



CMS Member Profile /  
Profil membre de la SMC  
Tim Alderson

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# CMS NOTES de la SMC

June / juin  
2016

## Vice-President's Notes / Notes du Vice-président

**David Pike**, (Memorial University)

Vice-President – Atlantic / Vice-président - Atlantique

## NSERC Funding for Research Activities



NSERC (the Natural Sciences and Engineering Research Council of Canada) plays a substantial role in supporting the research that drives our discipline's

advancements and scientific innovations. Over the years I've had the opportunity to engage with NSERC in various ways, the most recent of which is that I am now one year into a three year term on the Scholarships and Fellowships Selection Committee for Mathematical Sciences. Faculty members are likely be most familiar with Discovery Grants, but NSERC supports a whole pipeline of research activity, several parts of which I'll now share some comments on.

I'll begin at the undergraduate level, for which NSERC offers USRAs (Undergraduate Student Research Awards) to enable students to engage in 16-week long research projects under the supervision of a faculty mentor. Typically these projects take place in the summer time. As each university has its own allocation of USRAs to award there may be some variation in the internal processes for selecting award winners at each school, but if most universities are like mine then students must submit applications in advance of a January deadline. Students can submit applications to several schools and then spend the summer at any school that offers an award to them. As for what actually happens during the 16 weeks of an award, it depends heavily on the specific

## Financement du CRSNG pour la recherche

Le CRSNG (Conseil de recherches en sciences naturelles et en génie du Canada) joue un rôle important dans le financement des recherches qui stimulent le progrès de notre discipline et les innovations scientifiques. Au fil des ans, j'ai eu l'occasion de collaborer avec le CRSNG de diverses façons. En ce moment, par exemple, je viens de terminer la première année d'un mandat de trois ans au Comité de sélection des bourses en mathématiques. Les professeurs connaissent probablement bien les Subventions à la découverte, mais le CRSNG finance de nombreuses autres activités de recherche, dont j'en commenterai ici quelques éléments.

Au premier cycle, le CRSNG offre le Programme de bourses de recherche de 1er cycle, qui permet aux étudiants de prendre part à des projets de recherche de 16 semaines sous la direction d'un professeur. En général, ces projets se déroulent en été. Comme chaque université dispose d'un certain nombre de bourses à offrir, les processus internes de sélection des boursiers peuvent varier d'un établissement à l'autre. Toutefois, si la plupart des universités sont comme la mienne, les étudiants doivent faire leur demande avant une date limite fixée en janvier. Les étudiants peuvent faire une demande dans plusieurs établissements, puis choisir de passer l'été à n'importe quel des établissements qui leur offre une bourse. Quant à ce que font ces étudiants pendant ces 16 semaines, cela varie grandement selon le projet de recherche auquel ils participent (mathématiques informatiques avancées, exploration de conjectures, prouver de nouveaux théorèmes, etc.). Ces bourses

## When It Feels Inclined

**Robert Dawson**, *Department of Mathematics and Computer Science, Saint Mary's University*



**T**he great (and often mathematical) Danish poet Piet Hein once wrote enviously about the carefree life of the multiplication table, which, he said approximately, “simply applies when it feels inclined.” This frivolous little thought hides (or at least ornaments) an important truth. The power of mathematics to describe the world depends on the choice of mathematics. The

world contains a myriad of veins of mathematics: things that behave (at least in some ways) like integers under addition, things that behave like real three-vectors, things that behave like complex numbers or infinite-dimensional vector spaces. With so many different models to choose from, it's not surprising that scientists can often walk into the storeroom and come out with something that fits. But if you try to model something with the wrong mathematics, don't expect nice results.

A fine example of this is provided in the contrast between the visual arts and music. In the visual arts, simple quadratic surds tend to be “beautiful proportions,” and this is said to cut across cultures to some extent. We may wonder whether this is a result of our culture's ubiquitous use of right angles; as soon as the right angle is granted special status, hypotenuses appear, and Pythagoras' theorem causes square roots and other quadratic surds to materialize as proportions of simple natural designs. Be that as it may, ratios like the Golden Ratio (approximated by everything from the Parthenon to credit cards), and the  $\sqrt{2}$ :1 ratio of European A-series stationery, are ubiquitous and successful. Since these ratios are so beautiful to the eye, should they not be pleasing to the ear as well? Would we not expect exquisite tunes, golden harmonies, if we used these ratios in music?

The most traditional scales, in the West attributed (on scant evidence) to Pythagoras, are based on simple rational numbers. An octave is a ratio of 2:1, a perfect fifth 3:2, a major second 9:8, and so on: those are the white notes, as immortalized in the song “Do, a deer.” If we want to be able to modulate from key to key, we need sharps and flats, the black keys – a scale starting on G needs F# to be its second-last note, and so on. Unfortunately, in a pure Pythagorean tuning, this beautiful pattern does not close up: D# should be Eb, and it isn't. The solution was to make all twelve intervals equal, to make the ratios of adjoining notes all exactly  $^{12}\sqrt{2}$ :1.

So the interval of  $\sqrt{2}$ :1 actually exist – it is six semitones, from C to F#. We rush to the piano, count the keys carefully if we are not pianists, and play – what?

A diminished fifth, the “tritone,” the sourest-sounding interval in the octave! It's so dissonant that the scale based on it, the “Locrian mode,” was supposedly banned from church music and is still hardly ever used. Johann Sebastian Bach did use it – and has been jokingly referred to as the “Father of Death Metal” for doing so.

Our lesson? Mathematics applies to all sorts of things – when it feels inclined.

## Quand elles le veulent bien...

**L**e grand poète danois (et mathématicien à ses heures) Piet Hein a écrit avec envie sur la vie insouciance de la table de multiplication, qui, écrit-il à peu près en ces termes, « s'applique tout simplement quand elle le veut bien ». Cette petite pensée frivole cache (ou du moins, agrmente) une vérité importante. La puissance des mathématiques pour décrire le monde dépend des mathématiques que l'on choisit. Le monde contient une myriade de courants mathématiques : les choses qui se comportent (au moins à certains égards) comme des nombres entiers sous l'addition; les choses qui se comportent comme des vecteurs à trois composantes réelles; les choses qui se comportent comme des nombres complexes ou comme des espaces vectoriels de dimension infinie. Avec tant de modèles différents à choisir, il n'est pas surprenant que les scientifiques arrivent souvent à fouiller dans l'entrepôt et en ressortir avec quelque chose qui fait l'affaire. Mais si vous essayez de modéliser quelque chose en utilisant les mauvaises mathématiques, ne vous attendez pas à un résultat élégant.

Le contraste entre les arts visuels et la musique illustre bien cette affirmation. En arts visuels, les nombres sourds quadratiques simples ont généralement de « belles proportions », ce qui est, dans une certaine mesure, valide dans une perspective transculturelle. On peut se demander s'il s'agit du résultat de l'utilisation omniprésente des angles droits dans notre culture; dès que l'angle droit reçoit un statut spécial, l'hypoténuse apparaît, et le théorème de Pythagore amène les racines carrées et d'autres racines sourdes à se matérialiser comme des proportions de concepts naturels simples. Quoi qu'il en soit, les ratios tels que le ratio d'or (dont on a fait l'approximation pour avec à peu près tout, du Parthénon aux cartes de crédit) et le ratio  $\sqrt{2}$ :1 de la série A de la papeterie européenne, sont omniprésents et fort utiles. Comme ces ratios sont visuellement attrayants, ne devraient-ils pas aussi être agréables à l'oreille? Ne serions-nous pas en droit de nous attendre à des airs exquis et à des harmonies élégantes si nous utilisions ces ratios en musique?

Les gammes les plus traditionnelles, attribuées en Occident à Pythagore (sur la base de preuves insuffisantes), sont basées sur des nombres rationnels simples. Une octave est un ratio de 2:1, une quinte parfaite, de 3:2, une seconde majeure, de 9:8, et ainsi de suite : ce sont les « touches blanches », telles qu'immortalisées dans la chanson Do, le do, il a bon dos. Pour moduler d'une tonalité à l'autre, nous avons besoin des dièses et des bémols, les touches noires : une gamme partant du sol a besoin d'un fa dièse comme avant-dernière note, etc. Malheureusement, en mode purement pythagoricien, ce beau modèle ne marche pas : le ré dièse devrait être un mi bémol, ce qui n'est pas le cas. Il a donc fallu rendre les douze intervalles égaux de manière à ce que les ratios des notes contiguës soient tous exactement  $^{12}\sqrt{2}$ :1.

L'intervalle  $\sqrt{2}$ :1 existe donc réellement, il correspond à six demitons, de do à fa dièse. Nous nous précipitons donc au piano, comptons les touches soigneusement si nous ne sommes pas pianistes, pour jouer... jouer quoi?

Une quinte diminuée, aussi appelée triton, l'intervalle à la consonance la plus étrange de toute l'octave! Cette quinte est si dissonante que la gamme basée sur celle-ci, soit le « mode locrien », aurait été bannie de la musique d'église et demeure peu employée à ce jour. Jean Sébastien Bach l'a bien utilisée, ce qui lui a valu - à la blague - la réputation de « père du *death metal* ».

Notre leçon? Les mathématiques s'appliquent à toutes sortes de choses - quand elles le veulent bien.

## Booksale Fundraiser

Over 1500 used books from all areas of pure and applied mathematics, including some computer science, statistics, and mathematical physics, are for sale at [www.mathstat.dal.ca/~dilcher](http://www.mathstat.dal.ca/~dilcher)

“This is a continuing fundraiser, and prices are moderate. Books were donated by active and retired colleagues, and also include library discards. All proceeds go, in equal parts, to the Canadian Mathematical Society and to the Department of Mathematics & Statistics of Dalhousie University.”

- Karl Dilcher (Dalhousie)



### NOTES DE LA SMC

Les Notes de la SMC sont publiées par la Société mathématique du Canada (SMC) six fois par année (février, mars/avril, juin, septembre, octobre/novembre et décembre).

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Les Notes de la SMC, les rédacteurs et la SMC ne peuvent pas être tenus responsables des opinions exprimées par les auteurs.

### CMS NOTES

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La Société mathématique du Canada appuie l'avancement, la découverte, l'apprentissage et l'application des mathématiques. L'exécutif de la SMC encourage les questions, commentaires et suggestions des membres de la SMC et de la communauté.

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The CMS promotes the advancement, discovery, learning and application of mathematics. The CMS Executive welcomes queries, comments and suggestions from CMS members and the community.

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## A Response to the Vice-President's Notes in Vol. 47, Number 6, December 2015

### Gaps in the “Defence of the Classroom”

I welcome Rahim Moosa's invitation in the December Notes to talk about online courses. It was good to see that he acknowledged “Such courses do have a limited place in the university...” (p. 4). However, a personal concern is that the limitations on that place may be too strong. I should also make it clear that, as the current president, I am not representing the views of the Ontario Association of Mathematics Educators.

My concerns are exactly in line with the value of online courses as a “technologically updated form of what used to be called correspondence courses” (p. 4). While this is exactly the “limited place” that is referred to, it is limited to a fault. As a person who gained employment in the private sector working in applied mathematics, I was unable to find sufficient opportunity in that “limited place” that would have allowed me to complete a graduate degree and that constrained employment opportunities.

I was willing to blend my learning and attend some face-to-face classes. I did commute for a semester in order to take a course face-to-face. Subsequently, the institution had a schedule that did not allow me to continue my studies while working to support a young family. There were no online options and, in spite of a professor who had encouraged me to trust the program, I found a lack of options. Hence, I was also reluctant to accept another programmatic option, at another university, that was based on meeting on the first day of classes to work out a class time amenable to the students – perhaps that would have worked for a course or two, but would it have worked for a full program? Past schedules indicated not. My assessment of the situation generally was that there was a hidden requirement of face-to-face availability, at some point, on a regular basis between 10am and 2pm.

