CMS NOTES de la SMC

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Canadian Mathematical Society
Société mathématique du Canada

Vice-President's Notes / Notes du Vice-président

Rahim Moosa *(University of Waterloo)*
Vice-President Ontario / Vice-président, Ontario

Academic Freedom

In this issue of the *CMS Notes* (page 4) you will find a letter written by the Executive Committee of the CMS to the President of Turkey expressing our concern about a sustained campaign his government is waging against professors who signed a peace petition in January of 2016. The group of signatories, which calls itself Academics for Peace, reports 125 faculty dismissals, including 68 who were banned from public service by cabinet decrees on September 1 and October 29. Earlier in the year, 41 professors were placed in police custody, three of them remaining in pre-trial detention for 40 days. One of those three was a mathematician, Kivanç Ersoy of Mimar Sinan University in Istanbul, who soon after his release gave a seminar entitled *A theorem that was proven in prison.*

Academic freedom is by no means a settled principle, and its exact limitations are a matter of controversy all over the world. The core idea is that freedom of inquiry and expression is a precondition to the existence of a properly functioning and intellectually productive academia. The Canadian Association of University Teachers (CAUT) describes it as "the right to teach, learn, study and publish free of orthodoxy or threat of reprisal and discrimination." But I expect that most of us would go further, interpreting academic freedom to include political activity and expression even if it was not directly connected to one's academic work. This would fall under what the the American

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La liberté universitaire

Dans ce numéro des *Notes de la SMC* (page 4), vous trouverez une lettre rédigée par le Comité exécutif et destinée au président de la Turquie, exprimant son inquiétude au sujet de la campagne soutenue menée par son gouvernement contre les professeurs qui ont signé une pétition pour la paix en janvier 2016. Le groupe de signataires, qui s'appelle les Universitaires pour la paix, rapporte 125 licenciements de professeurs dont 68 ont été exclus de la fonction publique par décret du Conseil des ministres le 1 septembre et le 29 octobre. Plus tôt cette année, 41 professeurs ont été placés en garde à vue et trois d'entre eux sont restés en détention provisoire pendant 40 jours. L'un d'entre eux était un mathématicien, Kivanç Ersoy, de l'Université Mimar Sinan à Istanbul, qui peu de temps après sa libération a donné un séminaire intitulé *Un théorème qui a été prouvé en prison*.

La liberté universitaire n'est en aucun cas un principe bien établi, et ses limites exactes prêtent à la controverse partout dans le monde. L'idée de base est que la liberté d'enquête et d'expression est une condition préalable à l'existence d'un milieu universitaire intellectuellement productif et fonctionnel. L'Association canadienne des professeures et professeurs d'université (ACPPU) définit cette liberté ainsi : « [le] droit d'enseigner, d'apprendre, d'étudier et de publier sans craindre l'orthodoxie ou la menace de représailles et la discrimination. » Mais je pense que la plupart d'entre nous iraient plus loin et que notre interprétation de la liberté universitaire inclurait les activités et l'expression politiques, même si elles ne sont pas directement liées au travail universitaire de

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Special Cases

Robert Dawson, *Department of Mathematics and Computer Science, Saint Mary's University*

Much of mathematics is about generalities - theorems that apply to huge sets of things. Metaphorically, these results are like the Northern forests, the Prairies, and the oceans - vistas that recede forever. For obvious reasons, studying such properties requires a level of abstraction.

But there are also many results that deal with small numbers of cases. For instance, there are only five Platonic polyhedra, and many of the most beautiful results about them are even more specialized. There are exactly two ways - sometimes called the "true cross" and the "skew cross" - in which the twelve vertices of an icosahedron can be partitioned into three sets, each the vertex set of a golden rectangle. There's a similar but different result for the dodecahedron: its twenty vertices can be partitioned into the vertex sets of a cube and three rectangles, each with the proportions $1:\tau^2$. But these beautiful results do not generalize to anything else. It's not even easy to say what that would mean.

In first-year calculus, we teach students to compute things like areas under curves and arc lengths. Now, integral problems do not, in general, come with a guarantee of solubility; but if your function is relatively simple, the chances of being able to find its integral, and thus the area under its graph, are fairly good.

Arc lengths, however, are different. As you probably remember, the arc length element for the graph of $y = f(x)$ is $\sqrt{1+f^t(x)^2}dx$. If we are to integrate this, either $f^t(x)$ must be very simple indeed, or we must be able to take the square root before integrating. This is an extremely strong condition, so strong that it is not preserved even by multiplying by a constant. For instance, the graph of $y=\sqrt{1-x^2}$ is a semicircle, with arc length easily computed. But scale it in any way and you need (of course!) elliptic integrals.

As a final example, if you start adding consecutive squares, $1^2+2^2+\cdots+n^2$, the first partial sum, 1, is of course a perfect square. The 24th partial sum, 4900, is also a perfect square. As G.N. Watson showed in 1918, it's the *only* other one. Beautiful - and useless? Not so - this strange little summation turns out to be enormously important in the theory of high-dimensional lattices.

Mathematics is not just forests and oceans - it's also the Reversing Falls, the hot springs, the natural arches and balancing rocks: the weird things that just happen.

Des cas particuliers

Une bonne partie des mathématiques concerne des généralités – des théorèmes qui s'appliquent à d'énormes ensembles d'éléments. Par métaphore, on pourrait dire que ces résultats sont comme les forêts nordiques, les Prairies et les océans – des vues qui se perdent dans le lointain. Pour des raisons évidentes, l'étude de telles propriétés nécessite un certain degré d'abstraction.

Mais il y a aussi de nombreux résultats qui portent sur de petits nombres de cas. Par exemple, il n'existe que cinq polyèdres platoniques, et bon nombre des plus beaux résultats les concernant sont encore plus spécialisés. Il existe précisément deux façons de séparer les 12 sommets d'un icosaèdre en trois ensembles, chacun constituant les sommets d'un rectangle d'or. Le résultat est semblable, mais différent pour le dodécaèdre : ses 20 sommets peuvent être séparés en ensembles de sommets formant un cube et trois rectangles, chacun respectant les proportions $1:\tau^2$. Toutefois, ces résultats élégants ne se généralisent pour rien d'autre. Il est même difficile d'exprimer ce que cela voudrait dire.

Dans les cours de calcul différentiel et intégral de première année, nous enseignons aux étudiants à calculer, par exemple, l'aire sous les courbes et la longueur des arcs. Pourtant, les problèmes de calcul intégral ne sont pas tous solvables; mais dans le cas d'une fonction assez simple, la possibilité de trouver son intégrale et, ainsi, l'aire sous sa courbe, est assez bonne.

Il n'en va pas de même pour la longueur des arcs. Vous vous souviendrez peut-être que l'élément de la longueur de l'arc pour le graphique de $y = f(x)$ est $\sqrt{1+f^t(x)^2}dx$. Pour trouver l'intégrale, il faudrait soit que $f^t(x)$ soit très simple, soit pouvoir prendre la racine carrée avant de calculer l'intégrale. Cette condition est si forte qu'elle n'est pas préservée même si on la multiplie par une constante. Par exemple, le graphique de $y=\sqrt{1-x^2}$ est un demi-cercle dont la longueur d'arc est facile à calculer. Mais si on change l'échelle, n'importe comment, on a besoin (bien sûr!) des intégrales elliptiques.

Et un dernier exemple : si vous commencez à additionner des carrés consécutifs, $1^2+2^2+\cdots+n^2$, la première somme partielle, 1, est bien sûr un carré parfait. La 24e somme partielle, 4 900, est aussi un carré parfait. Comme l'a démontré G.N. Watson en 1918, c'est *le seul autre* carré parfait. Élégant, mais inutile? Absolument pas, cette étrange petite sommation étant en fait très importante dans la théorie des treillis de grande dimension.

Les mathématiques ne sont pas que des forêts et des océans; elles sont aussi des chutes réversibles, des sources thermales, des arches naturelles et des rochers en équilibre : des phénomènes étranges qui se produisent sans autre explication.

Letters to the Editors

The Editors of the NOTES welcome letters in English or French on any subject of mathematical interest but reserve the right to condense them. Those accepted for publication will appear in the language of submission. Readers may reach us at the Executive Office or at **notes-letters@cms.math.ca**

Lettres aux Rédacteurs

Les rédacteurs des NOTES acceptent les lettres en français ou en anglais portant sur n'importe quel sujet d'intérêt mathématique, mais ils se réservent le droit de les comprimer. Les lettres acceptées paraîtront dans la langue soumise. Les lecteurs peuvent nous joindre au bureau administratif de la SMC ou à l'adresse suivante : **notes-lettres@smc.math.ca**.

NOTES DE LA SMC

Les Notes de la SMC sont publiés par la Société mathématique du Canada (SMC) six fois par année (février, mars/avril, juin, septembre, octobre/novembre et décembre).

Les rédacteurs des Notes de la SMC accueillent vos articles, lettres et notes. Indiquer la section choisie pour votre article et le faire parvenir à l'adresse courriel appropriée ci-dessus.

Les Notes de la SMC, les rédacteurs et la SMC ne peuvent pas être tenus responsables des opinions exprimées par les auteurs.

CMS NOTES

The CMS Notes is published by the Canadian Mathematical Society (CMS) six times a year (February, March/April, June, September, October/November and December).

The editors welcome articles, letters and announcements. Indicate the section chosen for your article, and send it to CMS Notes at the appropriate email address indicated above.

No responsibility for the views expressed by authors is assumed by the CMS Notes, the editors or the CMS.

La Société mathématique du Canada appuie l'avancement, la découverte, l'apprentissage et l'application des mathématiques. L'exécutif de la SMC encourage les questions, commentaires et suggestions des membres de la SMC et de la communauté.

The CMS promotes the advancement, discovery, learning and application of mathematics. The CMS Executive welcomes queries, comments and suggestions from CMS members and the community.

Canadian Mathematical Society - Société mathématique du Canada
209-1725 St. Laurent Blvd., Ottawa, ON, Canada K1G 3V4 tel 613-733-2662

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October 12, 2016

H.E. Recep Tayyip Erdoğan
President of the Republic of Turkey
Cumhurbaşkanlığı Külliyesi
06560 Beştepe-Ankara
Turkey

Dear President Erdoğan:

I am writing on behalf of the Executive Committee of the Canadian Mathematical Society to express our deep concern about the recent treatment of some academic staff at Turkish universities, and the effect on academic freedom in Turkey.

We are particularly alarmed by your government's cabinet decree of September 1, made within the context of the state of emergency rule, which bans from public service over 40 academics who had signed a peace petition in January.

We have also seen reports of earlier retaliations against other signatories, including dismissal and imprisonment.

We regard these actions against professors who are publicly expressing their political views as serious violations of the intellectual freedoms that are fundamental to any university system.

We urge the government of Turkey to reinstate the dismissed faculty, and to ensure the future civil liberties, job security, and personal safety of all Turkish academics.

Michael Bennett
President of the Canadian Mathematical Society
Professor of Mathematics, University of British Columbia

and the members of the CMS Executive Committee:
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Bekir Bozdağ, Justice Minister of the Republic of Turkey
İsmail Kahraman, Speaker of the Turkish National Grand Assembly
Selçuk Ünal, Turkish Ambassador to Canada
Yekta Saraç, President of the Council of Higher Education
Prime Minister Binali Yıldırım, Office of the Prime Minister, Republic of Turkey
İsmet Yılmaz, Minister of National Education, The Grand National Assembly of Turkey

The Calendar brings current and upcoming domestic and select international mathematical sciences and education events to the attention of the CMS readership. Comments, suggestions, and submissions are welcome.

Denise Charron, *Canadian Mathematical Society, (managing-editor@cms.math.ca)*

Le calendrier annonce aux lecteurs de la SMC les activités en cours et à venir, sur la scène pancanadienne et internationale, dans les domaines des mathématiques et de l'enseignement des mathématiques. Vos commentaires, suggestions et propositions sont le bienvenue.

Denise Charron, *Société mathématique du Canada (redacteur-gerant@smc.math.ca)*

DECEMBER 2016 DÉCEMBRE

Nov 27- 2	AARMS Workshop: Fifth Parallel-in-time Integration Workshop, BIRS, Banff, Alta.
2-5	2016 CMS Winter Meeting / Réunion d'hiver de la SMC 2016, Sheraton on the Falls, Niagara Falls, Ont.
4-9	BIRS Workshop: Analytic versus Combinatorial in Free Probability, BIRS, Banff, Alta.
5-9	Workshop on Combinatorial Moduli Spaces and Intersection Theory, Fields Institute, Toronto, Ont.
7-9	2016 Montreal-Toronto Workshop in Number Theory, CRM, Montreal, Que.
11-14	12th International Conference on Web and Internet Economics (WINE), InterContinental Hotel, Montreal, Que.
14-15	CRM Workshop: 13th Workshop on Algorithms and Models for the Web-graph, CRM, Montreal, Que.

