CMS NOTES de la SMC

October / November
octobre / novembre
2016

IN THIS ISSUE / DANS CE NUMÉRO

CMS SMC
Canadian Mathematical Society
Société mathématique du Canada

Vice-President's Notes / Notes du Vice-président

Florin Diacu *(Victoria)*
Vice-President Pacific / Vice-président, Pacifique

On Mathematical Language

All those who have attended an International Congress of Mathematicians may have felt disappointed at realizing how little mathematics they know. Most titles of the talks, without even mentioning the abstracts, are usually incomprehensible. This problem occurs not only because our field has grown larger than life in the past century, but also because of the language barrier that we encounter. And by this I mean mathematical language, since every participant at such events has at least some rudimentary knowledge of English.

Whether we like it or not, we have developed hundreds of mathematical dialects that stem from the same basic language that we learned as undergraduates, each dialect employing concepts and symbols that are foreign to the uninitiated. Since most of us are experts in only one or two research directions, it's no wonder that we reached this point. How did we get here, and is this good or bad for mathematics?

A comprehensive source that traces the history of mathematical notations is the recent book *Enlightening Symbols* by my friend Joe Mazur. But in trying to answer the above questions, I am taking a slightly different and much shorter route than he did in his beautiful book.

If we open, say, the 3rd edition of The History of Mathematics by Victor J. Katz, we quickly learn that, except for various notations for

Continued on page 4

Le langage mathématique

Si vous avez déjà assisté à un Congrès international des mathématiciens, vous en êtes peut-être revenu déçu de la maigreur de vos connaissances mathématiques. La plupart des titres des conférences, sans parler des résumés, sont généralement incompréhensibles. Ce problème tient non seulement au fait que notre domaine a connu une expansion phénoménale au cours du dernier siècle, mais aussi à la barrière de la langue. Je parle bien sûr du langage mathématique, puisque tous les participants à de tels événements ont au moins une connaissance rudimentaire de l'anglais.

Que cela nous plaise ou non, nous avons développé des centaines de dialectes mathématiques qui découlent du même langage de base que nous avons appris en tant qu'étudiants de premier cycle, chaque dialecte employant des concepts et des symboles qui sont étrangers aux non-initiés. Comme la plupart d'entre nous sont des experts dans un ou deux domaines de recherche, il n'est pas étonnant que nous en soyons arrivés là. Mais comment cela s'est-il produit, et est-ce bon ou mauvais pour les mathématiques?

Le livre *Enlightening Symbols* publié récemment par mon ami Joe Mazur est une source riche qui retrace l'histoire des notations mathématiques. Mais en essayant de répondre aux questions précédentes, je prends une trajectoire légèrement différente et bien plus courte que l'auteur de ce bel ouvrage.

Si l'on ouvre, par exemple, la 3e édition de l'histoire des mathématiques de Victor J. Katz, on apprend rapidement que, sauf pour diverses notations de nombres, les symboles mathématiques sont apparus très lentement et seulement à la Renaissance. Avant cela, tout

Suite à la page 5

Having the Gang Over

Robert Dawson, *Department of Mathematics and Computer Science, Saint Mary's University*

This summer I attended three conferences; one of them I helped to arrange. As many of you know, it's a bit different when you're an organizer.

The conference wasn't huge, under ninety people, but it was still a lot of work. Getting funding, spreading the word, booking rooms, arranging accommodation, turning the abstracts into a conference booklet: even before people arrive, there's a lot to be done. And once they arrive, of course, things don't slow down until the final talk is over.

Of course, something always goes wrong. This time it was the outing on Wednesday afternoon. We'd offered three options, but by far the most popular was the boat trip to McNab's Island in Halifax Harbour, a five-kilometer-long island with hiking trails and old fortifications. It had been so popular that we'd arranged a second sailing. The weather was forecast to be dry and sunny, as it had been for weeks.

Then, Tuesday morning, an email came from the boat owner. Because of all the dry weather, the forest fire risk had been set to "extreme", and the province had implemented a wilderness travel ban, including McNab's Island. We decided that we could - just - accommodate everybody on the other option that involved hiking, and made the announcement.

Then, later that day, there was an update. The boat company had checked, and we could go - if we obtained a special wilderness travel permit from the Department of Natural Resources, and took along a trained guide with each boatload. Arrangements were made, the conference goers were told that the trip was back on, and the afternoon was a resounding success. The guides, who knew the history of the island in detail, made the visit even more interesting.

If you've never organized a conference before, it's an experience worth trying. Start off by working with people who've done it before, get involved in as many different aspects as you can, and be prepared for surprises. Your university will be able to help with many aspects; if you use hotels for accommodation, university caterers for breaks, or a restaurant for your banquet, they are all used to this sort of thing and will be able to help, too. It can be disconcerting to see how much coffee and cookies for a hundred people, ten times, comes to - but once you divide that a hundred ways, it will fit into the conference fee without scaring people away.

Another hidden element in most successful conferences is the student helpers. If you're organizing a conference, there are a surprising number of places where your students can help out. If you're a student, volunteering is a great way to learn about how conferences work - and later, when you're on your first organizing committee, it'll seem more familiar.

Organizing a conference is a big project, and does take time. But you don't have to do it every year, and insuperable problems are rare. Good luck!

Recevoir ses collègues chez soi

Cet été, j'ai assisté à trois congrès et participé à l'organisation de l'un d'entre eux. Comme vous êtes nombreux à le savoir, c'est un peu différent quand on est organisateur.

Le congrès n'était pas très gros (moins de 90 personnes), mais c'était tout de même beaucoup de travail. Sécuriser le financement, faire connaître l'événement, réserver les salles, publier les résumés dans le programme du congrès : il y a beaucoup à faire avant même l'arrivée des participants. Et une fois qu'ils arrivent, bien sûr, les choses ne ralentissent pas jusqu'à la fin de la dernière conférence.

Évidemment, il y a toujours des imprévus. Cette fois-là, c'était la sortie du mercredi après-midi. Nous avions offert trois options, mais l'activité de loin la plus populaire était la sortie en bateau à l'île McNab, dans le port d'Halifax, une île de 5 km de long abritant des sentiers pédestres et d'anciennes fortifications. L'activité avait été si populaire que nous avions prévu un deuxième bateau. Le temps devait être sec et ensoleillé, comme il l'avait été pendant des semaines.

Puis, le mardi matin, nous avons reçu un courriel du propriétaire du bateau. En raison de la sécheresse, le risque d'incendie de forêt venait de passer à « Extrême », et la province avait interdit l'accès à certaines zones naturelles, dont l'île McNab. Nous avons alors décidé que nous pouvions - tout juste - accueillir tous les participants pour l'autre option offrant de la randonnée, ce que nous avons annoncé.

Puis, plus tard ce jour-là, nous avons reçu d'autres nouvelles. Le propriétaire des bateaux avait vérifié, et il était possible de se rendre sur l'île à condition d'obtenir un permis spécial de déplacement en nature du ministère des Ressources naturelles et de retenir les services d'un guide d'expérience pour chaque bateau. Nous avons pris les arrangements nécessaires et informé les participants que la sortie en bateau était de nouveau offerte, et l'après-midi a connu un succès retentissant. Les guides, qui connaissaient à fond l'histoire de l'île, ont rendu la visite encore plus intéressante.

Si vous n'avez jamais organisé de congrès, le jeu en vaut la chandelle. Commencez par travailler avec des gens qui l'ont déjà fait, participez à autant d'aspects de l'organisation que possible et attendez-vous à des surprises. Votre université sera en mesure de vous aider à de nombreux égards; si vous confiez l'hébergement à des hôtels, les pauses-santé aux traiteurs de l'université ou le banquet à un restaurant, tous ces partenaires, habitués à ce genre d'organisation, seront aussi en mesure de vous accompagner. Il peut être déconcertant de voir la quantité de café et de biscuits à prévoir pour une centaine de personnes (multiplier par dix!), mais une fois le coût divisé par 100, il s'intégrera sans peine aux droits d'inscription et ne fera reculer personne.

Il ne faut pas non plus sous-estimer l'aide des étudiants, cet élément caché de la plupart des congrès réussis. En effet, vous serez surpris de constater à combien d'endroits vos étudiants peuvent vous venir en aide. Si vous êtes étudiant, le bénévolat est un excellent moyen de connaître le fonctionnement d'un congrès et, le jour où ferez partie pour la première fois d'un comité d'organisation, tout vous semblera plus familier.

L'organisation d'un congrès est une grande entreprise qui demande du temps. Mais rien ne vous oblige à le faire chaque année, et les problèmes insurmontables sont rares. Bonne chance!

Letters to the Editors

The Editors of the NOTES welcome letters in English or French on any subject of mathematical interest but reserve the right to condense them. Those accepted for publication will appear in the language of submission. Readers may reach us at the Executive Office or at **notes-letters@cms.math.ca**

Lettres aux Rédacteurs

Les rédacteurs des NOTES acceptent les lettres en français ou en anglais portant sur n'importe quel sujet d'intérêt mathématique, mais ils se réservent le droit de les comprimer. Les lettres acceptées paraîtront dans la langue soumise. Les lecteurs peuvent nous joindre au bureau administratif de la SMC ou à l'adresse suivante : **notes-lettres@smc.math.ca**.

Canadian Mathematical Society - Société mathématique du Canada
209-1725 St. Laurent Blvd., Ottawa, ON, Canada K1G 3V4 tel 613-733-2662

notes-articles@cms.math.ca | smc.math.ca | cms.math.ca
ISSN :1193-9273 (imprimé/print) | 1496-4295 (électronique/electronic)

Continued from cover

numbers, mathematical symbolism started developing very slowly and only as late as the Renaissance. Until then every mathematical statement was expressed in common language. For example, in the *Moscow Mathematical Papyrus*, which dates from around 1700 BC in Egypt, the scribe gave the formula for the volume of a truncated square pyramid in the following words (Katz, p. 10): "If someone says to you: a truncated pyramid of 6 for the height by 4 on the base by 2 on the top, you are to square this 4; the result is 16. You are to double 4; the result is 8. You are to square this 2; the result is 4. You are to add the 16 and the 8 and the 4; the result is 28. You are to take 1/3 of 6; the result is 2. You are to take 28 two times; the result is 56. Behold, the volume is 56. You will find that this is correct." What a complicated way to state a familiar formula, $V=(a^2+ab+b^2)\dfrac{h}{3}$, which every schoolchild learns today!