The substantial issue that is lurking is that this is not about online courses implying that mathematics “can really be taught as well without professors.” (p. 4), nor is it about “automation” (p.4). The issue is about synchronicity and the ability to engage, for a large portion of the time, in an asynchronous manner. I am quite willing to accept a course space set up to protect the intellectual property that video recordings of lectures represent. I am also willing to buy course books and invest time in working through material that requires effort to digest. However, I am not sure why I cannot do that on my own schedule so long as I accept that any question I pose may not receive a response from the professor until the next business day.

The challenge of that limited space is that I agree that many undergraduate courses should be face-to-face and I accept the pragmatic considerations of convening traditional lectures and tutorials. However, I fear that this furthers the limitation of the space afforded to people like myself, whether looking at graduate or undergraduate studies, who are excluded because they are working or supporting a family.

In my particular case, to clarify for readers, I did succeed in pursuing graduate studies in education and did not have to leave my high school teaching role. It was rewarding and, for all the limitations of online courses, there was a personal benefit that came out of many late nights. It was not the “...severely diminished educational value...” (p. 4) that Rahim conjectures. I still look at graduate studies in mathematics, periodically, and find the limited place is still far too limited for me to have followed that route though I would have, if the opportunity had been available.

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## Letters to the Editors

The Editors of the NOTES welcome letters in English or French on any subject of mathematical interest but reserve the right to condense them. Those accepted for publication will appear in the language of submission. Readers may reach us at the Executive Office or at [notes-letters@cms.math.ca](mailto:notes-letters@cms.math.ca)

## Lettres aux Rédacteurs

Les rédacteurs des NOTES acceptent les lettres en français ou en anglais portant sur n'importe quel sujet d'intérêt mathématique, mais ils se réservent le droit de les comprimer. Les lettres acceptées paraîtront dans la langue soumise. Les lecteurs peuvent nous joindre au bureau administratif de la SMC ou à l'adresse suivante : [notes-lettres@smc.math.ca](mailto:notes-lettres@smc.math.ca).

*The Calendar brings current and upcoming domestic and select international mathematical sciences and education events to the attention of the CMS readership. Comments, suggestions, and submissions are welcome.*

**Denise Charron**, Canadian Mathematical Society,  
([managing-editor@cms.math.ca](mailto:managing-editor@cms.math.ca))

*Le calendrier annonce aux lecteurs de la SMC les activités en cours et à venir, sur la scène pancanadienne et internationale, dans les domaines des mathématiques et de l'enseignement des mathématiques. Vos commentaires, suggestions et propositions sont le bienvenue.*

**Denise Charron**, Société mathématique du Canada  
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## JUNE 2016 JUIN

- 2-4** FIELDS Workshop on Nonlinear Optimization Algorithms and Industrial Applications, Fields Institute, Toronto, Ont.
- 2-8** CRM Atelier: Cycles algébriques et modules, CRM, Montréal, Qué.
- 3-7** CMESG (Canadian Mathematics Education Study Group), Queen's University, Kingston, Ont.
- 6-10** FIELDS Conference on Recent Trends on Elliptic Nonlocal Equations, Fields Institute, Toronto, Ont.
- 13-16** FIELDS Conference on Geometry, Algebra, Number Theory, and their Information Technology Applications (GANITA), Fields Institute, Toronto, Ont.
- 13-17** PIMS Workshop on Nonlocal Variational Problems and PDEs, University of British Columbia, Vancouver, B.C.
- 13-17** CRM 44<sup>e</sup> Symposium annuel canadien des algèbres d'opérateurs et leurs applications (COSy), CRM, Montréal, Qué.
- 19-22** FIELDS Workshop on Digital Online Mathematics Education (DOME), Fields Institute, Toronto, Ont.
- 20-24** Canadian Number Theory Association, 14<sup>th</sup> Meeting, University of Calgary, Calgary, Alta.
- 21-30** CRM Workshop: Partial Order in Materials: Analysis, Simulations and Beyond, CRM, Montreal, Que.
- 24-27** 2016 CMS Summer Meeting / Réunion d'été de la SMC 2016, University of Alberta, Edmonton, Alta.
- 26-30** CAIMS 2016 Annual Meeting, University of Alberta, Edmonton, Alta.
- 27-Jul 1** CRM École : L'abécédaire de SIDE (ASIDE), CRM, Montréal, Qué.
- 27-Jul 2** 17<sup>th</sup> International Conference on Fibonacci Numbers and Their Applications, University of Caen, France.

## JULY 2016 JUILLET

- 3-9** CRM 12<sup>th</sup> International Conference on Symmetries and Integrability of Difference Equations (SIDE12), Hotel Le Chantecler, Sainte-Adèle, Que.
- 4-8** CRM Workshop: Complex Boundary and Interface Problems: Theoretical models, Applications and Mathematical Challenges, CRM, Montreal, Que.
- 4-8** Formal Power Series and Algebraic Combinatorics, Simon Fraser University, Vancouver, B.C.
- 4-14** CRM Summer School: Spectral Theory and Applications, Université Laval, Quebec City, Que.
- 5-9** CRM Conference on Differential Geometry, CRM, Montreal, Que.
- 11-15** FIELDS World Congress of Probability and Statistics, Fields Institute, Toronto, Ont.

- 11-Aug 5** AARMS Summer School in Category Theory, Combinatorics and Number Theory, Dalhousie University, N.S.
- 18-22** CRM Workshop: Computational Optimal Transportation, Montreal, Que.
- 18-22** Conference on Geometry, Representation Theory and the Baum-Connes Conjecture, Fields Institute, Toronto, Ont.
- 18-22** EMS 7<sup>th</sup> European Congress of Mathematics, Technische Universität Berlin, Berlin, Germany
- 19** PIMS Marsden Memorial Lecture: The Constraint Manifold of General Relativity (Richard Schoen), BIRS, Banff, Alta.
- 24-31** 13<sup>th</sup> International Congress on Mathematics Education (ICME-13), University of Hamburg, Hamburg, Germany
- 25-28** New Trends in Approximation Theory: A Conference in Memory of André Boivin, Fields Institute, Toronto, Ont.
- 25-Aug 19** Statistical Causal Inference and Its Applications to Genetics, CRM Montreal, Que.
- 27-Aug 5** PIMS Summer School and Workshop on Geometric and Topological Aspects of the Representation Theory of Finite Groups, University of British Columbia, Vancouver, B.C.

## AUGUST 2016 AOÛT

- 1-7** CIFAR/CRM Deep Learning Summer School 2016, CRM, Montreal, Que.
- 3-6** MAA MathFest 2016, Columbus, Ohio
- 15-19** FIELDS 2016 Industrial Problem Solving Workshop, Fields Institute, Toronto, Ont.
- 15-26** Two Weeks in Vancouver - A Summer School for Women in Math, University of British Columbia, Vancouver, B.C.
- 21-26** 24<sup>th</sup> International Congress of Theoretical and Applied Mechanics (ICTAM 2016), Palais des congrès, Montreal, Que.
- 22-26** CRM Aisenstadt Chair 2016 - Speaker: Nalini Anantharaman (Strasbourg), CRM, Montreal, Que.
- 22-26** FIELDS/CRM Conference on Methods of Modern Mathematical Physics, Fields Institute, Toronto, Ont.
- 28-Sep 1** CRM/FIELDS Frontiers in Mathematical Physics: In honour of Barry Simon's 70<sup>th</sup> Birthday, CRM, Montreal, Que.

## SEPTEMBER 2016 SEPTEMBRE

- 1-7** CRM Aisenstadt Chair 2016 - Speakers: Yuval Peres (Microsoft Research) and Scott Sheffield (MIT), CRM, Montreal, Que.
- 2-6** CRM Workshop: Random Growth Problems and Random Matrices, CRM, Montreal, Que.
- 11-16** BIRS Workshop: Bridges Between Noncommutative Algebra and Algebraic Geometry, BIRS, Banff, Alta.
- 25-30** BIRS Workshop: Modular Forms in String Theory, BIRS, Banff, Alta.

*Continued from cover*

research project that the student is involved with, with some examples including advanced mathematical computing, exploring conjectures, and proving new theorems. USRA experiences often lead to presentations at events such as the annual CUMC (Canadian Undergraduate Mathematics Conference), and can also lead to the publication of journal articles. I highly encourage students who are contemplating going on to graduate school to consider USRA opportunities (including during the summer in between their undergraduate and graduate studies). Not only do USRAs give students first-hand exposure to the world of research but they also provide students with experiences that can help them to compete for scholarships with which to pursue graduate studies.

For early graduate studies, NSERC offers CGS (Canada Graduate Scholarship) awards at the Master's level, valued at \$17,500 for a single year. It used to be the case that students would submit a single scholarship application for Master's awards around November to their home institution, regardless of where they intended to take up their Master's studies about 10 months later; each school would then forward its best applications on to NSERC, discipline-specific committees of NSERC would assess the forwarded applications, and scholarships (which could be taken to any school in the country) were then awarded. About two or three years ago things changed. What now happens is that around October of each year, prospective Master's students submit a scholarship application using the NSERC Research Portal, and as part of the process they designate up to five universities to which their application will be forwarded for evaluation. The evaluation is no longer performed centrally by NSERC, but instead each university is provided an allocation of awards that it is able to make to those students whose applications it has received for evaluation. A scholarship offered to a student by one university is not transferable to another one; instead the student must accept or decline offers (which begin to be made in early April) on a school by school basis. Since each university performs its own ranking, and also since the competition may be more intense at some schools than at others, it may be that a student is offered a scholarship by some schools while remaining on a waiting list of runners-up at others. When a student accepts an offer, she/he is then removed from the competition at each of the other four schools to which her/his application had been sent for consideration (including those for which the student may be on a waiting list). Declined offers permit schools to move down their waiting lists and extend new offers. The whole concept of the accept/decline process is very dynamic and is something that students may wish to take into consideration when deciding which five schools to select at the time that their applications are submitted.

At the Doctoral level, it is still the case that an NSERC committee oversees the selection of scholarship winners. The top-ranked applicants are offered lucrative CGS-Doctoral awards valued at \$35,000 annually for up to three years, whereas PGS (Postgraduate Scholarship) awards valued at \$21,000 per year go to the next tier of scholarship recipients. This past year the Scholarships and Fellowships Committee for the Mathematical Sciences considered

over 100 applications. As for the assessment process, each application is assigned to two committee members for evaluation, who score each application on basis of the student's academic excellence, potential research ability as well as communication and leadership skills. These scores feed into a collective ranking of all of the applicants, with those ranking highest being offered scholarships.

The Scholarships and Fellowships Committee also evaluates applications for Post-Doctoral Fellowships. After an initial screening, this year the Mathematical Sciences committee narrowed the field down to about 25 applications that were then evaluated by each committee member (other than those for whom a conflict of interest might exist). The scoring of PDF applications is similar to that of CGS-Doctoral ones, except that the academic transcripts of the applicants are not taken into consideration.

As this was my first year on the committee, I was struck by some of the differences when compared to how the Evaluation Group for Discovery Grant applications operates. The most significant difference was the extent to which each application was assessed in comparison with others. For scholarships and fellowships, the critical task of the committee is to produce a total order so that no matter how many (or few) awards are ultimately awarded, it is indeed the most meritorious applicants who are successful. When assessing applications for research grants, which vary in their value, the task is to assign scores that combine to determine a bin into which an application is placed. There is still a sense of a ranking, in that Bin A's applications are stronger than those in Bin B, which are stronger than those in Bin C, etc. But within a bin there isn't as much of a need for a comparative ranking; although it is important for the committee to be able to distinguish C- versus C+ applications they both ultimately are in Bin C. On the other hand, within the grant review committee there was often discussion of borderline cases such as applications that might be C- versus D+ and which would ultimately be in Bin C or Bin D.