JANUARY 2017 JANVIER

4-7	AMS/MAA 2017 Joint Mathematics Meeting, Hyatt Regency Atlanta and Marriott Atlanta Marquis, Atlanta, GA
22-27	BIRS Workshop: String and M-theory geometries: Double Field Theory, Exceptional Field Theory and their Applications, BIRS, Banff, Alta.
22-27	BIRS Workshop: Combinatorial Reconfiguration, BIRS, Banff, Alta.
28	**Math Education Forum**, Fields Institute, Toronto, Ont.
29-Feb 3	BIRS Workshop: Data-Driven Methods for Reduced-Order Modeling and Stochastic Partial Differential Equations, BIRS, Banff, Alta.

FEBRUARY 2017 FÉVRIER

5-10	BIRS Workshop: Newton-Okounkov Bodies, Test Configurations, and Diophantine Geometry, BIRS, Banff, Alta.
12-17	BIRS Workshop: Mathematical Approaches to Evolutionary Trees and Networks, BIRS, Banff, Alta.
13-17	FIELDS Workshop: Heights and Applications to Unlikely Intersections, Fields Institute, Toronto, Ont.
19-24	BIRS Workshop: Validating and Expanding Approximate Bayesian Computation Methods, BIRS, Banff, Alta.
26-Mar 3	BIRS Workshop: Brain Dynamics and Statistics: Simulation versus Data, BIRS, Banff, Alta.

MARCH 2017 MARS

5-10	BIRS Workshop: Optimization and Inference for Physical Flows on Networks, BIRS, Banff, Alta.
12-17	BIRS Workshop: New Trends in Arithmetic and Geometry of Algebraic Surfaces, BIRS, Banff, Alta.
13-17	FIELDS Workshop: Efficient Congruencing and Translation-invariant Systems, Fields Institute, Toronto, Ont.
17	PIMS/ UBC Distinguished Colloquium: Michel Brion, University of British Columbia, Vancouver, B.C.
18-19	FIELDS Workshop: Algebraic Varieties With a Special Emphasis on Calabi-Yau Varieties and Mirror Symmetry, Fields Institute, Toronto, Ont.
18-22	Nirenberg Lectures in Geometric Analysis at the CRM: Gunther Uhlmann (University of Washington), CRM, Montreal, Que.
19-24	BIRS Workshop: Communication Complexity and Applications, II, BIRS, Banff, Alta.
20-24	Nirenberg Lectures in Geometric Analysis at the CRM: Min-max Theory and Geometry, CRM, Montreal, Que.
24-28	Nirenberg Lectures in Geometric Analysis at the CRM: Camillo De Lellis (Universität Zürich), CRM, Montreal, Que.

Continued from cover

Association of University Professors (AAUP) calls **extramural speech**. The AAUP's 1915 *Declaration of Principles on Academic Freedom and Academic Tenure* defines academic freedom as "freedom of inquiry and research; freedom of teaching within the university or college; and freedom of extramural utterance and action". While their 1940 *Statement of Principles on Academic Freedom and Tenure* does mention the "special obligations" that the social position of professorship entails, and the restraint that should go with it, the document also reaffirms that "when [college and university teachers] speak or write as citizens, they should be free from institutional censorship or discipline."

It is instructive to look back at a defining episode in the history of academic freedom in Canada. In the 1930s, many academics from Toronto and Montreal faced significant reprisals for their involvment in the formation of the leftist League for Social Reconstruction (LSR) and the socialist party Cooperative Commonwealth Federation (CCF). Historian Frank Underhill, a founder of the LSR and an outspoken activist with the CCF, was publicly admonished and nearly dismissed by the University of Toronto. Another LSR founder, King Gordon of the United Theological College in Montreal, lost his position.[1]

Today, our attachment to a broadly interpreted notion of academic freedom is much more robust than it was then. But violations in Canada are by no means a thing of the past; the CAUT, for example, is involved in a number of ongoing high profile cases that are described in some detail on their web pages. One major recent controversy south of the border is worth considering. Steven Salaita, an American of Jordanian and Palestinian descent, was offered a tenured position in American Indian studies at the University of Illinois at Urbana Champaign (UIUC) in 2013, but the offer was withdrawn in 2014 after he made incendiary comments on Twitter about the Israel-Gaza War of that year. There was a public outcry against the university's decision, with the AAUP censuring UIUC and several academic bodies calling for boycotts.[2]

There are also examples from the other side of the cultural divide, for lack of a better way to put it. In 2005 Lawrence Summers, then president of Harvard, faced withering criticism and a lack of confidence vote from the faculty for his provocative suggestion that the possibility of intrinsic differences in aptitude between the genders should be explored as an explanation for the underrepresentation of women in science at top universities. In September of this year, University of Toronto psychology professor Jordan Peterson caused an uproar for attacking current trends in gender identity politics in a YouTube post, saying, for example, that he does not recognise an individual's right to be addressed by a gender neutral pronoun. Peterson is being pilloried in the press, though as of now it does not look like he will be facing any official sanctions.

This is not to say that academic freedom should protect a university professor from every kind of speech and action. Certainly professorship does come with obligations, and to use one's academic position to spread hate and bigotry is unacceptable. I understand that there will always be borderline cases and that drawing this line may be difficult and contentious. I am arguing for a broad interpretation of academic freedom (to the extent that these ramblings can be construed as an argument), while hoping that such a position can have meaning even without a careful analysis of what the exact limitations of academic freedom should be.

It is true that there is relatively little controversy around mathematical research and teaching, and that we are unlikely to have to seek the protection of academic freedom on these matters. But all of us have political opinions, many of us make extramural utterances, and some might even use strong and controversial language. Frank Underhill or Steven Salaita could easily have been mathematicians. Kivanç Ersoy is one.

1 See M. Horn, "Professors in the Public Eye: Canadian Universities, Academic Freedom, and the League for Social Reconstruction", *History of Education Quarterly* 1980 (20), number 4, pp. 425–447.

2 The university stuck to its decision not to hire Salaita, but it did award him an out of court settlement worth $875,000 (US).

Suite de la couverture

chacun. Cela relèverait de ce que l'Association américaine des professeurs d'université (AAUP) appelle le **discours extramuros**. La *Déclaration de principes sur la liberté universitaire et la titularisation* de 1915 faite par l'AAUP définit la liberté universitaire comme étant la « liberté d'enquête et de recherche; la liberté de l'enseignement à l'université ou au collège; et la liberté de parole et d'action extramuros ». Même si l'*Énoncé de principes sur la liberté universitaire et la titularisation* de 1940 mentionne les « obligations spéciales » que suppose le statut social du professeur ainsi que la retenue qui devrait l'accompagner, le document réaffirme également que « lorsque [les professeurs des collèges et des universités] parlent ou écrivent en tant que citoyens, ils devraient être libres de toute censure ou discipline institutionnelle. »

C'est instructif de revenir sur un épisode déterminant de l'histoire de la liberté universitaire au Canada. Dans les années trente, de nombreux universitaires de Toronto et de Montréal ont subi d'importantes représailles à cause de leur participation à la formation de la League for Social Reconstruction (LSR), un organisme de gauche, et à la Cooperative Commonwealth Federation (CCF), un parti socialiste. L'historien Frank Underhill, un des fondateurs de la LSR qui militait ouvertement avec la CCF, a été publiquement réprimandé et presque exclu de l'Université de Toronto. Un autre fondateur de la LSR, King Gordon, du Séminaire uni à Montréal, a perdu son poste[1].

Aujourd'hui, notre attachement à la notion de liberté universitaire interprétée au sens large est beaucoup plus solide qu'à l'époque. Cependant, au Canada, les violations ne sont nullement chose du passé. Par exemple, l'ACPPU est impliquée dans un certain nombre de cas très médiatisés en cours qui sont décrits en détail dans les pages de son site Web. De plus, une récente controverse importante qui a lieu au sud de la frontière mérite d'être examinée. Steven Salaita, un Américain d'origine jordanienne et palestinienne, s'est vu offrir un poste permanent en études amérindiennes à l'Université de l'Illinois à Urbana-Champaign (UIUC) en 2013, mais l'offre a été retirée en 2014 après qu'il ait publié des commentaires incendiaires sur Twitter au sujet de la guerre d'Israël à Gaza cette année-là. La décision de l'Université a soulevé un tollé général, l'AAUP a censuré l'UIUC et plusieurs organismes universitaires ont appelé au boycottage[2].

Il existe également des exemples de la fracture culturelle au sud de notre frontière, si je puis m'exprimer ainsi. En 2005, Lawrence Summers, alors président de Harvard, a subi des critiques méprisantes et a fait l'objet d'une motion de défiance de la part du corps enseignant parce qu'il avait suggéré d'étudier la relation entre les aptitudes et les différences intrinsèques entre les sexes afin d'expliquer la sous-représentation des femmes dans les sciences au sein des meilleures universités, suggestion jugée provocante. En septembre de cette année, le professeur de psychologie de l'Université de Toronto, Jordan Peterson, a provoqué un tollé parce qu'il a attaqué les tendances actuelles en matière de politique sur l'identité sexuelle dans une vidéo sur YouTube, en disant par exemple qu'il ne reconnaissait pas le droit à une personne d'être désignée par un pronom neutre. M. Peterson a été cloué au pilori dans la presse, mais pour le moment, il semble qu'il ne fera pas l'objet de sanctions officielles.

Cela ne veut pas dire que la liberté universitaire devrait protéger un professeur, quels que soient ses paroles et ses actes. Le métier de professeur comporte des obligations, et il est inacceptable d'utiliser son statut d'universitaire pour répandre la haine et l'intolérance. Je comprends qu'il y aura toujours des cas limites et qu'il peut être difficile et discutable de tracer une ligne claire. Je plaide en faveur d'une interprétation élargie de la liberté universitaire (dans la mesure où ces divagations peuvent être interprétées comme étant un argument), tout en espérant qu'une telle position peut avoir un sens, même en l'absence d'une analyse minutieuse de ce que devraient être les limites exactes de cette liberté.

Il est vrai qu'il y a assez peu de controverse autour de la recherche et de l'enseignement en mathématique et que nous risquons assez peu d'avoir à demander la protection de la liberté universitaire dans ces domaines. Cependant, chacun d'entre nous a des opinions politiques, beaucoup d'entre nous font des déclarations extramuros, et certains pourraient même utiliser des mots très forts et controversés. Frank Underhill ou Steven Salaita pourraient facilement avoir été des mathématiciens. Kivanç Ersoy en est un.

1 Voir M. Horn, « Professors in the Public Eye: Canadian Universities, Academic Freedom, and the League for Social Reconstruction », *History of Education Quarterly* 1980 (20), numéro 4, pp. 425–447.

2 L'Université a maintenu sa décision de ne pas embaucher M. Salaita, mais elle s'est entendue avec lui hors cour et lui a accordé 875 000 $ (USD).

Book Reviews brings interesting mathematical sciences and education publications drawn from across the entire spectrum of mathematics to the attention of the CMS readership. Comments, suggestions, and submissions are welcome.

Karl Dilcher, *Dalhousie University (notes-reviews@cms.math.ca)*

Les comptes-rendus de livres présentent aux lecteurs de la SMC des ouvrages intéressants sur les mathématiques et l'enseignement des mathématiques dans un large éventail de domaines et sous-domaines. Vos commentaires, suggestions et propositions sont le bienvenue.

Karl Dilcher, *Dalhousie University (notes-critiques@smc.math.ca)*

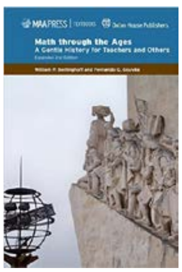
Math through the Ages: A Gentle History for Teachers and Others

By William P. Berlinghoff and Fernando Q. Gouvêa

Expanded 2nd Edition, MAA, 2015

ISBN 978-1939512123

Reviewed by *Glen Van Brummelen*, Quest University Canada



For motivating mathematical ideas, you can't beat history. To illuminate why a subject exists and what makes it important, what better place to go than the place where it came from? This is why, in many American colleges, math majors aiming at teaching careers are required to take courses in the history of mathematics. However, as a result, most textbooks are aimed at the upper undergraduate level. This leaves younger audiences out of luck; and when the majors eventually become teachers, the old textbook sitting on the shelf assumes more sophistication than their students have.

