



Learn Leap Fly

8

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CMS
SMC

Canadian Mathematical Society
Société mathématique du Canada

CMS NOTES de la SMC

September /
septembre
2017

President's Notes / Notes du président

Michael Bennett (UBC)

CMS President / Président de la SMC



In the summer of 2017, the breadth of mathematical achievement in Canada was on display for all to see. From July 24th to 28th, the CMS played

host to the second Mathematical Congress of the Americas (MCA), in Montreal. This highly anticipated meeting of mathematicians and students from across North, Central and South America, and the Caribbean was the largest meeting to be held in Canada since the International Congress of Mathematicians in Vancouver in 1974. It featured a truly first-rate collection of speakers, including plenary lectures by Manuel del Pino, Shafira Goldwasser, Peter Ozsvath, Yuval Peres and Kannan Soundararajan, and public lectures by Eric Demaine and Etienne Ghys.

Immediately preceding this congress, the International Mathematical Olympiad took place in Rio de Janeiro, Brazil, featuring teams of talented high school mathematicians from more than one hundred countries around the world. One year after the CMS-sponsored Canadian team completed their sixth straight top twelve finish, featuring both the youngest gold medalist and the highest scoring female in the Olympiad.

At the same time that Canada is showcasing our successes in mathematics, a confluence of political events in North America and Europe has presented us with unprecedented possibilities for recruitment at the undergraduate, graduate, postdoctoral and professorial levels. Applications from international students, both undergraduate and graduate, have risen across the country, a trend that shows no sign of abating; which begs the question: can we capitalize on

Cet été, un événement a véritablement mis en relief toute la portée de la recherche en mathématiques qui s'effectue au Canada. Du 24 au 28 juillet avait lieu à Montréal le deuxième Congrès mathématique des Amériques (MCA), dont la Société mathématique du Canada (SMC) était l'hôte. Ce rassemblement très attendu de mathématiciens et d'étudiants des trois Amériques et des Antilles était le plus grand événement du genre à se tenir au Canada depuis le Congrès international des mathématiciens de 1974, à Vancouver. D'éminents conférenciers y ont participé, parmi lesquels Manuel del Pino, Shafira Goldwasser, Peter Ozsvath, Yuval Peres et Kannan Soundararajan, qui ont présenté des conférences plénières, ainsi qu'Eric Demaine et Étienne Ghys, qui ont quant à eux prononcé des conférences publiques.

Deux semaines plus tôt se tenait à Rio de Janeiro, au Brésil, l'Olympiade internationale de mathématiques, où se sont mesurées des équipes de brillants élèves d'une centaine de pays. À l'édition 2016 de cet événement, l'équipe canadienne, parrainée par la SMC, s'était classée parmi les douze premières positions une sixième année de suite. Elle pouvait s'enorgueillir de compter dans ses rangs le plus jeune médaillé d'or de l'événement et la détentrice du score le plus élevé de toute l'Olympiade chez les dames.

Au moment même où rayonnent les réalisations canadiennes en mathématiques, une confluence d'événements politiques en Amérique du Nord et en Europe nous place devant des possibilités sans précédent de recrutement d'étudiants de premier cycle, des cycles supérieurs et de niveau postdoctoral, de même que de professeurs. Le nombre d'étudiants étrangers de premier cycle et des cycles supérieurs qui présentent des demandes est en hausse partout au pays, et la

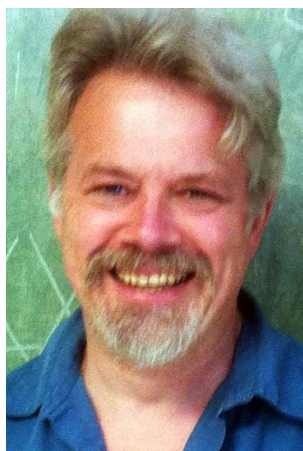
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A Week Away

Robert Dawson, *St. Mary's*

CMS Notes Editor-in-Chief



I'm writing this from a small but comfortable residence room at the University of British Columbia. It's the fifth day of a conference (category theory, as it happens, but imagine your own specialty.) The talks have been excellent, the food superb, and the company stimulating. Even the weather has cooperated. And I find myself wondering - how do we get away with doing this and calling it work? Shouldn't we just stay home and read journal articles?

