CMS NOTES de la SMC

Oct. - Nov. 2015

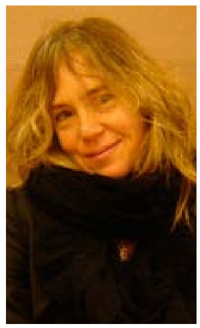
IN THIS ISSUE / DANS CE NUMÉRO

CMS SMC

Canadian Mathematical Society
Société mathématique du Canada

Vice-President's Notes / Notes du Vice-président

Chantal David, *(Concordia), Vice-President - Quebec / Vice-Présidente - Québec*



C'est avec plaisir que je prends la plume (ou le clavier...) pour écrire un texte pour les Notes de la SMC. La SMC est présente dans ma vie de mathématicienne depuis presque 30 ans, tout d'abord en tant qu'étudiante graduée à l'université McGill, et maintenant en tant que professeur à l'université Concordia. Les publications, les prix et autres activités de la SMC sont bien sur des outils essentiels pour le rayonnement de la communauté mathématique canadienne, mais ce sont pour moi les rencontres de la SMC qui sont le symbole de notre société. Pour ma part, et c'est une expérience que je sais que plusieurs de mes collègues canadiens partagent, ces rencontres régulières avec mes collègues canadiens de théorie des nombres, et d'autres domaines, ont marqué le rythme de mes années mathématiques, entre autres par les échanges réguliers avec certains collègues, et la découverte des nouveaux travaux de la relève mathématique canadienne.

La participation des étudiants gradués aux rencontres de la SMC a toujours été importante, et de plus en plus dans les derniéres années, ce dont nous pouvons nous réjouir. Au cours des années, la structure de ces rencontres a changé, avec entre autres la formation d'un comité scientifique fait d'experts canadiens et internationaux, en plus du comité scientifique régional. Nul doute que ces mesures grandiront encore l'impact des rencontres de la SMC, et leur popularité, mais sans en changer l'esprit.

Suite à la page 4

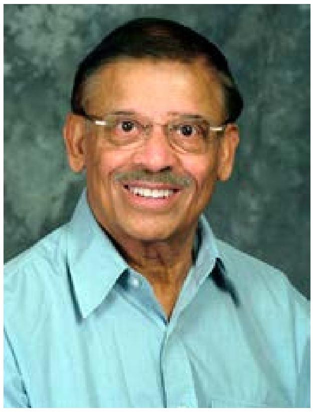
I am very pleased by this opportunity to write a text for the CMS Notes. As a mathematician, the CMS has been part of my life for almost 30 years, first as a graduate student from McGill University, and today as a professor at Concordia University. The publications, awards and other CMS activities are important and essential tools for the promotion of the Canadian mathematical community, but I personally think of the CMS meetings as the Society symbol. With regular discussions with some of my number theorist colleagues from all over Canada, and the discovery of the work from young Canadian mathematicians, these meetings marked the rhythm of my mathematician life, and I know that this is a feeling shared by many.

Graduate students attendance at CMS meetings has always been significant, especially in the past few years, and we can justifiably be pleased about that. Over the years, there were some changes in the format of the meetings, with, among others things, the creation of a scientific committee made of Canadian and international experts, in addition to the regional scientific committee. No doubt that these actions will increase the CMS meetings' impact as well as their popularity while keeping to the substance.

The 2015 CMS Winter Meeting in Montreal already promises to be a great success, reflecting the strength of the Canadian mathematical community. A new initiative will also be launched in Montreal, and child care will be offered to the attendees, which will allow a greater participation to the CMS scientific activities. Given the quality of the plenary lectures, and the record number of scientific sessions planned for the Montreal meeting, this initiative will be appreciated by many. To conclude, I would like to welcome all those attending the Montreal meeting, and looking forward to see you in Montreal!

Competitions

Srinivasa Swaminathan, *Dalhousie University*



Many people classify solving math problems right up there with jumping out of airplanes, camping in sub-zero weather, and speaking in public as activities they would studiously avoid. But each of these activities has its fans, not least solving math problems.

In order to encourage those students who do have an aptitude towards solving math problems which are not part of any prescribed course, contests are held periodically in schools, colleges and university departments. The Canadian Mathematical Society supports this and holds math competitions annually. It encourages regional and international competitions by providing a wide variety of resources for students to prepare for competitions.

The most prestigious competition at the high school level is the International Mathematical Olympiad (IMO), which has been held annually since 1959. The IMO features six questions over two days; three on each day, for more than four hours of competition. The 56th competition was held at Chiang Mai, Thailand in July this year, with more than 600 high school competitors from 104 countries. Canada finished ninth overall, ending up in the top ten for the third time in the past four years. Some Canadian individual results were even more impressive: four participants earned bronze medals, and Zhuo Qun (Alex) Song and Kevin Sun both earned gold medals. Alex, indeed, achieved a rare perfect score.

Among those who coached Team Canada this year was Jacob Tsimerman of the University of Toronto, who himself had written this competition in 2004 and finished first. Before the competition the six-member coaching team assembled at the Banff International Research Station (BIRS) for a couple of weeks for intensive training.

There has been debate for years about whether this is "real" mathematics, or an artificial spinoff, no more closely related to mathematical research that Olympic target shooting is to hunting. It is certainly true that some excellent mathematicians have had little time for this style of problem solving, and that some high-ranked contestants have ended up in non-mathematical careers. But previous participants in the IMO have gone on to win prestigious international awards such as the Fields medal; if there were no correlation, it's unlikely in the extreme that two such small sets would intersect. It is reported that Alex Song will begin college studies at Princeton University and hopes to study pure mathematics further. We wish him - and all the contestants - well.

Concours

Srinivasa Swaminathan, *Université Dalhousie*

Beaucoup de gens comparent la résolution de problèmes mathématiques au parachutisme, au camping à des températures sous zéro et aux exposés oraux, telles des activités qu'ils prennent soin d'éviter. Toutefois, chacune d'entre elles compte des adeptes et la résolution de problèmes mathématiques n'y échappe pas.

Des concours sont régulièrement organisés dans les écoles, les cégeps et les départements universitaires pour encourager les étudiants d'autres programmes d'études qui démontrent une aptitude en résolution de problèmes mathématiques. La Société mathématique du Canada appuie ce mouvement et organise des concours de mathématiques chaque année. Elle encourage les concours régionaux et internationaux en fournissant une grande diversité de ressources aux étudiants afin qu'ils s'y préparent.

Le concours le plus prestigieux au niveau secondaire est l'Olympiade internationale de mathématiques (OIM), qui a lieu chaque année depuis 1959. L'OIM se déroule sur deux jours et comprend six questions; trois chaque jour, pendant plus de quatre heures. Le 56e concours s'est tenu à Chiang Mai, Thaïlande en juillet dernier et comptait plus de 600 concurrents d'écoles secondaires originaires de 104 pays. Le Canada a terminé au neuvième rang, se classant dans les dix meilleurs pour la troisième fois au cours des quatre dernières années. Certains participants canadiens ont obtenu des résultats encore plus impressionnants : quatre ont gagné des médailles de bronze, et Zhuo Qun (Alex) Song et Kevin Sun ont chacun remporté une médaille d'or. Alex a effectivement présenté une note exceptionnellement parfaite.

Jacob Tsimerman de l'Université de Toronto compte parmi ceux qui ont formé l'équipe canadienne cette année, lui-même avait terminé premier lors de ce concours en 2004. Avant le concours, les six membres de l'équipe d'accompagnement se sont réunis à la Banff International Research Station (BIRS) pendant quelques semaines pour une formation intensive.

Un débat a cours depuis des années à savoir s'il s'agit « vraiment » de mathématiques ou d'un dérivé artificiel, qui ne serait pas plus lié à la recherche mathématique qu'est le tir sportif des Jeux olympiques à la chasse. Il est vrai que certains excellents mathématiciens ont consacré peu de temps à cette méthode de résolution de problèmes et que certains participants très bien classés n'ont pas poursuivi une carrière en mathématiques. Par contre, d'anciens participants de l'OIM ont remporté des prix internationaux prestigieux tels que la médaille Fields. S'il n'existe aucune corrélation, il semble extrêmement improbable que de tels cas se croisent. Nous avons appris qu'Alex Song commencera des études universitaires à l'Université de Princeton en vue d'étudier les mathématiques pures. Nous lui souhaitons, ainsi qu'à tous les participants, la meilleure des chances.

Letters to the Editors

The Editors of the NOTES welcome letters in English or French on any subject of mathematical interest but reserve the right to condense them. Those accepted for publication will appear in the language of submission. Readers may reach us at the Executive Office or at **notes-letters@cms.math.ca**

Lettres aux Rédacteurs

Les rédacteurs des NOTES acceptent les lettres en français ou en anglais portant sur n'importe quel sujet d'intérêt mathématique, mais ils se réservent le droit de les comprimer. Les lettres acceptées paraîtront dans la langue soumise. Les lecteurs peuvent nous joindre au bureau administratif de la SMC ou à l'adresse suivante : **notes-lettres@smc.math.ca**.