Enter *Math through the Ages: A Gentle History for Teachers and Others*. This lovely little gem displays a pedagogical versatility well beyond virtually every other book available. It's an informative light read for advanced students and professors; a diverse collection of project ideas and classroom episodes for history of mathematics instructors; and a brilliant source of inspiration for high school/ university mathematics teachers hoping to uncover the answers to the "why" questions.

Math through the Ages is not a stand-alone textbook. It opens with a summary of the entire history of mathematics in a mere 60 pages – a quixotic enterprise rather like trying to summarize functional analysis on the back of a napkin. Those looking for an encyclopedic reference text would be better served, for instance, by Victor Katz's *History of Mathematics*. This is simply a skeleton, a framework upon which a course can be built.

The heart of the book is a collection of 30 five-to-ten page episodes, complete with suggested readings and (in the expanded edition) mathematical and historical exercises. They may be used for a variety of purposes: as student projects, as bases for single-class lectures on historical topics, or even as a quick background for a teacher on some mathematical subject. The authors write in a conversational, almost colloquial style – but they achieve surprising depth.

In the units on negative and complex numbers, for instance, we read about 17th-century struggles with the reality, or lack thereof, of imaginary numbers. Conceptual struggles resulted in famous mathematicians like John Wallis claiming that negative numbers are larger than infinity. We learn how parallels between trigonometric identities and the multiplication of complex numbers led DeMoivre and Euler to the discovery of relations like $e^{ix} = cos\ x + i\ sin\ x$.

The unit concludes with Argand and Gauss's invention of the complex plane.

On Fermat's Last Theorem, of course we hear the familiar account of Fermat, his inspiration from Diophantus, and Wiles's proof. But the authors also describe intermediate results by Sophie Germain, Gabriel Lamé, and others. One of the five new units in this second edition describes irrational numbers, respecting the foreign (to our minds) views of the ancient Greeks. The unification of their views with a broader conception of numbers inherited from Arabic scholars in Simon Stevin's work brought rational and irrational numbers together under one umbrella. The story continues through Descartes, Liouville, Hermite, Cauchy, Dedekind, Kronecker, and Cantor.

Although *Math through the Ages* manages to reach a wider audience, these snapshots illustrate that it makes few compromises with its content. Do not expect deep, detailed treatments. But you can count on reliable episodes that reflect recent research and current perspectives from the scholarly practice of the history of mathematics. Freed from having to cover technical details, the book is able to touch on the important stories while remaining sensitive to the original contexts. Whether you're looking for a ten-minute refresher on the origins of Boolean algebra, want to send an undergraduate student toward sources for a paper in projective geometry, or are seeking historical colour in a lecture on conic sections, *Math through the Ages* is a great starting point.

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Leçons d'analyse classique

Exposition d'un cours fait par Paul Koosis

Par Philippe Poulin

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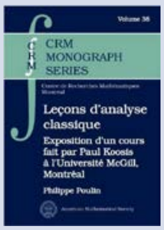
Revue du livre par *Jean-Pierre Kahane*, Université Paris-Sud



Leçons d'analyse classique

Ce livre est d'une profonde originalité. Je ne lui en connais d'équivalent ni de modèle en aucune langue. Le cours de Paul Koosis a été donné en anglais en 2005-2006. Sur sa demande, il a été rédigé en français par Paul Poulin, remanié par Koosis et par Poulin, et revu par plusieurs lecteurs avant sa sortie en 2015.

Le titre, Leçons d'analyse classique, correspond bien au contenu et au style, et pourtant il est trompeur. Il s'agit bien de sujets d'analyse classique, fonctions analytiques d'une variable complexe, approximation pondérée par polynômes ou sommes d'exponentielles, quasi-analyticité, fonctions et mesures harmoniques, problème de Dirichlet, et l'on y trouve de grands et beaux théorèmes dus à Hadamard, Jensen, Harnack, Serge Bernstein, Carleman, Titchmarsh, Cartwright, Caratheodory, Paley et Wiener, Levinson, Akhiezer, Szolem Mandelbrojt, Beurling et Alhfors, entre autres.

Il s'agit bien de leçons. De temps en temps, une petite remarque historique ou une anecdote sur un auteur. A la fin, douze pages de compléments et dix « devoirs » qui, il faut le dire, débordent largement le contenu des leçons. Par exemple, les fonctions moyenne-périodiques, ici appelées (pourquoi pas ?) fonctions moyen-périodiques, font l'objet d'un devoir. Et surtout, il y a un « devoir supplémentaire », très bien venu, avec une jolie démonstration du théorème de Frédéric et Marcel Riezs sur la continuité absolue des mesures dont le spectre est vide sur une demi-droite.

En quoi le titre est-il trompeur ? C'est qu'il ne rend pas compte de l'actualité des sujets et des méthodes. Koosis renouvelle les sujets en insistant sur des aspects peu connus (disons, les apports de Levinson et de Beurling à la quasi-analyticité) et surtout sur des preuves récentes ou des méthodes d'attaque puissantes (je pense au beau chapitre sue les mesures extrémales). Ici le guide est simplement le goût der Koosis, et le lecteur que je suis est plus entraîné par ce goût que par l'architecture d'un traité.

Education Notes brings mathematical and educational ideas forth to the CMS readership in a manner that promotes discussion of relevant topics including research, activities, and noteworthy news items. Comments, suggestions, and submissions are welcome.

Jennifer Hyndman, *University of Northern British Columbia (hyndman@unbc.ca)*
John McLoughlin, *University of New Brunswick (johngm@unb.ca)*

Notes pédagogiques présentent des sujets mathématiques et des articles sur l'éducation aux lecteurs de la SMC dans un format qui favorise les discussions sur différents thèmes, dont la recherche, les activités et les nouvelles d'intérêt. Vos commentaires, suggestions et propositions sont le bienvenue.

Jennifer Hyndman, *University of Northern British Columbia (hyndman@unbc.ca)*
John McLoughlin, *University of New Brunswick (johngm@unb.ca)*

In keeping with the spirit of this issue of the CMS Notes, this issue of Education Notes features words from Jonathan Borwein. Jonathan participated in several Canadian Mathematics Education Study Group (CMESG) meetings over the years. He was one of the plenary lecturers at the 2002 meeting held at Queen's University. An effort has been made here to extract portions of the plenary as a means of informing the readership about some of Jonathan Borwein's work.

The Experimental Mathematician: The Pleasure of Discovery and the Role of Proof

(excerpts from a 2002 CMESG Plenary Lecture given by Jonathan Borwein)

Introduction

Ten years ago I was offered the signal opportunity to found the Centre for Experimental and Constructive Mathematics (CECM) at Simon Fraser. On our website (www.cecm.sfu.ca) I wrote:

At CECM we are interested in developing methods for exploiting mathematical computation as a tool in the development of mathematical intuition, in hypotheses building, in the generation of symbolically assisted proofs, and in the construction of a flexible computer environment in which researchers and research students can undertake such research. That is, in doing 'Experimental Mathematics'.

CECM

The decision to build CECM was based on:

i) more than a decade's personal experience, largely since the advent of the personal computer, of the value of computing as an adjunct to mathematical insight and correctness;

ii) on a growing conviction that the future of mathematics would rely much more on collaboration and intelligent computation;

iii) that such developments needed to be enshrined in, and were equally valuable for, mathematical education; and

iv) that experimental mathematics is *fun*.

Ten years later, my colleagues and I are even more convinced of the value of our venture — and the 'mathematical universe is unfolding' much as we anticipated. Our efforts and philosophy are described in some detail in the forthcoming book and in the survey articles. Ten years ago the term 'experimental mathematics' was often treated as an oxymoron. Now there is a highly visible and high quality journal of the same name.

Fifteen years ago, most self-respecting research pure mathematicians would not admit to using computers as an adjunct to research. Now they will talk about the topic whether or not they have any expertise.

The centrality of information technology to our era and the growing need for concrete implementable answers suggests why we have attached the word 'Constructive' to CECM.

Plus ça change

While some things have happened much more slowly than we guessed (e.g., good character recognition (OCR) for mathematics, any substantial impact on classroom parole), others have happened much more rapidly (e.g., the explosion of the world wide web, the quality of graphics and animations, the speed and power of computers).

Crudely, the tools with broad societal or economic value arrive rapidly, those that are interesting primarily in our niche do not.

Research mathematicians for the most part neither think deeply about nor are terribly concerned with either pedagogy or the philosophy of mathematics.

The Aesthetic Impulse

Nonetheless, aesthetic and philosophical notions have always permeated (pure and applied) mathematics. And the top researchers have always been driven by an aesthetic imperative:

> We all believe that mathematics is an art. The author of a book, the lecturer in a classroom tries to convey the structural beauty of mathematics to his readers, to his listeners. In this attempt, he must always fail. Mathematics is logical to be sure, each conclusion is drawn from previously derived statements. Yet the whole of it, the real piece of art, is not linear; worse than that, its perception should be instantaneous. We have all experienced on some rare occasions the feeling of elation in realizing that we have enabled our listeners to see at a moment's glance the whole architecture and all its ramifications. (Emil Artin, 1898-1962)

Aesthetics and Utility

Elsewhere, I have similarly argued for aesthetics before utility. The opportunities to tie research and teaching to aesthetics are almost boundless—at all levels of the curriculum. This is in part due to the increasing power and sophistication of visualization, geometry, algebra and other mathematical software.

That said, in my online lectures and resources and in many of the references one will find numerous examples of the utility of experimental mathematics—such as gravitational boosting.

My Present Aim

In this setting, my primary concern is to explore the relationship between proof (*deduction*) and experiment (*induction*). I shall borrow shamelessly from my earlier writings.

There is a disconcerting pressure at all levels of the curriculum to derogate the role of proof. This is in part motivated by the aridity of some traditional teaching (e.g., Euclid), by the alternatives offered by good software, by the difficulty of teaching and learning the tools of the traditional trade, and perhaps by laziness. My own attitude is perhaps best summed up by a cartoon in a book on learning to program in APL (a very high level language). The blurb above it reads:

> *Remember ten minutes of computation is worth ten hours of thought.*

The blurb below it reads:

> *Remember ten minutes of thought is worth ten hours of computation.*

Just as 'the unlived life is not much worth examining' (Charles Krauthammer et al.), proof and rigour should be in the service of things worth proving.

And equally foolish, but pervasive, is encouraging students to 'discover' fatuous generalizations of uninteresting facts.

Gauss, Hadamard, & Hardy

Three of my personal mathematical heroes, very different men from different times, all testify interestingly on these points and on the nature of mathematics.

Gauss

Carl Friedrich Gauss (1777–1855) once wrote, "I have the result, but I do not yet know how to get it."

One of Gauss's greatest discoveries, in 1799, was the link between the *lemniscate sine* function and the *arithmetic-geometric mean* iteration. This was based on a purely computational observation. The young Gauss wrote in his diary that the result "will surely open up a whole new field of analysis"...

Hadamard

A constructivist, experimental, and aesthetic driven rationale for mathematics could hardly do better than to start with an insight from Jacques Hadamard:

> The *object* of mathematical rigor is to sanction and legitimize the conquests of intuition, and there was never any other object for it .

Hadamard (1865-1963) was perhaps the greatest mathematician to think deeply and seriously about cognition in mathematics.

Hadamard is quoted as saying "... in arithmetic, until the seventh grade, I was last or nearly last" which should give encouragement to many young students. He was both the author of *The psychology of invention in the mathematical field* (1945), a book still worth close inspection, and co-prover of the *Prime Number Theorem* (1896):

The number of primes less than n tends to ∞ as does $n/log\ n. \ldots$

Hardy's Apology

Correspondingly, G.H. Hardy (1877–1947), the leading British analyst of the first half of the twentieth century, was also a stylish author who wrote compellingly in defense of pure mathematics. In his apologia, *A Mathematican's Apology*, Hardy writes, "All physicists and a good many quite respectable mathematicians are contemptuous about proof". The *Apology* is a spirited defense of beauty over utility: "Beauty is the first test. There is no permanent place in the world for ugly mathematics".