Well, I do try to limit my travel these days. But a lot goes on at conferences that doesn't happen anywhere else. Apart from the talks - five hours or so per day of intensive learning - I've also discussed possible applications of my own work with researchers interested in the area, and found out from others what they're planning to do next. (Formal lectures are about what's already been done: the future belongs to the coffee breaks.) I've found out that a paper I coauthored many years ago is finally becoming relevant, and seen other strands of category theory coming together. (I even spent one evening working with a colleague on something for the CMS.)

People around me have been writing collaborative papers, recruiting graduate students and postdocs, and engaging in the sort of vague what-if conjecturing that rarely makes it into research papers. Some of this could be done in other circumstances, but would it have happened? The value of putting a bunch of researchers together in one spot, away from departmental pressures and other commitments, should not be underestimated.

Research conferences are a big part of our lives. Sometimes, like this week, everything goes smoothly. Sometimes the airlines are uncooperative, the facilities are spartan, the data projector has a mind of its own or the chalk's missing, and it rains most days. But, always, there's a sense of excitement, of big things afoot, and of community. Long may it be so!

Une semaine à l'écart

Robert Dawson, *(St. Mary's)*

rédacteur en chef

Me voici à rédiger dans une salle confortable d'une résidence de l'Université de Colombie-Britannique, au terme du cinquième jour d'un congrès sur la théorie des catégories (mais vous êtes libres d'y substituer votre propre spécialité). Les présentations sont très intéressantes, la nourriture est excellente et la compagnie, stimulante. Même le temps coopère. D'où mes interrogations : peut-on parler de « travail »? Le travail, ce ne serait pas plutôt rester chez soi à lire des articles savants?

Je tente de limiter mes déplacements, mais les congrès sont indubitablement riches en échanges qui ne sont possibles nulle part ailleurs. En l'occurrence, outre les présentations, soit quelque cinq heures par jour d'apprentissage intensif, j'ai pu discuter des applications de mon propre travail avec des chercheurs qui partagent mes champs d'intérêt et découvrir les projets des autres. (En effet, alors que les présentations rappellent ce qui a été fait, c'est à la pause café que l'avenir se révèle.) J'ai appris par ailleurs qu'un article rédigé avec un collègue voilà de nombreuses années montre enfin son applicabilité, et j'ai pu constater la convergence de divers axes de la théorie des catégories. (J'ai même passé une soirée à travailler avec un collègue pour la SMC!)

Pendant ce temps, les autres participants rédigent en collaboration, recrutent des étudiants des cycles supérieurs et des études postdoctorales, en plus de se livrer à de vagues conjectures dont peu feront leur chemin dans des rapports de recherche. Certes, ces personnes auraient pu en faire autant dans d'autres circonstances. Quoique... Il ne faut surtout pas sous-estimer le potentiel d'une réunion de chercheurs loin des pressions de leur département et d'autres engagements.

Les congrès tiennent une large place dans nos vies de chercheurs. Et si, cette semaine, tout se déroule sans heurt, il arrive que les transporteurs aériens nous mettent des bâtons dans les ailes, que les installations soient spartiates, que le projecteur n'en fasse qu'à sa tête, que la craie reste introuvable et qu'il pleuve pratiquement chaque jour. Pourtant, on sent chaque fois l'enthousiasme, l'esprit de corps et une grande fébrilité à l'idée qu'il se prépare quelque chose de grand. Puisse-t-il en être ainsi pendant longtemps encore!

Letters to the Editors

The Editors of the NOTES welcome letters in English or French on any subject of mathematical interest but reserve the right to condense them. Those accepted for publication will appear in the language of submission. Readers may reach us at the Executive Office or at notes-letters@cms.math.ca

Lettres aux Rédacteurs

Les rédacteurs des NOTES acceptent les lettres en français ou en anglais portant sur n'importe quel sujet d'intérêt mathématique, mais ils se réservent le droit de les compresser. Les lettres acceptées paraîtront dans la langue soumise. Les lecteurs peuvent nous joindre au bureau administratif de la SMC ou à l'adresse suivante : notes-lettres@smc.math.ca.