NOTES DE LA SMC

Les Notes de la SMC sont publiés par la Société mathématique du Canada (SMC) six fois par année (février, mars/avril, juin, septembre, octobre/novembre et décembre).

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Note aux auteurs : indiquer la section choisie pour votre article et le faire parvenir au Notes de la SMC à l'adresse postale ou de courriel ci-dessous.

Les Notes de la SMC, les rédacteurs et la SMC ne peuvent être tenus responsables des opinions exprimées par les auteurs.

CMS NOTES

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No responsibility for the views expressed by authors is assumed by the CMS Notes, the editors or the CMS.

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CMS Notes
Volume 47 No. 5, October 2015

La rencontre de Montréal en décembre 2015 promet déjà d'être un grand succès, témoignant de la vitalité de la communauté mathématique canadienne. Une nouvelle initiative sera aussi lancée à Montréal, soit des services de garderie offerts aux participants, mesure qui permettra une plus large participation aux activités scientifiques de la SMC. Étant donné la qualité des conférences plénières, et le nombre records de sessions scientifiques prévues pour la rencontre de Montréal, nul doute que cette mesure sera appréciée par plusieurs participants.

Je termine donc en souhaitant d'avance la bienvenue à tous ceux et celles qui participeront à la rencontre de Montréal, et au plaisir de se voir à Montréal!

CMS Member Profile / Profil membre de la SMC

Anna Stokke

HOME: University of Winnipeg, Winnipeg, Manitoba

CMS MEMBER SINCE: 2003

RESEARCH: Representation theory, algebraic combinatorics

SELECTED ACHIEVEMENTS: I recently received two awards for my public advocacy and community work in math education: the YMCA-YWCA Women of Distinction Award in the Community Activism and Social Enterprise category (2015) and the University of Winnipeg's Clarence Atchison Award for Excellence in Community Service (2015). I co-founded an advocacy group, WISE Math, and a non-profit organization, Archimedes Math Schools.

HOBBIES: Walking, knitting, piano

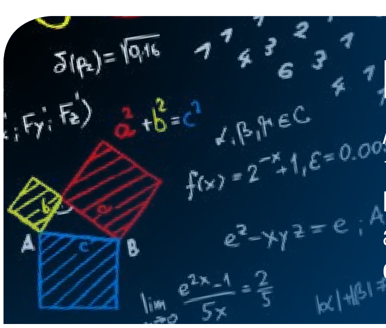
LATEST BOOK READ: The Girl on the Train by Paula Hawkins

LATEST PUBLICATION: Increasing tableaux, Narayana numbers and an instance of the cyclic sieving phenomenon, with Timothy Pressey and Terry Visentin (to appear).

WHAT I WOULD CHANGE (ABOUT THE CMS): It would be great to have more funding opportunities available for math outreach activities. It might also be helpful to establish a mentorship program where members offer advice and give feedback to those applying for NSERC Discovery Grants.

CMS ROLES: Member of the Board of Directors, 2011 - 2015

WHY I BELONG TO THE CMS: I think it is important to support Canada's national math organization, which brings mathematicians across the country together. The CMS has been very supportive of my work, too! We have received financial support from the CMS for Archimedes Math Schools, which has allowed us to offer math camps in inner-city Winnipeg.



Calendar Notes brings current and upcoming domestic and select international mathematical sciences and education events to the attention of the CMS readership. Comments, suggestions, and submissions are welcome.

Johan Rudnick, *Canadian Mathematical Society, (director@cms.math.ca)*

Le calendrier des événements announce aux lecteurs de la SMC les activités en cours et à venir, sur la scène pancanadienne et internationale, dans les domaines des mathématiques et de l'enseignement des mathématiques. Vos commentaires, suggestions et propositions sont le bienvenue.

Johan Rudnick, *Société mathématique du Canada (directeur@smc.math.ca)*

OCTOBER 2015 OCTOBRE

2-4 FIELDS **5th International Conference on Mathematical Modeling and Analysis of Populations in Biological Systems (ICMA-V)** University of Western Ontario, London, Ont.

5-6 FIELDS **Industrial-Academic Workshop on Optimization in Finance and Risk Management**, The Fields Institute, Toronto, Ont.

10-11 CRM 2015 **Montreal-Toronto Workshop in Number Theory**, CRM, Montreal, Que.

12-14 **SIAM Conference on Geometric and Physical Modeling (GDSPM15)** Salt Lake City, Utah

16-17 CRM **59e Congrès de l'AMQ**, Montreal, Que.

19-22 **FIELDS Medal Symposium**, The Fields Institute, Toronto, Ont.

19-23 CRM **Workshop: Applications of AdS/CFT to QCD and condensed matter physics**, CRM, Montreal, Que.

19-23 CRM **GRASTA 2015 - 7th Workshop on GRAph Searching, Theory and Applications & 5th workshop on Moving and Computing (MAC)**, CRM, Montreal, Que.

21 **Panorama of Mathematics: A Conference of the Hausdorff Center**, Bonn, Germany

23 **PIMS/UBC Distinguished Colloquium: Yakov Sinai**, University of British Columbia, Vancouver, B.C.

25-26 CRM-CANSSI **Workshop on Statistical inference for complex surveys with missing observations**, CRM, Montreal, Que.

26-31 FIELDS **Workshop on Linear Computer Algebra and Symbolic-Numeric Computation**, The Fields Institute, Toronto, Ont.

NOVEMBER 2015 NOVEMBRE

1-6 BIRS **Women in Geometry** Banff, Alta.

22-27 **BIRS First Nations Math Education** Banff, Alta.

DECEMBER 2015 DÉCEMBRE

4-7 **2015 CMS Winter Meeting / Réunion d'hiver de la SMC 2015**, Hyatt Regency, Montreal, Que.

7-10 SIAM **Conference on Analysis of Partial Differential Equations**, Scottsville, Arizona

7-11 **39th Australasian Conference on Combinatorial Math & Combinatorial Computing**, Brisbane, Australia

7-16 FIELDS **Workshop on Algebra, Geometry and Proofs in Symbolic Computation**, The Fields Institute, Toronto, Ont.

14-18 **Geometric & Categorical Representation Theory**, Mooloolaba, Queensland, Australia

JANUARY 2016 JANVIER

6-9 **AMS/MAA Joint Mathematics Meeting**, Washington State Convention Centre, Seattle, WA

9-13 CRM **Workshop: Moduli spaces, integrable systems, and topological recursions**, Montreal, Que.

10-15 BIRS **Creative Writing in Mathematics and Mathematical Sciences**, Banff, Alta.

10-16 BIRS **19th Conference on Quantum Information Processing**, Banff, Alta.

31-Feb 5 CANSSI **Mathematical and Statistical Challenges in Neuroimaging Data Analysis**, BIRS, Banff, Alta.

Book Review Notes brings interesting mathematical sciences and education publications drawn from across the entire spectrum of mathematics to the attention of the CMS readership. Comments, suggestions, and submissions are welcome.

Karl Dilcher, *Dalhousie University (notes-reviews@cms.math.ca)*

Les critiques litéraires présent aux lecteurs de la SMC des ouvrages intéressants sur les mathématiques et l'enseignement des mathématiques dans un large éventail de domaines et sous-domaines. Vos commentaires, suggestions et propositions sont le bienvenue.

Karl Dilcher, *Dalhousie University (notes-critiques@smc.math.ca)*

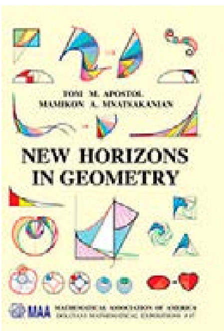
New Horizons in Geometry

by Tom M. Apostol and Mamikon A. Mnatsakanian

Mathematical Association of America, 2013

ISBN: 978-0-88385-354-2

Reviewed by ***Robert Dawson****, Saint Mary's University*



Over the last two decades, the authors of this book have written many geometry papers in popular mathematics journals such as the *American Mathematical Monthly, Mathematics Magazine,* and the *Mathematical Gazette.* This book collects and expands – hugely – upon these.

Apostol is, of course, widely known for his classic calculus textbooks, described by many as "the books I'd like to teach out of, but not *this* class." In a world of proverbially isomorphic calculus textbooks – the joke is that an instructor can pick up the wrong Calc I textbook, take it to class, give a lecture from Section 4.2, and neither instructor nor students will notice – Apostol's book famously flouts tradition – and follows history – by introducing integration before differentiation. An immediate corollary of this is that his textbook does not immediately reduce integration to antidifferentiation and proceed as if they had always been one and the same thing throughout history.

His collaborator, Mamikon Mnatsakanian (who goes professionally by his given name) is an Armenian-born mathematician who was stranded in the United States as a result of the collapse of the Soviet Union. He had, many years before, become interested in "Cavalieri-style" proofs of theorems about areas between curves. His collaboration with Apostol has resulted in many beautiful theorems that Archimedes might have appreciated and that will surprise the modern mathematician with their elegance. The book under review collects these – almost 500 pages of them.