That said, his comment that "Real mathematics ... is almost wholly 'useless'" has been overplayed and is now to my mind very dated, given the importance of cryptography and other pieces of algebra and number theory devolving from very pure study. But he does acknowledge that, "If the theory of numbers could be employed for any practical and obviously honourable purpose ...", even Gauss would be persuaded.

The existence of Amazon or Google means that I can be less than thorough with my bibliographic details without derailing anyone who wishes to find the source. ...

My Own Methodology

As a computational and experimental pure mathematician my main goal is *insight*. Insight demands speed and increasingly parallelism. Extraordinary speed and enough space are prerequisite for rapid verification and for validation and falsification ('proofs and refutations'). One cannot have an 'aha' when the 'a' and 'ha' come minutes or hours apart.

What is 'easy' changes as computers and mathematical software grow more powerful. We see an exciting merging of disciplines, levels, and collaborators. We are more and more able: to marry theory and practice, history and philosophy, proofs and experiments; to match elegance and balance to utility and economy; and to inform all mathematical modalities computationally (analytic, algebraic, geometric, and topological).

This has led us to articulate an *Experimental* Math*odology*, as a philosophy and in practice, based on:

i) meshing computation and mathematics (intuition is acquired not natural);

ii) visualization (even three is a lot of dimensions). Nowadays we can exploit pictures, animations, *emmersive reality*, sounds and other haptic stimuli; and

iii) 'caging' and 'monster-barring' (Imre Lakatos's terms for how one rules out exceptions and refines hypotheses).

Two particularly useful components are:

- *graphic checks*: comparing $2\sqrt{y} - y$ and $\sqrt{y}\ln(y)$, $0 < y < 1$ pictorially is a much more rapid way to divine which is larger than traditional analytic methods.
- *randomized checks*: of equations, linear algebra, or primality can provide enormously secure knowledge or counter-examples when deterministic methods are doomed.

All of which is relevant at every level of learning and research. My own works depend heavily on:

i) *High Precision* (computation of object(s) for subsequent examination);

ii) *Pattern Recognition of Real Numbers* (e.g., using CECM's Inverse Calculator and 'RevEng'), or *Sequences* (e.g., using Salvy & Zimmermann's 'gfun' or Sloane and Plouffe's *Online Encyclopedia*); and

iii) Extensive use of *Integer Relation Methods*. *PSLQ* & *LLL* and *FFT*.

Integer Relation methods are an integral part of a wonderful test bed for experimental mathematics. Ruling out things ('exclusion bounds') is, as always in science, often more useful than finding things.

To make more sense of all this it is helpful to discuss the nature of experiment. Peter Medawar usefully distinguishes four forms of scientific experiment:

i) The *Kantian* example: generating "the classical non-Euclidean geometries (hyperbolic, elliptic) by replacing Euclid's axiom of parallels (or something equivalent to it) with alternative forms".

ii) The *Baconian* experiment is a contrived as opposed to a natural happening, it "is the consequence of 'trying things out' or even of merely messing about".

iii) *Aristotelian* demonstrations: "apply electrodes to a frog's sciatic nerve, and lo, the leg kicks; always precede the presentation of the dog's dinner with the ringing of a bell, and lo, the bell alone will soon make the dog dribble".

iv) The most important is *Galilean*: "a critical experiment—one that discriminates between possibilities and, in doing so, either gives us confidence in the view we are taking or makes us think it in need of correction".

The first three forms of experiment are common in mathematics, the fourth (Galilean) is not. The Galilean Experiment is also the only one of the four forms that has the promise of making Experimental Mathematics a serious replicable scientific enterprise. I'll illustrate this point with some examples.

Editor's note: A significant amount of mathematical discussion takes place here prior to shifting into a subsequent phase of the lecture.

TRUTH versus PROOF

By some accounts Percival's web-computation of π is one of the largest computations ever done. It certainly shows the possibility to use inductive engineering-like methods in mathematics, if one keeps one's eye on the ball.

To assure accuracy the algorithm can be run twice starting at different points—say, starting at 40 trillion minus 10. The overlapping digits will differ if any error has been made. If 20 hex-digits agree we can argue heuristically that the probability of error is roughly 1 part in 10^{25}. While this is not a proof of correctness, it is certainly much less likely to be wrong than any really complicated piece of human mathematics.

Fermat's Margins

For example, perhaps 20 people alive can, given enough time, digest all of Andrew Wiles' extraordinarily sophisticated proof of *Fermat's Last Theorem* and it relies on a century-long program. If there is even a 1% chance that each has overlooked the same subtle error—probably in prior work not explicitly in Wiles' corrected version—then, clearly, many computation-based ventures are much more secure.

This would seem to be a good place to address another common misconception:

> *No amount of simple-minded case checking constitutes a proof.*

Four Colours Suffice

The 1976-7 'proof' of the

> Four Colour Theorem. *Every planar map can be coloured with four colours so adjoining countries are never the same colour*

was a proof because prior mathematical analysis had reduced the problem to showing that a large but finite number of bad configurations could be ruled out.

The proof was viewed as somewhat flawed because the case analysis was inelegant, complicated and originally incomplete. In the last few years, the computation has been redone after a more satisfactory analysis.

Though many mathematicians still yearn for a simple proof in both cases, there is no particular reason to think that all elegant true conjectures have accessible proofs. Nor indeed, given Goedel's work, need they have proofs at all.

> The message is that mathematics is quasi-empirical, that mathematics is not the same as physics, not an empirical

science, but I think it's more akin to an empirical science than mathematicians would like to admit. (Greg Chaitin, 2000)

Kuhn & Planck

Much of what I have described in detail or in passing involves changing set modes of thinking. Many profound thinkers view such changes as difficult:

> The issue of paradigm choice can never be unequivocally settled by logic and experiment alone. ... in these matters neither proof nor error is at issue. The transfer of allegiance from paradigm to paradigm is a conversion experience that cannot be forced. (Thomas Kuhn)

... and

> a new scientific truth does not triumph by convincing its opponents and making them see the light, but rather because its opponents die and a new generation grows up that's familiar with it. (Albert Einstein quoting Max Planck)

However hard such paradigm shifts and whatever the outcome of these discourses, mathematics is and will remain a uniquely human undertaking.

Hersh's Humanist Philosophy

Indeed Reuben Hersh's arguments for a humanist philosophy of mathematics, as paraphrased below, become more convincing in our setting:

i) *Mathematics is human*. It is part of and fits into human culture. It does not match Frege's concept of an abstract, timeless, tenseless, objective reality.

ii) *Mathematical knowledge is fallible*. As in science, mathematics can advance by making mistakes and then correcting or even re-correcting them. The "fallibilism" of mathematics is brilliantly argued in Lakatos's *Proofs and Refutations*.

iii) *There are different versions of proof or rigor*. Standards of rigor can vary depending on time, place, and other things. The use of computers in formal proofs, exemplified by the computer-assisted proof of the *Four Colour Theorem* in 1977, is just one example of an emerging nontraditional standard of rigor.

iv) *Empirical evidence, numerical experimentation and probabilistic proof all can help us decide what to believe in mathematics*. Aristotelian logic isn't necessarily always the best way of deciding.

v) *Mathematical objects are a special variety of a social-cultural-historical object*. Contrary to the assertions of certain post-modern detractors, mathematics cannot be dismissed as merely a new form of literature or religion. Nevertheless, many mathematical objects can be seen as shared ideas, like *Moby Dick* in literature, or the Immaculate Conception in religion.

To this I would add that for me now mathematics is not ultimately about proof but about secure mathematical knowledge.

Note: A concluding section separates this part from the summary comments.

In Summary

- Good software packages can make difficult concepts accessible (e.g., *Mathematica* and *SketchPad*) and radically assist mathematical discovery. Nonetheless, introspection is here to stay.
- "We are Pleistocene People" (Kieran Egan). Our minds can subitize, but were not made for modern mathematics. We need all the help we can get.
- While proofs are often out of reach to students or indeed lie beyond present mathematics, understanding, even certainty, is not.
- "It is more important to have beauty in one's equations than to have them fit experiment" (Paul Dirac).
- And surely: "You can't go home again" (Thomas Wolfe).

Note: Many notes and references appear in the manuscript. These have not been included here. Rather readers are encouraged to go directly to pages 15-29 of the link below.

www.cmesg.org/wp-content/uploads/2015/01/CMESG2002.pdf

Research Notes brings mathematical research ideas forth to the CMS readership in a generally accessible manner that promotes discussion of relevant topics including research (both pure and applied), activities, and noteworthy news items. Comments, suggestions, and submissions are welcome.

Patrick Ingram, *York University (notes-research@cms.math.ca)*

Les articles de recherche présentent des sujets mathématiques aux lecteurs de la SMC dans un format généralement accessible qui favorise les discussions sur divers sujets pertinents, dont la recherche (pure et appliquée), les activités et des nouvelles dignes de mention. Vos commentaires, suggestions et propositions sont le bienvenue.

Patrick Ingram, *York University (notes-recherche@smc.math.ca)*

Jonathan M. Borwein's Extraordinary Mathematical Career

David H. Bailey, *Lawrence Berkeley National Lab (retired), Berkeley, and University of California, Davis, Dept. of Computer Science*

The death of Jonathan M. Borwein on 2 August 2016, in London, Ontario, where he had been visiting from the University of Newcastle, Australia, came as a huge shock to the mathematical community.

Since his death, the present author and Nelson H. F. Beebe of the University of Utah have been collecting Borwein's many published papers, books, reports and talks, as well as a number of articles written by others (such as book reviews) about Jon and his work. Our current catalog (available at **www.jonborwein.org/jmbpapers/** lists over 1700 items! Even if one focuses only on his published books and refereed articles, there are over 500 items. And they are heavily cited — the Google citation tracker finds over 22,000 citations; one paper on optimization [1] has been cited over 1300 times. In examining this catalogue, what is most striking is its breadth. In an era when academic researchers in general, and mathematicians in particular, focus ever more tightly on a single specialty, Borwein did significant research in a wide range of fields, ranging from optimization and experimental mathematics to biomedical imaging and mathematical finance.

One notable paper in the optimization arena is [1], mentioned above, which presents what is now known as the Barzilai-Borwein algorithm for large-scale unconstrained optimization. It is a gradient method that includes modified step sizes, stemming from imitating Newton's method, but does not require the relatively expensive computation of the Hessian of the function. In particular, for an objective function $f(x)$ (for a vector x) and iterates x_k, define $g_k(x_k) = \nabla f(x_k)$ and $F_k = \nabla^2 f(x_k)$. The standard gradient method iterates $x_{k+1} = x_k - \alpha_k g_k(x_k)$, where α_k is typically calculated based on a fixed line search procedure. It is simple to use, but it makes no use of second order information, and sometimes zig-zags rather than converges. Newton's method is to iterate $x_{k+1} = x_k - (F_k(x(k))^{-1} g_k(x_k)$. This utilizes second-order information and typically converges quite rapidly near the solution. Its disadvantage is that the computation of the matrix $(F_k(x(k))^{-1}$ is often very expensive, and for some applications the scheme requires additional custom modifications to ensure convergence.

The Barzilai-Borwein method mimics the gradient method, in that it selects α_k so that $\alpha_k g_k(x_k)$ approximates $F_k(x_k)^1 g_k(x_k)$, without actually computing $(F_k(x_k))^{-1}$. Thus this scheme often converges nearly as fast as the Newton method, but at significantly lower computational cost. Due to its simplicity and efficiency, variations of this method have been applied in a variety of applications, including sparse optimization and image-signal processing.

Jonathan Borwein is perhaps best known for deriving, with his brother Peter, quadratically and higher order convergent algorithms for π, including p-th order convergent algorithms for any prime p, and similar quadratically convergent algorithms for certain other fundamental constants and functions [3, 4, 5]. Here "quadratically convergent" means that each iteration of the algorithm approximately *doubles* the number of correct digits in the result, with a similar definition for higher-order convergence.

One of their algorithms is the following: Set $a_0 = 6 - 4\sqrt{2}$ and $y_0 = \sqrt{2} - 1$. Then iterate

$$\begin{aligned} y_{k+1} &= \frac{1-(1-y_k^4)^{1/4}}{1+(1-y_k^4)^{1/4}} \\ a_{k+1} &= a_k(1+y_{k+1})^4 - 2^{2k+3} y_{k+1}(1+y_{k+1}+y_{k+1}^2). \end{aligned} \qquad (1)$$

Then a_k converge *quartically* to $1/\pi$: each iteration approximately *quadruples* the number of correct digits. This algorithm, together with a quadratically convergent algorithm due to Brent and Salamin, were employed in several large computations of π by Kanada and others.