As this article is already becoming somewhat lengthy, I won't dwell on the operations of the Evaluation Group for Discovery Grants. I will, however, note that staff from NSERC regularly make an effort to participate in at least one CMS conference each year, to meet with researchers, hear our concerns and to provide advice and information. For instance, they came to our Winter 2015 meeting in Montréal and held a lunchtime Town Hall gathering. Personally, I found it interesting and refreshing to hear them speak with what (to me) seemed to be greater freedom and candour than in past years. I left that meeting with a sense of optimism that the funding envelope for our discipline might fare better in the coming years than it has on past occasion, although I'm also confident that it will take a sustained effort to convince the appropriate decision-makers that increased investment in long-term curiosity-driven and fundamental research will yield positive dividends.



*Suite de la couverture*

mènent souvent à des communications dans le cadre d'activités comme le Congrès canadien des étudiants en mathématiques, ou encore à la publication d'articles de revues. J'incite fortement les étudiants qui songent à poursuivre leurs études au 2<sup>e</sup> cycle à faire une demande au Programme de bourses de recherche de 1<sup>er</sup> cycle (qui comprend l'été entre le 1<sup>er</sup> et le 2<sup>e</sup> cycle). Ce programme offre non seulement aux étudiants de l'expérience pratique en recherche, mais aussi des expériences qui pourront les aider à obtenir d'autres bourses pour la suite de leurs études.

Pour le 2<sup>e</sup> cycle, le CRSNG offre le Programme de bourses d'études supérieures du Canada (BESC) au niveau de la maîtrise, d'une valeur de 17 500 \$ pour une année. Auparavant, les étudiants n'avaient qu'à faire une demande de bourse de maîtrise au mois de novembre à leur établissement, peu importe où ils pensaient faire leurs études de maîtrise 10 mois plus tard. Chaque établissement faisait ensuite suivre les meilleurs dossiers au CRSNG, où des comités propres à la discipline les évaluaient et attribuaient les bourses (qui pouvaient être transférées dans n'importe quel établissement du pays). Tout cela a changé il y a deux ou trois ans. En ce moment, les étudiants de maîtrise présentent une demande de bourse en octobre via le portail de recherche du CRSNG, où ils choisissent jusqu'à cinq universités auxquelles leur demande sera transmise à des fins d'évaluation. L'évaluation ne se fait plus au niveau central par le CRSNG; chaque université reçoit plutôt un lot de bourses qu'elle est en mesure d'offrir aux étudiants dont elle a reçu une demande à évaluer. Une bourse offerte à un étudiant par une université n'est pas transférable à l'autre. L'étudiant doit plutôt accepter ou refuser les offres (qui commencent à lui parvenir au début d'avril) de chaque établissement. Comme chaque université fait son propre classement, et comme la concurrence peut être plus vive dans certains établissements que d'autres, un étudiant peut recevoir une offre de bourse de quelques endroits tout en demeurant sur une liste d'attente ailleurs. Quand un étudiant accepte une offre, il est retiré du concours dans les autres établissements où sa demande avait été envoyée (y compris ceux pour lesquels l'étudiant était peut-être sur une liste d'attente). Les offres refusées permettent aux établissements d'avancer dans leurs listes d'attente et de faire de nouvelles offres. Le concept global du processus d'acceptation/refus est très dynamique, et les étudiants voudront peut-être en tenir compte au moment de décider à quels établissements ils veulent que leur demande soit acheminée.

Au niveau du doctorat, c'est toujours un comité du CRSNG qui détermine la sélection des boursiers. Les candidats les mieux classés se voient offrir de grosses bourses du Programme de BESC au niveau du doctorat (BESC D), d'une valeur de 35 000 \$ par année pour un maximum de trois ans, et les autres reçoivent une ES D (bourse d'études supérieures) de 21 000 \$ par année. L'année dernière, le Comité de sélection des bourses en mathématiques a évalué plus de 100 demandes. Quant au processus d'évaluation, chaque demande est remise à deux membres du comité pour évaluation; chacun attribue une note à la demande en fonction de l'excellence de l'étudiant, de son potentiel en recherche et de ses compétences en communications et en leadership. Ces notes sont ensuite regroupées

au sein d'un classement collectif, et les candidats ayant les meilleures notes reçoivent les bourses les plus élevées.

Le Comité évalue également les demandes de bourses postdoctorales. Cette année, après une sélection initiale, le comité a ramené son choix à environ 25 demandes, qui ont ensuite été évaluées par chaque membre du comité (sauf ceux qui auraient pu être en conflit d'intérêts). La notation des demandes de bourses postdoctorales est semblable à celle des demandes au niveau du doctorat, sauf que les relevés de notes des candidats ne sont pas pris en considération.

Comme c'était ma première année au comité, j'ai été frappé par les différences entre ce processus et celui du Groupe d'évaluation des demandes de Subvention à la découverte, la principale différence entre les deux processus étant le degré d'évaluation de chaque demande par rapport aux autres. Dans le cas des bourses, le comité a la responsabilité d'établir un classement de sorte que, sans égard au nombre total de bourses accordées, les meilleurs candidats seront réellement les plus récompensés. Dans le cas des demandes de subventions de recherche, dont la valeur est variable, la tâche consiste à attribuer des notes qui, combinées, déterminent dans quel « classeur » la demande est placée. Il y a encore un certain classement, en ce sens que les demandes placées dans le classeur A sont meilleures que celles du classeur B, qui sont à leur tour meilleures que celles du classeur C, etc. Toutefois, dans un même classeur, il n'est pas aussi nécessaire de faire un classement comparatif; bien qu'il soit important que le comité puisse faire une distinction entre une demande notée C et une C+, ces deux demandes se retrouvent toutes les deux dans le classeur C. Par contre, au sein du comité d'évaluation des demandes de subventions, on a souvent discuté de cas limites, entre C- et D+, par exemple, qui se sont retrouvées soit dans le classeur C, soit dans le D.

Comme cet article commence à s'allonger, je ne reviendrai pas sur le fonctionnement du Groupe d'évaluation des demandes de Subvention à la découverte. Je soulignerai toutefois que le personnel du CRSNG fait régulièrement des efforts pour participer à au moins une Réunion de la SMC par année, rencontrer des chercheurs, entendre nos préoccupations et donner des conseils et de l'information. Des représentants du CRSNG ont notamment assisté à la Réunion d'hiver 2015 de la SMC à Montréal et y ont organisé une séance de discussion à l'heure du dîner. Personnellement, j'ai trouvé intéressant et rafraîchissant de les entendre parler avec ce qui m'a semblé une liberté de parole et une candeur supérieures à ce qu'on avait vu ces dernières années. Au sortir de cette Réunion, j'avais espoir que le financement consacré à notre discipline serait en meilleure posture dans les années à venir que ce que nous avons vu dernièrement, même si je suis aussi convaincu qu'il faudra y mettre des efforts soutenus pour convaincre les bons décideurs qu'une hausse des investissements à long terme axée sur la curiosité et la recherche fondamentale produira des résultats positifs.

*Book Reviews brings interesting mathematical sciences and education publications drawn from across the entire spectrum of mathematics to the attention of the CMS readership. Comments, suggestions, and submissions are welcome.*

**Karl Dilcher**, *Dalhousie University* (notes-reviews@cms.math.ca)

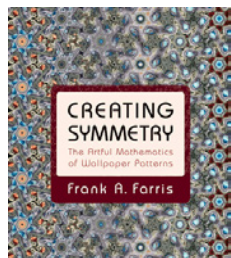
## Creating Symmetry: The Artful Mathematics of Wallpaper Patterns

by Frank A. Farris

Princeton University Press, 2015, 248 pp.

ISBN 978-0-691-16173-0

Reviewed by *Anthony C. Thompson*



This is a beautiful book – in both the mathematical and the more usual aesthetic sense of the word. The mathematical content and exposition are lovely and the illustrations are wonderful. There are not many mathematical coffee-table books and it is even more unusual to find one with solid mathematical content. In this volume we get (almost) all the details behind the compelling pictures. Typos, although some exist, are few and far between and none is serious.

The first five chapters deal with curves (among other things). They are linked by the following example:

$$f(t) := e^{it} + 1/2 e^{6it} + i/3 e^{-14it}$$

whose picture appears already on p.3 with the question “why does it have 5-fold symmetry?”. Also included in these five chapters are: (1) a quick introduction to complex numbers (so that the curve, originally expressed in parametric form involving sines and cosines, can be rewritten as above); (2) a quick introduction to vector spaces and groups; and (3) a quick introduction to Fourier series. If that seems a lot for 33 pages it is a tribute to the author’s skill that it is not. These fundamental ideas are conveyed clearly and precisely.

Chapter 6 begins the main purpose of the book by extending the domain of functions like  $f$  from the unit circle to the whole complex plane (perhaps excluding the origin):

$$f(z) := z + 1/2 z^6 + i/3 z^{-14}$$

(analytic but with a pole at 0). Alternatively,

$$f(z) := z + 1/2 z^6 + i/3 \bar{z}^{14},$$

(defined everywhere but only continuous). The question now arises as to how to visualize functions from  $\mathbf{C}$  to  $\mathbf{C}$ . The domain colouring algorithm [1] (see also [2] where the underlying idea is attributed to Larry Crone and Hans Lundmark) requires that we first colour each point of the complex plane with a unique colour

*Les comptes-rendus de livres présentent aux lecteurs de la SMC des ouvrages intéressants sur les mathématiques et l’enseignement des mathématiques dans un large éventail de domaines et sous-domaines. Vos commentaires, suggestions et propositions sont le bienvenue.*

**Karl Dilcher**, *Dalhousie University* (notes-critiques@smc.math.ca)

and then portray  $f$  by colouring each point  $z$  with the colour for  $f(z)$ . One might arrive at a suitable colouring of  $\mathbf{C}$  by using three primary colours (red, yellow or green, blue) coded with the vectors  $(1, 0, 0)$ ,  $(0, 1, 0)$ ,  $(0, 0, 1)$  and other colours represented by points in the unit cube from  $(0, 0, 0)$  – white, to  $(1, 1, 1)$  – black. This is the water-colourists view in which one adds colour to white paper: alternatively, the RGB (additive) scheme uses  $(0, 0, 0)$  as the absence of colour and  $(1, 1, 1)$  as the full spectrum. Then colour the plane by projecting the surface of the unit cube onto the plane – first tip the cube up so that  $(1, 1, 1)$  is vertically above  $(0, 0, 0)$ , and then project stereographically. Thus zeroes of the function appear as white dots and poles as black. The author, however, eschews this somewhat mechanical approach in favour of much more artistic freedom. He uses his own photographs as “palettes” to colour a sufficiently large area of the complex plane. This gives a somewhat more limited range of colours but produces much more visually attractive patterns.

Chapter 7 is headed “Rosettes as plane functions” and deals with functions which have rotational symmetry. For example, the function  $f$  above has 5-fold rotational symmetry because the exponents 1, 6 and -14 are all congruent to 1 mod 5. Here we first encounter vector spaces of such functions and group actions on those functions. The next chapter deals with friezes and develops naturally from the preceding one by transforming rotational invariance into translational invariance.

The next ten chapters are the heart of the book and deal with wallpaper patterns of all possible types. One major aim is, of course, to show that there are precisely 17 such types. These are catalogued according to their symmetry groups: a translational subgroup generated by two linearly independent translations together with subgroups of rotations and/or reflections. After two chapters dealing with wave functions, the author begins with wallpaper groups with 3-fold rotational symmetry. The pace here is rather leisurely and everything is explained in detail. Chapter 13 proves the “crystallographic restriction” that the order of any rotational symmetry must be one of 2,3,4,6 – no order 5 symmetries. The proof is elegantly presented. After another theoretical section dealing with dual lattices, the rest of the wallpaper groups based on square, rhombic and rectangular lattices are enumerated. It is here that the pace is somewhat accelerated and some details are left to the reader.

All this material can be summarized by saying that it concerns functions  $f$  for which  $f(\sigma z) = f(z)$  for a group of isometries  $\sigma$ . Next we get an even broader view with functions for which (a)  $f(\sigma z) = -f(z)$ ; and then (b)  $f(\sigma z) = \omega f(z)$  where  $\omega$  is a primitive cube root of 1. These properties are made beautifully,



visually apparent by using (a) an involution, and (b) a cyclic group of order 3, on the set of colours. For the first we have the artist's well-known idea of complementary colours in which colour  $(a, b, c) \mapsto (1 - a, 1 - b, 1 - c)$ ; and for the second, a cyclic permutation of the primary colours:  $(a, b, c) \mapsto (b, c, a)$ . The author calls the resulting patterns "colour reversing" and "colour turning" respectively.