Some twenty-three centuries later, nothing had changed. All mathematical statements were still expressed in plain words. The Indian mathematician Brahmagupta, for instance, provided the formula of the solution $x=\dfrac{\sqrt{4ac+b^2}-b}{2a}$ for the quadratic equation $ax^2+bx=c$ as follows (Katz, p. 242). Calling b, as well as x, the "middle number," a "the square," and c "the rūpas," he wrote: "Diminish by the middle number the square root of the rūpas multiplied by four times the square of the middle number; divide the remainder by twice the square. The result is the middle number."

Common local language continued to be the mathematical language all over the world. The first signs of symbolism appeared in Europe in the early part of the 15th century. Some Italian abacists began to introduce some crude notations. Concepts like *radice* (root), *censo* (square), and *cubo* (cube) were abbreviated to R, ce, and cu, respectively. The expression x^2x^3, for instance, appeared as ce cu. The fifth power, x^5, was later written as *p.r.* (primo relato), and the sixth power, x^6, as *s.r.* (secondo relato). But most other things still appeared in words (Katz, p. 386).

Newton's *Principia Mathematica*, published in 1687, finally contained algebraic expressions, but differentials and integrals were expressed with the help of geometric figures. As a result, even experts in celestial mechanics find this book diffcult to read today. An edition published by The Folio Society in 2008 appeared in two volumes, one of them being a guide to the other, which contains the original work.

The basic modern symbolism we use now started to show up in the memoirs of Euler, Lagrange, and Laplace, i.e. as late as the 18th century, but even some books from the first part of the 19th century were lacking in proper notation. A few decades later, however, things changed. The works of Poincaré, for instance, were written in a style easy to follow, with clear explanations and inspired formalism.

The invention of an adequate mathematical language triggered a rapid development of our field, which boomed in the 20th century and has grown exponentially to this day. The language for a mathematician is like the toolbox for a mechanic or the medical instruments for a surgeon. The problem, however, is that it has become more and more diffcult to communicate between various branches of mathematics since each of them is now so specialized. It is no wonder that work that lays bridges between apparently very different research areas is held in high regard.

After all, mathematics is not a chest of drawers, one labeled geometry, another analysis, and yet another algebra, but a huge and complicated web that is both disconnected and unified. So we should not be disappointed for not understanding most titles of talks at the International World Congress of Mathematicians, but keep researching what we like and try to learn new mathematical dialects in the course of our lives. The more such languages we master, the more we will be able to understand.

New CRM System Coming to CMS!

The CMS has undertaken a review of its IT operations and is now implementing a new Customer Relationship Management (CRM) system to better serve you. This will mark a substantial improvement and streamlining in the way members can renew and donate directly to the Society, receive receipts automatically by email and allow them to manage their own profiles and preferences of communication, and much more!

All members will receive an email with instructions on how to set up their own username and password and update their profile. We encourage you to review/update your profile as soon as possible after receiving this email. A few days following this email, the 2017 renewals will be emailed to members.

We ask for your patience and understanding during this significant transition.

Should you encounter any difficulties setting up your profile or renewing your membership, please contact us at **memberships@cms.math.ca**.

Suite de la couverture

énoncé mathématique était exprimé dans un langage commun. Par exemple, dans le *Papyrus de Moscou*, qui date d'environ 1700 av. J.-C. en Égypte, le scribe a donné la formule pour trouver le volume d'une pyramide carrée tronquée dans les termes suivants (Katz, p. 10) : « Si quelqu'un vous dit : une pyramide tronquée de 6 pour la hauteur sur 4 pour la base sur 2 pour le dessus, vous devez mettre ce 4 au carré; le résultat est 16. Vous devez ensuite doubler le 4, ce qui donne 8. Vous devez mettre le 2 au carré, ce qui donne 4. Vous devez additionner 16, 8 et 4, ce qui donne 28. Vous devez multiplier 6 par 1/3, ce qui donne 2. Vous devez multiplier 28 par 2, ce qui donne 56. Le volume de cette pyramide est donc de 56. Vous constaterez que ce calcul est correct. » Quelle façon compliquée d'énoncer une formule courante, $V=(a^2+ab+b^2)\frac{h}{3}$, que connaissent la plupart des élèves de nos jours!

Quelque 23 siècles plus tard, rien n'avait changé. Tous les énoncés mathématiques étaient encore exprimés en termes simples. Le mathématicien indien Brahmagupta, par exemple, a fourni la formule suivante de la solution $x=\dfrac{\sqrt{4ac+b^2}-b}{2a}$ pour l'équation quadratique $ax^2+bx=c$ (Katz, p. 242). En utilisant b, ainsi que x étant « le nombre du milieu », a étant « le carré » et c, « les rūpas », il a écrit : « Diminuez du nombre du milieu la racine carrée des rūpas multipliée par quatre fois le carré du nombre du milieu; divisez le reste par deux fois le carré. Vous obtiendrez ainsi le nombre du milieu. »

Le langage mathématique était la langue locale commune, et ce, toujours partout dans le monde. Les premiers signes du symbolisme sont apparus en Europe au début du 15e siècle. Certains abacistes italiens ont commencé à introduire quelques notations brutes. Des concepts comme *radice* (racine), *censo* (carré) et *cubo* (cube) ont été abrégés en R, ce et cu, respectivement. L'expression x^2x^3, par exemple, était notée « $ce\ cu$ ». La puissance cinq, x^5, a ensuite été écrite « *p.r.* » (primo relato), et la puissance six, x^6, « *s.r.* » (secondo relato). Mais la plupart des autres concepts étaient encore notés en mots (Katz, p. 386).

Les *Principia Mathematica* de Newton, publiés en 1687, contenaient enfin des expressions algébriques, mais les différentielles et les intégrales étaient exprimées à l'aide de figures géométriques. Par conséquent, même les experts en mécanique céleste trouvent ce livre difficile à lire aujourd'hui. Une édition publiée par The Folio Society en 2008 est parue en deux volumes, l'un d'entre eux étant un guide de l'autre, qui contient l'œuvre originale.

Le symbolisme moderne de base que nous utilisons maintenant a commencé à voir le jour dans les mémoires d'Euler, Lagrange et Laplace, soit aussi tard qu'au 18e siècle; toutefois, certains ouvrages de la première partie du 19e siècle n'utilisaient pas la notation appropriée. Quelques décennies plus tard, cependant, les choses ont changé. Les travaux de Poincaré, par exemple, ont été écrits dans un style facile à suivre, avec des explications claires et un formalisme inspiré.

L'invention d'un langage mathématique adéquat a déclenché une évolution rapide de notre domaine, qui a explosé au 20e siècle et a connu une croissance exponentielle à ce jour. Le langage d'un mathématicien est comme le coffre à outils d'un mécanicien ou les instruments chirurgicaux d'un chirurgien. Le problème, toutefois, c'est qu'il est devenu de plus en plus difficile de communiquer entre les différents domaines des mathématiques, chacun étant devenu si spécialisé. Il n'est pas étonnant que l'on tienne en haute estime les travaux qui jettent des ponts entre des domaines de recherche très différents.

Après tout, les mathématiques ne sont pas une commode à tiroirs, l'un marqué « géométrie », l'autre « analyse » et un autre encore « algèbre », mais une toile immense et complexe à la fois déconnectée et unifiée. Nous ne devrions donc pas être déçus de ne pas comprendre la plupart des titres des conférences du Congrès international des mathématiciens; nous devrions continuer à pousser les recherches dans les domaines que nous aimons et essayer d'apprendre de nouveaux dialectes mathématiques tout au long de notre vie. Plus nous maîtriserons ces langues, plus nous serons en mesure de comprendre.

Bientôt un nouveau système de GRC à la SMC!

La SMC revoit en ce moment ses activités informatiques et met en place un système de gestion de la relation client (GRC) pour mieux vous servir. Ce système améliorera et rationalisera substantiellement les processus et permettra aux membres de renouveler leur adhésion et de faire un don directement à la Société, de recevoir automatiquement des reçus par courriel et de gérer leurs propres profils et préférences de communication, et bien plus encore!

Tous les membres recevront un courriel contenant la marche à suivre pour créer leur propre nom d'utilisateur et mot de passe et mettre à jour leur profil. Nous vous encourageons à consulter ou à mettre à jour votre profil le plus tôt possible après avoir reçu ce courriel. Quelques jours après l'envoi de ce courriel, les avis de renouvellement 2017 seront envoyés par courriel aux membres.

Nous implorons votre patience et votre compréhension pendant cette importante transition.

Si vous éprouvez des difficultés à créer votre profil ou à renouveler votre adhésion, écrivez-nous à **adhesions@smc.math.ca**.

CMS Notes
Volume 48 No. 5, October/November 2016

The Calendar brings current and upcoming domestic and select international mathematical sciences and education events to the attention of the CMS readership. Comments, suggestions, and submissions are welcome.

Denise Charron, *Canadian Mathematical Society, (managing-editor@cms.math.ca)*

Le calendrier annonce aux lecteurs de la SMC les activités en cours et à venir, sur la scène pancanadienne et internationale, dans les domaines des mathématiques et de l'enseignement des mathématiques. Vos commentaires, suggestions et propositions sont le bienvenue.

Denise Charron, *Société mathématique du Canada (redacteur-gerant@smc.math.ca)*

OCTOBER 2016 OCTOBRE

2-7	BIRS Workshop: Painlevé Equations and Discrete Dynamics, BIRS, Banff, Alta.
3-7	Workshop on Convexity in Algebraic Geometry, Fields Institute, Toronto, Ont.
3-7	CRM Workshop: Probabilistic Methods in Dynamical Systems and Applications, CRM, Montreal, Que.
8-9	2016 Québec-Maine Number Theory Conference, Université Laval, Qué.
11-14	CRM 14th RECOMB Comparative Genomics Satellite Workshop, ITHQ (Institut de tourisme et d'hôtellerie du Québec), Montreal, Que.
14-15	60e Congrès de l'Association de Mathématique du Québec (AMQ), Cégep Garneau, Québec, Qué.
16	AARMS Workshop: Partial Differential Equations and Numerical Analysis at Atlantic Universities Conference, Cape Breton University, Sydney, N.S.
16-24	BIRS Workshop: New Trends in Graph Coloring, BIRS, Banff, Alta.
20	PIMS-UManitoba Distinguished Lecture: Faith Ellen, University of Manitoba, Winnipeg, Man.
23-28	BIRS Workshop: Workshop in Analytic and Probabilistic Combinatorics, BIRS, Banff, Alta.
28	PIMS-CRM-FIELDS Lecture: Daniel Wise, University of British Columbia, Vancouver, BC
30-Nov 4	BIRS Workshop: Theoretical and Computational Aspects of Nonlinear Surface Waves, BIRS, Banff, Alta.