However, Jonathan Borwein's most enduring legacy is his contribution to experimental mathematics, in particular his advocacy to utilize high-performance computing technology to *discover* new principles and formulas of mathematics. Just one of many examples of this methodology in action was his analysis (in conjunction with the present author and the late Richard Crandall) of the following three classes of integrals that arise in mathematical physics: C_n are connected to quantum field theory, D_n arise in Ising theory, while the E_n integrands are derived from D_n:

$$
\begin{aligned}
C_n &:= \frac{4}{n!}\int_0^\infty \cdots \int_0^\infty \frac{1}{\left(\sum_{j=1}^n (u_j + 1/u_j)\right)^2} \frac{du_1}{u_1} \cdots \frac{du_n}{u_n} \\
D_n &:= \frac{4}{n!}\int_0^\infty \cdots \int_0^\infty \frac{\prod_{i<j}\left(\frac{u_i - u_j}{u_i + u_j}\right)^2}{\left(\sum_{j=1}^n (u_j + 1/u_j)\right)^2} \frac{du_1}{u_1} \cdots \frac{du_n}{u_n} \qquad (2) \\
E_n &= 2\int_0^1 \cdots \int_0^1 \left(\prod_{1\le j<k\le n} \frac{u_k - u_j}{u_k + u_j}\right)^2 dt_2\, dt_3 \cdots dt_n,
\end{aligned}
$$

In the last line $u_k = t_1 t_2 \cdots t_k$.

One early observation was that the C_n integrals can be converted to one-dimensional integrals involving the modified Bessel function $K_0(t)$:

$$C_n = \frac{2^n}{n!}\int_0^\infty t K_0^n(t)\, dt. \qquad (3)$$

It was quickly evident that high-precision numerical values of this sequence, computed using (3) and tanh-sinh quadrature, approach a limit. For example:

$$C_{1024} = 0.630473503374386796122040192710878904354587078 7\,.$$

When the first 50 digits of this constant were copied into the online Inverse Symbolic Calculator-2 (ISC-2) at **https://isc.carma.newcastle.edu.au** (which Jon was instrumental in developing and deploying), the result was:

$$\lim_{n\to\infty} C_n = 2e^{-2\gamma}, \qquad (4)$$

where γ denotes Euler's constant, a result which was then proved.

Subsequently high-precision computations, in conjunction with Ferguson's PSLQ algorithm [6], were applied to find experimental evaluations of numerous other specific instances of these integrals, including:

$$
\begin{aligned}
D_3 &= 8 + 4\pi^2/3 - 27\,\mathrm{L}_{-3}(2) \\
D_4 &= 4\pi^2/9 - 1/6 - 7\zeta(3)/2 \\
E_2 &= 6 - 8\log 2 \\
E_3 &= 10 - 2\pi^2 - 8\log 2 + 32\log^2 2 \qquad (5) \\
E_4 &= 22 - 82\zeta(3) - 24\log 2 + 176\log^2 2 - 256(\log^3 2)/3 + 16\pi^2\log 2 - 22\pi^2/3 \\
E_5 &= 42 - 1984\,\mathrm{Li}_4(1/2) + 189\pi^4/10 - 74\zeta(3) - 1272\zeta(3)\log 2 + 40\pi^2\log^2 2 \\
&\quad - 62\pi^2/3 + 40(\pi^2\log 2)/3 + 88\log^4 2 + 464\log^2 2 - 40\log 2,
\end{aligned}
$$

where $\mathrm{L}_{-3}(2)$ is a Dirichlet L-function constant, $\zeta(x)$ is the Riemann zeta function and $\mathrm{Li}_n(x)$ is the polylogarithm function. The formula for E_5, which was initially found by Borwein (and which he was quite proud of), remained a numerically discovered but open conjecture for several years, but was finally proven in 2014 by Erik Panzer [7]. Resolution of the general case is still open.

Jonathan Borwein's prodigious output in optimization and experimental mathematics is certainly his singular contribution to modern mathematics. But beyond his technical accomplishments, he was a master of mathematical communication (his lectures were always paragons of well-organized and visually appealing mathematics and graphics), mathematical education (part of his interest in π was to bring the joy of mathematical discovery to students), and in promoting science, mathematics and computing to the general public. To this end, he wrote and lectured tirelessly. By one reckoning he presented an average of one lecture per week for decades, and wrote well over 100 articles targeted at the general public. His death is a loss to all those who treasure modern mathematics, science and clear thinking.

Acknowledgement: The author wishes to thank Michel Théra and Karl Dilcher for their assistance in the preparation of this article.

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Quantum State Transfer

Sarah Plosker, *Department of Mathematics & Computer Science, Brandon University*

Xiaohong Zhang, *Department of Mathematics, University of Manitoba*

In a quantum computer, there is a large network of quantum registers for computing and storing information; the ability to transfer information reliably and efficiently from one register to another is vital in such large and networked systems. For short distance quantum communication, Bose [1] proposed a scheme for using a spin chain to accomplish this task. A quantum spin chain can be defined physically as a collection of interacting qubits (quantum bits) forming a quantum wire whose dynamics are governed by a suitable Hamiltonian (essentially, a matrix describing the total amount of energy of a quantum system). Mathematically, we can think of a quantum spin chain as a path on n vertices, where n is the number of qubits.

The goal is to transfer a quantum state from vertex s (sender) to vertex r (receiver) perfectly, in that the state being sent at time $t = 0$ is identical (up to an overall scalar multiple $e^{i\theta}$) to the state received at time $t = t_0$. Typically $s = 1$ and $r = n$, so that the state is sent the maximum possible distance within the graph. Because of the inherent difficulty in the manipulation of coherent quantum mechanical objects, setups for quantum state transfer that employ a time-independent Hamiltonian (without any modulation of couplings between qubits in the process and of the external fields) and no interaction with the system except at initialization and readout, are the most appealing. Stated another way: it is desirable to simply set up the network and allow the transfer of quantum states to occur only via the natural dynamics of the system. Such a setup avoids possible errors arising from dynamical control of interqubit interactions. Unfortunately, it was found [2] that unweighted paths only exhibit perfect state transfer (PST) for $n \leq 3$ vertices through this “hands off” approach, so one cannot use unweighted spin chains to transfer states very far. Several immediate questions can then be posed: what graphs other than chains exhibit PST? what if we do not require that the state transfer be perfect? what about weighted chains?

Some graph theoretic definitions and notations are required before answering these questions. In this note, a *simple graph* is an undirected graph in which both multiple edges and loops (edges that connect a vertex to itself) are disallowed, while a *graph* allows for loops.

To any graph G, we can associate a matrix that represents all information about the graph, and thus we can approach problems in graph theory from a matrix analysis point of view. The *adjacency matrix* $A = (a_{jk})$ of a graph G on n vertices is an $n \times n$ matrix satisfying $a_{jk} = a_{kj} = w(j, k)$ if vertices j and k are adjacent (and zero otherwise), where $w(j, k)$ is the weight of the edge between vertices j and k for weighted graphs (and $w(j, k) = 1$ for unweighted graphs).

Another, related, matrix associated to a graph is its Laplacian matrix:

Given a weighted simple graph G on n vertices, its *Laplacian matrix* L is the $n \times n$ matrix defined by $L = D - A$, where D is the diagonal matrix of row sums of A, i.e., $D = diag(A\mathbf{1})$ with $\mathbf{1}$ being the all-ones vector, and A is the adjacency matrix of G.

Because quantum state transfer within a quantum system is so important, there is a strong need to quantify the fidelity of the transfer, and the occurrence of *perfect* state transfer is of special significance. We now formally define the fidelity, as well as perfect and pretty good state transfer.

Definition 1 Let $U(t) = e^{itM}$, where M is the adjacency matrix of a weighted graph or the Laplacian of a weighted simple graph. The *fidelity or probability of state transfer* from vertex s (sender) to vertex r (receiver) at time t is given by $p(t) = |u(t)_{sr}|^2$, where $u(t)_{sr}$ is the (s, r)-th entry of $U(t)$.

The fidelity is a number between 0 and 1; it is a measurement of the closeness between two quantum states and is used to determine the accuracy of the quantum state transfer occurring between quantum registers and/or processors.

Definition 2 If s and r are distinct vertices in G, we say *perfect state transfer* (PST) from s to r occurs if there is time $t_0 (\in \mathbb{R})$ such that fidelity $p(t_0)$ between the two vertices is 1; we say *pretty good state transfer* (PGST) from s to r occurs if, for each $\epsilon > 0$, there exists a time $t_0 (\in \mathbb{R})$ such that $1 - p(t_0) < \epsilon$, i.e., the fidelity of state transfer between the two vertices can be made arbitrary close to 1.

There are many different binary operations on graphs that produce a third graph, such as the union of two graphs, the intersection of two graphs, the join of two graphs, and the Cartesian product of two graphs. These operations can be used in clever ways to create new graphs exhibiting PST. For example, if two graphs G and H have PST at the same time t_0, then their Cartesian product also has PST at time t_0. In particular, the n-fold Cartesian product of either a path on two vertices (P_2) or a path on 3 vertices (P_3) with itself has PST. In [6], we introduce a new weighted binary operation on graphs called *merge*, and give a characterization of when the merge of two graphs that are diagonalizable by the same Hadamard matrix has PST.

Perfect state transfer is more of a theoretical idea than a physical reality: in a lab setting, small errors may occur in readout time (i.e, the received state will be read out by a mechanical device at time $t_0 + h$, for some $h \in \mathbb{R}$ with small $|h|$, rather than at time t_0), and small errors may also occur on the edge weights (all edge weights need to be manufactured, and so edge weight errors are often called manufacturing errors)—for this type of error, we consider an $n \times n$ matrix E whose (j, k)-th entry is the error for the weight $w(j,k)$; the Hamiltonian of the system is then either $A+E$ or $L-E+diag(E\mathbf{1})$. The sensitivity of the probability of state transfer with respect to readout time and with respect to manufacturing errors was analysed in [7], [8], and [5]. In short, minimizing the sensitivity of the probability of state transfer with respect to either small changes in readout time or with respect to manufacturing errors typically involves careful analysis of the spectrum of the Hamiltonian, and bounds on the probability of state transfer for the perturbed setting can be produced.

As previously stated, PST does not occur on an unweighted chain with more than three vertices. If we allow for different coupling strengths between qubits in a chain, then perfect state transfer can be achieved over arbitrarily long distances [2]. But this weighting schema gives more chances for errors. Since an unweighted chain minimizes the amount of physical and technological resources required compared to other, more complicated graphs, it is natural to consider PGST in an unweighted setup. In [4], a complete characterization of the parameters (length) of the unweighted chain for which there is PGST was given; this characterization was in terms of prime numbers, and hinted at a deeper connection between quantum communication and primality testing in number theory.

A good resource on quantum state transfer is the survey article [3], which emphasizes the mathematics rather than the physics of the topic.

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Call for Nominations – CMS Research Prizes

The CMS Research Committee is inviting nominations for three prize lectureships. These prize lectureships are intended to recognize members of the Canadian mathematical community.