The final two chapters deal with the symmetries of the platonic solids (almost exclusively, the tetrahedron) and with how wallpaper patterns behave on the hyperbolic half-plane. Of necessity, these are rather brief introductions to these subjects but, nevertheless, are clearly presented and, as always, accompanied by beautiful pictures.

Throughout the book there is a rich selection of exercises which range from routine calculations to open ended explorations. The first type are often presented ahead of where the calculation comes in the text, at which point the solution to the exercise is given. Many of the second type would make excellent starting points for a class project or an undergraduate thesis. Throughout the book also, there are comments of an aesthetic nature. Why are some patterns more attractive than others? Why do some colour schemes work better than others? Why is the curve defined in the second paragraph above more appealing than some others with similar symmetries?

Let me close with a quotation from the epilog which shows not only the essential contents of the book but also its overall philosophy and spirit. "I hope I have increased your sensitivity to a strange property of mathematics: that every part of it can find application in every other part... Vector spaces need not be literal spaces of vectors, but rather spaces of functions from which artists can choose patterns. Groups need not appear as abstract sets with generators and relations, but as symmetries of those patterns. Complex numbers are not just for doing contour integrals, but for moving the plane about and even controlling the symmetries of polyhedra. Fourier series are good for more than predicting the progress of heat in the legendary laterally insulated bar; they can help wallpaper patterns dance."

## References

- [1] F. Farris *Review of Visual Complex Analysis*, by Tristan Needham Amer. Math. Monthly, 105 (1998) 570–76.
- [2] [https://en.wikipedia.org/wiki/Color\\_wheel\\_graphs\\_of\\_complex\\_functions](https://en.wikipedia.org/wiki/Color_wheel_graphs_of_complex_functions)



## TARGET SURPASSED

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**Thank you!**



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La Société remercie toutes les personnes qui ont contribué à la campagne de don de milles SMC-Air Canada, et en particulier Gerri Jensen d'avoir coordonné cette initiative très fructueuse. Le fait d'atteindre notre objectif nous a valu 30 000 milles supplémentaires d'Air Canada.

Les milles que vous donnerez serviront au transport d'étudiants qui participeront par exemple à l'Olympiade internationale de mathématiques ou aux Réunions de recherche. Plus amples renseignements sont disponible à [beyondmiles.aeroplan.com/fra/charity/941](http://beyondmiles.aeroplan.com/fra/charity/941)

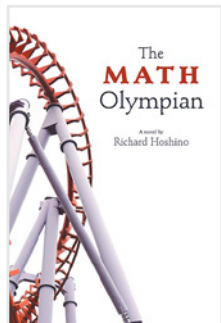
**Merci !**

## The Math Olympian

by Richard Hoshino

Friesen Press, 2015, ISBN: 978-1460258736

Reviewed by *Marc Chamberland*, Grinnell College



The book opens with an atypical scene: a teenage girl waits nervously as she is about to participate in the Canadian Mathematical Olympiad. Moments later, the reader is presented with a set of five math problems. As someone who has delighted in math contest problems since I was a student, my attention was immediately ensnared. Indeed, I stopped reading and plunged into solving the problems! After some success, it dawned on me that the author probably didn't expect his readers to put down his novel, pull out a pad of paper, and start doing math. Or did he?

This is the fictional story of a teenage girl named Bethany who, after seeing a news report about national and international math contests, realizes that there is a community of far-flung young people who, like her, unabashedly love mathematics. She wants to be one of them: a math olympian. From the outset, the context reveals that she will succeed, but her path is anything but clear. How does one become a math olympian? The author Richard Hoshino would know, having competed for Canada in the International Math Olympiad in 1996.

Hoshino has written a human story about Bethany's journey from being a naive girl who knows little about math or herself to someone who comes to know both much better. The chapters are structured around the five problems that Hoshino presents at the beginning of the book.

Hoshino does a masterful job of weaving the mathematics into the narrative. For each of the five main contest problems, he gently motivates the techniques that will be helpful, develops the components of the solution, and lastly offers an economical, polished write-up. Hoshino makes the problem-solving process accessible by having the characters ask questions, explore and surmount dead-ends, and

express their mathematical epiphanies. We witness Bethany's thought processes, an informative lesson for both students and teachers alike.

Don't be misled, however, into thinking that this is principally a problem-solving book in disguise. It is a novel detailing how an esteem-challenged teenager, through diligence, support and serendipity, becomes a star at math contests. One reads not only about how the world of mathematics is opened up to her, but also about Bethany's normal experiences as a young person, including the complexity of teenage relationships (with friends, enemies, lovers, coaches and parents), overcoming her anxieties, and overall, navigating the challenges of growing up.

Hoshino also does not limit the mathematics to the problems that are solved. While one encounters standard material — always presented in an engaging way — such as summing consecutive integers to produce the triangular numbers, telescoping series, the use of symmetry to solve algebraic equations, and some classical planar geometry, the reader is also exposed to Newcomb's Law (usually called Benford's Law), the idea of groups via Rubik's Cube (even a mention of Burnside's Problem), and dimensional analysis. Hoshino presents mathematics as it should be seen for the uninitiated: a taste of fascinating mathematical ideas without too much symbolic clutter.

As a Canadian who has lived outside the country for almost twenty years, it was also a treat to "tour" Canada. Bethany is from Sydney, Nova Scotia, and, until her involvement with math contests, has never left her home province. She soon brings us along to Ottawa and for a longer visit to Hoshino's current province, British Columbia. It has been said that an author's first novel is usually autobiographical and there is enough evidence to suggest that "The Math Olympian" is no exception. Hoshino takes us to his current institution (Quest University), escorts us into the world of competitive math contests, and, I suspect, shows us other elements of his personal life, all embodied in Bethany's experience. If you're looking for a young person's embrace of mathematics as she navigates the uneven road of growing up, you'll find an enjoyable read here. And some math problems to solve along the way.

*Marc Chamberlain is author of "Single Digits - In Praise of Small Numbers", which was reviewed in CMS Notes Vol. 47 No. 6.*



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*Education Notes brings mathematical and educational ideas forth to the CMS readership in a manner that promotes discussion of relevant topics including research, activities, and noteworthy news items. Comments, suggestions, and submissions are welcome.*

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## Looking Back on Problem Solving and Pedagogy from Waterloo Days

**John McLoughlin**, University of New Brunswick

Today, April 4th, I learned of the passing of Ross Honsberger, a retired teacher and professor well known for his problem solving talents. The timing of this news reminded me of a course taken with Ross while an undergraduate at University of Waterloo, in addition to more recent communications concerning a mutual colleague and friend, Jim Totten. Broadly speaking it is fair to say that my train of thought in preparing this issue of the *Education Notes* has been derailed. So I have decided to shift the focus to a retrospective of my own path linking mathematical problem solving and pedagogy with a focus on my experiences at University of Waterloo from 1980 through 1985. Initially I will share some pivotal moments before shifting the focus of attention to Ross Honsberger's place in the picture.

### The 100 Problems Course

The so-called *100 Problems* course opened with a stapled collection of problems being handed out the first day. This was effectively the content of the course in a nutshell. The professor, Dean Hoffman, taught arguably the most memorable course of my undergraduate days there. Typically class began with an open invitation to the class of seventy or so students along the lines, "Is there any problem that you would be willing to present a solution to in the next class?" Upon answering, it may be that the numbers 13, 28, 61, and 79 would be written on the board. Implicitly this meant our assignment was to attempt them before the next class. It was not that the solutions were to be collected but that the presentation of solutions would become more meaningful through grappling with the problems in advance.

Student presentations of solutions brought forth greater awareness of the range of perspectives and approaches taken in solving mathematical problems. Further, the smorgasbord of problems highlighted many mathematical techniques and ideas. Lulls in volunteerism suggested a need to bring forth more mathematics to the table. Judiciously it would fall upon Dean Hoffman then to introduce some relevant ideas in geometry such as isometries, or perhaps insights into primes and divisibility tests

*Notes pédagogiques présentent des sujets mathématiques et des articles sur l'éducation aux lecteurs de la SMC dans un format qui favorise les discussions sur différents thèmes, dont la recherche, les activités et les nouvelles d'intérêt. Vos commentaires, suggestions et propositions sont le bienvenue.*

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in number theory. The ideas could then be drawn into problem solving efforts with problems that had not yet been discussed from the collection. The back-and-forth nature of seeing solutions and learning mathematical concepts broadened my personal repertoire for problem solving in a manner that had lasting effects.

The importance of mathematical argument and proof was emphasized in this course in a novel way. That is, the need to concisely present an argument in solution to a problem was magnified by the peer involvement, as it was no longer a case of simply being marked on a paper. Instead the presence of peer input in the process had a way of sharpening the thinking and presentation. One such problem that I recall from the course has been used in my own teaching on numerous examples for this reason. It is not that the answer is difficult to obtain, but rather that there is a need to present a convincing argument – a skill needing development and frequently overlooked with respect to problems that may not appear to have a solution. A paraphrased recollection of the problem is shared below:

**In a certain classroom, there are 25 students seated in a square arrangement with 5 rows each having 5 seats. Each student is to change seats by going to the seat immediately in front or behind, or immediately to the left or right. (No diagonal moves are permitted.) Is it possible for all of the students to change seats in this manner? Explain.**

### Pedagogy and Student Experience

The significance of the above course extends into my teaching as the approach resonated with me. As someone who has been teaching over 30 years, there has been a conscious effort on my part to toss ideas out for consideration with a motivation for learning that goes beyond submitting work to be graded. Permission to muddle with mathematics and not necessarily successfully solve problems is valuable. Hence, I have tried to enable such opportunities for students. Pedagogically I am reminded of a couple other striking examples from my undergraduate experience that have figured into my own teaching.

Chris Springer taught typical lecture style classes on Mondays and Fridays (the days more likely to have attendance concerns in his experience). It was the midweek Wednesday *problems class* that made an impression. The class began with three or four problems written on the board concerning current course topics in probability and statistics. That would be the content for the day as the fifty



minutes were devoted to solving the problems. Chris would facilitate the process by moving about in the class periodically discussing a solution, offering a suggestion, or calling forth insights from the class. Pedagogically the merits were threefold: healthy student-professor interactions positively affected teaching & learning; conceptual understanding was developed through problem solving; and, further, experiences in working on problems collectively enhanced confidence in working on them alone. I use problems classes in two forms in my own teaching. First, there is the model discussed here. Second, I may pose problems a day or two in advance of class with answers (not solutions). The intention is to remove those students who can independently solve problems, thus, allowing for a smaller tutorial-like model to be employed with others in the class. Commonly I have a quiz in the following class to quickly temper any thoughts of simply making it a class to miss. I use the term “problem” loosely in this particular context as there may be problems and/or exercises integrated into the model depending upon topics.

Rob Brown taught several actuarial science courses in my undergraduate degree. An example was *Theory of Interest*, much of which involved doing a range of calculations, proofs, and problems involving financial mathematics. The course evaluation was simple with four tests worth 10% each and a final exam worth 60% of the course grade. Basically any questions appearing in the text at the end of the units or review were fair game for the tests. That is, all of the test questions were there already. The final exam would be so-called original questions in nature. There it is - do all of the questions, if you wish, or don't bother doing a, b, and c, if you can handle d, e, and f that are likely more difficult. Use your judgment to do a good cross-section of questions knowing that there are no tricks to be found on the tests. I found that refreshing and have used this in various courses as a professor. It is freeing to all of us as the students feel it is fair and as a teacher it opens up space for being more creative, in my opinion. Since there is a clear understanding of expectations with respect to the tests it appears that the students are much more receptive to doing questions in class that broaden the scope for learning. Hence, I find it much easier to integrate problems into my teaching. The perennial dilemma of doing problems in class and then having tests that differ in that they are not so problem-based is diminished in these classes.