NOVEMBER 2016 NOVEMBRE

1-4	2016 Fields Medal Symposium, Fields Institute, Toronto, Ont.
4	PIMS/UBC Distinguished Colloquium: Yakov Sinai, University of British Columbia, Vancouver, BC
6-11	BIRS Workshop: Random Geometric Graphs and Their Applications to Complex Networks, BIRS, Banff, Alta.
13-18	BIRS Workshop: Permutation Groups, BIRS, Banff, Alta.
14-18	CRM Workshop: Probabilistic Methods in Topology, CRM, Montreal, Que.
14-18	Workshop on Hall Algebras, Enumerative Invariants and Gauge Theories, Fields Institute, Toronto, Ont.
27-Dec 2	AARMS Workshop: Fifth Parallel-in-time Integration Workshop, BIRS, Banff, Alta.

DECEMBER 2016 DÉCEMBRE

2-5	2016 CMS Winter Meeting / Réunion d'hiver de la SMC 2016, Sheraton on the Falls, Niagara Falls, Ont.
4-9	BIRS Workshop: Analytic versus Combinatorial in Free Probability, BIRS, Banff, Alta.
5-9	Workshop on Combinatorial Moduli Spaces and Intersection Theory, Fields Institute, Toronto, Ont.
11-14	12th International Conference on Web and Internet Economics (WINE), InterContinental Hotel, Montreal, Que.
14-15	CRM Workshop: 13th Workshop on Algorithms and Models for the Web-graph, CRM, Montreal, Que.

JANUARY 2017 JANVIER

4-7	AMS/MAA 2017 Joint Mathematics Meeting, Hyatt Regency Atlanta and Marriott Atlanta Marquis, Atlanta, GA

Notes de la SMC
Tome 48, numéro 5, octobre/novembre 2016

Book Reviews brings interesting mathematical sciences and education publications drawn from across the entire spectrum of mathematics to the attention of the CMS readership. Comments, suggestions, and submissions are welcome.

Karl Dilcher, *Dalhousie University (notes-reviews@cms.math.ca)*

Les comptes-rendus de livres présentent aux lecteurs de la SMC des ouvrages intéressants sur les mathématiques et l'enseignement des mathématiques dans un large éventail de domaines et sous-domaines. Vos commentaires, suggestions et propositions sont le bienvenue.

Karl Dilcher, *Dalhousie University (notes-critiques@smc.math.ca)*

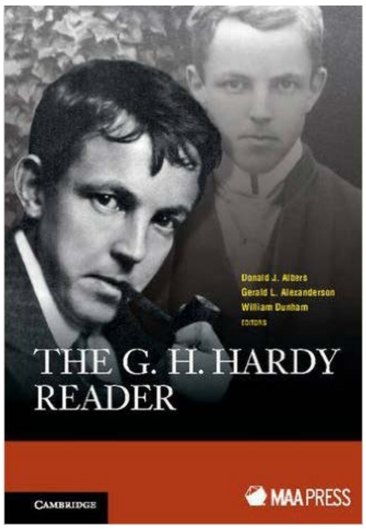
The G. H. Hardy Reader

Edited by Donald J. Albers, Gerald L. Alexanderson, and William Dunham

MAA Press and Cambridge Univ. Press, 2015

ISBN: 978-1-10759-464-7

Reviewed by *Kenneth S. Williams*, Carleton University



The editors are to be congratulated on putting together this beautiful "reader" with material from so many different sources, which illustrates so well the life, character and work of one of the great mathematicians of the twentieth century, Godfrey Harold Hardy (1877-1947). Even if you are familiar with Hardy's masterpiece "A Mathematician's Apology" or his book on Ramanujan, "Ramanujan: Twelve Lectures on Subjects Suggested by His Life and Work" you will find a wealth of new and fascinating material in this "reader" about Hardy.

The first chapter of the "reader" is a biography of Hardy written by the editors. Illustrating this chapter are eight photographs of Hardy. Of these I had only seen two of them before, the one which appeared on the cover of "A Mathematician's Apology" showing Hardy sitting in a wicker chair in his rooms at Trinity College, Cambridge, peering over his glasses and looking very scholarly and aristocratic, and the other one showing Hardy bundled in a coat and scarf watching a rugby match on a chilly day in Cambridge in 1941 during the grim days of World War II. The other six photographs were new to me, and the editors have performed a valuable service in tracking them down. They show Hardy at various times of his life (1895, 1900, 1910 and 1924).

Chapter 2 consists of the famous letter dated 16 January 1913 written from India by Ramanujan to introduce himself and some of his work to Hardy. It begins with that well-known first line "I beg to introduce myself to you as a clerk in the Accounts Department of the Port Trust Office at Madras on a salary of only £20 per annum." This was the very beginning of the famous Hardy-Ramanujan collaboration, which led eventually to their asymptotic formula for the partition function.

Chapter 3 is a letter from Bertrand Russell to Lady Ottoline Morrell, who was a member of the intellectual and artistic community known as the Bloomsbury Group. In this letter the excitement of Hardy and Littlewood over the discovery of the work of Ramanujan is described.

Chapter 4 contains the text of a lecture delivered by Hardy at Harvard University on August 31, 1936 celebrating the 300th year of Harvard. In this lecture Hardy discusses his relationship with Ramanujan as well as many of the formal identities Ramanujan had sent to Hardy.

Chapter 5 consists of the epilogue from Robert Kanigel's biography of Ramanujan entitled "The Man Who Knew Infinity" on which the recent film by the same name was based. The editors emphasize that although this material is from a biography of Ramanujan, it describes the collaboration between Hardy and Ramanujan during their years together at Cambridge University (1914-19).

Although Hardy is mainly remembered as a Cambridge mathematician, he did in fact spend eleven years (1920-1931) at Oxford University as Savilian Professor of Geometry. Chapter 6 comprises six posters on "Hardy's Years at Oxford" prepared by Robin Wilson, Emeritus Professor of Pure Mathematics, The Open University, UK.

Another famous collaboration of Hardy was with J. E. Littlewood. This collaboration dominated the English mathematical scene for the first half of the twentieth century. Theirs was perhaps the most remarkable and successful mathematical partnership ever. This collaboration gave mathematics the famous Hardy-Littlewood method in analytic number theory. Chapter 7 contains excerpts from Robin Wilson's article "Hardy and Littlewood" which appeared in the book "Cambridge Scientific Minds", published by Cambridge University Press in 2002.

Chapter 8 contains a letter from the mathematical physicist Freeman Dyson to C. P. Snow reflecting on being a student in Hardy's classes and providing insights into his sense of humour and approach to learning.

Chapter 9 describes the life of Hardy's sister Gertrude Hardy. Chapters 10, 11 and 12 are devoted to writings by and about Hardy.

For those readers looking for real pure mathematics in the "reader", you will not be disappointed. It is in Chapters 13-28, where some of Hardy's mathematics is given. I particularly enjoyed Chapter 28 on the integral $\int_0^\infty \frac{\sin x}{x} dx$, where Hardy discusses and marks several proofs of the evaluation of this integral. By this point in the "reader" it will be clear to the reader that Hardy loved to list and rank. I published a simple proof of $\int_0^\infty \frac{\sin x}{x} dx = \frac{\pi}{2}$ in Mathematics Magazine in 1971 and I cannot help wondering how Hardy would mark it if he were alive today.

Chapters 29-32 consist of obituaries written by Hardy celebrating the lives of the famous mathematicians J. W. L. Glaisher, D. Hilbert, E. Landau and G. Mittag-Leffler. Chapters 33-38 give some book reviews written by Hardy.

This book is a must for every mathematician's library. If you have a spare moment, or are stuck proving a lemma you need, you can pull it down from your bookshelf and be sure to find something of interest in it to you.

The MAA Press and Cambridge University Press have done as usual a fine job in producing this book. The photo collage on the cover is very attractive, the print throughout is clear and easy to read and there is an extensive list of sources as well as a comprehensive index. I have only two tiny comments. Robert A. Rankin was a student of Hardy and he wrote an article "G. H. Hardy as I knew him" which was published in the Australian Mathematical Society Gazette in 1998 (Volume 25, pages 73-81). I think that a reference to this article should have been included in the "reader" since it includes first hand information about Hardy not included in the "reader" as well as lists of Hardy's students both at Cambridge and Oxford. Secondly I think that the editors stating on page 228 that Hardy was "obsessive" about evaluating definite integrals was a little strong. He was indeed fascinated by integrals and a master at evaluating them, but no more obsessed with them than he was with other topics in mathematics, such as inequalities.

Readers may be interested to know that Hardy's signed personal copy of "Quadratic Partitions" by A. J. Cunningham is housed in the Special Collections area at the University of Alberta Library.

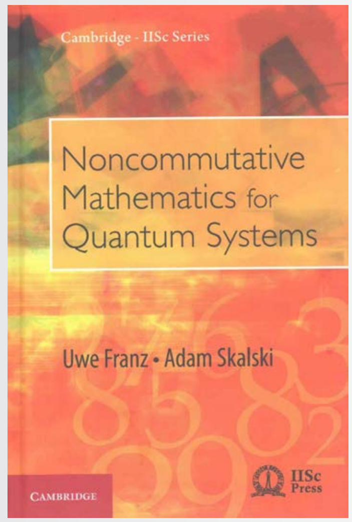
Short Review

Noncommutative Mathematics for Quantum Systems

by Uwe Franz and Adam Skalski

Cambridge University Press, 2016

ISBN: 978-1-107-14805-5



From the publisher's book description: "Noncommutative mathematics is a significant new trend of mathematics. Initially motivated by the development of quantum physics, the idea of 'making theory noncommutative' has been extended to many areas of pure and applied mathematics. This book is divided into two parts. The first part provides an introduction to quantum probability, focusing on the notion of independence in quantum probability and on the theory of quantum stochastic processes with independent and stationary increments. The second part provides an introduction to quantum dynamical systems, discussing analogies with fundamental problems studied in classical dynamics. The desire to build an extension of the classical theory provides new, original ways to understand well-known 'commutative' results. On the other hand the richness of the quantum mathematical world presents completely novel phenomena, never encountered in the classical setting. This book will be useful to students and researchers in noncommutative probability, mathematical physics and operator algebras."