The **Coxeter-James Prize** Lectureship recognizes young mathematicians who have made outstanding contributions to mathematical research. The recipient shall be a member of the Canadian mathematical community. Nominations may be made up to ten years from the candidate's Ph.D. A nomination can be updated and will remain active for a second year unless the original nomination is made in the tenth year from the candidate's Ph.D. For more information, visit: **https://cms.math.ca/Prizes/cj-nom**

The **Jeffery-Williams Prize** Lectureship recognizes mathematicians who have made outstanding contributions to mathematical research. The recipient shall be a member of the Canadian mathematical community. A nomination can be updated and will remain active for three years. For more information: **https://cms.math.ca/Prizes/jw-nom**

The **Krieger-Nelson Prize** Lectureship recognizes outstanding research by a female mathematician. The recipient shall be a member of the Canadian mathematical community. A nomination can be updated and will remain active for two years. For more information: **https://cms.math.ca/Prizes/kn-nom**

The deadline for nominations, including at least three letters of reference, is **September 30, 2017**. Nomination letters should list the chosen referees and include a recent curriculum vitae for the nominee. Some arms-length referees are strongly encouraged. Nominations and the reference letters from the chosen referees should be submitted electronically, preferably in PDF format, to the corresponding email address and **no later than September 30, 2017**:

Coxeter-James: **cjprize@cms.math.ca**
Jeffery-Williams: **jwprize@cms.math.ca**
Krieger-Nelson: **knprize@cms.math.ca**

CSHPM Notes brings scholarly work on the history and philosophy of mathematics to the broader mathematics community. Authors are members of the Canadian Society for History and Philosophy of Mathematics (CSHPM). Comments and suggestions are welcome; they may be directed to either of the column's co-editors:

Amy Ackerberg-Hastings, *University of Maryland University College (aackerbe@verizon.net)*

Hardy Grant, *York University [retired] (hardygrant@yahoo.com)*

Les articles de la SCHPM présentent des travaux de recherche en histoire et en philosophie des mathématiques à la communauté mathématique élargie. Les auteurs sont membres de la Société canadienne d'histoire et de philosophie des mathématiques (SCHPM). Vos commentaires et suggestions sont le bienvenue; ils peuvent être adressées à l'une des co-rédacteurs:

Amy Ackerberg-Hastings, *University of Maryland University College (aackerbe@verizon.net)*

Hardy Grant, *York University [retraité] (hardygrant@yahoo.com)*

Exploring Epistemology of Applied Math: Where Mathematicians and Philosophers Meet

Robert H. C. Moir, *Western University*

In the spring of this year I and six other scholars from France, Belgium, Germany, Norway and Canada met at the research centre of the historic Université Paris-Sorbonne to discuss a common interest in the burgeoning field of philosophy of applied mathematics. We were joined by three prominent Parisian applied mathematicians who gave presentations on their research. Holding together this group of researchers is an interest in understanding the function of mathematics in the modeling of real-world phenomena.

Such an interest in the practice of applied mathematicians is part of a growing shift in philosophy of mathematics toward the activities and results of research mathematics. This is in contrast to the considerably more conservative focus of traditional philosophy of mathematics, which has tended to focus largely on the foundational role of arithmetic and set theory and has placed great emphasis on the semantic and ontological issues thus raised.

Attempts both to articulate a distinctive role for a philosophy of *applied* mathematics and to identify its central concerns, such as Chris Pincock's "Toward a Philosophy of Applied Mathematics" [1], reserve a place for new kinds of epistemic, semantic and ontological issues raised specifically by applied mathematics, and leave open a wide range of approaches to fruitful philosophical investigation. The participants in the Paris meeting, however, share an emphasis on deriving philosophical insight from a detailed engagement with the procedures and products of practising mathematicians, and a common interest in understanding the plurality and flexibility of mathematical concepts and methods that emerge through the challenging process of making mathematics applicable.

The topics addressed in the papers, along with some of the ensuing discussion, provide a window into what interests philosophers about applied mathematics and how philosophical insights are derived from an investigation of practice. The first four papers focused on epistemology of different aspects of applied math, and the final two dealt with issues closer to more general philosophy of mathematics.

First to present was Johannes Lenhard of Bielefeld University in Germany, speaking on "Holism, Or the Erosion of Modularity." Complex simulation models are usually designed and evaluated in a modular fashion, in order to track and control model behaviour and error. Lenhard argued, however, that the heavy reliance on tuneable parameters, and the frequent kludges in simulation code, systematically undermine that modularity, resulting in forms of (dis)confirmation holism. The concern with such holism is that it obscures identification of the source of model behaviour, in terms of both success and failure.

This led to a discussion of whether a similar kind of holism undermines the independence of verification and validation procedures, in particular due to changes in these processes resulting from the ubiquity of high-performance computing. Is it thus possible that the traditional rational picture of mathematization cannot account for the rational function of mathematical practice? That altered perspective would suggest—since mathematical modeling is surely still a rational process—that the computer is changing not only how mathematics is used but potentially the structure of scientific rationality itself.

As the second to present, I spoke about "The Practical Computational Character of (Pure and Applied) Mathematical Inference." The need to compute solutions to mathematical problems when there is some kind of inferential obstacle (for instance, closed-form solutions are unavailable, accurate solutions are needed quickly, proofs are too complex, etc.) gives rise to a recursive strategy of transforming the problem to a (nearly) equivalent one and back-interpreting accessible solutions to overcome the obstacle and complete the inference. This pattern is replicated surprisingly widely, with examples ranging from data handling in applications, to numerical solution of differential equations, to proofs in modern algebraic topology.

Some commenters pointed out that my argument offers a contrast between traditional foundational epistemology, which focuses on so-called pure methods associated with the axiomatic method, and feasible epistemology, which focuses on the "impure" methods available in practice. The existence of clear general patterns in the epistemology of approximation methods traditionally regarded as impure suggests that a different conception of purity of method is needed as a result of the engagement between mathematics and its application in practice, by which I mean the existence of new forms of inference that can only be identified by empirical investigation of that engagement.

Third to present was Vincent Ardourel, a postdoc at Université catholique de Louvain, speaking on "Irreversibility in the Derivation of the Boltzmann Equation." This most technical of the conference papers investigated the origin of time-asymmetry in the derivation of the Boltzmann equation (BE) from the time-symmetric Hamiltonian equations of motion of a hard-sphere gas, arguing that both the asymptotic Boltzmann-Grad (B-G) limit and the assumption of uncorrelated velocities before collision introduce time-asymmetric ingredients. This paper is based in part on the work on the BE by Laure Saint-Raymond, one of the mathematicians who spoke at the event.

Aside from the clear philosophical interest of using contemporary mathematical research to provide insights into the origin of time-asymmetry in physics, the paper shows a number of ways that formal/analytical mathematical inference is relevant in the philosophy of applied mathematics. There are issues here of fully rigorous versus heuristic derivations—the former not always being the most insightful—the role of mathematical constraints in derivations in physics, the role of asymptotic reasoning (B-G limit) in foundations of physical theories, and the notion of 'solution' versus 'formal solution'.

The fourth paper, "Recurring Models and Sensitivity to Computational Constraints," was given jointly by Anouk Barberousse of Université Paris-Sorbonne, the organizer of the event, and Cyrille Imbert of Université de Lorraine. A recognized pattern of a limited number of models being used across scientific disciplines (e.g., the simple harmonic oscillator (SHO), a few Hamiltonians, the Lotka-Volterra equations) has not been given a satisfactory general explanation. Barberousse and Imbert argued that the general pattern can be accounted for by the beliefs among mathematicians about how best to generate knowledge given a perception that novel methods are very difficult to develop and a commonly experienced pressure for exactly solvable models and timely definite results.

Though this is an epistemic explanation, because it looks to the belief states of mathematical researchers, it appears to be compatible also with an explanation in terms of objective computational constraints on mathematical inference. We concluded in the discussion of their paper that, assuming that the beliefs do indeed reflect an objective situation of limited mathematical tractability, their explanation shows a role for belief in methodology as an aggregate of mathematical experiences, providing a naturalistic rather than sociological role for belief.

The fifth paper, given by Nicolas Fillion of Simon Fraser University, shifted to more general issues in philosophy of mathematics with a consideration of "Conceptual and Computational Mathematics." Fillion regards recurrent debates between constructive/computational and non-constructive/conceptual views of mathematics as an impediment to the subject's progress. Accordingly, he argued that contemporary uses of computation in applied mathematics show paths toward a synthesis of broadly complementary views. He further stated that the computer has changed the practice of mathematics but has not transformed mathematics itself, as some philosophers claim. Thus the use of computers helps us to better understand the integration of conceptual and computational mathematics.

An important example of this kind of interaction, outside the well-known examples of computer-assisted proof and numerical solution of equations, is the use of computers in what is now called "experimental mathematics," of the sort developed by the late Jon Borwein, David Bailey, and others, to detect and explain mathematical phenomena. A tendency to regard conceptual mathematics as superior to computational certainly exists, but experimental mathematics helps to show how computation can be a source of new ideas and results that in due course can be

Appel de mises en candidature – Prix de recherche de la SMC

Le Comité de recherche de la SMC lance un appel de mises en candidatures pour trois de ses prix de conférence. Ces prix ont tous pour objectif de souligner l'excellence de membres de la communauté mathématique canadienne.

Le **Prix Coxeter-James** rend hommage aux jeunes mathématiciens qui se sont distingués par l'excellence de leur contribution à la recherche mathématique. Cette personne doit etre membre de la communauté mathématique canadienne. Les candidats sont admissibles jusqu'à dix ans après l'obtention de leur doctorat. Toute mise en candidature est modifiable et demeurera active l'année suivante, à moins que la mise en candidature originale ait été faite la 10e année suivant l'obtention du doctorat. Pour les renseignements, voir : **https://cms.math.ca/Prix/cj-nom**

Le **Prix Jeffery-Williams** rend hommage aux mathématiciens ayant fait une contribution exceptionnelle à la recherche mathématique. Cette personne doit être membre de la communauté mathématique canadienne. Toute mise en candidature est modifiable et demeurera active pendant trois ans. Pour les renseignements, voir : **https://cms.math.ca/Prix/jw-nom**

Le **Prix Krieger-Nelson** rend hommage aux mathématiciennes qui se sont distinguées par l'excellence de leur contribution à la recherche mathématique. La laureate doit etre membre de la communauté mathématique canadienne. Toute mise en candidature est modifiable et demeurera active pendant deux ans. Pour les renseignements, voir : **https://cms.math.ca/Prix/info/kn**

La date limite pour déposer une candidature, qui comprendra au moins trois lettres de référence, est **le 30 septembre 2017**. Le dossier de candidature doit comprendre le nom des personnes données à titre de référence ainsi qu'un curriculum vitae récent du candidat ou de la candidate. Veuillez faire parvenir les mises en candidature et lettres de référence par voie électronique, de préférence en format PDF, avant la date limite, à l'adresse électronique correspondante et **au plus tard le 30 septembre 2017** :

Coxeter-James : **prixcj@smc.math.ca**
Jeffery-Williams : **prixjw@smc.math.ca**
Krieger-Nelson : **prixkn@smc.math.ca**

proved rigorously and explained theoretically, allowing conceptual and computational mathematics to interact synergistically.

The final paper, "The Miracle of Applicability? The Curious Case of the Simple Harmonic Oscillator," was given by Sorin Bangu of the University of Bergen. Concerning Wigner's famous puzzlement over the so-called "miracle of applicability" of mathematics to physics, philosophers have largely dissolved the mystery in its general form, but there remain *particular* puzzling cases, such as, Bangu argues, the mathematical explanation of the periodicity of the SHO using complex numbers.

In response to queries as to what really is mysterious in this case, Bangu replied that he was presenting this simple example in an abstract way to bring into focus what is surprising when an earlier development in mathematics has an unexpected application in physics. In this case, the tools for a general explanation of the behaviour of the SHO were available in mathematics without any expectation of their applicability, so a surprising applicability resulted from parallel developments in mathematics and in physics. Since the puzzling nature of this case may not hold up to historical scrutiny, it was debated whether such examples can be considered abstractly and independently of historical details or whether the actual development must be considered. The fact that there could be a path between these extremes raises the question of what is the correct framework for addressing such questions of applicability.

Led by Barberousse, the participants drew a number of conclusions from the meeting. One general theme among the papers was a recognition of the importance of the role of constraints on feasibility in shaping the way that mathematics is applied in science and industry. By better understanding these constraints we can learn what underlies methodological and epistemological patterns in mathematical practice. Also acknowledged was the importance of balancing systematic general theories for conceptual understanding with computational tools for accessible inference, which fosters subtler views of the character of mathematics.

Additionally, the heterogeneity of applied mathematics reveals a plurality of conceptions of what constitutes good mathematics, suggesting that lack of agreement results from value judgements. This heterogeneity also makes applied math less rigid than traditionally conceived—its conceptual flexibility makes it adaptable to different purposes, and multiple perspectives on a given bit of mathematics expand its content and facilitate its use in problem-solving. This conceptual pluralism and adaptability of applied mathematics contrasts strikingly with the homogeneous and rigid nature of traditional philosophical views of mathematics based largely on considerations of axiomatic foundations.

The meeting was a great success in consolidating a group of philosophers of applied mathematics who are committed to revealing the many philosophical insights hiding in the practice of applied mathematicians. Indeed, among the group there is a shared interest in connecting with the interests and work of practising mathematicians and elucidating the epistemology of applied mathematics. Part of the special place of applied mathematics in this regard is its central role in science, providing a bridge between disciplines. We as a group aim to build a network of philosophers and mathematicians who are interested in exploring the epistemology of mathematics in its research and modeling contexts, the territory where mathematics and philosophy meet.