## Remembering Ross Honsberger



Ross Honsberger

University of Waterloo offered two courses in Problem Solving that were numbered something like C&O 380 and 381 at the time. I took both of these courses, the first described above and the second with Ross Honsberger. The two offerings complemented each other in that the styles were different yet the learning rich in both instances.

Ross Honsberger loved mathematics and problems. His passionate engagement with the ideas and the smiling sense of

amusement he brought to the sharing of problems was special. Ross focused attention on problems he found to be rich. Many of these problems would be found in his expository books such as *Mathematical Gems* or *Mathematical Morsels* – enriching resources in themselves. He tended to spend most of an entire class on a particular problem or a piece of mathematics such as the Butterfly Theorem. It was enjoyable to be in the midst of a mathematician who demonstrated such a passion for the field. Fortuitously this would be subsequently experienced again in later studies during my Master's degree at University of Toronto. The wonder and passion became evident once again with Ed Barbeau's teaching of a mathematical problem-solving course.

## A Curious Connection

Jim Totten was a colleague with whom I shared problems through collaborations with the BC Mathematics Contests. The relationship deepened as a member of the Editorial Board during Jim's tenure as Editor of *Crux Mathematicorum with Mathematical Mayhem*. One thing learned through communications was how Jim was deeply influenced by Ross Honsberger. He considered Ross to be a mentor and an inspiration with respect to mathematical problem solving, in particular.

Jim Totten had a *Problem of the Week* feature through his years of teaching undergraduates, briefly at St. Mary's University in Halifax and subsequently in Cariboo College (and its subsequent renamings to finally Thompson Rivers University) in Kamloops. Upon Jim's sudden passing I was given a gift of the red binders containing the different problems that had been used by Jim over 25+ years with this feature. The collection was given to me with an intention of having it written up and shared, as it had been his intention in retirement to develop a resource. In fact, Jim authored *Volume VII* of the *ATOM Series* entitled *Problems of the Week* containing 80 of the problems. This was his start of a bigger project. Later myself along with Joseph Khoury and Bruce Shawyer co-edited *Jim Totten's Problems of the Week* (World Scientific Publishing, 2013). An excerpt from the Preface is offered here in tribute to Ross Honsberger as a person.

**Jim never pretended that the problems were original. The problems come from many sources, including several brought to his attention during his graduate studies at University of Waterloo from 1968 to 1974. It was there that Jim became acquainted with Ross Honsberger. Jim described himself as a willing listener when Ross wanted to share interesting problems or solutions with someone. This excitement for gems was contagious to Jim, and he proceeded to carry forth his own love of problems with a commitment to sharing that spirit of his own.**

My final personal communications with Ross Honsberger surrounded Jim's passing in different respects. There was a special issue of *Crux Mathematicorum with Mathematical Mayhem* dedicated to Jim Totten. Ross graciously received a request to make a contribution of a problem for this issue, and went well beyond

the request to prepare a seven-page article discussing a particular problem, namely, *The Tanker Problem*. The complete article is accessible at the link below. The problem is also shared here for those who wish to solve first and read later.

[https://cms.math.ca/crux/v35/n5/public\\_page283-289.pdf](https://cms.math.ca/crux/v35/n5/public_page283-289.pdf)

A security patrol boat repeatedly circles a supertanker that is a gigantic rectangular box 450 metres long and 50 metres across. The ocean is calm and the tanker travels at a constant speed along a straight path. The patrol boat goes up the left side, across the front, down the right side, and across the back, and keeps doing it over and over.

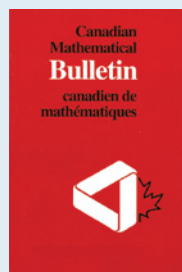
The patrol boat travels in only two directions of the compass – when going parallel to the path of the tanker, it travels in straight lines parallel to the tanker, one on each side at a distance of 25 metres from it, and when crossing in front or behind, it goes straight across perpendicular to the path of the tanker.

Neglecting the dimensions of the patrol boat (that is, considering it to be represented geometrically by a point) and given that it goes constantly at twice the speed of the tanker and that its turns are instantaneous, *what is the shortest distance that the patrol boat must travel in completing one cycle around the tanker?*

### Closing remarks

I am grateful for the presence of Ross Honsberger along my mathematical path. I continue to play with the content of some of his problem books. Upon reconnecting with Ross concerning the *Crux* article, I mentioned that we had held a conference honouring Jim Totten earlier in 2009, and that he may like a copy of the proceedings. I close with mention of this as the gracious spirit of Ross Honsberger was exemplified in his later communication: "The mailman delivered the Tribute to Jim Totten this morning. I am looking forward to reading it all. It's my kind of thing! I am delighted to have a copy. My sincerest thanks."

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Patrick Ingram, Colorado State (notes-research@cms.math.ca)

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## On Solving Bilevel Optimization Problems

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In an optimization problem, one looks for a point in a feasible region to optimize an objective function. A bilevel optimization problem consists of an upper level optimization problem where the constraint region is defined implicitly through another optimization problem called the lower level problem. Bilevel optimization problems can be used to model situations where there is a hierarchy in decision making and hence is often referred to as a leader-follower game or a Stackelberg game. There are various forms of bilevel optimization problems and they belong to a class of extremely difficult optimization problems. In this note we use the following simple version of the bilevel program:

$$\min_{x \in R^n, y \in R^m} F(x, y) \quad \text{s.t.} \quad y \in S(x), \quad (\text{BP})$$

where  $S(x)$  is the set of global minimizers of the optimization problem parameterized by  $x$ :

$$P_x : \min_{y \in Y} f(x, y)$$

where  $Y$  is a closed subset of  $R^m$  to illustrate challenges and some recent research results.

In the case where the solution set of the lower level problem  $S(x) = \{y(x)\}$  is a singleton for each  $x$ , (BP) is equivalent to minimizing the function  $F(x, y(x))$  over all  $x$ . Moreover if the solution  $y(x)$  can be explicitly solved as a function of  $x$  then hopefully the problem can be solved by using a certain optimization strategy.

In general, however, the solution set  $S(x)$  is not a singleton. For simplicity assume that the lower level solution always lies in the interior point of the set  $Y$ . Then any minimizer of the problem  $P_x$  is a stationary point, that is  $\nabla_y f(x, y) = 0$ . The classical first order approach is to consider the one level optimization problem

$$\min_{x, y} F(x, y) \quad \text{s.t.} \quad \nabla_y f(x, y) = 0. \quad (\text{FP})$$

But when the function  $f(x, y)$  is not convex in  $y$ , the stationary points may not be minimizers and hence the solutions to the lower level problem (FP) may not be a solution to the bilevel problem (BP). Moreover even a local optimal solution of the problem (BP) may not be a stationary point of the problem (FP). Hence one may never find the true optimal solution of (BP) by solving the relaxed problem (FP). Instead of requiring  $y \in S(x)$ , one may equivalently require that the objective function  $f(x, y)$  attains the optimal value at  $y$ . Then it is obvious that problem (BP) can be reformulated as the following single level optimization problem:

$$\min_{x, y \in Y} F(x, y) \quad \text{s.t.} \quad f(x, y) - V(x) = 0, \quad (\text{VP})$$

where  $V(x) := \inf_{y \in Y} f(x, y)$  is the value function of the lower level problem. In general the value function may not be differentiable. However when the set  $Y$  is compact, Danskin's theorem guarantees Lipschitz continuity and gives a formula for the Clarke generalized gradient which is a replacement of the classical gradient when a function is not differentiable. Denote by  $\partial V(x)$  the Clarke generalized gradient of the function  $V$  at  $x$ . An optimal solution  $(x^*, y^*)$  of (BP) where  $y^*$  lies in the interior of  $Y$  is a Clarke stationary point of problem (VP) if there exists some Lagrange multiplier  $\lambda$  such that

$$0 \in \nabla F(x^*, y^*) + \lambda(\nabla f(x^*, y^*) - \partial V(x^*) \times \{0\}).$$

The above condition, however, may be too strong since the above equation implies that  $\nabla_y F(x^*, y^*) = 0$  which is rarely true. Observe that an optimal solution  $(x^*, y^*)$  of (BP) where  $y^*$  lies in the interior of  $Y$  is also optimal for the combined program:

$$\min_{x, y \in Y} F(x, y) \quad \text{s.t.} \quad f(x, y) - V(x) = 0, \nabla_y f(x, y) = 0. \quad (\text{CP})$$

The generalized Lagrange multiplier rule for the problem (CP) is

$$0 \in \nabla F(x^*, y^*) + \lambda(\nabla f(x^*, y^*) - \partial V(x^*) \times \{0\}) + \nabla_{x, y}(\nabla_y f)(x^*, y^*)^T \eta$$

for some Lagrange multiplier  $\lambda, \eta$  and it is much more likely to hold at an optimal solution of (BP).



Now we discuss how to find stationary points for problems (VP) and (CP) numerically. We suppose that the set  $Y$  is nonempty and compact with a nonzero outer measure. For given  $\rho > 0$  and an integrable function  $f(x, y)$ , we define the integral entropy function as

$$\gamma_\rho(x) := -\rho^{-1} \ln \left( \int_Y \exp[-\rho f(x, y)] dy \right).$$

The integral entropy function is a smooth function. It turns out that the integral entropy function is a useful smooth approximation of the value function in the following sense.

**Theorem.** *Let  $f(x, y)$  be continuous in  $(x, y)$  and continuously differentiable in  $x$ . Then*

$$\lim_{z \rightarrow x, \rho \uparrow \infty} \gamma_\rho(z) = V(x) \text{ for any fixed } x.$$

Assume that  $f$  is a continuously differentiable function. Then the gradient consistent property holds. That is,

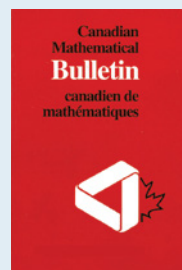
$$\emptyset \neq \limsup_{z \rightarrow x, \rho \uparrow \infty} \nabla \gamma_\rho(z) \subseteq \partial V(x) \text{ for any fixed } x.$$

The smoothing and the gradient consistent property allow us to design a numerical algorithm for finding stationary points of the problems (VP) and (CP); see [1, 3]. Recently an algorithm for globally solving (BP) where all functions involved are polynomials and the constraint set  $Y$  is semialgebraic was proposed in [2].

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## The Steklov Eigenvalue Problem: Some Open Questions

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Let  $\Omega$  be a compact Riemannian manifold of dimension  $n \geq 2$  with smooth boundary  $M = \partial\Omega$ . The Steklov problem on  $\Omega$  is

$$\Delta u = 0 \text{ in } \Omega, \quad \partial_\nu u = \sigma u \text{ on } M, \quad (1)$$

where  $\Delta$  is the Laplace operator acting on functions on  $\Omega$ , and  $\partial_\nu$  is the outward normal derivative along the boundary  $M$ . This problem was introduced by Vladimir Steklov in 1902. It is well known that the spectrum of the Steklov problem is discrete: the eigenvalues form a sequence  $0 = \sigma_0 < \sigma_1 \leq \sigma_2 \leq \dots \nearrow \infty$ . The Steklov eigenvalues can be interpreted as the eigenvalues of the Dirichlet-to-Neumann operator  $\mathcal{D} : C^\infty(M) \rightarrow C^\infty(M)$  which maps a function  $f \in C^\infty(M)$  to  $\mathcal{D}f = \partial_\nu(Hf)$ , where  $Hf$  is the harmonic extension of  $f$  to  $\Omega$ . The study of the Dirichlet-to-Neumann operator (also known as the voltage-to-current map) is essential for applications to electrical impedance tomography, which is used in medical and geophysical imaging.

Over the past few years, the Steklov eigenvalue problem has been actively investigated from the viewpoint of spectral geometry. In this note, which is based on our joint survey paper [7], we would like to discuss some recent developments and present some open problems on Steklov eigenvalues and eigenfunctions that could be of interest to a general mathematical audience.