The first significant theorem in the book (a few pages in) is *Mamikon's Theorem for Oval Rings.* This gives the flavor of the whole book nicely. It states that if a segment of length R moves with one endpoint on a given curve and tangent to it, the area of the oval ring it sweeps out is independent of the curve and is equal to πr^2. This is proved here by Cavalieri-style methods, equating one set of infinitesimal wedges that make up the ring to another that make up a disc.

These "protocalculus" techniques might seem to be of limited applicability, but not so. Because they work with no algebraic description of the curves they work on, they are very robust and generally applicable. The authors use the same techniques to deal with curves – cycloids and trochoids – generated by rolling circles, then in the next chapter go on to use the same techniques to study the piecewise-smooth nonalgebraic curves traced by rolling polygons: "cyclogons" and "trochogons." (If you're unfamiliar with these names, don't worry. This book is full of neologisms, for the very good reason that it deals, extensively, with shapes and constructions unusual enough to have no established names. The reader may expect to make the acquaintance of autogons, tanvolutes, and fiber-elliptic domes!)

There are also chapters on "Circumgons and Circumsolids," "Punctured Containers," (extending Archimedes' work on volumes) "Unwrapping Curves," "New Descriptions of Conics," "Trammels," "Isoperimetric and Isoparametric Problems," "Arclength and Tanvolutes," "Centroid," "Sums of Squares" and other topics. Each chapter is introduced with a selected "teaser" problem and has its own detailed table of contents.

The book is amply – no, lavishly – illustrated, with (according to the back cover, and an *ad hoc* sample supports this) a thousand colored illustrations. The color is not just there to be pretty, as is sometimes the case; colour-coding is used really well to illustrate corresponding elements.

I found little to criticize in this book. In Section 2.7, areas of trochoidal regions are given to only two significant figures: *e.g.,* 3.4, 2.6. The reader may legitimately wonder whether these are exact rational numbers (definite integrals surprise us this way sometimes) or (as seems to be the case) unusually imprecise decimal approximations.

At $75US (list) this book is a little more expensive than some Dolciani publications; but its length, thoroughness and novelty makes this a worthwhile purchase for many individuals. Researchers in geometry will surely find something worthwhile between its covers, as will calculus teachers. University and high school libraries should certainly have a copy. But its very elementary and visual approach to what might be seen as advanced problems in integral calculus, integral geometry and differential geometry make it a worthwhile book for the keen undergraduate mathematician, or even the particularly able high school student. This could be an ideal competition prize.

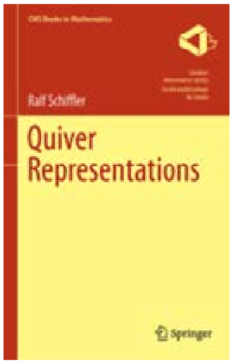
Quiver Representations

by Ralf Schiffler

CMS Books in Mathematics, Springer, 2014

ISBN: 978-3-319-09204-1

Reviewed by ***Graham Leuschke****, Syracuse University*



When we use two different names for the same thing, it's often just a linguistic hiccup, but sometimes because we are thinking about the same thing in two fundamentally different ways. The morning star Phosphorus and the evening star Hesperus are identical, being both just the planet Venus, but they serve different purposes in the world. A *quiver* is nothing but a finite directed graph, but the questions we ask about quivers, and the uses we put them to, are essentially disjoint from digraph theory. Instead, quivers are a source of examples and a model for representation theory, as well as being a tool in their study. Using only the standard ingredients of linear algebra, the theory of quiver representations contains the entire study of modules over finite-dimensional algebras over a field. Quiver representations have applications in algebraic geometry, Lie algebras and quantum groups, and even string theory in mathematical physics, but one could teach a semester course on quivers to first- or second-year graduate students.

This book is intended for exactly that use. Based on a course given by the author at a graduate/advanced undergraduate summer school of the Atlantic Association for Research in the Mathematical Sciences (AARMS), it picks up from where a first course in abstract linear algebra might leave off. There are other textbooks covering the representation theory of finite-dimensional algebras, but as far as I know this is the first one to use the quiver setting as the central focus. The language of algebras and modules is not used at all until Chapter 4.

The text is clearly and engagingly written, with many examples. In fact, examples serve as an organizing principle for the book, since the author makes a point of getting as quickly as possible to examples of the construction of the Auslander-Reiten quiver of a given quiver. The Auslander-Reiten quiver is the main organizational tool for describing the module theory of a finite-dimensional algebra, or equivalently the indecomposable representations of a quiver. In addition to the modules, the Auslander-Reiten quiver gives explicit information about the morphisms between them and the extensions among them, and furthermore, in the special case where the quiver has only finitely many indecomposable representations up to isomorphism, contains all possible information about them. Chapter 2 contains the relevant definitions (projective and injective representations, the Nakayama functor, the Auslander-Reiten translation), as well as a direct, explicit proof that representations of a quiver form a hereditary category, and Chapter 3 presents several different algorithms to compute the Auslander-Reiten quiver of two classes of quivers.

These first three chapters, which constitute Part I of the book, could be used for independent study or a seminar for students with almost no background in the area. There are a couple of mysterious things like meshes and the Nakayama automorphism which must either be taken on faith or would benefit from the involvement of a knowledgeable instructor. Explaining these concepts more completely and justifying the computations in Chapter 3 requires quite a bit more algebraic machinery, so Part II circles back around to recast the theory in terms of algebras and modules. Chapter 4 covers the basics required from the theory of finite-dimensional algebras, while Chapter 5 introduces bound quiver algebras and proves that studying quiver representations is the same thing as studying modules over bound quiver algebras.

The level of the material increases steadily as the book proceeds. Chapter 6 presents several constructions of new algebras (and hence quivers) from old, ranging from the classical trivial extensions to the relatively cutting-edge cluster tilting. A lovely feature of the book is that at this point some of the examples of Chapter 3, which seemed a bit *ad hoc* at the time, turn out to be exactly what is required. Specifically, cluster-tilted algebras of type $\mathbb{A}_n$ (resp., $\mathbb{D}_n$) are bound quiver algebras associated to triangulations of the (n+3)-gon (resp. of the punctured n-gon), as described in Chapter 3.

Chapter 7 contains the theorems and proofs justifying the examples of Auslander-Reiten theory appearing in Chapter 3. This material is more advanced, but the focus remains on examples, so that this material too could appear in an introductory graduate course. Chapter 8, which is largely independent of the rest of Part II, contains a complete proof of Gabriel's Theorem, which classifies those quivers Q having only finitely many indecomposable representations up to isomorphism: they are those for which the underlying (undirected) graph is one of the Dynkin diagrams $\mathbb{A}$, $\mathbb{D}$, and $\mathbb{E}$.

This book takes a unique perspective on the representation theory of finite-dimensional algebras, and is the most accessible introduction I have seen. A student, or a mathematician from another field who is interested in applications of quiver representations to their own field without spending a year learning background, can very quickly get to the modern forefront of the area. Along the way, the reader can pick up the first steps in category theory, which are introduced when required, making the text almost completely self-contained. Throughout, the author takes full advantage of the concreteness of the subject, which makes it an excellent bridge to present-day research.

CMS Notes
Volume 47 No. 5, October 2015

Education Notes brings mathematical and educational ideas forth to the CMS readership in a manner that promotes discussion of relevant topics including research, activities, and noteworthy news items. Comments, suggestions, and submissions are welcome.

Jennifer Hyndman, *University of Northern British Columbia (hyndman@unbc.ca)*

John McLoughlin, *University of New Brunswick (johngm@unb.ca)*

Les articles sur l'éducation présente des sujets mathématiques et des articles sur l'éducation aux lecteurs de la SMC dans un format qui favorise les discussions sur différents thèmes, dont la recherche, les activités et des nouvelles d'intérêt. Vos commentaires, suggestions et propositions sont le bienvenue.

Jennifer Hyndman, *University of Northern British Columbia (hyndman@unbc.ca)*

John McLoughlin, *University of New Brunswick (johngm@unb.ca)*

As I write this article I am recovering from the second day of a five day intensive dance camp for adults. I have just experienced being a student with four different teachers covering six different styles of movement. These teachers are at four different points in their careers but they all provided amazing experiences for the dancers. My development as a teacher has been strongly influenced by my experiences in the dance classroom and my growth continues to happen as I watch what the teachers do and how the students respond, and how I respond. I then translate that into how I see mathematics teaching and our students' experiences.

Who are our teachers?

Jennifer Hyndman, *University of Northern British Columbia, hyndman@unbc.ca*

Here I describe X different mythical mathematicians you might find in a university or college and invite you to decide if you fit in my classification system.

