Williams
Zinovy Reichstein (University of British Columbia)

Doctoral Prize | Prix de doctorat
Marc Ryser (Duke University)

Graham Wright Award for Distinguished Service
Prix Graham Wright pour service méritoire
Harold Edward Alexander (Eddy) Campbell (University of New Brunswick)

Adrien Pouliot Award | Prix Adrien-Pouliot
John Grant McLoughlin (University of New Brunswick)

G. de B. Robinson Award | Prix G. de B. Robinson
Kenneth Davidson (University of Waterloo)
Alex Wright (University of Chicago)

Public Lectures | Conférences publiques

Manuel A. Báez (Carleton University)
John Conway (Princeton)
David Sankoff (University of Ottawa)
Robert Smith? (University of Ottawa)

Plenary Speakers | Conférences plénières

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Alexander Karp (Columbia)
Stephen Kudla (University of Toronto)
Mikhail Lyubich (SUNY Stony Brook)
Norbert Schappacher (Université de Strasbourg)
Daniel Wise (McGill University)

Scientific Director | Directeur scientifique

Vadim Kaimanovich (University of Ottawa)

Related Events | Événements liés

CMS **Town Hall Meeting and NSERC** will be present to address any questions from CMS members

The Canadian Mathematical Society invites you to their **awards banquet** to highlight exceptional performance in the area of mathematical research and education. Prizes will be awarded during the event. During the dinner you will be entertained by Daniel Richer, The **official town crier** of the national capital region, on a short history of the Outaouais.

There will be a **Saturday luncheon** hosted by the **CMS Women in Mathematics Committee** and is open to all women registered in the conference. There is no charge and no preregistration is required.

STUDC will host **two student workshops** on Friday, Dec 6th. The first session will be **Academic CV** Writing by Amanda Malloch from 1-2:30pm. Following a coffee break, there will be another session on **Intermediate/Advanced LaTeX** run by Léo Belzile from 3-4 pm.

SMC 2013
Réunion d'hiver
6 - 9, décembre 2013 • Ottawa Marriott, Ottawa

Regular Sessions | Sessions générales

Banach Algebras and Abstract Harmonic Analysis
Algèbres de Banach et analyse harmonique abstraite
Mehrdad Kalantar, Matthew Kennedy and Matthias Neufang (Carleton)

Complex Analysis and Complex Geometry
Analyse complexe et géométrie complexe
Damir Kinzebulatov(Fields), Rasul Shafikov (Western)

Connections Between Noncommutative Algebra and Geometry | Liens entre l'algèbre non commutative et la géométrie
Jason Bell (Waterloo), Colin Ingalls (New Brunswick)

Functional Differential Equations and Applications
Équations différentielles fonctionnelles et applications
Victor Leblanc (Ottawa)

Geometric Group Theory and Low Dimensional Topology | Théorie des groupes géométrique et topologie en basse dimension
Adam Clay (Manitoba), Mikael Pichot (McGill),
Eduardo Martinez-Pedroza (Memorial)

Groups and Algorithms | Groupes et algorithmes
Inna Bumagin (Carleton)

Harmonic Analysis on Groups over Local Fields
Analyse harmonique sur des groupes définis sur des corps locaux
Monica Nevins (Ottawa), Hadi Salmasian (Ottawa)

History and Philosophy of Mathematics
Histoire et philosophie des mathématiques
Tom Archibald (SFU)

Holomorphic dynamics and related topics
Dynamique holomorphe et sujets reliés
Ilia Binder and Michael Yampolsky (University of Toronto)

Infectious Disease Modelling
Modélisation de maladies infectieuses
Robert Smith? (Ottawa)

Inverse Problems in Imaging
Problèmes inverses en imagerie
Peter Gibson (York)

Lie Algebras, Representations and Cohomological Invariants | Algèbres de Lie, représentations et invariants cohomologiques
Yuli Billig (Carleton), Alistair Savage (Ottawa), Kirill Zaynullin (Ottawa)

Mathematical Genomics | Génomique mathématique
David Sankoff (Ottawa)

Matrix Theory in Quantum Information
Théorie des matrices en information quantique
David Kribs (Guelph), Rajesh Pereira (Guelph),
Sarah Plosker (Brandon)

Modular Forms and Physics
Physique et formes modulaires
Abdellah Sebbar (Ottawa)

Number Theory | Théorie des nombres
Damien Roy (Ottawa)

Operator Algebras | Algèbres d'opérateurs
Benoit Collins (Ottawa), Thierry Giordano (Ottawa),
Mehrdad Kalantar (Carleton), Matthew Kennedy (Carleton)

Partial Differential Equations and Biological Applications | Équations aux dérivées partielles et leurs applications en biologie
Frithjof Lutscher (University of Ottawa),
Xingfu Zou (Western University)

Random Walks and Geometry
Marches aléatoires et géométrie
Giulio Tiozzo (Harvard)

Representations of Algebra
Représentations d'algèbres
Vlastimil Dlab (Carleton), Ragnar-Olaf Buchweitz (Toronto),
Shiping Liu (Sherbrooke)

Symmetric Functions and Generalizations
Fonctions symétriques et généralisations
Angele Hamel (Laurier), Stephanie van Willigenburg (UBC)

Contributed Papers | Communications libres
Jason Levy (Ottawa)

AARMS-CMS Student Poster Session | Présentations par affiches pour étudiants
Leo Belzile (McGill), Cristina Rosu (Waterloo)

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