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The 2018 membership renewal will begin soon! Please renew your membership online as soon as possible at portal.cms.math.ca by logging into your member account. Should you have any questions, please email us at memberships@cms.math.ca

Le renouvellement pour l'an 2018 va commencer bientôt! S'il vous plaît renouveler votre adhésion en ligne dès que possible à portail.smc.math.ca et en vous connectant à votre compte de membre. Si vous avez des questions, s'il vous plaît écrivez-nous à adhesions@smc.math.ca

NOTES DE LA SMC

Les Notes de la SMC sont publiées par la Société mathématique du Canada (SMC) six fois par année (février, mars/avril, juin, septembre, octobre/novembre et décembre).

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Les rédacteurs des Notes de la SMC accueillent vos articles, lettres et notes. Indiquer la section choisie pour votre article et le faire parvenir à l'adresse courriel appropriée ci-dessus.

Les Notes de la SMC, les rédacteurs et la SMC ne peuvent pas être tenus responsables des opinions exprimées par les auteurs.

CMS NOTES

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La Société mathématique du Canada appuie l'avancement, la découverte, l'apprentissage et l'application des mathématiques. L'exécutif de la SMC encourage les questions, commentaires et suggestions des membres de la SMC et de la communauté.

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Continued from cover

these opportunities, and deal with the corresponding challenges? While much of the onus to do so will undoubtedly rely upon quick and flexible thinking from administrators at individual universities and colleges, they will need to depend on financial backing from provincial authorities. Additionally, there remains an urgent need for federal government support as well.

With the current political climate, there are reasons to believe that such support might even be forthcoming, which brings us to the subject of NSERC, the Natural Sciences and Engineering Research Council.

NSERC provides the majority of all centralized research funding for much of university mathematics in Canada. Indeed, it is essentially the only source of such funding for applicants in many fields of Pure Mathematics. Despite the central role that NSERC plays in Canadian mathematics, the relationship between the Council and the mathematics community has not always been an optimal one. On one side, there are complaints that mathematicians have failed to persuasively articulate our discipline's fundamental value to society; on the other, one frequently hears concerns about NSERC's very substantial allocation of scarce resources to what many outside the council would describe as vanity projects and corporate welfare. Massive ongoing financial allocations to acronym-based faculty positions in areas of current fashion are unlikely to be in the best interests of either our community or, long term, that of Canadian society as a whole.

So how can the CMS, together with the CRM, the Fields Institute, PIMS, AARMS and CAIMS, best work with NSERC and other granting agencies, and more generally with the federal government, to ensure that the voice of mathematics is heard in this country? How do we capitalize on chances to recruit outstanding mathematicians at all levels, attracted by Canada's welcoming, multicultural society? The stakes are too high and the opportunities too abundant to leave this to chance. Please let me know your ideas at president@cms.math.ca.

Suite de la couverture

tendance ne semble pas vouloir s'essouffler. Une question s'impose alors : sommes-nous en mesure d'exploiter ces possibilités et de relever les défis qui les accompagnent? Certes, c'est en grande partie aux administrateurs d'universités et de collèges qu'il incombe de déterminer rapidement comment la réponse peut s'articuler, mais ceux-ci doivent aussi pouvoir compter sur le soutien financier des autorités provinciales. Et les besoins de deniers publics fédéraux restent également criants.

Le contexte politique actuel donne à penser que l'aide souhaitée pourrait très bien être au rendez-vous, ce qui m'amène à vous parler du Conseil de recherches en sciences naturelles et en génie, le CRSNG.

Le CRSNG est le principal organe de financement centralisé de la recherche universitaire en mathématiques au Canada. Dans de nombreux champs des mathématiques pures, il est pratiquement l'unique source de ce type de financement. Malgré ce rôle de pivot du CRSNG, les rapports entre le Conseil et le milieu canadien des mathématiques ne sont pas toujours optimaux. D'un côté, on reproche aux mathématiciens de ne pas savoir expliquer de façon convaincante l'apport fondamental de leur discipline à la société; de l'autre, on reproche souvent au NSERC d'affecter une proportion substantielle de ses ressources limitées à ce que beaucoup qualifieraient de projets « flatteurs d'ego » ou de subventions à des entreprises parasites. Ce n'est pas en continuant de subventionner massivement des chaires aux désignations abscones dans les domaines en vogue que le Conseil servira au mieux les intérêts de notre milieu ni, à long terme, ceux de l'ensemble de la société canadienne.