The Technical Expert

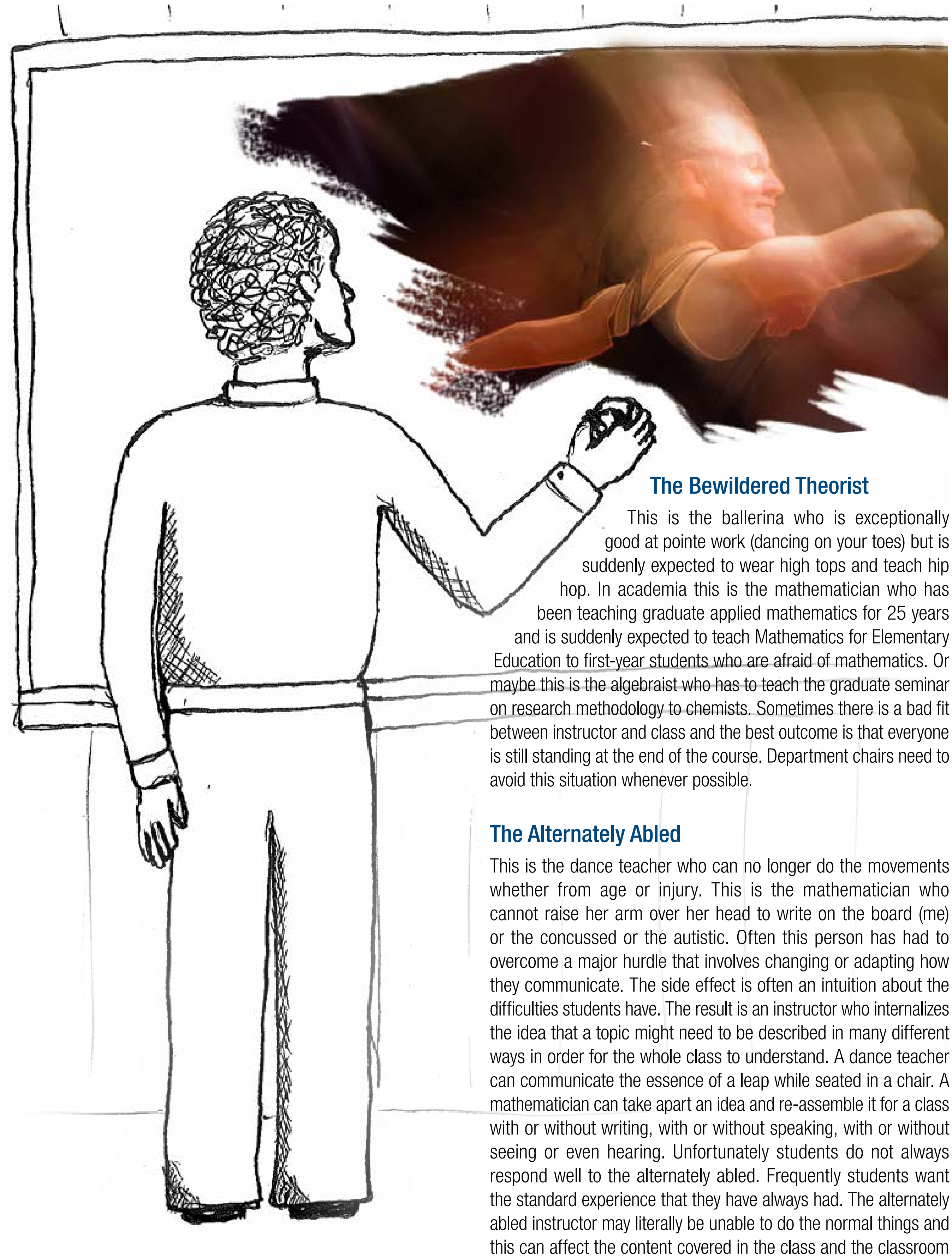
This teacher knows the subject inside out and backwards and at all levels. They might be teaching the same material at several levels. For example, in Precalculus we add integers, in Calculus we add derivatives, in Ring Theory we add polynomials or matrices, in Universal Algebra we decide what adding should mean. The technical expert in a dance class can demonstrate a complicated movement and then talk a novice into doing it. The advantages of being a technical expert include never having to say "I don't know." More importantly, the technical expert can tell a student where the material is going, when it might be useful, what it will develop into, and who the important people in the area are. In fact, the technical expert probably knows the important people and might even be an important person. The disadvantages include being easily sidetracked into the marvelous little details that are really important but do not really belong in the particular course being taught.

The Anxious Researcher

This is the dancer who really wants a career on the stage but needs to earn a living. Most tenure-track/tenured professors have days when they fall into this category. The most amazing problem would have been solved if they had had five more minutes before class. The anxious researcher's passion is doing mathematics. And they are extremely passionate about mathematics. They will tell almost anyone about their favourite project. Unfortunately it is usually mathematics at a level well above what they have to teach. The anxious researcher bounces about at the front of the classroom trying very hard to focus on the day's topic but often skips the small details that the average student needs. Sometimes the anxious researcher gets to teach an upper year class or a graduate class in their own area. In that case the students have the opportunity to see live mathematics happen. If the students are good enough they understand what is happening and also learn to do it. A side effect is that these students often turn into anxious researchers.

The Exhausted Multitasker

There is a certain joy to standing in front of a classroom and participating in the learning experience of a room full of students. The exhausted multitasker gets energy from the classes they teach no matter what else is happening elsewhere in their life. This is the dancer who also runs the studio, or is an in-demand choreographer, or has children, or even has a second job. In the academic world this might be the department chair or someone with a complicated family life. Sometimes it is the post-doc who is continually looking for a new job. The exhausted multitasker teaches because they really want to be teaching. The downside of the exhausted multitasker is that they might miss a panicked email from a student or forget to give assignments to a marker. However, while they are teaching they are also multitasking. They are disseminating information, responding to questions, choosing what direction to go next, reading the anxiety level of the class, watching the barometer students, and getting excited as they go. (A barometer student is a person who usually matches the understanding level of the whole class.) The level of attention the multitasker pays to the class means they see all the little light bulbs going on and off above the students' heads. They know when someone has had a moment of insight. Those "I get it" moments of the students give the exhausted multitasker the energy to go out into the less structured world and tick things off the never-ending task list.



The Bewildered Theorist

This is the ballerina who is exceptionally good at pointe work (dancing on your toes) but is suddenly expected to wear high tops and teach hip hop. In academia this is the mathematician who has been teaching graduate applied mathematics for 25 years and is suddenly expected to teach Mathematics for Elementary Education to first-year students who are afraid of mathematics. Or maybe this is the algebraist who has to teach the graduate seminar on research methodology to chemists. Sometimes there is a bad fit between instructor and class and the best outcome is that everyone is still standing at the end of the course. Department chairs need to avoid this situation whenever possible.

The Alternately Abled

This is the dance teacher who can no longer do the movements whether from age or injury. This is the mathematician who cannot raise her arm over her head to write on the board (me) or the concussed or the autistic. Often this person has had to overcome a major hurdle that involves changing or adapting how they communicate. The side effect is often an intuition about the difficulties students have. The result is an instructor who internalizes the idea that a topic might need to be described in many different ways in order for the whole class to understand. A dance teacher can communicate the essence of a leap while seated in a chair. A mathematician can take apart an idea and re-assemble it for a class with or without writing, with or without speaking, with or without seeing or even hearing. Unfortunately students do not always respond well to the alternately abled. Frequently students want the standard experience that they have always had. The alternately abled instructor may literally be unable to do the normal things and this can affect the content covered in the class and the classroom

Photo : Lisa Dickson
Drawing : Heather Grant McLoughlin

dynamics. If you want to experience being alternately abled, teach a class with your hands tied behind your back or with a blindfold on.

The Lost Soul

Sometimes life takes its toll and things happen. It might be an accident, ill health, a divorce or an unexpected death. Often this is a temporary state that lasts for a term or two. Maybe assignments get lost or the instructor forgets to go to a lecture. Eventually either things return to "normal" or the lost soul becomes an alternately abled teacher. At the time, this teacher needs the support of those around him but he doesn't always tell people what is happening. The usual type A academic has difficulty accepting help but this is the time colleagues need to step up and be there.

The Professional Pedagogue

Some dancers grow up wanting to teach rather than be on the stage. The same goes for some mathematicians. She has dreamed of teaching since she was small. She is fascinated by the process of learning and gets joy from watching others learn. There is now a place for the professional pedagogue at universities with the development of tenure-track teaching streams that require pedagogical research. She is usually current with the latest teaching techniques and is willing to try things in the classroom on the possibility that the students will learn more. Students might go into shock over what they are expected to do because "no one has ever asked them to do *that* before." Sometimes a teaching innovation is actually a bad idea and the professional pedagogue is the instructor who finds out how bad it is. Think-pair-share, inverted classrooms, clickers are all phrases that occur in a casual conversation with the professional pedagogue.

The Unintended Pedagogue

Some dancers grow up thinking they want to be on stage. They love the limelight and the razzle dazzle of performance. Then they are put in front of a class because of their stage skills and something odd happens to them. They discover that the performance they give in the classroom is more interesting than that on stage and they want to know why. Mathematically the unintended pedagogue really wants to be a star researcher but gets side-tracked by a conversation in a hallway or a moment in a class. He might go to a teaching workshop out of curiousity and discover himself teaching a workshop at the next event. Students rate his classes highly and nominate him for a teaching award. Talking to people about teaching starts to distract him from mathematical research and he's OK with that.