Let us start with some isoperimetric type problems.

**Open Problem 1.** *What is the maximal value of  $\sigma_1(\Omega)$  among Euclidean domains  $\Omega \subset \mathbb{R}^n$  with boundary of fixed  $(n-1)$ -dimensional volume? On which domain (or in the limit of which sequence of domains) is the supremum realized?*

In the class of two dimensional simply connected domains, it was proved by Weinstock that the unique maximizer is a disk. However, the simple connectedness assumption cannot be removed, and therefore the problem above is open even for planar domains.

The question to maximize  $\sigma_k$  for arbitrary index  $k$  among Euclidean domains of given volume of the boundary is also interesting. It was proved in [3] that for any fixed  $k$ , the quantity  $\sigma_k(\Omega) \text{Vol}_{n-1}(\partial\Omega)^{\frac{1}{n-1}}$  is bounded above among all  $n$ -dimensional bounded Euclidean domains. Similar problems have also been considered on surfaces, where upper bounds in terms of the genus and the number of boundary components were proved in [4, 6, 8]. On higher dimensional Riemannian manifolds, eigenvalue estimates involving the isoperimetric ratio were obtained in [3].

One could also consider extremal problems for Steklov eigenvalues with the volume of the boundary constraint replaced by the assumption that the total volume of  $\Omega$  is fixed. For  $k = 1$ , it was proved by Brock that the ball  $B \subset \mathbb{R}^n$  is the unique maximizer among all Euclidean domains of given volume. The following conjecture for planar domains and arbitrary  $k \geq 1$  has recently been proposed in [1, 2].

**Open Problem 2.** *For each  $k \geq 2$ , prove the existence of a unique maximizer for  $\sigma_k$  among planar domains of fixed area. Show that it is connected and has the symmetry of a regular  $k$ -gon.*

Strong numerical evidence in favour of this conjecture is provided in both [1] and [2], based on different methods of computation.

We now turn to questions concerning isospectrality, which have been intensively studied for the Laplace operator following Kac's celebrated question "Can one hear the shape of a drum?"

**Open Problem 3.** *Is a planar domain uniquely determined by its Steklov spectrum?*

Note that this is not the case for Laplace eigenvalues: there exist isospectral but not isometric planar domains. However, none of the known methods for producing Laplace isospectral domains can be easily adapted to construct Steklov isospectral planar domains.

At the same time, it is known that the number and the individual lengths of the boundary components of a smooth domain are determined by its Steklov spectrum [5]. Moreover, smooth domains having the same connectivity and the same set of lengths of boundary components have Steklov spectra asymptotically very close to each other. For instance, the Steklov eigenvalues of a smooth simply connected planar domain  $\Omega$  of perimeter  $L$  are asymptotically "almost indistinguishable" from the eigenvalues of a disk of the same perimeter:

$$\sigma_{2j} = \sigma_{2j-1} + \mathcal{O}(j^{-\infty}) = \frac{2\pi}{L}j + \mathcal{O}(j^{-\infty}),$$

where  $\mathcal{O}(j^{-\infty})$  means that the error decays faster than any power of  $j$ . This makes the inverse spectral problem particularly difficult.

**Open Problem 4.** [9] *Let  $\Omega \subset \mathbb{R}^n$  be a domain which is isospectral to a ball of radius  $r$ . Show that it is a ball of radius  $r$ .*

In dimension two, the result holds by the combination of the equality case in Weinstock's inequality mentioned above, and the fact that the perimeter and connectedness are determined by the Steklov spectrum. In three dimensions, it has been proved in [9] that any smooth Euclidean domain with connected boundary which is Steklov isospectral to a ball is a ball of the same radius. In dimensions four and higher the question is completely open. Note that a similar problem is also open in the context of the Laplace operator: in dimension  $n \geq 7$  it is not known whether a round sphere is uniquely determined by its Laplace eigenvalues.

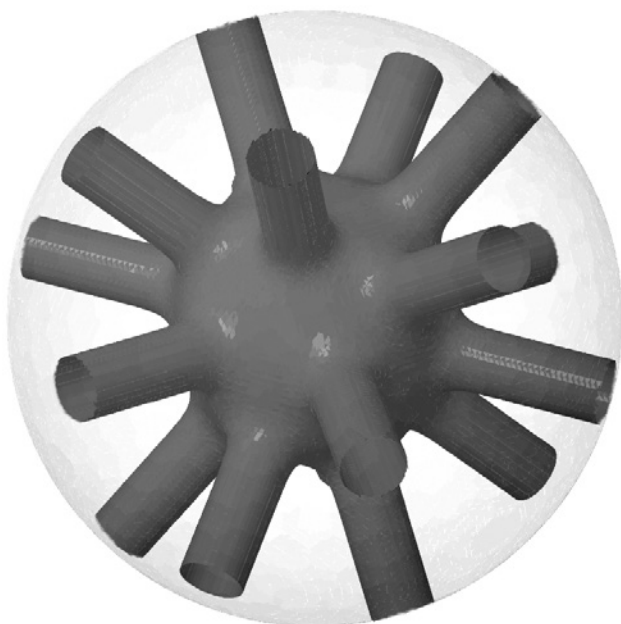
Let us now discuss some properties of Steklov eigenfunctions. The study of nodal domains and nodal sets of eigenfunctions is

probably the oldest topic in geometric spectral theory, going back to the experiments of E. Chladni with vibrating plates. The fundamental result in the subject is Courant's nodal domain theorem which states that the  $k$ -th eigenfunction of the Dirichlet boundary value problem has at most  $k$  nodal domains. The same result holds for Steklov eigenfunctions  $u : \Omega \rightarrow \mathbb{R}$ .

Apart from the interior nodal domains and nodal sets of Steklov eigenfunctions, one could also investigate the boundary nodal domains and nodal sets of  $\phi = u|_M$ , that is, the nodal domains and the nodal sets of the eigenfunctions of the Dirichlet-to-Neumann operator.

**Open Problem 5.** Let  $\Omega$  be a Riemannian manifold with boundary  $M$ . Find an upper bound for the number of nodal domains of the  $k$ -th eigenfunction of the Dirichlet-to-Neumann operator on  $M$ .

For surfaces, a simple topological argument shows that the bound on the number of the interior nodal domains implies an estimate on the number of the boundary nodal domains of a Steklov eigenfunction. In higher dimensions, the number of the interior nodal domains does not control the number of the boundary nodal domains.



Another fundamental problem in nodal geometry is to estimate the size of the nodal set. It was conjectured by S. T. Yau that for any Riemannian manifold of dimension  $n$ ,  $C_1\sqrt{\lambda} \leq \mathcal{H}_{n-1}(\mathcal{N}(\phi_\lambda)) \leq C_2\sqrt{\lambda}$ , where  $\mathcal{H}_{n-1}(\mathcal{N}(\phi_\lambda))$  denotes the  $n - 1$ -dimensional Hausdorff measure of the nodal set  $\mathcal{N}(\phi_\lambda)$  of a Laplace eigenfunction  $\phi_\lambda$ , and the constants  $C_1, C_2$  depend only on the geometry of the manifold. Similar questions can be asked for Steklov eigenfunctions:

**Open Problem 6.** Let  $\Omega$  be an  $n$ -dimensional Riemannian manifold with boundary  $M$ . Let  $u_\sigma$  be an eigenfunction of the Steklov problem on  $\Omega$  corresponding to the eigenvalue  $\sigma$  and let  $\phi_\sigma = u_\sigma|_M$  be the corresponding eigenfunction of the Dirichlet-to-Neumann operator on  $M$ . Show that

$$(i) \quad C_1\sigma \leq \mathcal{H}_{n-1}(\mathcal{N}(u_\sigma)) \leq C_2\sigma,$$

$$(ii) \quad C'_1\sigma \leq \mathcal{H}_{n-2}(\mathcal{N}(\phi_\sigma)) \leq C'_2\sigma,$$

where the constants  $C_1, C_2, C'_1, C'_2$  depend only on the Riemannian metric.

The upper bound in (ii) was conjectured by Bellova–Lin and proved in [11] for real analytic Riemannian manifolds with boundary. A number of partial results on this open problem has been obtained in the last couple of years. In particular, in [10], both upper and lower bounds in (i) were proved for real analytic Riemannian surfaces. The upper bound was later extended in [12] to real analytic Riemannian manifolds of arbitrary dimension.

For other open questions and further discussion on the spectral geometry of the Steklov problem we refer the reader to the recent survey article [7].

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## Mathematics in “Jazz Age” Toronto

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Mathematics figured largely in the early international scientific congresses held in Canada, starting in 1857 in Montreal with the American Association for the Advancement of Science (AAAS), which returned in 1882. The British Association for the Advancement of Science (BAAS) also began in Montreal (1884), and then moved west, to Winnipeg, in 1909. The first in Toronto was the AAAS (1889), followed by the BAAS in 1897 and the International Geological Congress in 1913. After World War I, the University of Toronto hosted the Americans again, in 1921; the British, in 1924; and the 1924 International Mathematical Congress (IMC, August 11–16). The programs of the last three meetings provide a view of mathematics and science—and their histories—in “Jazz Age” Toronto!

My source material comes from the University of Toronto (UofT) Library system, both archival (the complete minutes of the 1924 IMC Organizing Committee) and published (a complete set of IMC *Proceedings*, including the magnificent two volumes for 1924, which were produced by the UofT Press in 1928 and edited by UofT Mathematics Professor and impresario of scholarship John Charles Fields).

Fields deserves foremost credit for Toronto hosting these 1920s meetings. He was President of the Royal Canadian Institute (the co-host, along with UofT, of the 1921 AAAS), as well as the Chairman of the Local Arrangements Committee and the Local Representative for Section A, Mathematics.

Following that success Fields, in partnership with Physics Professor John Cunningham McLennan, acquired the 1924 BAAS. Then, Fields was on the Council of the American Mathematical Society (AMS) when it had to withdraw its offer to host the 1924 IMC because the International Mathematics Union adamantly refused to admit German and Austrian mathematicians in the aftermath of World War I. With the prearranged support of his President, Robert Falconer, Fields scooped up the opportunity on behalf of Canada and UofT.

Dean Alfred Tennyson DeLury convened IMC's Section VI: History, Philosophy, Didactics in University College at 2:30 p.m. on Monday, August 11. The Chairman was Florian Cajori, and the Secretary was Louis Karpinski. The participants returned for a second session the next day at 9:30 a.m. and then were left free to pursue their other mathematical interests on the remaining days of the Congress. During the two days, sixteen papers were delivered by fourteen different

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speakers in four different languages: French, English, Italian and “Latino sine flexione.” None of the speakers was Canadian. Nine papers were historical, five philosophical, and only two didactic. Three papers never made it into the *Proceedings*, six appeared as Abstracts, and the remaining seven were printed as full Communications.

Giuseppe Peano's “*De Aequalitate*” (“On Equality”) appeared as a two-page abstract but represented essentially the entire paper. Peano notes, in Italian, that “The article published here is written in ‘Latino sine flexione’ in which all the words are Latin, in thematic form (ablative or imperative); it's not grammatical.” Peano, from the University of Turin, remains a renowned philosopher of mathematics, responsible for Peano's Postulates, a basis for our number system. By 1924 he was also a leader in the movement for establishing a Universal Artificial Language. The other Italian present, Ettore Bortolotti (University of Bologna), discussed Evangelista Torricelli's *De Infinitis Hyperbolis*.

Three professors hailed from Switzerland; they all presented in French. Bern's Louis Jacques Crelier discussed “Observations pratiques de méthodologie”, focusing on universities preparing high school mathematics teachers with “their professional education needing to advance in tandem with their scientific education. . . . Teaching, and above all mathematics teaching, isn't a trade but a calling.” Henri Fehr, University of Geneva, similarly discussed “L'Université et la préparation des professeurs de mathématiques”. Later, Fehr published extensively on the 1924 Toronto IMC in his journal *L'Enseignement mathématiques*.