This relatively short monograph (of 180 pages) arose from lectures delivered by the authors during a graduate school that was part of the meeting "Recent Advances in Operator Theory and Operator Algebras" (Dec. 31, 2012 – Jan. 13, 2013). Its two chapters, "Independence and Lévy processes in quantum probability", and "Quantum dynamical systems from the point of view of noncommutative mathematics", written by the first and the second author, respectively. The book also contains a brief introduction, with some historical remarks, which puts the topic of this book into perspective.

Education Notes brings mathematical and educational ideas forth to the CMS readership in a manner that promotes discussion of relevant topics including research, activities, and noteworthy news items. Comments, suggestions, and submissions are welcome.

Jennifer Hyndman, *University of Northern British Columbia (hyndman@unbc.ca)*
John McLoughlin, *University of New Brunswick (johngm@unb.ca)*

Notes pédagogiques présentent des sujets mathématiques et des articles sur l'éducation aux lecteurs de la SMC dans un format qui favorise les discussions sur différents thèmes, dont la recherche, les activités et les nouvelles d'intérêt. Vos commentaires, suggestions et propositions sont le bienvenue.

Jennifer Hyndman, *University of Northern British Columbia (hyndman@unbc.ca)*
John McLoughlin, *University of New Brunswick (johngm@unb.ca)*

Collaborative efforts across the spectrum of mathematics and education are welcome contributions. This issue features a first of sorts for Education Notes, in that it highlights an interdisciplinary collaboration from secondary school that begins in a history classroom. The teacher, Armand Doucet, invites students to delve into areas that they are passionate about. Jata MacCabe, a student, is passionate about mathematics. Upon hearing of this initiative an invitation was extended to share their story with the readership. The co-authorship enriches the value. Armand's writing (in italics below) will introduce the context and the background with regards to Passion Projects. Jata will then share her experience with the project and what it meant to discover that she wanted to continue pursuing mathematics as a career path.

Developing a Passion for Mathematics through History

Armand Doucet *and* **Jata MacCabe**, *Riverview High School, Riverview, NB (Armand.Doucet@nbed.nb.ca)*

My goal as a teacher in the classroom is to develop skills intertwined with curriculum content. Social and Emotional Learning (SEL) and 21st century skills need to be developed not haphazardly, but purposefully. For this to happen, the culture and design of my classroom and how I approach curriculum outcomes and standards, as well as skills development, is with a combination growth mindset (Carol Dweck) and design thinking process (IDEO-Tim Brown). I try to foster and develop divergent thinking (Sir Ken Robinson) in students who will embrace the problems of the world instead of fearing them because in reality: "The world doesn't care what you know. What the world cares about is what do you do with it (Tony Wagner)."

So, I believe that connecting the curriculum to what the students are passionate about is a great way to develop my classroom. With Passion Projects students realize the joys of learning again by following their own path. As you can see in Jata's statement below, when allowed to pursue their own goals in education, students struggle at first. I try to let them explore before giving them stricter guidelines for their creative piece. We conference in order for them to discover what it is they would really like to pursue. As they embrace the core problem of their Passion Projects, resiliency precedes enthusiasm and then enthusiasm leads to pride as students create and subsequently showcase their projects. History comes alive as students gather information and collaborate with the international community. The experience is unique to each student. Tony Wagner (Creating Innovators) states, "the most important thing is allowing students to ask questions and then give them the space to find the answers."

With Jata, she wanted to pursue something in math and women's rights. Her project revolved around proving that women played a key role in WW2 with Bletchley Park and this was one of the main reasons that the Allied forces had won the war. At first, she researched a lot on Bletchley Park's role itself, realizing that Mavis Batty played a major part. As her project progressed, she decided to create her own Enigma scavenger hunt. This got her looking at the way the Code breakers were using math to break codes and build the Enigma machine. She ended up being able to utilize her pre-calculus class to help her develop the scavenger hunt and Enigma machine (which was made up of tin foil and some boxes). However, what she really developed was an understanding of how math, as well as history, are both connected from a perspective of the skills that are needed such as problem solving, critical thinking and creativity. Those higher order skills were pushed to the limit as she continuously tried, innovated and ultimately created her supportive creative piece, namely, the scavenger hunt.

Giving students like Jata a chance to pursue their passion, math in this case, in combination with other subjects like history, within a safe environment to take a chance on a project, analyze, improve and try again, gives them the opportunity to realize if they truly want to chase down those dreams in the future. Jata's project garnered attention from CBC once we posted the results on Social Media. They attended her presentation, interviewed her and it was shared over 200 times. Also, she received praise from Sue Black, O.B.E. and computer scientist, who was one of the people who helped to save Bletchley Park. Sue was able to share with Jata her connection with Mavis Batty, having known her before she passed away. All these things combined to solidify for Jata that she wanted to continue pursuing math and that it was going to give her numerous avenues for an interesting career.

That, I believe, is my job as a teacher, to help students develop skills while finding who they are so they can succeed in the future. You can visit my template for this type of classroom and other Passion Project examples at **www.lifelessonlearning.com**

My approach to history has always been impersonal. Dates and names have never stuck in my head for longer than they took to go in one ear and out the other. I was kind of into that Roman unit, but my friends tell me watching The Gladiator doesn't actually count as studying. I was obviously not looking forward to an entire semester of memorization and regurgitation of a subject I didn't particularly care about.

Within the first week of Mr. Doucet's class we were introduced to the Passion Project. The idea was to connect something we were passionate about to a revolution in Modern History. I was terrified. That very helpful premise narrowed the possible topics down to relatively everything, and the only concrete thing I understood was the deadline. When Mr. Doucet suggested researching code breaking during World War Two, I finally had some small lifeline to grasp onto. This was a way to explore my passion for mathematics in a course that I would have otherwise loathed. And besides, what kid isn't intrigued by spies and code breaking?

For almost the entire history of the world, battles have been the epitome of concrete and physical. Obey him, protect them, bash and whack the enemy. The major action occurred directly on the battlefield; you simply had to roll with the punches as they came — literally. Espionage had always been field agents infiltrating enemy divisions, overhearing important information and accessing critical documents. However painstakingly won, this information hardly ever majorly impacted the outcome of a battle.

Communication was slow and unreliable; a messenger could be delayed or a letter could be intercepted. Even if the information should have reached someone who might have been able to act upon it, the information was often as unreliable as the methods to send it. In matters of life and death, confusion is not always the preference. Our Modern History course taught us of major innovations that were catalysts for revolution.

Very few modern innovations had such a profound effect on military communication, and the world, as radio transmission. During the Second World War, communication was decidedly less tangible. Encrypted messages could pass through brick walls, over enemy camps and across borders. In a game of interceptions, the best encryption won. As tensions and conflicts mounted, it was clear that the Germans had it.

Originally, the Enigma machine was a commercial product designed for businesses or firms to encrypt their financial data. The creators were quick to see the machine's potential military use and began approaching federal governments with the product. Ironically, the encryption machines were even presented to the British government, who chose not to invest. The German government was interested in the product, however. After ramping up the security, the German Enigma resembled the simple commercial product solely on a superficial level. The machine had a standard German keyboard, like a typewriter, and an additional alphabet with illuminated keys. It included a series of rotors that encode letters and rotated with each additional letter. It also had a switchboard that added an extra layer of security by switching the coded letters for other —seemingly random— letters. By changing the settings of the rotors every night at midnight, the Germans had created a nearly invincible fortress of security. In a total blackout of information, the Allied forces would be subject to almost certain defeat. Britain's Government Code and Cypher School's base for Axis decryptions was at Bletchley Park.

Perhaps the most famous name to come out of Bletchley Park is Alan Turing. Before the war, the Polish had developed a method of deciphering Enigma codes with the use of "Bomba" machines. These functioned by checking all possibilities using a series of sheets. The machine was slow, inconsistent and fickle, but it was progress. After being introduced to the Bomba, the concept that a machine could do the quantitative work of a human mind would stay with Turing for the rest of his life. Turing was a theorist, but he couldn't achieve his objective of creating a more efficient version of the Bomba alone. He and Gordon Welchman combined with an Oxford engineering team and created the first Bombe machine. It was not precisely a computer as one still needed to feed the machine a section of code guessed at manually, but Bombe machines could check thousands of possibilities in minutes.

In high school, math seems almost completely unrelated to the world at large. You can barely step into a Pre-Calculus classroom without hearing "How will this help me in the real world?" We all want our hard work to mean something more than a number on a test. It was amazingly coincidental that while I was researching how probability was used at Bletchley Park, we had just begun our Combinatorics and Probability unit in Pre-Calculus. At Bletchley Park, mathematics and problem solving meant lives saved. Churchill believed that the work conducted at Bletchley shortened the war by two years. Many others believe that the war could not have been won without the park. It is often said that Bletchley was present at every famous battle in the Second War, stealthily swaying the balance.

The most confusing part of this project was the creative part. For mine, groups of five had to use a tin foil and pool noodle "Enigma machine" to decipher the location of their next checkpoint. It was exactly The Amazing Race and it certainly wasn't life at Bletchley Park, but teams had to work together to solve problems under pressure, which was my goal.

The world is composed of dichotomies. You're a naive child or a sophisticated adult. You're a dreamer or a realist. You're a mathematician or an artist. Bletchley Park unabashedly disregarded these constraining labels. Academics, translators, debutantes,

2016 Excellence in Teaching Award

The CMS Excellence in Teaching Award Selection Committee invites nominations for the 2016 Excellence in Teaching Award.