The Excited Newbie

In a big dance studio the up-and-coming young dancers get to teach the little children so dancers can be relatively young the first time they are put in charge of others. This is changing for academics. In the past, a faculty member's first class might be part of their first tenure-track position. Now there is more mentoring and training to get graduate students and post-docs into the classroom. However, most of us probably remember the first time we were given a course to teach and had complete control of the course. Terror might have been the primary emotion followed closely by excitement and a sense of being a fraud. Some award-winning teachers still feel like excited newbies on the first day of every course. The true excited newbie doesn't now when to stop and is at risk of burning out in a semester or two. They want every lecture to be perfect and every student to get an A. An excited newbie's first experience with a student who truly doesn't care about the course can be deeply disheartening for them. Most people who get to the excited newbie stage of being an academic have an underlying passion for their subject but have not yet learned that some people are not passionate about anything – let alone education.

The Passionate Novice

She's been teaching for a few years and is really starting to find her feet as an instructor. The intense joy a dancer feels upon a fleeting moment of perfection can be easily recalled and described by the passionate novice. The excitement of new choreography (or of new mathematics) generates a resonance within the classroom and the passionate novice can bring it to a crescendo. Unfortunately the passionate novice is likely to reinvent the wheel and make the same mistakes that others have already made and learned from. She might not actually be an expert and sometimes unknowingly makes a mistake that has to be undone the next day. Or she hasn't yet learned to say *I don't know*. But the students love to be in her class as they are carried along by the enthusiasm and can't help learning something. Classes are never boring and students want this teacher for their next course.

What Is The Point Of Classifying Teachers?

Aside from being an amusing thing to do and satisfying the usual compulsion of mathematicians to classify objects, it can be useful. Teachers in the different categories have very different needs and can offer very different levels of help to others. Pairing a passionate novice with a professional pedagogue is clearly going to be beneficial. The novice will learn concrete skills and the pedagogue will have an eager pupil. An anxious researcher might turn into an unintended pedagogue with the right guidance. A lost soul needs to be led back to mathematical happiness.

At different stages of our academic careers all teachers have been in one of these categories. I have probably been in all of these categories at some point which is why I can describe these teachers. Other teachers that I have not described include the reluctant retiree, the gleeful retiree, the old deadwood, the happy hinter and the inspiring coach. I encourage you to decide who you are right now and who you have been. Are you where you want to be? What is the description of the teacher that you want to be? How do you get there from here?

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The n-body problem and its symmetries

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Consider n point masses m_i with position vectors $\boldsymbol{q}_i$, $i = 1 \dots n$, in an empty euclidean space $\mathbb{R}^d$ with $d > 0$, i.e. consider a point $\boldsymbol{q}$ in the configuration space $\mathbb{F}_n(\mathbb{R}^d)$. The n-body problem describes the dynamics of the n points subject to forces given by an interaction potential of type $U(\boldsymbol{q}) = \sum_{i,j} m_i m_j \varphi(\boldsymbol{q}_i, \boldsymbol{q}_j)$ for a smooth function φ. More precisely, it can be formulated as the Cauchy initial value problem of the Newton-Lagrange equations

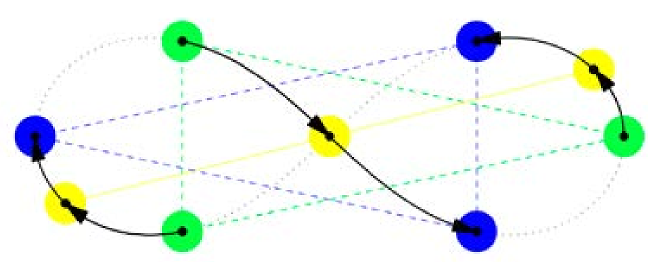
$$m_i \boldsymbol{q}_i = \frac{\partial U}{\partial \boldsymbol{q}_i}, \; i = 1, \dots, n.$$

The case with $\varphi(\boldsymbol{q}_i, \boldsymbol{q}_j) = |\boldsymbol{q}_i - \boldsymbol{q}_j|^{-1}$ models the gravitational interaction of n celestial bodies. Other choices of φ give rise to other potentials: homogeneous of degree $-\alpha$, or sum of such, or anisotropic, to name a few that are less famous, nevertheless not less interesting. At the crossroad of diverse fascinating fields and research areas, the n-body problem has been studied from many different angles and perspectives, and extensively described and analyzed in a vast literature. I am going to highlight only a tiny fraction of it, the one I happened to work on in the past years.

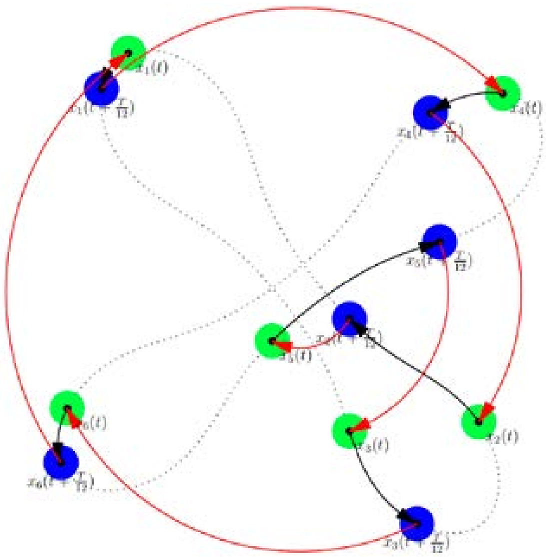
Periodic orbits and symmetries. If the mutual interaction potential depends only on the mutual distances, that is $\varphi(\boldsymbol{q}_i, \boldsymbol{q}_j) = \psi(|\boldsymbol{q}_i - \boldsymbol{q}_j|)$, then the problem, or more precisely the force field, the potential function and therefore the Lagrangian, has some natural symmetries: isometries of the euclidean space, isometries in the time line, and permutations of the particles (for those with equal masses). Such symmetries, when containing 1-dimensional Lie groups, yield first integrals and conservation laws by Noether's theorem. But the role of symmetries does not end here. Suitable symmetric orbits, namely those such that $\boldsymbol{q}(t + T) = \boldsymbol{q}(t)$ for a period $T > 0$, are called T-periodic orbits and show an elusive nature. Elusive since only a few have been proved to exist. Here "a few" means "significantly fewer than one would like to have," since the common believe that periodic orbits are countless can be traced back to the well-known, yet less famed, Poincaré conjecture for the restricted 3-body problem.

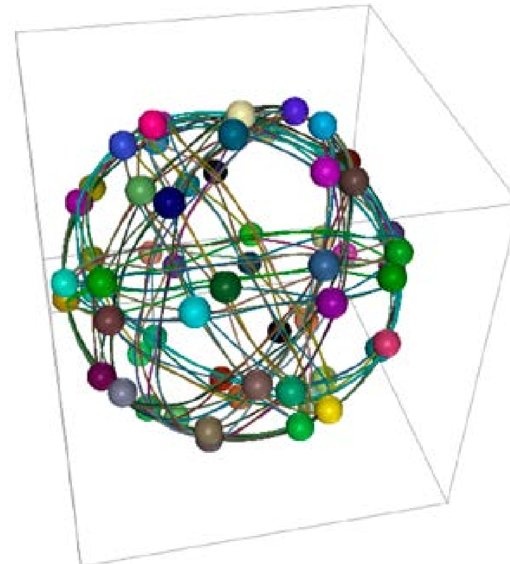
The 2-body problem is integrable and has been completely solved. For the Newtonian case, solutions are conical trajectories, ellipses, parabolas and hyperbolas; they are called Keplerian orbits. Already for the 3-body problem things are of a different nature. Even if the concept of "solution" has somehow changed over the years, the understanding of its dynamics is far from complete. Nevertheless, a first family of periodic solutions can be shown to exist: the homographic orbits. Examples of such orbits are named after Lagrange and Euler, and are configurations following distinct Keplerian orbits while at the same time being self-similar with respect to time. Such orbits can be imagined as the most symmetrical of all orbits, since they are orbits in the sense of classical mechanics, and also orbits of a $SO(2)$-action on the configuration space. In order to find such orbits, one has to solve a set of nonlinear equations in the $\boldsymbol{q}_i$, whose solutions are termed *central configurations*. So, the first type of symmetric periodic orbits in the n-body problem actually is given by central configurations. And, unsurprisingly, the problem persists in being not solved. We don't know even if, for given n bodies, the number of central configurations are finite or not, a question listed by Smale in his list of problems for the 21st century; a few recent remarkable breakthroughs in fact shed some light only on a part of the landscape.

The figure-eight. In 1998, Chenciner and Montgomery demonstrated the existence of a remarkable eight-shaped closed orbit for the three-body problem with equal masses [2], as shown in the following figure.