Reference

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Robert H.C. Moir (robert@moir.net) is a doctoral candidate in the department of applied mathematics at Western University. He also holds a PhD in philosophy from Western. His interests in philosophy involve mapping scientific inference by constructing epistemic models of the application of mathematics in scientific practice. His interests in applied mathematics concern the use of combined symbolic-numeric methods for the integration problem in computer algebra.

CSHPM Student Award

Since 2013, the Canadian Society for History and Philosophy of Mathematics has awarded a prize to the best student paper presented at its Annual Meeting and published in its *Proceedings*. The award is a stipend of several hundred dollars intended to defray the costs of presenting the paper or additional work at another academic conference. For more information, see **www.cshpm.org/meeting/CSHPM Award Guidelines.pdf**.

Thus far, two students have received the award:

- **Robert H. C. Moir** in 2013 for "Rational Discovery of the Natural World: An Algebro-Geometric Response to Steiner" in *Proceedings of the Canadian Society for History and Philosophy of Mathematics*, ed. Tom Archibald, Vol. 26, Thirty-Ninth Annual Meeting, 1–3 August 2013, 43–59.
- **Sylvia Nickerson** in 2014 for "Mathematics for the World: Publishing Mathematics and the International Book Trade, Macmillan and Co." in *Research in History and Philosophy of Mathematics: The CSHPM 2014 Annual Meeting in St. Catharines, Ontario*, ed. Maria Zack and Elaine Landry (Birkhäuser, 2015), 121–137.

Both scholars have also shared their current work with CMS *Notes* readers; Nickerson with "How Objects Reveal Mathematical Culture" in the September 2016 issue, and Moir with "Exploring Epistemology of Applied Math: Where Mathematicians and Philosophers Meet" in this issue.

CSHPM invites students who are researching in history or philosophy of mathematics to submit proposals for its 2017 Annual Meeting at the HSSFC Congress in Toronto, May 28–30, no later than **February 1**. See **www.cshpm.org/meeting/**.

Jonathan Borwein and the Canadian Mathematical Society

We are writing to express the pleasure we have known in working with Jonathan for the Canadian Mathematical Society as well as to express our gratitude for his many and varied contributions. Jonathan was passionate, and he combined a phenomenal energy together with a vision for the CMS and, more broadly, for mathematics at the international level. The CMS was at the forefront of the move to electronic services and electronic publishing. We were one of the first publishers to move to an electronic version of its journals and Jonathan strongly supported the digitization and subsequent appearance of the complete history of the Journal and Bulletin online using the TeX Office at the University of Manitoba. It was in the 1990's that the CMS established its first professional website, Camel, as a resource for the Canadian mathematical community: the name was chosen because the site had two humps, the main one at the Center for Experimental and Computational Mathematics (CECM) at SFU, and the second one at the CMS office in Ottawa. The driving forces behind that project were Michel Delfour, Eddy Campbell and Jonathan Borwein, and the first version of the website was developed at CECM by Jonathan and his staff, until the Society developed sufficient expertise to take over the website.

Jonathan dreamed of digitizing the world mathematical literature. He pushed CMS in that direction and he also brought this dream to the international level where he served for 10 years on the Committee for Electronic Information and Communication of the International Mathematical Union (IMU), including six years as chair. He and Michael Doob brought the expertise they had gathered in the CMS as a model of the right way of doing things. Jon left his marks on IMU due to his involvement in the Committee of Electronic Information and Communication (CEIC). According to Martin Groetchel of the IMU, Jonathan was a driving force in preparing Best Current Practices Documents on what we call collectively Open Access today, an activity that was championed in mathematics before the term was coined and has become a hot topic in the sciences and humanities.

Jonathan worked hard for the Canadian mathematical community both at the national and international level. He had the idea of the Education Fora of 2003 and 2005. He is also the one who suggested the first Canada-France congress in Toulouse in 2004. He also was involved in the CMS hosting a reception for the Fields Medal winners at the International Congress of Mathematicians. The first such reception as held at ICM'1998 in Berlin in cooperation with the Canadian Embassy, and the tradition has continued at each subsequent ICM.

Jonathan was also very passionate for mathematical books and literature. A prolific writer himself, he did immense service to the CMS on the publications side. He served as Editor-in-Chief of the CMS/Wiley Book Series (1995-1998), Editor-in-Chief of the CMS/Springer Books in Mathematics (1998 - 2005), Associate Publisher of CMS Books and Rich Media (2004-2007) and he was the founder of the CMS Treatises in Mathematics in cooperation with A.K. Peters (2008-2013). He was very active in the search of authors for CMS book series and he is the one who convinced Klaus Peters to serve on the CMS Board and publish the CMS Treatises in Mathematics.

Jonathan Borwein was deeply committed to build a healthy CMS, strong in its service to the community as well as financially sound. During his term as CMS President he spent a significant portion of his time in fund-raising, and also in negotiation with the University of Ottawa, which housed the CMS Executive Office at the time. He had foreseen the decline of revenues from the journals and was actively looking for new sources of revenues.

The three of us had the pleasure of working with Jonathan.

I, Christiane Rousseau, had the pleasure to work two years with Jonathan when he was president and I president-elect, and the next year when I was president and he was past president. It has been a very rewarding experience. When I was nominated as president-elect I was Christiane Who?, and I did not know so much of CMS. Jonathan was very generous of his ideas and he was working for Canada and the CMS. One recipe of Jonathan's incredible efficiency is that he would delegate the work. But when he was doing so, he was also giving the credit.

I, Graham Wright, during my years as Executive Director (1979 to 2009), had the distinct pleasure of working with Jonathan on many different fronts. His counsel, enthusiasm, dedication and vision brought about many initiatives that showcased the CMS to both the national and international communities. Jonathan was tireless in his efforts to increase the scope of the Society and he succeeded in involving many players to promote the CMS. I regarded Jonathan as a great friend of the CMS and as valued personal friend. It was with great sadness that I learned that the Canadian and international mathematical community had lost a shining light.

I, Eddy Campbell, shared the pleasure expressed by Christiane and Graham, of working with Jonathan over a period of many years at the CMS. He had many virtues as discussed above, but I particularly remember his amazing energy and enthusiasm. He simply never stopped, and he was very good at attracting others to work with him on his projects, and these covered a very wide range. One shared friend and colleague described the experience of writing a paper with him as a wholly different and unique experience, this was back in the days of modems. "I'm 128K baud," he said, "and Jonathan is 2048K baud." That was the way it was for most of us, I am sure: Jonathan was always moving at high relative speed. It was great fun to work on the first CMS web site with Michel and Jonathan, as described above. Michel had the "big picture" vision and knew Ottawa very well, Jonathan had the CECM, its staff and expertise, and many, many ideas for the digital services we might provide. Jonathan was also instrumental in setting up the prestigious CMS David Borwein Distinguished Career Award in honour of his father. I was very surprised and saddened to learn of his death, far too soon. He will be remembered with fondness and affection.

Eddy Campbell
Christiane Rousseau
Graham P. Wright

2017 CMS Winter Meeting

December 8-11, 2017
University of Waterloo, Waterloo, Ontario

CALL FOR SESSIONS

The Canadian Mathematical Society (CMS) welcomes and invites proposals for sessions for the 2017 CMS Winter Meeting in Waterloo. Proposals should include a brief description of the focus and purpose of the session, the expected number of speakers, as well as the organizer's name, complete address, telephone number, e-mail address, etc. All sessions will be advertised in the CMS Notes, on the web site and in the AMS Notices. Speakers will be requested to submit abstracts, which will be published on the web site and in the meeting program. Those wishing to organize a session should send a proposal to the scientific directors.

Scientific Directors:

Ken Davidson, University of Waterloo, **krdavids@uwaterloo.ca**
Cam Stewart, University of Waterloo, **cstewart@uwaterloo.ca**

Réunion d'hiver de la SMC 2017

8-11 décembre 2017
Université de Waterloo, Waterloo, Ontario

NOUS VOUS INVITONS

La Société mathématique du Canada (SMC) vous invitent à proposer des séances pour la Réunion d'hiver de la SMC 2017 qui se tiendra à l'Université de Waterloo du 8 au 11 décembre 2017. Ces propositions de séances doivent presenter une brève description de l'orientation et des objectifs de la séance, le nombre de conférenciers prévu, de même que le nom, l'adresse complète, le numéro de téléphone et l'adresse électronique de l'organisateur. Toutes les séances seront annoncées dans les Notes de la SMC, sur le site Web SMC et dans les AMS Notices. Les conférenciers devront présenter un résumé, qui sera publié sur le site Web SMC et dans le programme de la réunion. Toute personne qui souhaiterait organiser une séance est priée de faire parvenir une proposition aux directeurs scientifiques.

Directeurs scientifiques:

Ken Davidson, Université de Waterloo, **krdavids@uwaterloo.ca**
Cam Stewart, Université de Waterloo, **cstewart@uwaterloo.ca**

Mathematics of Planet Earth

MPE

Mathematics of Planet Earth (MPE) invites the scientific community to enter the Second International Competition to design museum exhibits on Mathematics of Planet Earth. This new competition is supported by the International Mathematical Union, UNESCO, the International Commission of Mathematical Instruction and IMAGINARY.

The competition will enrich the MPE Open Source Exhibition (https://imaginary.org/exhibition/mathematics-of-planet-earth), which provides material that can be used by museums and schools around the world. Exhibits can take the form of an interactive program, a hands-on physical exhibit, a short film, or an image gallery; and must address some planetary theme where mathematics plays a role. A special call is made for topics related to the African continent.

An international jury will award four prizes to the best modules:

- 1st, 2nd and 3rd prize: 5 000 USD, 2 000 USD, and 1 000 USD respectively.
- A special prize of 2 000 USD for the best exhibit addressing an African topic.

Participation is open until **June 30, 2017**. More information at

www.mathofplanetearth.org/competition.

International Mathematical Union IMU

UNESCO United Nations Educational, Scientific and Cultural Organization — Natural Sciences Sector

ICMI International Commission on Mathematical Instruction

IMAGINARY open mathematics

The CMS Annual Giving Campaign Underway

Last year due to your generous support we were able to expand the funds available to support CMS meeting sessions; to provide travel expenses for students and graduate students at CMS meetings; and support Math Competitions such as the Canadian Open Math Competition and Math Team Canada at the International Mathematical Olympiad.

This year we again are seeking donations from members and friends of CMS for:

- Next Summer's Mathematical Congress of America Conference
- CMS Meeting Sessions
- Travel expenses for students to attend training, the MCA and CMS meetings.
- Endowment Grants for Research
- Educational Programs

You can help by:

Contributing to the CMS. Your donation makes a difference in the future of Mathematics in Canada, Donate Online at **Canada Helps**

Donating Air Miles at: **Aeroplan**

More Ways to Help:

- Identifying mathematicians who work in private industry who we can contact to seek donations
- Volunteering for CMS committees
- Estate Planning Gifts
- Encouraging colleagues to join the CMS

Working together, we can promote and advance the discovery, learning, and application of mathematics. If you have questions or want more information, please contact Gerri Jensen **gjensen@cms.math.ca**.

La campagne de financement annuelle de la SMC est lancée!

L'an dernier, votre générosité nous a permis d'augmenter le financement consacré aux sessions des Réunions de la SMC; d'accorder plus d'allocations de déplacement aux étudiants pour qu'ils assistent aux Réunions; et de financer des concours mathématiques comme le Défi ouvert canadien de mathématiques et la participation d'Équipe Math Canada à l'Olympiade internationale de mathématiques.

Cette année, nous sollicitons à nouveau l'appui de nos membres et des amis de la SMC pour financer :

- Le Congrès mathématique des Amériques (CMA) de l'été prochain
- Les sessions des Réunions de la SMC
- Les frais de déplacement des étudiants assistant à une formation, au CMA ou à une Réunion de la SMC
- Les bourses du fonds de dotation destinées à la recherche
- Des programmes éducatifs

Comment pouvez-vous aider?

En faisant un don à la SMC. Votre don contribuera à améliorer l'avenir des mathématiques au Canada. Faites un don en ligne sur **CanadaDon.org**.

En donnant vos milles aériens sur le site d'**Aeroplan**.