Alors, je vous le demande. Comment la SMC – de concert avec le CRM, l'Institut Fields, le PIMS, l'AARMA et la SCMAI – peut-elle le mieux intervenir auprès du CRSNG et d'autres organismes subventionnaires, et plus généralement auprès du gouvernement fédéral, pour faire entendre la voix du milieu des mathématiques en ce pays? Comment saisir les occasions qui se présentent de recruter ces mathématiciens d'exception, de tous les niveaux, qu'attire notre société accueillante et multiculturelle? Les enjeux sont trop grands et les possibilités, trop nombreuses, pour que nous laissions tout cela au hasard. Faites-moi part de vos idées. Écrivez-moi à president@smc.math.ca.

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The Calendar brings current and upcoming domestic and select international mathematical sciences and education events to the attention of the CMS readership. Comments, suggestions, and submissions are welcome.

Denise Charron, Canadian Mathematical Society,
(managing-editor@cms.math.ca)

Le calendrier annonce aux lecteurs de la SMC les activités en cours et à venir, sur la scène pancanadienne et internationale, dans les domaines des mathématiques et de l'enseignement des mathématiques. Vos commentaires, suggestions et propositions sont le bienvenue.

Denise Charron, Société mathématique du Canada
(redacteur-gerant@smc.math.ca)



SEPTEMBER 2017 SEPTEMBRE

Aug 21-Sep 7	CRM Aisenstadt Chair: Claudia Klüppelberg, CRM, Montreal, Que.
Aug 27-Sep 1	BIRS Workshop: The Analysis of Gauge-Theoretic Moduli Spaces , BIRS, Banff, Alta.
Aug 28-Sep 1	PIMS Combinatorics of Group Actions and its Applications Workshop , St. John's, N.L.
3-8	BIRS Workshop: Future Targets in the Classification Program for Amenable C*-Algebras , BIRS, Banff, Alta.
10-15	BIRS Workshop: Photonic Topological Insulators , BIRS, Banff, Alta.
10-17	BIRS Workshop: Stochastic Lattice Differential Equations and Applications , BIRS, Banff, Alta.
11-14	CRM Workshop: Risk Measurement and Regulatory Issues in Business , CRM, Montreal, Que.
11-15	11th International Conference on Words , UQAM, Montreal, Que.
11-15	Semaine d'Étude Maths-Entreprises , Université Paris Descartes, Paris, France
17-22	BIRS Workshop: Lattice walks at the Interface of Algebra, Analysis and Combinatorics , BIRS, Banff, Alta.
18-22	Conference on Big Data and Information Analytics , Fields Institute, Toronto, Ont.
18-Oct 5	CRM Aisenstadt Chair: Alexander J. McNeil, CRM, Montreal, Que.
24-29	BIRS Workshop: Symmetries of Surfaces, Maps and Dessins , BIRS, Banff, Alta.
25-28	CRM Workshop: Measurement and Control of Systemic Risk , CRM, Montreal, Que.
29-Oct 1	BIRS Workshop: Open Source Computation and Algebraic Surfaces , BIRS, Banff, Alta.
30	MathEd Forum: September 30, 2017 , Fields Institute, Toronto, Ont.

OCTOBER 2017 OCTOBRE

1-6	BIRS Workshop: p-adic Cohomology and Arithmetic Applications , BIRS, Banff, Alta.
2-5	CRM Workshop: Dependence modeling tools for risk management , CRM, Montreal, Que.
8-13	BIRS Workshop: Computational Uncertainty Quantification , BIRS, Banff, Alta.
9-13	CRM Workshop: The Beauty of Discrete Mathematics , CRM, Montreal, Que.
13-14	61^e Congrès de l'Association de Mathématique du Québec (AMQ) , Cégep de l'Outaouais, Gatineau, Qué.
15-20	BIRS Workshop: New Perspectives in Representation Theory of Finite Groups , BIRS, Banff, Alta.
20-22	Retreat for Young Researchers in Stochastics , BIRS, Banff, Alta.
21-22	Canadian Western Algebraic Geometry Symposium , University of Alberta, Edmonton, Alta.
22-27	BIRS Workshop: Stochastic Analysis and its Applications , BIRS, Banff, Alta.
29-Nov 3	BIRS Workshop: Automorphic Forms, Mock Modular Forms and String Theory , BIRS, Banff, Alta.