The CMS Excellence in Teaching Award focuses on the recipient's proven excellence as a teacher at the undergraduate level, including at universities, colleges and cégeps, as exemplified by unusual effectiveness in the classroom and/or commitment and dedication to teaching and to students. The dossier should provide evidence of the effectiveness and impact of the nominee's teaching. The prize recognizes sustained and distinguished contributions in teaching at the post-secondary undergraduate level at a Canadian institution. Only full-time teachers or professors who have been at their institution for at least five years will be considered. The first award was presented in 2004.

The deadline for nominations, including at least three letters of reference, is **November 15, 2016**. Nomination letters should list the chosen referees and include a recent curriculum vitae for the nominee, if available. Nominations and reference letters should be submitted electronically, preferably in PDF format, to: **etaward@cms.math.ca** no later than **November 15, 2016**.

Prix d'excellence en enseignement 2016

La Comité de sélection du Prix d'excellence en enseignement de la SMC sollicite des mises en candidature pour le Prix d'excellence en enseignement 2016.

Le Prix d'excellence en enseignement de la SMC récompense l'excellence reconnue d'un enseignant ou d'un professeur de niveau postsecondaire (universités, collèges et cégeps), telle qu'illustrée par son efficacité exceptionnelle en classe et/ou son engagement et son dévouement envers l'enseignement et les étudiants. Le dossier de candidature doit montrer l'efficacité et les effets de l'enseignement du candidat ou de la candidate. Ce prix récompense des contributions exceptionnelles et soutenues en enseignement collégial et de premier cycle universitaire dans un établissement canadien. Seules les candidatures d'enseignants et de professeurs à temps plein qui travaillent dans le même établissement depuis au moins cinq ans seront retenues. Ce prix a été décerné pour la première fois en 2004.

Les proposants doivent faire parvenir au moins trois lettres de référence à la SMC au plus tard **le 15 novembre 2016**. Le dossier de candidature doit comprendre le nom des personnes données à titre de référence ainsi qu'un curriculum vitae récent du candidat ou de la candidate, dans la mesure du possible. Veuillez faire parvenir les mises en candidature et lettres de référence par voie électronique, de préférence en format PDF, à : **prixee@smc.math.ca** avant la date limite du **15 novembre 2016**.

actors, novelists, athletes and even chess enthusiasts were recruited to aid their country. Major operations included but weren't limited to university students or graduates. Some of the greatest breakthroughs during World War Two were interdisciplinary collaborations of many kinds of thinkers.

I knew before this project that I wanted to be a mathematician. I knew that I loved numbers and I knew that solving a difficult problem made me irrationally happy. This project taught me that I already was a mathematician. Math never was about numbers or formulas on a page. Math has always been about humans solving problems. All of those countless symbols, complex equations and abstract theories have been about the human race learning to understand and manipulate the world around them. So maybe it was weird that I found my passion for mathematics studying people and civilizations and revolutions, but maybe it wasn't that weird at all.

Armand Doucet is a passionate and award winning Educator, Leader and Business Professional with a unique combination of entrepreneurial, teaching and motivational speaking experience. He recently received the Prime Minister's Award for Teaching Excellence as well as a Meritorious Service Medal from the Governor General. He is the creator of **www.lifelessonlearning.com** *which leads the way in placing skills development on equal footing to curriculum content in the classroom.*

Jata MacCabe is a self-proclaimed math dork who is equally talented in the classroom as on the improv stage or rugby field. As a grade 12 student this year, she is looking to pursue a career in math while still being passionate about many other subjects.

CMS Notes
Volume 48 No. 5, October/November 2016

Research Notes brings mathematical research ideas forth to the CMS readership in a generally accessible manner that promotes discussion of relevant topics including research (both pure and applied), activities, and noteworthy news items. Comments, suggestions, and submissions are welcome.

Patrick Ingram, *York University (notes-research@cms.math.ca)*

Les articles de recherche présentent des sujets mathématiques aux lecteurs de la SMC dans un format généralement accessible qui favorise les discussions sur divers sujets pertinents, dont la recherche (pure et appliquée), les activités et des nouvelles dignes de mention. Vos commentaires, suggestions et propositions sont le bienvenue.

Patrick Ingram, *York University (notes-recherche@smc.math.ca)*

The geometry of social networks

Anthony Bonato, *Ryerson University, abonato@ryerson.ca*

We propose a broad geometric view of real-world complex networks, inspired by seminal work on Blau space for social networks. We describe how the examination of mathematical models for on-line social networks led to a hypothesis regarding their dimension. A thesis is presented on how all complex networks possess an underlying geometry related to but distinct from their underlying graph distance metric space.

1. Introduction

Online social networks (or OSNs) such as Facebook, Twitter, and Instagram are highly prevalent in our daily lives and commerce. The availability of social networking data has witnessed an active multi-disciplinary trend towards the modelling and mining of these big data sets.

Not unlike earlier studied complex networks such as the web graph (see [2]), OSNs exhibit several common evolutionary properties. OSNs possess the *small world property*, which demands that there is a short path joining any two nodes, and if two nodes share a common neighbor, they are more likely to be adjacent. OSNs have power law degree distributions. In a graph G, let N_k denote the number of nodes of degree k. The degree distribution of G follows a *power law* in some range of k if N_k is proportional to k^{-b}, for a fixed *exponent* $b > 2$. Other properties include densification power laws (where the network becomes more dense over time), and constant or even shrinking distances over time.

Several stochastic models have emerged simulating the evolution of OSNs. Such models serve to simulate known properties of social networks, and suggest new ones. We focus in this note on *geometric models* of OSNs; that is, models which posit nodes as points in fixed metric space, and edges are determined by various probabilistic rules along with their relative proximity in the space. We present the view that the geometry is indeed a fundamental construct of social networks, and is in alignment with other central role of geometric models in the physical or natural sciences.

2. Blau Space and models for OSNs

Sociologists considered a geometric view of social networks long before the advent of OSNs. In *Blau space* [7], nodes correspond to points in a multi-dimensional metric space, and the link structure is governed by the principle of *homophily*: nodes with similar socio-demographic attributes are more likely joined.

Blau space suggests a view towards random geometric graphs as natural objects for the modelling of OSNs. Random geometric graphs are well-studied within discrete mathematics and graph theory. In such stochastic models, nodes are points in a fixed metric space $\mathcal{S}$, and are each assigned a *ball of influence* in $\mathcal{S}$. Nodes are joined with some prescribed probability if they arise in each other's ball of influence. Several geometric models for OSNs and complex networks have independently and recently emerged; see [1, 5, 6, 10].

We focus here on the MGEO-P model which uses both geometry and ranking first presented in [3]. The MGEO-P model has five parameters: n the total number of nodes, m the dimension of the metric space, $0 < \alpha < 1$ the attachment strength parameter, $0 < \beta < 1 - \alpha$ the density parameter and $0 < p \leq 1$ the connection probability. We identify the agents of an OSN with points in $\mathbb{R}^m$ with the L_∞-metric, each chosen uniformly at random in the unit hypercube (with the torus metric to avoid boundary effects). Nodes are ranked by their popularity from 1 to n, where n is the number of nodes. Here, 1 is the highest ranked node with n the lowest. Each node has a ball of influence that is a function of their rank (along with α and β); the higher the rank, the larger the volume of the ball of influence. A node is joined to another with probability p if it is in its ball of influence. See Figure 1.

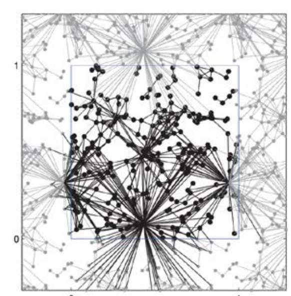


Figure 1. A simulation of the MGEO-P model resulting in graph with 250 nodes in the unit square with torus metric. Each figure shows the graph "replicated" in grey on all sides in order to illustrate the torus metric. The blue square indicates the unit square.

As shown in [3], MGEO-P model provably generates graphs satisfying the following properties with probability tending to 1 as n tends to ∞.

1. The degree distribution follows a power law with exponent $1 + \frac{1}{\alpha}$.
2. The average degree of node is $\frac{p}{1-\alpha} n^{1-\alpha-\beta}(1 + o(1))$.
3. The diameter (that is, length of a longest shortest path connecting vertices) is $n^{\Theta\left(\frac{1}{m}\right)}$.

3. The Logarithmic Dimension Hypothesis and the Feature Space Thesis

Property (3) of MGEO-P on its diameter suggests that, ignoring constants, for a network with n nodes and diameter D, the expected dimension based on the MGEO-P model is

$$m \approx \frac{\log n}{\log D}.$$

Hence, the model predicts a logarithmically scaled dimension of Blau space. Interestingly, this was proposed independently in at least two other models for OSNs including those in [5, 10}. The logarithmic relationship between the dimension of the metric space and the number of nodes has been called the *Logarithmic Dimension Hypothesis* (or *LDH*) [4].

The study [3] set out to experimentally verify LDH, using data sets from Facebook (the so-called "Facebook 100" data set [9]) and LinkedIn. Employing machine learning algorithms using motif counts (that is, counts of small subgraphs) and analysis of eigenvalue distributions, dimensions of real OSN data sets were predicted. While the results of [3] do not conclusively prove the LDH, they are suggestive of a logarithmic or sub-logarithmic dimension for Blau space. For future work, it would be useful to further validate the LDH with additional data sets. The challenge is to find representative samples of OSN data that scales with time (as found in the Facebook 100 data sets).

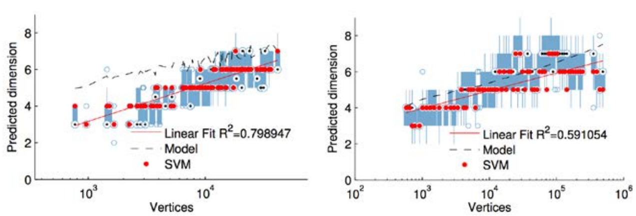


Figure 2. Facebook dimension at left, LinkedIn dimension at right. Each red dot is the predicted dimension computed via motif counts and a support vector machine classifier. MGEO-P predicts a dimension of $\frac{\log n}{\log D}$, which is plotted as the dashed line.