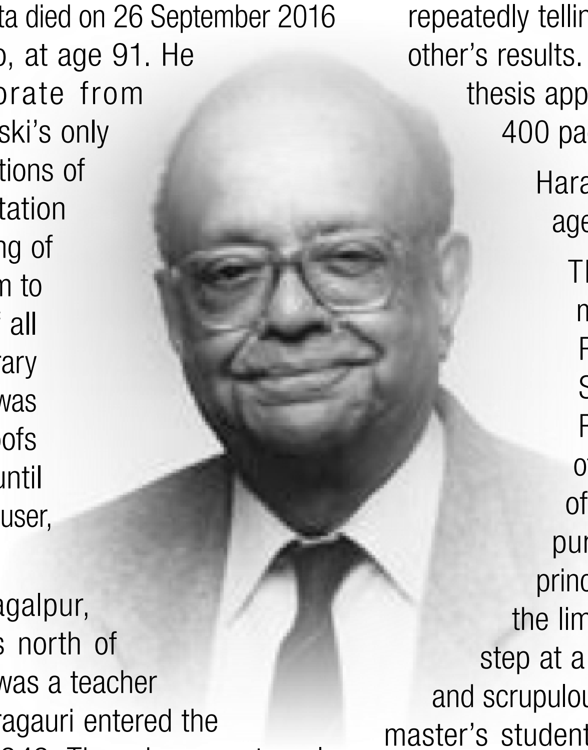
D'une des façons suivantes :

- Suggérez-nous des mathématiciens qui travaillent dans le secteur privé que nous pourrions contacter pour obtenir un don
- Devenez bénévole à l'un des comités de la SMC
- Faites un don testamentaire
- Encouragez vos collègues à adhérer à la SMC

En travaillant ensemble, nous pourrons poursuivre notre mission de favoriser la découverte, l'apprentissage et les applications des mathématiques. Pour toute question ou pour obtenir plus d'information, veuillez communiquer avec Gerri Jensen **gjensen@cms.math.ca**.

Notes de la SMC
Tome 48, numéro 6, décembre 2016

H. N. Gupta (1925 - 2016)



Haragauri Narayan Gupta died on 26 September 2016 in Waterloo, Ontario, at age 91. He earned the doctorate from Alfred Tarski in 1965: Tarski's only doctoral student in foundations of geometry. Gupta's dissertation was devoted to a weakening of Tarski's 1957 axiom system to incorporate geometries of all finite dimensions over arbitrary ordered coordinate fields. It was also the only source of proofs for Tarski's 1957 results until the 1983 book by Schwabhäuser, Szmielew, and Tarski.

Gupta was born in Bhagalpur, Bihar, four hundred miles north of Calcutta, India. His father was a teacher of English and History. Haragauri entered the University of Calcutta in 1942. There he was strongly influenced by the mathematics chair, Friedrich W. Levi (1999–1966), a German refugee who specialized in the interplay between geometry and algebra. Haragauri earned a master's degree in pure mathematics in 1945, began teaching at various locations, and earned a second master's in statistics in 1949. In 1952 he married Manjula Roy (1932–2012), also a teacher. In 1957 Haragauri became principal of Calcutta's Dum Dum Motijheel College.

Humboldt and Fulbright grants brought Haragauri to West Germany in 1959 to study logic, and a year later to Berkeley, a world center of logic research developed in the late 1940s by the renowned Polish emigre scholar Alfred Tarski. At Berkeley Haragauri was particularly influenced and aided by Tarski's former student Wanda Szmielew, with whom he maintained close ties until her death in 1976. Benjamin Wells, a contemporary student of Tarski, wrote in 2014 that Haragauri may have been

> ... one of the strongest examples of Tarski's tendency to overwork students. Wanda Szmielew's student Zenon Piesyk was also researching geometry based on Tarski's axioms. Tarski and Szmielew drove their two students crazy by repeatedly telling them that one had surpassed the other's results. The consequence was that Gupta's thesis approached a ream in length, well over 400 pages.

Haragauri finished after five years, at age forty.

The next year, Gupta joined the mathematics department at the Regina campus of the University of Saskatchewan, now the University of Regina, where he remained for the rest of his career. Gupta published a number of papers on foundations of geometry, pure logic, and teaching. His guiding principle was "to work constantly to extend the limits of our knowledge, if only one little step at a time." He was an effective, inspiring, and scrupulously fair teacher. He supervised several master's students. The present writer, his only PhD student, also in foundations of geometry, graduated in 1970.

Gupta was a polymath, an articulate discussant in history, linguistics, religion, and politics as well as logic and mathematics. He was fluent in five languages and capable in six others. He served his university in numerous ways, including as department chair. He became an elder statesman of the vibrant Regina Hindu community.

Haragauri is survived by his daughter Neena, born in Palo Alto in 1965, now an attorney in Waterloo, Ontario, and by one grandchild. Neena's wonderfully vivid and personal remembrance of her father has been published in the Regina Leader–Post: click here. A memorial service will be held in Regina at a later date. Haragauri often called from airports around the world, communicating only his excitement as a traveler. I still expect to hear him, when the phone rings at an odd hour!

James T. Smith
Professor Emeritus of Mathematics
San Francisco State University

2017 Doctoral Prize

The CMS Doctoral Prize recognizes outstanding performance by a doctoral student. The prize is awarded to a candidate who received a Ph.D. from a Canadian university in the preceding year (January 1st to December 31st) and whose overall performance in graduate school is judged to be the most outstanding. Although the dissertation is the most important criterion (the impact of the results, the creativity of the work, the quality of exposition, etc.) other publications, activities in support of students and other accomplishments will also be considered.

Nominations that were not successful in the first competition will be kept active for a further year (with no possibility of updating the file) and will be considered by the Doctoral Prize Selection Committee in the following year's competition.

The CMS Doctoral Prize consists of a $500 award, a two-year complimentary membership in the CMS, a framed Doctoral Prize certificate and a stipend for travel expenses to attend the CMS meeting to receive the award and present a plenary lecture.

Nominations

Candidates must be nominated by their university and the nominator is responsible for preparing the documentation described below, and submitting the nomination to the Canadian Mathematical Society. Universities may nominate more than one candidate.

The documentation shall consist of:

- A curriculum vitae prepared by the student.
- A resumé of the student's work written by the student and which must not exceed ten pages. The resumé should include a brief description of the thesis and why it is important, as well as of any other contributions made by the student while a doctoral student.
- Three letters of recommendation of which one should be from the thesis advisor and one from an external reviewer. A copy of the external examiner's report may be substituted for the latter. More than three letters of recommendation are not accepted.

The deadline for the receipt of nominations is **January 31, 2017**. All documentation, including letters of recommendation, must be submitted electronically to docprize@cms.math.ca.

Prix de doctorat 2017

La SMC a créé ce Prix de doctorat pour récompenser le travail exceptionnel d'un étudiant au doctorat. Le prix sera décerné à une personne qui aura reçu son dipôme de troisième cycle d'une université canadienne l'année précédente (entre le 1er janvier et le 31 décembre) et dont les résultats pour l'ensemble des études supérieures seront jugés les meilleurs.

La dissertation constituera le principal critère de selection (impact des résultats, créativité, qualité de l'exposition, etc.), mais ne sera pas le seul aspect évalué. On tiendra également compte des publications de l'étudiant, de son engagement dans la vie étudiante et de ses autres réalisations.

Les mises en candidature qui ne seront pas choisies dans leur première compétition seront considérées pour une année additionelle (sans possibilité de mise à jour du dossier), et seront révisées par le comité de sélection du Prix de doctorat l'an prochain.

Le lauréat du Prix de doctorat de la SMC aura droit à une bourse de 500 $. De plus, la SMC lui offrira l'adhésion gratuite à la Société pendant deux ans et lui remettra un certificate encadré et une subvention pour frais de déplacements lui permettant d'assister à la réunion de la SMC où il recevra son prix et présentera une conférence.

Candidatures

Les candidats doivent être nommés par leur université; la personne qui propose un candidat doit se charger de regrouper les documents décrits aux paragraphes suivants et de faire parvenir la candidature à la Société Mathématique du Canada. Les universités peut nommer plus d'un candidat.

Le dossier sera constitué des documents suivants :

- Un curriculum vitae rédigé par l'étudiant.
- Un résumé du travail du candidat d'au plus dix pages, rédigé par l'étudiant, où celui-ci décrira brièvement sa thèse et en expliquera l'importance, et énumérera toutes ses autres réalisations pendant ses études de doctorat.
- Trois lettres de recommandation, dont une du directeur de thèse et une d'un examinateur de l'extérieur (une copie de son rapport serait aussi acceptable). Le comité n'acceptera pas plus de trois lettres de recommandation.

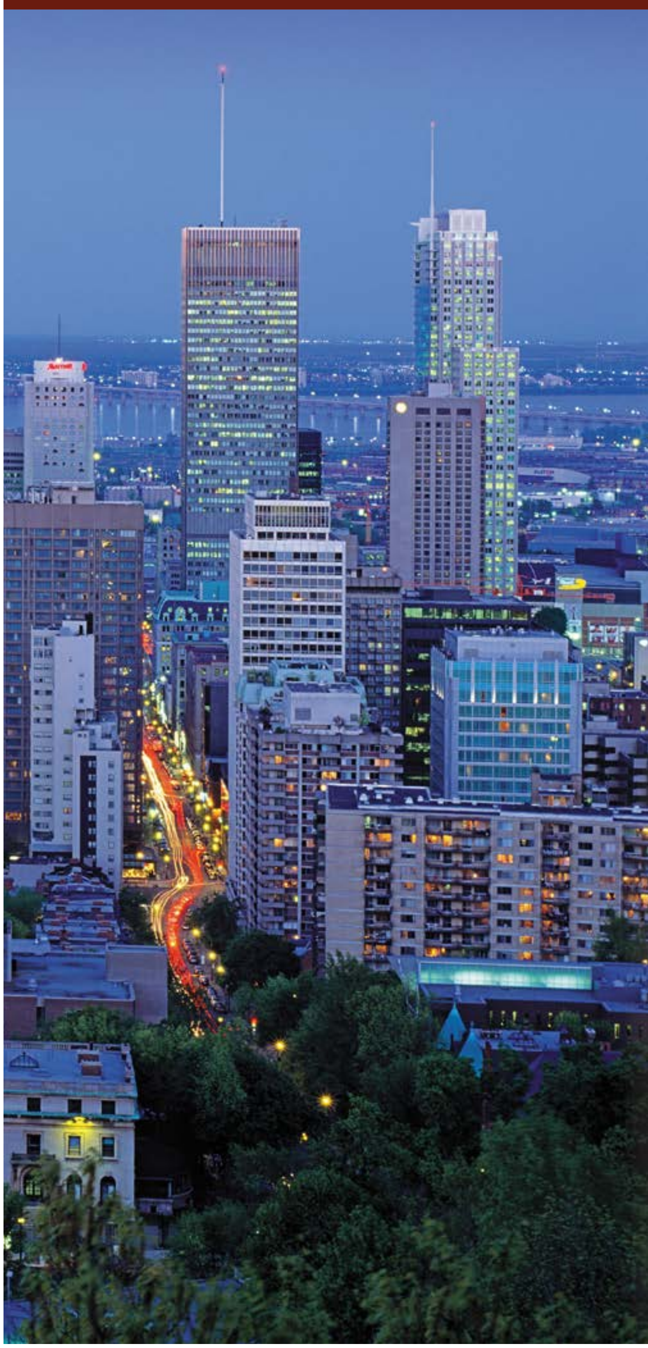
Les candidatures doivent parvenir à la SMC au plus tard le **31 janvier 2017**. Veuillez faire parvenir tous les documents par voie électronique avant la date limite à prixdoc@smc.math.ca.

Mathematical Congress of the Americas
Congrès Mathématique des Amériques

MCA 2017

CMS SMC

July 24-28 juillet Montréal, Canada



The second Mathematical Congress of the Americas (MCA) will take place from July 24-28, 2017, at Centre Mont-Royal and McGill University in Montreal, Canada.

MCA 2017 will highlight mathematical achievements of the Americas and will foster collaboration between the continents' mathematical communities.

There will be a large number of special sessions. Follow this link to view the confirmed scientific sessions: **https://mca2017.org/program/scientific-program**

Registration is now open!

mca2017.org

Le deuxième Congrès mathématiques des Amériques (CMA) aura lieu du 24 au 28 juillet 2017 au Centre Mont-Royal et l'Université McGill à Montréal, Canada.

Le CMA 2017 met en lumière les accomplissements mathématiques des Amériques et encourage la collaboration entre les différentes communautés mathématiques du continent.

Un grand nombre de sessions spéciales seront au programme. Consultez la page **https://mca2017.org/program/scientific-program** pour voir les sessions confirmées.

La période d'inscription est ouverte!

mca2017.org/fr

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