The result and, more importantly, the techniques used in its proof could open new pathways and perspectives in the search of periodic orbits and the understanding of some qualitative aspects of the n-body problem (see for example [3], [1], [4]) The sketch of the idea is the following. Consider the loop space Λ, consisting of the space of all T-periodic paths in $\mathbb{F}_n(\mathbb{R}^d)$, and partially define on it the Lagrangean action functional $\mathcal{A} : \Lambda \to \mathbb{R}$, having as potential the function U defined above. Regular solutions to the n-body problem are critical points of the action functional, hence they can be proven to exist by variational methods provided the action functional has at least some non-colliding minima. But in general it does not have them. Hence, some symmetry constraints are added, in order to achieve coercivity (locally or globally). In other words, if G is a subgroup of the full symmetry group of $\mathcal{A}$, one can restrict the action functional to $\Lambda^G = \{\boldsymbol{q}(t) \in \Lambda : \forall g \in G, g\boldsymbol{q}(t) = \boldsymbol{q}(gt)\}$. If A^G, the restriction of A to A^G, does not yet have the required properties (see [4]), then one can restrict $\mathcal{A}$ to a smaller subspace $\Lambda_0 \subset \Lambda^G$, by adding topological constraints (such as, degree or winding number, homology or homotopy classes of paths). In one restriction or the other, local minimizers are shown to exist.





But such trajectories may end in collision, and to understand when they do, a range of different techniques have to be carried out (Marchal averaging estimates, local or global level estimates, two-body regularizations, etc.). In the end, different choices of symmetry groups yield many symmetric periodic orbits by different types of methods and demonstrations. New problems arise about these peculiar, and often quite unrealistic orbits: stability, bifurcations, Morse indices and connections with the geometry and topology of configuration and loop spaces, to name just a few.

Experiments. The choice of possible groups and contraints is vast, and numerical experiments and computer-assisted proofs have been used successfully and extensively, in many cases and by many authors, for the search and analysis of the corresponding symmetric orbits or for testing new hypotheses.

Sometimes while watching such computer simulations, which show in a somehow aesthetically pleasing form the technical difficulties and intricacies underneath the mathematical problem, it surfaces a vague sense that all of it is in fact *the* "glass bead game," to quote Hermann Hesse, or maybe its origin for a literary future (see the figures): a *Glasperlenspiel*-problem with noble roots and much left to be told in the time to come.

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Approximation on Pyramids

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We recall the de Rham complex of L^2-based forms on a polyhedral domain $\Omega \in \mathbb{R}^3$: if $\mathcal{H}^{(s)}(\Omega)$ denotes the space of differentiable s-forms on Ω and d is the exterior derivative, then

$$\mathcal{H}^{(0)}(\Omega) \xrightarrow{d} \mathcal{H}^{(1)}(\Omega) \xrightarrow{d} \mathcal{H}^{(2)}(\Omega) \xrightarrow{d} \mathcal{H}^{(3)}(\Omega).$$

This sequence is exact. Some readers may be familiar with this sequence in a different form. We can identify $\mathcal{H}^{(0)}(\Omega)$ with the Sobolev space $H^1(\Omega)$ of $L^2(\Omega)$ functions whose gradients are also square integrable, and so on:

$$H^1(\Omega) \xrightarrow{grad} H(curl,\Omega) \xrightarrow{curl} H(div,\Omega) \xrightarrow{div} L^2(\Omega).$$

Now let $\mathcal{U}^{(s),k}$ be a finite-dimensional space of s-forms on Ω, and $\Sigma^{(s),k}$ be a set of linear functionals on $\mathcal{U}^{(s),k}$. We say the triplet $(\Omega, \mathcal{U}^{(s),k}, \Sigma)$ is *conforming, unisolvent and high-order approximating* provided (i) $\mathcal{U}^{(s),k} \subset \mathcal{H}^{(s)}(\Omega)$, (ii) the elements of $\mathcal{U}^{(s),k}$ are uniquely specified by prescribing *degrees of freedom (d.o.f.)* in Σ and (iii) and such that for sufficiently high k, $\mathcal{U}^{(s),k} \supseteq P^m(\Omega)$, all scalar (or vector valued) polynomials of degree $\leq m$. We can attempt to construct these triplets $(\Omega, \mathcal{U}^{(s),k}, \Sigma)$ in a unified manner so that the $\mathcal{U}^{(s),k}(\Omega)$ form a discrete complex which additionally satisfies a *commuting diagram* for the de Rham complex:

$$\begin{tikzcd} H^{0,r}(\Omega) \arrow[r, "d\equiv\nabla"] \arrow[d, "\Pi^{(0)}"'] & H^{1,r-1}(\Omega) \arrow[r, "d\equiv\nabla\times"] \arrow[d, "\Pi^{(1)}"'] & H^{2,r-2}(,\Omega) \arrow[r, "d\equiv\nabla\cdot"] \arrow[d, "\Pi^{(2)}"'] & H^{3,r-3}(\Omega) \arrow[d, "\Pi^{(3)}"'] \\ \mathcal{U}^{(0),k}(\Omega) \arrow[r, "d"] & \mathcal{U}^{(1),k}(\Omega) \arrow[r, "d"] & \mathcal{U}^{(2),k}(\Omega) \arrow[r, "d"] & \mathcal{U}^{(3),k}(\Omega) \end{tikzcd} \quad (1)$$

Here $\Pi^{(s)}$, $s = 0,1,2,3$, denote interpolation operators induced by the degrees of freedom, $\Sigma^{(s),k}$ and $\mathcal{H}^{s,r}(\Omega) \subset \mathcal{H}^s(\Omega)$, r are chosen so that the interpolants are well-defined. If Ω is a tetrahedron or a hexahedron, such constructions are well-known (for an excellent overview, see [2]).

Our goal is to construct triplets $(\Omega, \mathcal{U}^{(s),k}, \Sigma)$ when Ω is a pyramid with a square base and triangular faces: $\Omega = \{\boldsymbol{\xi} = (\xi,\eta,\zeta) \in \mathbb{R}^3 \quad | \quad \xi,\eta,\zeta \geq 0,\ \xi \leq 1-\zeta,\ \eta \leq 1-\zeta\}$. We want $(\Omega, \mathcal{U}^{(s),k})$ to be conforming, unisolvent, high-order approximating, and to satisfy the commuting diagram (1).

We insist that certain traces of $u \in \mathcal{U}^{(s),k}$ onto vertices, edges and faces are polynomials; and also that certain elements of $\Sigma^{(s),k}$ have a particular form: on triangular faces (and the edges of such faces), the d.o.f. must match those typically associated with a tetrahedron. On the square base (and the associated edges), the d.o.f. match those for a hexahedron. These specifications yield *compatibility conditions*.

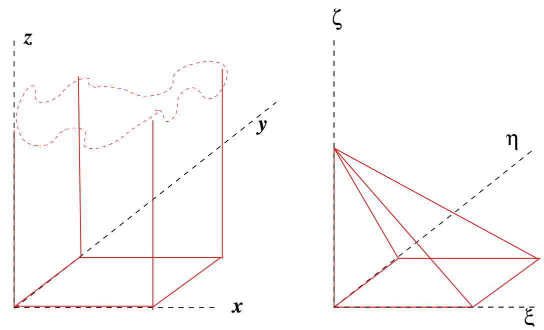

Figure 1: The infinite pyramid Ω_∞ and the finite pyramid Ω

This construction presents several challenges, and remained an open problem for well over 30 years. The difficulties in building $(\mathcal{U}^{(s),k}(\Omega), \Sigma^{(s),k})$ on a pyramid come primarily from the non-simplicial nature of the domain. For instance, the function $\dfrac{\xi\zeta(\xi+\zeta-1)(\eta+\zeta-1)}{1-\zeta} \in \mathcal{H}^{(0)}(\Omega)$ has polynomial traces on the faces and edges of $\partial\Omega$, but cannot be interpolated by a purely polynomial function. Therefore, $\mathcal{U}^{(0),k}$ *must* contain non-polynomial functions as well. In this case, then, what d.o.f. $\Sigma^{(0),k}$ should we specify inside Ω? Next, how does one construct the discrete 1-forms $\mathcal{U}^{(1),k}$ such that $d\mathcal{U}^{(0),k} \subset \mathcal{U}^{(1),k}$? One cannot construct $\mathcal{U}^{(s),k}(\Omega)$ by first constructing such spaces on a cube or tetrahedron and then using a Duffy transform to the pyramid.