Louis Gustave du Pasquier, from Neuchâtel, shared his “Propositions concernant l'unification de la terminologie dans la numération parlée”, which had been endorsed at the 1923 Bordeaux Congress of the *Association française pour l'avancement des sciences*. After a short summary of the mathematical basis for numbering systems, Du Pasquier noted that “[T]he words *million* for  $10^6$  and *milliard* for  $10^9$  are used by all civilised peoples.” The remaining European, A.W. Conway, University College Dublin, Irish Free State, patriotically recounted “The Mathematical Works of Sir W.R. Hamilton.”

All other papers came from the United States. The founder of General Semantics, Count Alfred Korzybski of New York City's Fifth Avenue Bank, was the only non-academic. His “Time-binding: the General Theory” introduced the Anthropometer. Another New Yorker, Columbia's Cassius Jackson Keyser, looked at “the Doctrinal Function's role in Mathematics and General Thought” with its “service in economising intellectual energy and the possibility of one supreme function embracing all others.”

Berkeley's Florian Cajori contributed two papers: "Past Struggles Between Symbolists and Rhetoricians in Mathematical Publications" and "Uniformity of Mathematical Notations—Retrospect and Prospect". The first contrasted William Oughtred's 1648 "translation of the tenth book of Euclid . . . using about forty new symbols" with Robert Simson's 1756 *Elements of Euclid* "presenting Euclid unmodified avoiding all mathematical signs". Meanwhile, Peano's more recent *Formulaire de mathématiques* "practically disposes with ordinary language, expressing all propositions with a small number of signs."

"Retrospect and Prospect" is both more programmatic and more algebraic, citing various uses of the letter D including 35 varieties of partial derivative notation: "Much might have been achieved with greater symbolic uniformity. Mathematicians must break with their extreme individualism and organize strong international committees to adapt new and reject outgrown symbols."

George Abram Miller, University of Illinois at Urbana, looked at the "History of Several Fundamental Mathematical Topics", including numbers, systems of axioms, functions, and groups (Miller's own speciality). Ann Arbor's Louis Karpinski surveyed "Colonial American Arithmetics", listing "all the arithmetics published in the new world before 1755."

Both Miller and Karpinski, and possibly Cajori, had been in Toronto in 1921 with the AAAS. Then, Miller delivered a paper on Group Theory

to Section A2, the AMS: "Substitutions which are Commutative with Every Substitution of an Intransitive Group." For section A3, the Mathematical Association of America, of which Miller was the President, he spoke on "New American Mathematical Periodicals". The Committee on the History of Science, including Karpinski and Cajori, held a short six-paper programme, including a paper by Karpinski on Hermann von Helmholtz. UofT Philosophy Professor George Sidney Brett served as Local Section Representative.

These congresses did not land in a History of Mathematics and Science vacuum in Toronto. Already by 1895, Ottawa School Inspector John Cadenhead Glashen was congratulating DeLury for including the historical perspective in his mathematics courses. In 1921 Glashen contributed a paper to the Mathematics Section. In 1924 DeLury discussed the "Life of Evariste Galois" at a meeting of the student Mathematical and Physical Society. Similarly, the descriptions here are glimpses of a much broader context for the reception of the history and philosophy of mathematics and science at the University of Toronto in the "Jazz Age". But that's another story!

*David Orenstein (david.orenstein@utoronto.ca) is in the fourth year of retirement from teaching mathematics at an inner city Toronto high school. Inclement days can now be spent in archival research into the history of Canadian mathematics and science, and more welcoming ones in the parks and on the patios.*

## CMS Member Profile / Profil membre de la SMC

### Tim Alderson

**HOME:** University of New Brunswick  
Saint John

**CMS MEMBER SINCE:** 2009

**RESEARCH:** Finite Geometries and  
Coding Theory

**SELECTED ACHIEVEMENTS:** Over the past 15 years I have been fortunate enough to have been funded by NSERC, to have traveled to many different countries to meet and work with many fantastic people. I have received several teaching awards and was promoted to professor before turning 40 (a personal goal). I've held elected positions on Senate, as Department Head, on the CMS BoD, and have taken turns as Acting Dean. I am, however, most proud of my (eventual) ability to strike a balance between teaching and research, and later between teaching, research, and family.

**MEMORABLE MATH MOMENT:** The "aha!" moment in my humid student

apartment in London, (Ont.), when the birth of an idea transformed my thesis from nothing to complete... In an instance! Those are the moments that I live for. I have been chasing the dragon ever since.

**HOBBIES:** Other than mathematics, I enjoy fishing, hiking (mostly to find good trout pools), tinkering in my work shop and most things tech.

**LATEST BOOK READ:** Math: Finite Geometry and Combinatorial Applications by my good friend Simeon Ball. Other: Candide by Voltaire.

**LATEST PUBLICATION:** "The partition weight enumerator and bounds on MDS codes" with Svenja Huntemann (an undergrad researcher at the time)

**WHAT I WOULD CHANGE (ABOUT THE CMS):** If I had a magic wand I would increase funding and expand membership. I would like to see more high school math teacher participation at our meetings.



**CMS ROLES:** I have served as an Atlantic Director and on the Endowments Committee (which I currently chair).

**WHY I BELONG TO THE CMS:** Because I am a Canadian mathematician. Our country is vast, our fields are diverse, we may work in isolation, but many of our challenges are common. It behooves us to come together in the CMS.

## Call for Nominations — CJM/CMB Associate Editors



The Publications Committee of the CMS solicits nominations for Associate Editors for the Canadian Journal of Mathematics (CJM) and the Canadian Mathematical Bulletin (CMB). The appointment will be

for five years beginning January 1, 2017. The current members (with their end of term) are below.

For over fifty years, the **Canadian Journal of Mathematics** (CJM) and the **Canadian Mathematical Bulletin** (CMB) have been the flagship research journals of the Society, devoted to publishing original research works of high standard. The CJM publishes longer papers with six issues per year and the CMB publishes shorter papers with four issues per year. CJM and CMB are supported by respective Editors-in-Chief and share a common Editorial Board.

Expressions of interest should include your curriculum vitae, your cover letter and sent electronically to: [cjmcmb-ednom-2016@cms.math.ca](mailto:cjmcmb-ednom-2016@cms.math.ca) before September 15th 2016.

### Current members of CJM/CMB editorial board:

Henry Kim (Toronto)	12/2016	Editor-in-Chief CJM
Robert McCann (Toronto)	12/2016	Editor-in-Chief CJM
Jie Xiao (Memorial)	12/2019	Editor-in-Chief CMB
Xiaoqiang Zhao (Memorial)	12/2019	Editor-in-Chief CMB
Louigi Addario-Berry (McGill)	12/2018	Associate Editor
Jason Bell (Waterloo)	12/2020	Associate Editor
Hans Boden (McMaster)	12/2020	Associate Editor
Alexander Brudnyi (Calgary)	12/2020	Associate Editor
Florin Diacu (Victoria)	12/2016	Associate Editor
Ilijas Farah (York)	12/2020	Associate Editor
Ailana Fraser (UBC Vancouver)	12/2020	Associate Editor
Skip Garibaldi (UCLA)	12/2016	Associate Editor
Dragos Ghioca (UBC Vancouver)	12/2018	Associate Editor
Eyal Goren (McGill)	12/2018	Associate Editor
Robert Leon Jerrard (Toronto)	12/2016	Associate Editor
Anthony To-Ming Lau (Alberta)	12/2016	Associate Editor
Alexander Litvak (Alberta)	12/2016	Associate Editor
Javad Mashreghi (Laval)	12/2020	Associate Editor
Marco Merkli (Memorial)	12/2020	Associate Editor
Assaf Naor (Princeton)	12/2018	Associate Editor
Erhard Neher (Ottawa)	12/2016	Associate Editor
Nilima Nigam (Simon Fraser)	12/2020	Associate Editor
McKenzie Wang (McMaster)	12/2016	Associate Editor
Juncheng Wei (UBC Vancouver)	12/2018	Associate Editor
Daniel Wise (McGill)	12/2018	Associate Editor
Efim Zelmanov (UCSD)	12/2016	Associate Editor

## Appel à candidatures — Rédacteur(trice) associé(e) pour le JCM et le BCM

Le Comité des publications de la SMC sollicite des mises en candidatures pour des rédacteurs associés pour le Journal canadien de mathématiques (JCM) et pour le Bulletin Canadien de mathématiques (BCM). Le mandat sera de cinq ans qui commencera le 1er janvier 2017. Les membres actuels (avec la fin de leur terme) sont ci-dessous.

Reuves phares de la Société depuis plus de 50 ans, le **Journal canadien de mathématiques** (JCM) et le **Bulletin canadien de mathématiques** (BCM) présentent des travaux de recherche originaux de haute qualité. Le JCM publie des articles longs dans ses six numéros annuels, et le BCM publie des articles plus courts quatre fois l'an. Le JCM et le BCM ont chacun leur rédacteur en chef et partagent un même conseil de rédaction.

Les propositions de candidature doivent inclure votre curriculum vitae, votre lettre de présentation et doivent être envoyé par courriel électronique à : [jcmbcm-rednom-2016@smc.math.ca](mailto:jcmbcm-rednom-2016@smc.math.ca) au plus tard le 15 septembre 2016.

### Membres Actuels du Conseil de rédaction scientifique pour le JCM et le BCM:

Henry Kim (Toronto)	12/2016	Rédacteur en chef JCM
Robert McCann (Toronto)	12/2016	Rédacteur en chef JCM
Jie Xiao (Memorial)	12/2019	Rédacteur en chef BCM
Xiaoqiang Zhao (Memorial)	12/2019	Rédacteur en chef BCM
Louigi Addario-Berry (McGill)	12/2018	Rédacteur associé
Jason Bell (Waterloo)	12/2020	Rédacteur associé
Hans Boden (McMaster)	12/2020	Rédacteur associé
Alexander Brudnyi (Calgary)	12/2020	Rédacteur associé
Florin Diacu (Victoria)	12/2016	Rédacteur associé
Ilijas Farah (York)	12/2020	Rédacteur associé
Ailana Fraser (UBC Vancouver)	12/2020	Rédactrice associée
Skip Garibaldi (UCLA)	12/2016	Rédacteur associé
Dragos Ghioca (UBC Vancouver)	12/2018	Rédacteur associé
Eyal Goren (McGill)	12/2018	Rédacteur associé
Robert Leon Jerrard (Toronto)	12/2016	Rédacteur associé
Anthony To-Ming Lau (Alberta)	12/2016	Rédacteur associé
Alexander Litvak (Alberta)	12/2016	Rédacteur associé
Javad Mashreghi (Laval)	12/2020	Rédacteur associé
Marco Merkli (Memorial)	12/2020	Rédacteur associé
Assaf Naor (Princeton)	12/2018	Rédacteur associé
Erhard Neher (Ottawa)	12/2016	Rédacteur associé
Nilima Nigam (Simon Fraser)	12/2020	Rédactrice associée
McKenzie Wang (McMaster)	12/2016	Rédacteur associé
Juncheng Wei (UBC Vancouver)	12/2018	Rédacteur associé
Daniel Wise (McGill)	12/2018	Rédacteur associé
Efim Zelmanov (UCSD)	12/2016	Rédacteur associé





Summer Meeting – June 24-27, 2016

University of Alberta, Edmonton, Alberta

[cms.math.ca/Events/summer16](http://cms.math.ca/Events/summer16)

## Prizes | Prix

Excellence in Teaching Award

Prix d'excellence en enseignement

Ian VanderBurgh (Waterloo)

Krieger-Nelson Prize Lecture

Conférence | Prix Krieger-Nelson

Malabika Pramanik (UBC)

## Public Lecture | Conférence publique

Yuval Peres (Microsoft)

## Confirmed Regular Sessions | Sessions générales confirmées

**Advances in Lyapunov Functions in Mathematical Biology | Percées dans les fonctions de Lyapunov en biologie mathématique**

Hongbin Guo (Ottawa), Connell McCluskey (Wilfred Laurier)