The study of Blau space inspires a geometric view not only to OSNs, but also other complex networks. We propose here the *Feature Space Thesis* which states that every complex network has an underlying metric (or *feature*) space, where nodes are identified with points in the feature space, and edges are influenced by node similarity and proximity in the space. As with Blau space, the metric of the feature space is not arbitrary, but is rather a hidden property of the network. For example, in the web graph there is an underlying *topic space*, with web pages more closely related in topics more likely to be linked. In protein-protein interaction networks, we may view these as embedded in a *biochemical space*; see [8] for more on the geometry of protein networks.

The Feature Space Thesis is not provable, per se. However, it should be used as a guide towards a deeper understanding of complex networks. For instance, while agents may be close via graph distance in an OSN, they may be far apart in Blau space; the latter metric better reflects their different sociological profile. How do we infer the underlying metric of Blau space? How are the graph structure and feature space geometry related? We propose these questions for future work.

References

[1] W. Aiello, A. Bonato, C. Cooper, J. Janssen, P. Pralat, A spatial web graph model with local influence regions, *Internet Mathematics* (2009) 5 175–196.

[2] A. Bonato, *A Course on the Web Graph*, American Mathematical Society Graduate Studies Series in Mathematics, Providence, Rhode Island, 2008.

[3] A. Bonato, D. F. Gleich, M. Kim, D. Mitsche, P. Pralat, A. Tian, S. J. Young. Dimensionality matching of social networks using motifs and eigenvalues, *PLOS ONE* 9 (9):e106052, 2014.

[4] A. Bonato, J. Janssen, P. Pralat, Geometric protean graphs, *Internet Mathematics* 8 (2012) 2–28.

[5] M. Kim, J. Leskovec, Multiplicative attribute graph model of real-world networks, *Internet Mathematics* 8 (2012) 113–160.

[6] D. Krioukov, F. Papadopoulos, M. Kitsak, A. Vahdat, M. Boguñá, Hyperbolic geometry of complex networks, *Phys. Rev. E*, 82:036106, 2010.

[7] J.M. McPherson, J. R. Ranger-Moore, Evolution on a dancing landscape: Organizations and networks in dynamic blau space, *Social Forces* 70 (1991) 19–42.

[8] V. Memišević, T. Milenković, N. Pržulj, An integrative approach to modeling biological networks, *Journal of Integrative Bioinformatics*, 7:120, 2010.

[9] A. L. Traud, P. J. Mucha, M. A. Porter, Social structure of facebook networks, *arXiv*, cs.SI:1102.2166, February 2011.

[10] Z. Zhang, F. Comellas, G. Fertin, L. Rong, Highdimensional apollonian networks, *Journal of Physics A: Mathematical and General*, 39:1811, 2006.

A Mathematical Framework for the Study of Biological Adaptation

Geoff Wild, *Department of Applied Mathematics, The University of Western Ontario*

A particular mathematical framework for the study of adaptation in living systems has proven to be important both practically and philosophically. My goal in this note is to lead the reader through a quick sketch of the framework, then to suggest, briefly, why the framework has been an important development.

I begin my sketch of the framework by considering two variants of a gene for some trait that, itself, takes some real-numbered value, denoted as $\alpha \in [a, b]$(e.g. height). The two gene variants are found together in a single-species population. Their arrangement and relationship to one another (e.g. spatial arrangement, hierarchical groupings), as well as the kind of individual in which they are found (e.g. young/old, male/female) determine the *state* of the population.

Next, I make the mildly restrictive assumption that the state of the population at a given time t can be represented by a vector of finite dimension, $\mathbf{n}(t)$. In most, if not all, of these situations, one finds that

$$\mathbf{n}(t+1) = \mathbf{F}(\mathbf{n}(t))\,\mathbf{n}(t), \tag{1}$$

where $\mathbf{F}$ is a sufficiently well-behaved matrix-valued function (this discussion can be modified to accommodate continuous-time models) [1]. Though the notation does not reflect it, $\mathbf{F}$ will also depend on variables like α. More accurately, $\mathbf{F}$ will depend on how gene variants combine to determine the real-valued traits of the individuals in the population.

To keep matters simple, I assume that equation (1) admits a steady-state solution (denoted $\bar{\mathbf{n}}$) that corresponds to a population in which only one of the gene variants (call it, *variant number 1*) is present. Given the right ordering of the elements of the state space, I can reflect my previous statement by writing $\bar{\mathbf{n}} = (\bar{\mathbf{n}}_1, \mathbf{0})$where $\bar{\mathbf{n}}_1 \neq \mathbf{0}$. The main question I face at this point concerns the local asymptotic stability of $\bar{\mathbf{n}}$. If $\bar{\mathbf{n}}$ is stable, then one would predict that the gene-variant number 1 is able to resist invasion by its counterpart (call it *variant number 2*, and persist in the population, on its own, over time. If it is unstable, then one would predict that variant 1 is either unable to resist invasion by its counterpart, or is unable to produce a trait value that would allow the genetically homogeneous population to persist over time (or both). Biologically speaking, I expect a trait value $\alpha = \alpha^*$ to be adaptive (a) if a population expressing the trait at level α^* persists over time, and (b) if, when the trait value is coded for by a single gene variant, that variant resists invasion by a second one that would alter α^*. To determine an adaptive value of the trait, therefore, it makes sense that I linearize (1) about the equilibrium $\bar{\mathbf{n}}$, focus my attention on a block-diagonal Jacobian matrix,

$$\mathbf{J} = \begin{bmatrix} \mathbf{J}_{11} & \mathbf{J}_{12} \\ \mathbf{0} & \mathbf{J}_{22} \end{bmatrix}, \tag{2}$$

then, finally, attempt to find an α^* that ensures eigenvalues of $\mathbf{J}$ are less than 1 in absolute value.

The square submatrix $\mathbf{J}_{11}$ in (2) corresponds to the linearization of a difference equation that, if specified, would describe the dynamics of a population made up entirely of individuals carrying variant 1 only. Neglecting the possibility of periodic or chaotic attractors in (1) for the sake of brevity, I can assume that the eigenvalues of $\mathbf{J}_{11}$ are less than one in absolute value. Hence, the stability of $\bar{\mathbf{n}}$ hinges only upon the eigenvalues of the square submatrix $\mathbf{J}_{22}$.

In general, $\mathbf{J}_{22}$ can be quite large – much larger than $\mathbf{J}_{11}$ – so determining its spectral radius is a non-trivial task, to say the least. That task is made easier if I recognize that $\mathbf{J}_{22}$ is non-negative and, as is often the case, satisfies other key requirements of the Perron-Frobenius Theorem. This theorem says that $\mathbf{J}_{22}$ will possess a real eigenvalue of multiplicity one (call it ρ), that ρ will be positive, and that ρ will be greater, in absolute value, than any other eigenvalue of the matrix. The theorem, therefore, reduces my task to one involving a single eigenvalue rather than several. Importantly, the theorem also gives me an expression for ρ, namely $\rho\, \mathbf{v}^T\mathbf{u} = \mathbf{v}^T\mathbf{J}_{22}\mathbf{u}$, where $\mathbf{u}$ and $\mathbf{v}$ are the positive right and left eigenvectors, respectively, associated with ρ. Note that with the right scaling of one or the other eigenvector I obtain $\mathbf{v}^T\mathbf{u} = 1$, which allows me write $\rho = \mathbf{v}^T\mathbf{J}_{22}\mathbf{u}$.

My assessment of local stability can be made easier still if I am willing to assume, when it is necessary to do so, that the two gene variants combine additively to influence the trait value of an individual. Essentially, this means that $\mathbf{F}$, and ultimately ρ, can be treated as (differentiable) functions of two trait values. The first trait value is the familiar α, while the second is a new value, β—all other trait values in the population can be considered to be averages of these two. It follows that, if $\alpha = \alpha^* \in (a, b)$ is adaptive, then choosing β to be anything other than α^* would lead to maladaptation on the part of those carrying the second gene variant. The eigenvalue ρ would be pushed below 1, in this case, and the second gene variant would come to be eliminated from the population. Based on this discussion, then, I expect the adaptive trait value α^* to satisfy the following equation,

$$\begin{gathered} \partial_\beta\, \rho\ \big]_{\beta=\alpha=\alpha^*} = 0 \quad \text{or equivalently} \\ \mathbf{v}^T(\partial_\beta\, \mathbf{J}_{22})\,\mathbf{u}\ \big]_{\beta=\alpha=\alpha^*} = 0, \end{gathered} \tag{3}$$

where I have used the fact that the derivative of $\mathbf{v}^T\mathbf{u}$ is zero.

The practical importance of the framework I have presented stems from the fact that the terms in (3) can be organized in a very convenient way. Specifically, I can organize these terms so that the most complicated aspects of the state space (i.e. those aspects associated with the way in which gene-variant 2 is distributed throughout the population) come to be summarized entirely using measures of genetic similarity [2]. The measures in question, here, are the same ones we think of when we say that we are related to a sibling by a factor of one-half. The key take-away, here, is not about genetic relatedness, however. Instead, the key take-away it is that the assumptions concerning the additivity of interactions between gene variants ultimately "telescopes" the state space, reducing it to something much more manageable.

The framework is also of philosophical importance, owing to the fact that (3) is a statement about a change in what is known as

New Funding Key to CMS Success

The CMS will undertake a new fundraising program in the fall of 2016 in hopes of expanding the funds available to support CMS meeting sessions; and to provide travel and accommodations for students and graduate students at CMS meetings. The **CMS Annual Giving Campaign** has been underway since mid-December. We are seeking donations from members and friends of CMS that can be used to support CMS programs.

Sponsorships and donations are also being sought from foundations and corporations, who often have programs that **match** their employee's donations. Fundraising efforts also continue to support on-going CMS programs such as the Math Competitions, Canada's IMO Team, Endowment Grants for Research, and CMS Meetings. We are also positioning the CMS to accept donations by **Planned or Estate Giving**.

You can help by:

Contributing to the CMS. Your donation can be designated for a specific program or activity, or can be made for general use. For more information about the programs the CMS supports, go to **http://funding.cms.math.ca**. Donate online at **cms.math.ca**.

- Identifying mathematicians who work in private industry who we can contact to seek donations.
- Volunteering for CMS committees.
- Encouraging colleagues to join the CMS.

Working together, we can promote and advance the discovery, learning, and application of mathematics. If you have questions or want more information, please contact **giving@funding.cms.math.ca**.