However, challenging questions ask for ingenious solutions. So, consider the *infinite pyramid* $\Omega_\infty := \{\mathbf{x} = (x,y,z) \in \mathbb{R}^3 \cup \infty \mid x,y,z \geq 0,\ x \leq 1,\ y \leq 1\}$. Define the bijection $\phi : \Omega_\infty \to \Omega$ as $\phi(x,y,z) = \left(\dfrac{x}{1+z}, \dfrac{y}{1+z}, \dfrac{z}{1+z}\right), \phi(\infty) = (0,0,1)$, which is a diffeomorphism on $\Omega_\infty \backslash \infty$. We first construct $\mathcal{U}^{(0),k}(\Omega_\infty)$. Define $Q_k^{l,m,n} := \{\frac{x^l y^m z^n}{(1+z)^k}\}$, a collection of rational polynomials. We define a subset of $Q_k^{k,k,k}$

$$\overline{\mathcal{U}^{(0),k}}(\Omega_\infty) = Q_k^{k,k,k-1} \oplus span\left\{\frac{z^k}{(1+z)^k}\right\}. \quad (2)$$

Constrain $\overline{\mathcal{U}^{(0),k}}$ to scalar functions which have the correct behaviour on faces and edges, to get $\mathcal{U}^{(0),k}(\Omega_\infty)$. Similar considerations are used to obtain $\mathcal{U}^{((s)),k}(\Omega_\infty)$ for $s = 1,2,3$. The relation $d\mathcal{U}^{((s)),k}(\Omega) \subset \mathcal{U}^{(s+1),k}(\Omega)$ is explicitly used in this construction. The desired conforming spaces on the finite pyramid are then simply

$$\mathcal{U}^{(s),k}(\Omega) := \left\{(\phi^{-1})^* u : u \in \mathcal{U}^{(s),k}(\Omega_\infty)\right\}, s = 0,1,2,3.$$

Each of these contains rational functions, and not simply polynomials. These spaces are shown to achieve approximation: $P^{k-1}(\Omega) \subset \mathcal{U}^{(3),k}(\Omega)$, $P^{k-1} \subset \mathcal{U}^{(2),k}(\Omega)$, $P^{k-1} \subset \mathcal{U}^{(1),k}(\Omega)$ and $P^k \subset \mathcal{U}^{(0),k}(\Omega)$. Therefore, given $m \in \mathbb{N}$, it is possible to select k such that $P^m \subset \mathcal{U}^{(s),k}$.

What about the d.o.f.s $\Sigma^{(s),k}$? For the interior d.o.f., we use the so-called *projection-based interpolatory d.o.f.*, where we specify certain integrals of $u \in \mathcal{U}^{(s),k}$. It can be shown that with these specifications $(\Omega, \mathcal{U}^{(s),k}, \Sigma^{(s),k})$ form conforming, unisolvent and high-order approximating triplets. The discrete forms $\mathcal{U}^{(s),k}$ form an exact sequence. Further, we can use the d.o.f.s to define local interpolants $\Pi^{(s)}$ so that

$$\Pi^{(s)}(u) \in \mathcal{U}^{(s),k}(\Omega) \quad \text{and}$$
$$m(u) = m(\Pi^{(s)}u) \quad \forall m \in \Sigma^{(s),k}.$$

With these interpolants, the triplets satisfy a commuting diagram property:

Theorem 1 Let $r > 0$ be chosen so that the interpolation operators $\Pi^{(s)}$ are well-defined. Then the diagram (1) commutes.

This construction [3] has interesting mathematical features. It also turns out to be important in application. For any readers familiar with the finite element method: occasionally problems in the physical sciences require hybrid meshes with both hexahedral and tetrahedral elements; pyramids are used as 'gluing' elements in this context. The compatibility constraints on the traces of $\mathcal{U}^{(s),k}(\Omega)$ and $\Sigma^{(s),k}$ come from the need for compatibility throughout the mesh. The benefits of exactness and approximability are obvious, and the commuting diagram property mega ensures mixed formulations using these finite elements are stable and resolve spectra of elliptic operators correctly.

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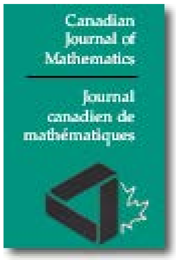

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Rédacteur(trice) associé(e) pour le JCM et le BCM

Le Comité des publications de la SMC sollicite des mises en candidatures pour cinq postes de rédacteurs associés pour le Journal canadien de mathématiques (JCM) et pour le Bulletin Canadien de mathématiques (BCM). **Le mandat sera de cinq ans qui commencera le 1er janvier 2016**. Les membres qui continuent (avec la fin de leur terme) sont ci-dessous.

Revues phares de la Société depuis plus de 50 ans, le **Journal canadien de mathématiques** (JCM) et le **Bulletin canadien de mathématiques** (BCM) présentent des travaux de recherche originaux de haute qualité. Le JCM publie des articles longs dans ses six numéros annuels, et le BCM publie des articles plus courts quatre fois l'an. Le JCM et le BCM ont chacun leur rédacteur en chef et partagent un même conseil de rédaction.

Les propositions de candidature doivent inclure votre curriculum vitae, votre lettre de présentation et doivent être envoyé par courriel électronique à : **jcmbcm-rednom-2015@smc.math.ca** **au plus tard le 15 novembre 2015**.

Membres Actuels du Conseil de rédaction scientifique pour le JCM et le BCM :

Henry Kim (Toronto)	12/2016	Rédacteur en chef JCM
Robert McCann (Toronto)	12/2016	Rédacteur en chef JCM
Jie Xiao (Memorial)	12/2019	Rédacteur en chef BCM
Xiaoqiang Zhao (Memorial)	12/2019	Rédacteur en chef BCM
Louigi Addario-Berry (McGill)	12/2018	Rédacteur associé
Florin Diacu (Victoria)	12/2016	Rédacteur associé
Ilijas Farah (York)	12/2015	Rédacteur associé
Skip Garibaldi (UCLA)	12/2016	Rédacteur associé
Dragos Ghioca (UBC Vancouver)	12/2018	Rédacteur associé
Eyal Goren (McGill)	12/2018	Rédacteur associé
Robert Leon Jerrard (Toronto)	12/2016	Rédacteur associé
Izabella Laba (UBC Vancouver)	12/2015	Rédactrice associée
Anthony To-Ming Lau (Alberta)	12/2016	Rédacteur associé
Alexander Litvak (Alberta)	12/2016	Rédacteur associé
Alexander Nabutovsky (Toronto)	12/2015	Rédacteur associé
Assaf Naor (Princeton)	12/2018	Rédacteur associé
Erhard Neher (Ottawa)	12/2016	Rédacteur associé
Frank Sottile (Texas A&M)	12/2015	Rédacteur associé
McKenzie Wang (McMaster)	12/2016	Rédacteur associé
Juncheng Wei (UBC Vancouver)	12/2018	Rédacteur associé
Daniel Wise (McGill)	12/2018	Rédacteur associé
Efim Zelmanov (UCSD)	12/2016	Rédacteur associé

CMS Notes
Volume 47 No. 5, October 2015

CMS
SMC

CMS Winter Meeting
Réunion d'hiver de la SMC

MONTRÉAL 2015

December 4-7
Montreal, Quebec

Hyatt Regency Montreal
cms.math.ca/events/winter15
#CMSwinter

4-7 décembre
Montréal, Québec

Hyatt Regency Montréal
smc.math.ca/reunions/hiver15
#hiverSMC

Photo : Tourisme Montréal / Stéphan Poulin

Public Lectures | Conférences publiques

John Baez (U. C. Riverside, California)
Isabelle Gallagher (Université Paris-Diderot)

Plenary Lectures | Conférences plénières

Gilles Brassard (Montréal)
Anna Gilbert (Michigan)
Martin Hairer (Warwick, U.K.)
Bernard Hodgson (Laval)
Caroline Series (Warwick, U.K.)
Jamie Tappenden (Michigan)

Prizes | Prix

Adrien Pouliot Award | Prix Adrien-Pouliot
Mark Mac Lean (UBC)

Doctoral Prize | Prix de doctorat
Yuval Filmus (Toronto), Hector H. Pasten Vasquez (Queen's)

G. de B. Robinson Award | Prix G. de B. Robinson
Philippe Gille (Université Claude Bernard, France)

Graham Wright Award for Distinguished Service | Prix Graham Wright pour service méritoire
recipient to be announced | lauréat à confirmer

Jeffery-Williams Prize | Prix Jeffery-Williams
Alejandro Adem (UBC)

Scientific Director | Directeur scientifique

Louigi Addario-Berry : louigi.addario@mcgill.ca

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TOURISME / MONTRÉAL
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Related Events | Événements liés

The **CMS Town Hall** meeting will occur on Saturday, December 5, from 12:30 - 14:00. All CMS members and meeting participants are invited to join the CMS Executive and to engage on upcoming plans and to discuss any interests or concerns that members of our community may have. | La **Séance de discussion de la SMC** aura lieu le samedi 5 décembre de 12 h 30 à 14h. Tous les membres de la SMC et participants à la réunion sont invités à se joindre à l'Exécutif de la SMC à s'engager sur les plans à venir et de discuter des préoccupations ou des intérêts que les membres de notre communauté peuvent avoir.

The Canadian Mathematical Society invites you to their **awards banquet** on Sunday, December 6, to highlight exceptional performance in the area of mathematical research and education. Prizes will be awarded during the event. La Société mathématique du Canada vous invite à son **banquet de prix** le dimanche 6 décembre pour souligner des contributions exceptionnelles en recherche mathématique et en enseignement des mathématiques. Des prix seront remis durant la soirée.