**Algebraic Design Theory | Théorie de la conception algébrique**

Hadi Kharigani (Lethbridge)

**Algebraic Graph Theory: including Cayley graphs, group actions on graphs, graph eigenvalues, graphs and matrices | Théorie des graphes : y compris les graphes de Cayley, les actions de groupe sur les graphes, les valeurs propres des graphes, les graphes et les matrices**

Joy Morris (Lethbridge)

**Analysis and Applications of Differential Equations using Symmetries, Conservation laws, and Integrability | Analyse et applications d'équations différentielles utilisant des symétries, les lois de la conservation et l'intégrabilité**

Stephen Anco (Brock), Anton Cheviakov (Saskatchewan)

**Analytic Number Theory and Diophantine Equations | Théorie analytique des nombres et équations diophantiennes**

Michael Bennett (UBC), Patrick Ingram (Colorado State)

**Computational Number Theory | Théorie algorithmique des nombres**

Kevin Hare (Waterloo), Patrick Ingram (Colorado State)

**Convex and Discrete Geometry, and Geometric Analysis | Géométrie convexe et discrète, et analyse géométrique**

Alexander Litvak, Anna Lytova and Vladyslav Yaskin (Alberta)

**Geometric Methods in Mechanics and Control with Applications | Méthodes géométriques en mécanique et contrôle avec applications**

Vakhtang Putkaradze (Alberta), Dmitry Zenkov (Math, NCSU)

Réunion d'été – 24-27 juin 2016

Université de l'Alberta, Edmonton, Alberta

[cms.math.ca/Reunions/ete16](http://cms.math.ca/Reunions/ete16)

## Plenary Speakers | Conférences plénières

Andrea Bertozzi (UCLA)

Andrew Granville (Montreal)

Rachel Kuske (UBC)

Tatiana Shubin (San Jose State University)

Yuval Peres (Microsoft)

## Scientific Director | Directeur scientifique

Anthony Quas (Victoria)

Marcelo Laca (Victoria)

**Industrial Mathematics | Mathématiques industrielle**

Huaxiong Huang (York), Michael Lamoureux (Calgary), Odile Marcotte (UQAM)

**Mathematics Outreach Programs: Reach Out, Reach Wide, Reach Deep | Programmes de sensibilisation aux mathématiques: promouvoir les maths partout et pour tous**

Gerda de Vries (Alberta), Malgorzata Dubiel (SFU), Veselin Jungic (SFU)

**Partial Differential Equations | Équations aux dérivées partielles**

Mostafa Fazly (Alberta), Juncheng Wei (UBC)

**Rational Points, Rational Curves, and Positivity of Projective Varieties | Points rationnels, courbes rationnelles et positivité des variétés projectives**

Xi Chen (Alberta), Nathan Grieve (UNB)

**Representation Theory | Théorie des représentations**

Thomas Creutzig and Nicolas Guay (Alberta)

**Special Session on Combinatorial Games to celebrate Richard K. Guy's 100<sup>th</sup> Birthday | Session spéciale sur les jeux combinatoires pour célébrer le 100<sup>e</sup> anniversaire de Richard K. Guy**

Richard Nowakowski (Dalhousie)

**Student Research Presentations | Présentations de recherche des étudiants**

Muhammad Khan (Calgary), Kyle MacDonald (McMaster)

**Theoretical and numerical methods in nonlinear analysis with real-world applications | Méthodes théoriques et numériques de l'analyse non linéaire et leurs applications**

Fabrice Colin and Albert Nina Sandjo (Laurentian)

**Contributed Papers | Communications libres**

**AARMS-CMS Student Poster Session | Présentations par affiches pour étudiants**

Svenja Huntemann (Dalhousie)

## Call for Nominations – CMS Research Prizes

The CMS Research Committee is inviting nominations for three prize lectureships. These prize lectureships are intended to recognize members of the Canadian mathematical community.

The **Coxeter-James Prize** Lectureship recognizes young mathematicians who have made outstanding contributions to mathematical research. The recipient shall be a member of the Canadian mathematical community. Nominations may be made up to ten years from the candidate's Ph.D. A nomination can be updated and will remain active for a second year unless the original nomination is made in the tenth year from the candidate's Ph.D. For more information, visit: <https://cms.math.ca/Prizes/cj-nom>

The **Jeffery-Williams Prize** Lectureship recognizes mathematicians who have made outstanding contributions to mathematical research. The recipient shall be a member of the Canadian mathematical community. A nomination can be updated and will remain active for three years. For more information: <https://cms.math.ca/Prizes/jw-nom>

The **Krieger-Nelson Prize** Lectureship recognizes outstanding research by a female mathematician. The recipient shall be a member of the Canadian mathematical community. A nomination can be updated and will remain active for two years. For more information: <https://cms.math.ca/Prizes/kn-nom>

The deadline for nominations, including at least three letters of reference, is **September 30, 2016**. Nomination letters should list the chosen referees and include a recent curriculum vitae for the nominee. Some arms-length referees are strongly encouraged. Nominations and the reference letters from the chosen referees should be submitted electronically, preferably in PDF format, to the corresponding email address and **no later than September 30, 2016** :

Coxeter-James: [cjprize@cms.math.ca](mailto:cjprize@cms.math.ca)  
Jeffery-Williams: [jwprize@cms.math.ca](mailto:jwprize@cms.math.ca)  
Krieger-Nelson: [knprize@cms.math.ca](mailto:knprize@cms.math.ca)

## Appel de mises en candidature – Prix de recherche de la SMC

Le Comité de recherche de la SMC lance un appel de mises en candidatures pour trois de ses prix de conférence. Ces prix ont tous pour objectif de souligner l'excellence de membres de la communauté mathématique canadienne.

Le **Prix Coxeter-James** rend hommage aux jeunes mathématiciens qui se sont distingués par l'excellence de leur contribution à la recherche mathématique. Cette personne doit être membre de la communauté mathématique canadienne. Les candidats sont admissibles jusqu'à dix ans après l'obtention de leur doctorat. Toute mise en candidature est modifiable et demeurera active l'année suivante, à moins que la mise en candidature originale ait été faite la 10<sup>e</sup> année suivant l'obtention du doctorat. Pour les renseignements, voir : <https://cms.math.ca/Prix/cj-nom>

Le **Prix Jeffery-Williams** rend hommage aux mathématiciens ayant fait une contribution exceptionnelle à la recherche mathématique. Cette personne doit être membre de la communauté mathématique canadienne. Toute mise en candidature est modifiable et demeurera active pendant trois ans. Pour les renseignements, voir : <https://cms.math.ca/Prix/jw-nom>

Le **Prix Krieger-Nelson** rend hommage aux mathématiciennes qui se sont distinguées par l'excellence de leur contribution à la recherche mathématique. La lauréate doit être membre de la communauté mathématique canadienne. Toute mise en candidature est modifiable et demeurera active pendant deux ans. Pour les renseignements, voir : <https://cms.math.ca/Prix/info/kn>

La date limite pour déposer une candidature, qui comprendra au moins trois lettres de référence, est le **30 septembre 2016**. Le dossier de candidature doit comprendre le nom des personnes données à titre de référence ainsi qu'un curriculum vitae récent du candidat ou de la candidate. Nous vous incitons fortement à fournir des références indépendantes. Veuillez faire parvenir les mises en candidature et lettres de référence par voie électronique, de préférence en format PDF, à l'adresse électronique correspondante et **au plus tard le 30 septembre 2016** :

Coxeter-James : [prixcj@smc.math.ca](mailto:prixcj@smc.math.ca)  
Jeffery-Williams : [prixjw@smc.math.ca](mailto:prixjw@smc.math.ca)  
Krieger-Nelson : [prixkn@smc.math.ca](mailto:prixkn@smc.math.ca)

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Winter Meeting – December 2-5, 2016 – Sheraton on the Falls, Niagara Falls, Ontario

## Call For Sessions

The Canadian Mathematical Society (CMS) welcomes and invites session proposals for the 2016 CMS Winter Meeting in Niagara Falls from December 2-5, 2016. Proposals should include a brief description of the focus and purpose of the session, the expected number of speakers, as well as the organizer's name, complete address, telephone number, e-mail address, etc. All sessions will be advertised in the CMS Notes, on the CMS web site and in the AMS Notices. Speakers will be

requested to submit abstracts, which will be published on the CMS web site and in the meeting program. Those wishing to organize a session should send a proposal to the Scientific Directors.

### Scientific Directors

Hans Boden, McMaster University, [boden@mcmaster.ca](mailto:boden@mcmaster.ca)  
Bartosz Protas, McMaster University, [bprotas@mcmaster.ca](mailto:bprotas@mcmaster.ca)



Réunion d'hiver – 2-5 décembre 2016 – Sheraton on the Falls, Niagara Falls (Ontario)

## Appel de propositions de séances

La Société mathématique du Canada (SMC) vous invite à proposer des séances pour la Réunion d'hiver de la SMC 2016 qui se tiendra à Niagara Falls du 2 au 5 décembre 2016. Ces propositions de séances doivent présenter une brève description de l'orientation et des objectifs de la séance, le nombre de conférenciers prévu, de même que le nom, l'adresse complète, le numéro de téléphone et l'adresse électronique de l'organisateur. Toutes les séances seront annoncées dans les Notes de la SMC, sur le site Web SMC et dans les AMS Notices.

Les conférenciers devront présenter un résumé, qui sera publié sur le site Web SMC et dans le programme de la réunion. Toute personne qui souhaiterait organiser une séance est priée de faire parvenir une proposition aux directeurs scientifiques.

### Directeurs scientifiques

Hans Boden, l'Université McMaster, [boden@mcmaster.ca](mailto:boden@mcmaster.ca)  
Bartosz Protas, l'Université McMaster, [bprotas@mcmaster.ca](mailto:bprotas@mcmaster.ca)





# MATHEMATICAL CONGRESS OF THE AMERICAS CONGRÈS MATHÉMATIQUE DES AMÉRIQUES



# MCA 2017



## JULY 24-28 JUILLET MONTRÉAL, CANADA



The second Mathematical Congress of the Americas (MCA) will take place on July 24-28, 2017, at Centre Mont-Royal and McGill University, Montreal, Canada. The congress is expected to attract mathematicians and students from throughout North America, Central America, South America and the Caribbean.

MCA 2017 highlights mathematical achievements of the Americas and fosters collaboration between the continents' mathematical communities. The congress is a collective initiative of the **Mathematical Council of the Americas** (MCofA). MCA 2017 is being supported by a Canadian organizing committee that includes the Pacific Institute for the Mathematical Sciences (PIMS), the Fields Institute (FIELDS), Le Centre de recherches mathématiques (CRM), the Atlantic Association for Research in the Mathematical Sciences (AARMS) and the CMS, which is staging the event.

There will be a large number of special sessions. The deadline for submitting proposals to organize is July 2016.

Le deuxième Congrès mathématique des Amériques (CMA) aura lieu du 24 au 28 juillet 2017 au Centre Mont-Royal et l'Université McGill, à Montréal, Canada. L'événement devrait attirer des mathématiciens et mathématiciennes ainsi que des étudiantes et étudiants de partout à travers l'Amérique du Nord, l'Amérique centrale, l'Amérique du Sud et les Caraïbes.

Le CMA 2017 met en lumière les accomplissements mathématiques des Amériques et encourage la collaboration entre les différentes communautés mathématiques du continent. Le congrès est une initiative collective du **Mathematical Council of the Americas** (MCofA). Le CMA 2017 est financé par un comité canadien incluant le Pacific Institute for the Mathematical Sciences (PIMS), l'institut Fields (FIELDS), le Centre de recherches mathématiques (CRM), l'Atlantic Association for Research in the Mathematical Sciences (AARMS) et la SMC, qui organise aussi l'événement.

Il y aura également de nombreuses sessions spéciales. La date limite de réception des propositions d'organisation est le juillet 2016.

[mca2017.org](http://mca2017.org)

[mca2017.org/fr](http://mca2017.org/fr)

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