Une nouvelle ronde de financement, clé du succès de la SMC

La SMC lancera une nouvelle ronde de financement en automne 2016 dans l'espoir d'augmenter les fonds disponibles afin de soutenir les conférences des réunions de la SMC et de subventionner les voyages et hébergements des étudiants et diplômés, aux réunions de la SMC. **La campagne annuelle de financement de la SMC** est en cours depuis la mi-décembre. Nous recherchons des dons de la part des membres et des amis de la SMC, qui peuvent être utilisés afin de soutenir les programmes de la SMC.

Nous recherchons aussi des parrainages et des dons de la part de fondations et d'entreprises qui offrent souvent des programmes de **dons équivalents** à ceux effectués par leurs employés. Les efforts de financement continuent aussi de soutenir les programmes en cours de la SMC, tels que les défis mathématiques, l'équipe canadienne de l'OIM, les fonds de dotations pour la recherche et les réunions de la SMC. Nous acceptons aussi les **dons planifiés ou les dons de biens**.

Vous pouvez aider en :

Contribuant à la SMC. Votre don peut être ciblé envers un programme ou une activité spécifique ou pour utlisation générale à la SMC. Pour plus d'informations à propos des programmes que la SMC soutient, consultez **http://funding.cms.math.ca/**. Donnez en ligne à **https://cms.math.ca/.f**.

- Identifiant des mathématiciens qui travaillent au privé et que nous pouvons contacter afin d'obtenir des dons de leur part.
- Étant bénévole sur un comité de la SMC.
- En encourageant des collègues à adhérer à la SMC.

En travaillant ensemble, nous pouvons promouvoir et favoriser la découverte, l'avancement, l'apprentissage et l'application des mathématiques. Si vous avez des questions ou souhaitez obtenir plus d'informations, veuillez contacter **giving@funding.cms.math.ca**.

inclusive fitness. An individual's inclusive fitness represents the contribution its genetic lineage makes to its species' gene pool in the very, very distant future. These contributions are made through both descendant and non-descendant genetic relatives. Equation (3) suggests that I can interpret $\alpha^* \in (a, b)$ as a trait that maximizes the inclusive fitness of the individual. In turn, these suggestions have prompted biologists to propose that adaptive traits be viewed as solutions to problems—problems confronted by individuals interested in improving their inclusive fitness, rather than the more standard Darwinian measure [3]. One can, therefore, look at a broader range of adaptations in "anthropic" goal-oriented terms, which has implications for our view of natural populations, and arguably the cosmos [4].

References

[1] J. A. J. Metz, R. M. Nisbet and S. A. H. Geritz, *How should we define 'fitness' for general ecological scenarios?*, Trends in Ecology and Evolution, 7 (1992), 198–202.

[2] G. Wild, *Inclusive fitness from multitype branching processes*, Bulletin of Mathematical Biology, 73 (2011), 1028–1051.

[3] A. Gardner, *Adaptation as organism design*, Biology Letters, 5 (2009), 861–864.

[4] A. Gardner and J. P. Conlon *Cosmological natural selection and the purpose of the universe*, Complexity, 18 (2013), 48–56.

CSHPM Notes brings scholarly work on the history and philosophy of mathematics to the broader mathematics community. Authors are members of the Canadian Society for History and Philosophy of Mathematics (CSHPM). Comments and suggestions are welcome; they may be directed to either of the column's co-editors.

Amy Ackerberg-Hastings, *University of Maryland University College (aackerbe@verizon.net)*

Hardy Grant, *York University [retired] (hardygrant@yahoo.com)*

Les articles de la SCHPM présentent des travaux de recherche en histoire et en philosophie des mathématiques à la communauté mathématique élargie. Les auteurs sont membres de la Société canadienne d'histoire et de philosophie des mathématiques (SCHPM). Vos commentaires et suggestions sont le bienvenue; ils peuvent être adressées à l'une des co-rédacteurs.

Amy Ackerberg-Hastings, *University of Maryland University College (aackerbe@verizon.net)*

Hardy Grant, *York University [retraité] (hardygrant@yahoo.com)*

Lessons from old textbooks: Introducing modern geometry to the Middle East

Gregg De Young, *American University, Cairo*

The history of mathematics textbooks is still in its infancy. Old textbooks are rarely preserved, which of course makes the historian's task more challenging. But those that do survive can reveal what mathematicians, as well as "consumers" of mathematics, thought were the essentials of the subject. They can show us how newer concepts competed with more traditional approaches; they can illuminate the process of assimilation; and they can highlight *problems* arising from the importation of modern mathematics into non-Western cultures. In particular the introduction to 19th-century Islamic societies of European approaches to geometry produced a confrontation with an indigenous tradition of Euclidean scholarship [De Young, 2012].

In the Islamic world one of the earliest textbooks of modern geometry was a 1797 Ottoman Turkish translation of John Bonnycastle's *Elements of Geometry* (first English edition, 1789) prepared by Ḥüseyn Rıfkı Tamânî in Belgrade. Rıfkı, who died in 1817, served for several years as director of the new engineering school (Mühendishane-i Cedid) founded by Ottoman Sultan Selim III in an attempt to develop a modern military corps. The translation was printed by the Bûlâk Press in Cairo in 1825, using typography. It was also printed in at least two lithographed editions, whose dates are unknown.

In Cairo the Italian priest Carlo Bilotti (died 1828) taught mathematics in another new engineering school, founded by Mohammed Ali, the Ottoman governor. Bilotti asked several students to translate their mathematics notes into Arabic for use as textbooks. Moḥammed Surūr translated his notes on geometry (1825), while Aḥmed al-Ṭaabbākh translated his notes on arithmetic and algebra (1824). Abdeljaouad [2011] showed that al-Ṭabbākh's notes were based almost entirely on Étienne Bézout, *Arithmétique De Bézout: à l'usage de le marine et de l'artillerie* (1771), which initially appeared as the first volume in his *Cours de mathématiques: à l'usage de la marine et de l'artillerie* (1764–1769). The source for Surūr's notes has not yet been identified.

These brief examples reveal that a powerful motivation for native speakers to translate European mathematics into local vernaculars was to provide resources for teaching the new mathematics, often at the behest of the political authorities. This new mathematics was often introduced in order to reform the education of military personnel who, it was hoped, would enable the state to confront the increasing pressure of the European colonial powers.

But it was not always native speakers who were active in translating mathematics. In 1857 Cornelius Van Dyck (1818–1895) published an Arabic translation of John Playfair's popular textbook *Elements of Geometry* (first English edition, 1795)—his Arabic translation was reprinted in 1889. Van Dyck served the American Board of Commissioners for Foreign Missions (ABCFM) as a physician attached to the Beirut mission in Ottoman Syria. By mid-century the missionary community, increasingly criticized by its supporters for failing to produce converts, developed a new strategy. It offered young people education that included generous amounts of the new science and mathematics—along with exposure to Christianity. But this endeavor necessitated relevant textbooks, and Van Dyck, a skilled linguist, set himself the task of supplying this need. Over the next two decades he would publish nearly a dozen Arabic textbooks on modern sciences and mathematics.

Although Van Dyck's translation was generally a faithful rendition of Playfair's textbook, he seems to have deliberately disregarded the long tradition of geometrical discourse in Arabic by introducing new technical vocabulary. He also altered the traditional system of assigning letters to points in geometrical diagrams. And he chose to orient his diagrams to be read from left to right, exactly like those in Playfair's textbook, rather than comply with the right-to-left character of Arabic prose. These changes seem intended to emphasize the discontinuities between traditional Arabic geometry and the new mathematics of the Western tradition [De Young, 2014].

Van Dyck's translation supported the missionary zeal for conversions by implicitly emphasizing that traditional learning was completely divorced from the wonders of modern science and mathematics. This approach also fit well with the colonial aims of Western powers: an enlightened populace would recognize the inadequacy of the decadent Ottoman state and might more readily embrace European models of society and government.

The first geometry textbooks printed in Persian were created, like those in Ottoman dominions, for use in a new educational institution (Dār al-Funūn, established in 1851) to train Persian military personnel in the new sciences that seemed to have given the European colonial powers a decided advantage on the battlefield. The earliest that I have discovered is a translation (dated 1273/1856-1857) of Adrien-Marie Legendre's influential *Éléments de géométrie*. One remarkable feature of this textbook is the use of Roman script to label the points in its diagrams. (Equations are also written in Roman script.) It appears that both the diagrams and the equations were considered as units and so remained unmodified in the translation process. The result is a curious "hybrid" in which some textual elements must be read left-to-right and others right-to-left [De Young, 2016].

The next year, another Persian textbook was printed. It had no title page, but modern catalogers have called it *`Ilm al-Misāḥat* (Science of Surveying / Measurement). A traditional introductory paragraph reports that the author was Augustus Kržiž (1814–1886), an artillery officer recruited from the Austrian Empire to teach in the Dār al-Funūn. Kržiž composed his textbook in French; it was then translated into Persian by Mīrzā Zakī Māzandarānī, an instructor in the Dār al-Funūn. Occasionally the translator could not find an appropriate Persian equivalent, so he transliterated the French term—usually inserting the original term in Roman script for clarity. The translator also preserved the diagrams and equations in Roman script, producing another "hybrid" textbook.

A new Persian translation of Legendre's textbook (as revised by Marie Alphonse Blanchet) was printed in 1318/1900-1901. Its translator, Ḥajj Najm al-Dawlah Mīrzā ʻAbd al-Ghaffār (1255/1839-1840–1326/1908-1909), was professor of mathematical sciences at the Dār al-Funūn. His translation abandoned the "hybrid" style and translated even labels of points in diagrams and numbers and letters within equations into Persian script, making his textbook easier for students to read and understand.

As these few vignettes show, the transmission of modern geometry into nineteenth-century Middle Eastern societies was a complex phenomenon involving players with different and sometimes competing agendas. In some cases, European textbooks were translated, more or less literally, into local vernaculars. In other cases, new textbooks were composed following European exemplars. The stories of these early textbooks are only beginning to be told. They can help us to appreciate mathematics as a living intellectual activity and not merely as a collection of abstract academic results.