Regular Sessions | Sessions générales

Algebraic Combinatorics | Combinatoire algébrique
Christophe Hohlweg (UQAM), Hugh Thomas (UNB), Franco Saliola (UQAM)

Algebraic Number Theory | Théorie algébrique des nombres
Antonio Lei (Laval)

Analysis on Singular Manifolds | Analyse sur des variétés singulières
Alexey Kokotov (Concordia), Frédéric Rochon (UQAM)

Analytic Number Theory | Théorie analytique des nombres
Daniel Fiorilli (Ottawa), Nathan Jones (UIC), Dimitris Koukoulopoulous (Montréal), Matilde Lalin (Montréal)

Bridging the Gap between Mathematical Approaches and Biological Problems | Combler le fossé entre les approches mathématiques et problèmes biologiques
Fred Guichard (McGill), Erik Cook (McGill), Lea Popovic (Concordia)

Cohomological Methods in Quadratic Forms and Algebraic Groups | Méthodes cohomologiques pour les formes quadratiques et les groupes algébriques
Stefan Gille (Alberta), Nikita Karpenko (Alberta)

Combinatorics on Words | Combinatoire des mots
Alexandre Blondin Massé (UQAM), Srecko Brlek (UQAM), Christophe Reutenauer (UQAM)

Complex Analysis and Operator Theory | Analyse complexe et théorie des opérateurs
Javad Mashreghi (Laval), Thomas Ransford (Laval)

Computational and Topological Methods in Dynamical Systems | Calcul et méthodes topologiques en systèmes dynamiques
Tomasz Kaczynski (Sherbrooke), JeanPhilippe Lessard (Laval)

Descriptive Set Theory | Théorie descriptive des ensembles
Marcin Sabok (McGill)

Differential Geometry | Géométrie différentielle
Ailana Fraser (UBC), Regina Rotman (Toronto)

Diophantine Equations and Harmonic Analysis | Équations diophantiennes et analyse harmonique
Scott Parsell (West Chester University of Pennsylvania), Craig Spencer (Kansas State)

Discrete and Continuous Optimization | Optimisation discrète et continue
Dan Bienstock (Columbia), Andrea Lodi (École Polytechnique de Montréal)

Continued on next page | Suite à la page suivante

Continued from previous page | Suite de la précédent page

Fibrations, Mirror Symmetry and Calabi-Yau Geometry | Fibrations, symétrie miroir et géométrie de Calabi-Yau
Charles Doran (Alberta), Andreas Malmendier (Colby College), Alan Thompson (Waterloo)

Geometric Spectral Theory | Théorie géométrie spectrale
Alexandre Girouard (Laval)

Graph Theory | Théorie des graphes
Hamed Hatami (McGill), Sergey Norin (McGill)

History and Philosophy of Mathematics | Histoire et philosophie des mathématiques
Tom Archibald (SFU)

Logic, Category Theory and Computation | Logique, théorie des catégories et calcul
Prakash Panangaden (McGill)

Low Dimensional Topology and Geometric Group Theory | Topologie en basse dimension et théorie géométrique des groupes
Mark Powell (UQAM), Piotr Przytycki (McGill), Adam Clay (Manitoba)

Mathematical Finance | Finance mathématique
Cody Hyndman (Concordia), Alexandre Roch (UQAM), Alexandru Badescu (Calgary)

Mathematics Education | Enseignement des mathématiques
France Caron (Montréal), Veselin Jungic (SFU)

Mathematics: Source of New Solutions to Old Problems in Pharmaceutical Research and Therapy | Mathématiques: source de nouvelles solutions à de vieux problèmes en recherché pharmaceutique et en pharmacothérapie
Fahima Nekka (Montréal), Jun Li (Montréal)

Measure-Valued Diffusions | Diffusions à valeurs mesurées
Xiaowen Zhou (Concordia)

Nonlinear Evolutionary Equations | Équations d'évolution non linéaires
Dong Li (UBC), Xinwei Yu (Alberta)

Operator Algebras | Algèbres d'opérateurs
Mikael Pichot (McGill)

Probability and Statistical Mechanics | Probabilités et statistique mécanique
Alex Fribergh (Montréal), LouisPierre Arguin (Montréal)

Representation Theory | Théorie des représentations
Clifton Cunningham (Calgary), David Roe (UBC)

Stochastic Partial Differential Equations | Équations aux dérivées partielles stochastiques
Lea Popovic (Concordia), Don Dawson (Carleton)

Student Research Presentations | Exposés de recherche d'étudiants
Svenja Huntemann (Dalhousie), Muhammad Khan (Calgary)

Symplectic Geometry, Moment Maps and Morse Theory | Géométrie symplectique, applications moment et théorie de Morse
Lisa Jeffrey (Toronto)

AARMS-CMS Student Poster Session | Présentations par affiches des étudiants - AARMS-SMC
Svenja Huntemann (Dalhousie), Aaron Berk (UBC)

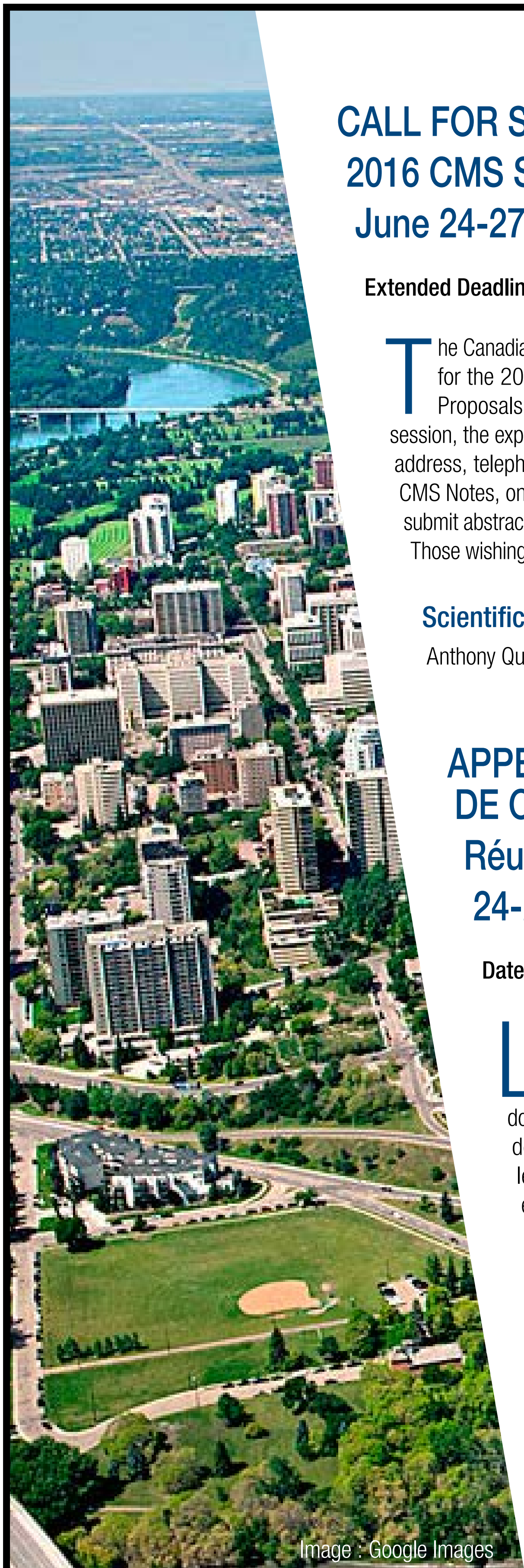

Image : Google Images

CALL FOR SESSIONS
2016 CMS Summer Meeting
June 24-27, 2016, Edmonton, Alberta

Extended Deadline: November 30, 2015

The Canadian Mathematical Society (CMS) welcomes and invites session proposals for the 2016 CMS Summer Meeting in Edmonton from June 24 to 27, 2016. Proposals should include a brief description of the focus and purpose of the session, the expected number of speakers, as well as the organizer's name, complete address, telephone number, e-mail address, etc. Sessions will be advertised in the CMS Notes, on the web site and in the AMS Notices. Speakers will be requested to submit abstracts, which will be published on the web site and in the meeting program. Those wishing to organize a session should send a proposal to the Scientific Director.

Scientific Director :

Anthony Quas : **aquas@uvic.ca**

APPEL DE PROPOSITIONS DE CONFÉRENCES
Réunion d'été de la SMC 2016
24-27 juin 2016, Edmonton, Alberta

Date limite prolongée : 30 novembre 2015

La Société mathématique du Canada (SMC) invite les gens à proposer des conférences pour la Réunion d'été de la SMC 2016 qui se tiendra à Edmonton du 24 au 27 juin 2016. Ces propositions doivent présenter une brève description de l'orientation et des objectifs de la conférence, le nombre de conférenciers prévu, de même que le nom, l'adresse complète, le numéro de téléphone et l'adresse électronique de l'organisateur. Tous les conférences seront annoncés dans les Notes de la SMC, sur le site Web et dans les AMS Notices. Les conférenciers devront présenter un résumé, qui sera publié sur le site Web et dans le programme de la réunion. Toute personne qui souhaiterait organiser un conférence est priée de faire parvenir une proposition au directeur scientifique.

Directeur scientifique :

Anthony Quas : **aquas@uvic.ca**

If undelivered, please return to:
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