References

[1] Abdeljaouad, M. (2011). The first modern Egyptian mathematics textbook. *International Journal for History of Mathematics Education*, 6:1–11.

[2] De Young, G. (2012). Nineteenth-century traditional Arabic geometry textbooks. *International Journal for History of Mathematics Education*, 7:1–35.

[3] De Young, G. (2014). Translating Playfair's Geometry into Arabic: Mathematics and missions. In G. van Brummelen and N. Sidoli (eds), *From Alexandria, Through Baghdad,* 502–527. New York: Springer.

[4] De Young, G. (2016). Early geometry textbooks printed in Persian. To appear in *Proceedings of the Fourth International Conference on History of Mathematics Education*.

Gregg De Young (gdeyoung@aucegypt.edu) has taught "Scientific Thinking" in the Core Curriculum Program at the American University in Cairo (Egypt) since 1990. His primary research focus is Euclidean geometry in medieval Mediterranean societies.

December 2-5, 2016

Site: Sheraton on the Falls, Niagara Falls (Ontario)

cms.math.ca/events/winter16

2-5 décembre 2016

Site: Sheraton on the Falls, Niagara Falls (Ontario)

smc.math.ca/reunions/hiver16

Prizes I Prix

Adrien Pouliot Award I Prix Adrien-Pouliot

To be announced

Coxeter-James Prize and Lecture I Prix Coxeter-James et conférence

Louigi Addario-Berry (McGill)

Doctoral Prize I Prix de doctorat

Vincent X. Genest (MIT)

G. de B. Robinson Award I Prix G. de B. Robinson

Jim Agler (UCSD) and John McCarthy (WUSTL)

Jeffery-Williams Prize I Prix Jeffery-Williams

Daniel Wise (McGill)

Plenary Lectures I Conférences plénières

Natasa Sesum (Rutgers), Diane Maclagan (Warwick)
Karol Zyczkowski (Jagiellonian), Mark Ablowitz (Colorado)

Public Lectures I Conférence publiques

Dror Bar-Natan (Toronto)

Scientific Directors I Directeurs scientifique

Hans Boden, McMaster University, boden@mcmaster.ca
Bartosz Protas, McMaster University, bprotas@mcmaster.ca

Supported by I Soutenu par

pims FIELDS CRM AARMS

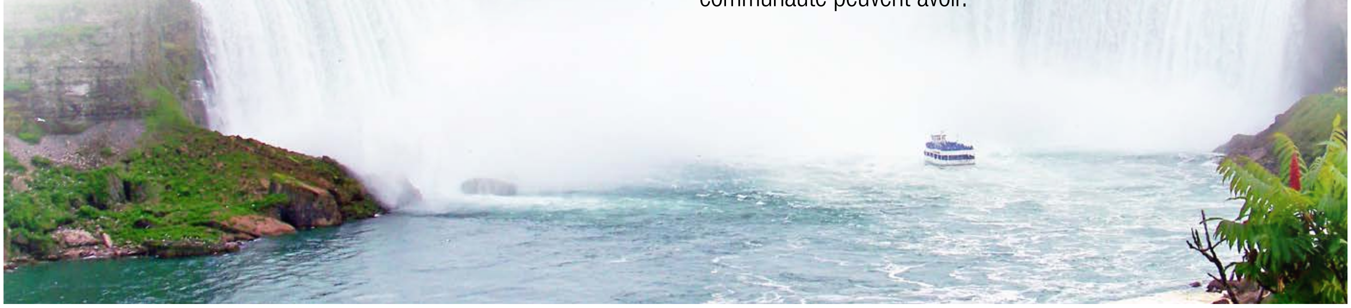
Related Events I Événements liés

The Canadian Mathematical Society invites you to their **awards banquet** on Saturday, December 3, to highlight exceptional performance in the area of mathematical research and education. Prizes will be awarded during the event taking place at Elements on the Falls Restaurant.

The **CMS Town Hall** meeting will occur on Sunday, December 4, from 12:30 - 14:00. All CMS members and meeting participants are invited to join the CMS Executive and to engage on upcoming plans and to discuss any interests or concerns that members of our community may have.

La Société mathématique du Canada vous invite à son **banquet** le samedi 3 décembre pour souligner des contributions exceptionnelles en recherche mathématique et en enseignement des mathématiques. Les prix seront remis durant la soirée, qui se déroulera au restaurant Elements on the Falls.

La **Séance de discussion de la SMC** auront lieu le samedi 4 décembre, à partir de 12h30-14h00. Tous les membres de la SMC et participants à la réunion sont invités à se joindre à l'exécutif de la SMC à s'engager sur les plans à venir et de discuter des préoccupations ou des intérêts que les membres de notre communauté peuvent avoir.



Regular Sessions | Sessions générales

Analytic number theory | Théorie analytique des nombres
Patrick Ingram and Youness Lamzouri (York)

Combinatorial Algebraic Geometry | Géométrie algébrique combinatoire
Kiumars Kaveh (Pittsburgh) and Frank Sottile (Texas A&M)

Combinatorial, Geometric, and Computational Aspects of Optimization | Optimisation: aspects combinatoire, géométrique et calculatoire
Antoine Deza (McMaster), Ricardo Fukasawa, Laura Sanita (Waterloo)

Complex Analysis and Applications | Analyse complexe et applications
Maxime Fortier Bourque (Toronto) and Malik Younsi (Stony Brook)

Financial Mathematics | Mathématiques financières
Petar Jevtic, Traian Pirvu (McMaster), Adam Metzler(Wilfred Laurier)

Fractal Geometry, Analysis, and Applications | Géométrie fractale
Franklin Mendivil (Acadia), József Vass (York)

Geometric Group Theory and Topology in Low Dimensions | Théorie des groupes géométriques et topologie en basses dimensions
Adam Clay (Manitoba) and Ying Hu (UQÀM)

Geometric PDEs, the Einstein equation, and mathematical relativity | Équations aux dérivées partielles géométriques, équation d'Einstein et relativité mathématique
Hari Kunduri (Memorial) and Eric Woolgar (Alberta)

Geometry and Topology in Low Dimensions: Interactions with Floer theory | Géométrie et topologie en basses dimensions : interactions avec la théorie de Floer
Cagatay Kutluhan (Buffalo) and Liam Watson (Sherbrooke)

History and Philosophy of Mathematics | Histoire et philosophie des mathématiques
Maritza M. Branker (Niagara)

Improving success rates in first year calculus | Améliorer le taux de réussite en première année de calcul différentiel et intégral
Jaimal Thind and Maria Wesslén (Toronto)

Integrable systems and applications | Systèmes intégrables et applications
Stephen Anco (Brock) and Gino Biondini (Buffalo)

Large Scale Optimization: Theory, Algorithms and Applications | Optimisation à grande échelle : théorie, algorithmes et applications
Heinz Bauschke (UBC, Kelowna) and Henry Wolkowicz (Waterloo)

Mathematical Logic | Logique mathématique
Bradd Hart, Matthew Valeriote (McMaster)

Mathematical Population Biology | Biologie mathématique des populations
Ben Bolker, Jonathan Dushoff and David Earn (McMaster)

Matrix Theory | Théorie matricielle
Hadi Kharaghani (Lethbridge) and Javad Mashreghi (Laval)

Nonlinear Evolution Equations | Équations d'évolution non linéaires
Almut Burchard, (Toronto), Marina Chugunova (CGU)

Optimization Techniques in Quantum Information Theory | Techniques d'optimisation en théorie de l'information quantique
Nathaniel Johnston (Mount Allison), Rajesh Pereira (Guelph) and Sarah Plosker (Brandon)

Recent Advances in Commutative Algebra | Percées récentes en algèbre commutative
Sara Faridi (Dalhousie), Federico Galleto and Adam Van Tuyl (McMaster)

Recent Advances in Dynamical Systems and Bifurcation Theory | Progrès récents en systèmes dynamiques et théorie des bifurcations
Pietro-Luciano Buono (UIUT), Victor LeBlanc (Ottawa)

Recent Advances in Structure-Preserving Discretizations | Percées récentes en discrétisations avec préservation des structures
Pietro-Luciano Buono (UIUT), Victor LeBlanc (Ottawa)

Stochastic Properties of Dynamical Systems | Propriétés stochastiques des systèmes dynamiques
Jacopo De Simoi, Ke Zhang (Toronto), HongKun Zhang (Massachusetts Amherst)

Symmetry, Dynamics, and Bifurcations in Functional Differential Equations | Symétrie, dynamique et bifurcations dans les équations différentielles fonctionnelles
Israel Ncube (Alabama A&M)

Teaching for Deeper Mathematical Engagement | Enseigner pour renforcer l'engagement en mathématiques
Caroline Junkins (Western), John Grant McLoughlin (UNB)

AARMS-CMS Student Poster Session | Présentations par affiches des étudiants - AARMS-SMC
Svenja Huntemann (Dalhousie), Aaron Berk (UBC)

Contributed Papers | Communications libres
to be announced | Org : à venir

Mathematical Congress of the Americas
Congrès Mathématique des Amériques

MC of A · Mathematical Council of the Americas

MCA 2017

CMS SMC

July 24-28 juillet Montréal, Canada

The second Mathematical Congress of the Americas (MCA) will take place from July 24-28, 2017, at Centre Mont-Royal and McGill University in Montreal, Canada.

MCA 2017 will highlight mathematical achievements of the Americas and will foster collaboration between the continents' mathematical communities.

There will be a large number of special sessions. Follow this link to view the confirmed scientific sessions: **https://mca2017.org/program/scientific-program**

Registration is now open!

mca2017.org

Le deuxième Congrès mathématiques des Amériques (CMA) aura lieu du 24 au 28 juillet 2017 au Centre Mont-Royal et l'Université McGill à Montréal, Canada.

Le CMA 2017 met en lumière les accomplissements mathématiques des Amériques et encourage la collaboration entre les différentes communautés mathématiques du continent.

Un grand nombre de sessions spéciales seront au programme. Consultez la page **https://mca2017.org/program/scientific-program** pour voir les sessions confirmées.

La période d'inscription est ouverte!

mca2017.org/fr

If undelivered, please return to:
Si NON-LIVRÉ, veuillez retourner à :

CMS Notes / Notes de la SMC
209 - 1725 St. Laurent Blvd Ottawa, ON K1G 3V4 Canada