NOVEMBER 2017 NOVEMBRE

12-17	BIRS Workshop: Approximation Algorithms and the Hardness of Approximation , BIRS, Banff, Alta.
26-Dec 1	BIRS Workshop: Partial Order in Materials: at the Triple Point of Mathematics , Physics and Applications, BIRS, Banff, Alta.

DECEMBER 2017 DÉCEMBRE

3-8	BIRS Workshop: Inferential Challenges for Large Spatio-Temporal Data Structures , BIRS, Banff, Alta.
8-11	2017 CMS Winter Meeting/Réunion d'hiver de la SMC 2017 , University of Waterloo, Waterloo, Ont.
10-15	BIRS Workshop: Mathematics for Developmental Biology , BIRS, Banff, Alta.
11-13	CRM Workshop: Risk Modeling, Management and Mitigation in Health Sciences , CRM, Montreal, Que.

Book Reviews brings interesting mathematical sciences and education publications drawn from across the entire spectrum of mathematics to the attention of the CMS readership. Comments, suggestions, and submissions are welcome.

Karl Dilcher, Dalhousie University (notes-reviews@cms.math.ca)

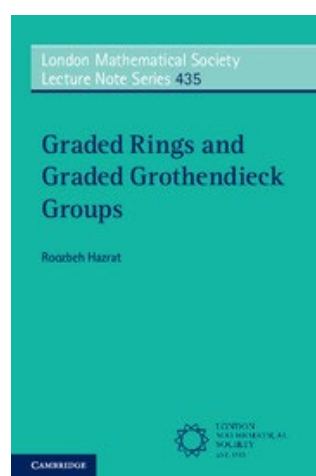
Graded rings and graded Grothendieck groups

By Roozbeh Hazrat

Cambridge University Press, 2016

ISBN 978-13166119582

Reviewed by Sorin Dăscălescu, University of Bucharest



The theory of group graded rings has developed as an independent research direction after 1970. The motivation came from examples in commutative algebra, algebraic geometry and group representation theory. The first monograph devoted to the topic was written by C. Năstăsescu and F. Van Oystaeyen in 1982. Since then, ideas and methods of graded ring theory have proved to be useful in algebra, representation theory,

combinatorics and category theory. The book under review is new evidence in this direction. The main aim of the book is to study the graded version of the Grothendieck group associated to a graded ring and to show that it is a powerful invariant in classification problems.

The titles of the six chapters are: 1. Graded rings and graded modules. 2. Graded Morita theory. 3. Graded Grothendieck groups. 4. Graded Picard groups. 5. Graded ultramatricial algebras, classification via K_0^{gr} . 6. Graded versus nongraded (higher) K -theory. Even if several results presented in the book are true for gradings by arbitrary groups, only gradings by abelian groups are considered. The author explains this option by the fact that for the main aims of the book, the abelian grading framework is large enough, and moreover, the presentation and proofs are often more transparent for such gradings.

Chapter 1 provides general concepts and results on graded rings and graded modules, with an impressive number of interesting examples. One of the examples is the Leavitt path algebra associated to a directed graph with the property that only finitely many arrows leave from any vertex; this algebra, endowed with a \mathbb{Z} -grading, plays a key role in the book. Chapter 2 develops the theory of equivalences between the categories of graded modules over two graded rings, and investigates the connection to the equivalence between the categories of modules over those rings.

Les comptes-rendus de livres présentent aux lecteurs de la SMC des ouvrages intéressants sur les mathématiques et l'enseignement des mathématiques dans un large éventail de domaines et sous-domaines. Vos commentaires, suggestions et propositions sont le bienvenue.

Karl Dilcher, Dalhousie University (notes-critiques@smc.math.ca)

In Chapter 3, a ring A graded by an abelian group Γ is considered, and the graded Grothendieck group of A , denoted by $K_0^{gr}(A)$, is constructed as the group completion of the monoid of isomorphism classes of finitely generated projective graded A -modules, with the direct sum as monoid operation. $K_0^{gr}(A)$ is more than an abelian group. It has a structure of a $\mathbb{Z}[\Gamma]$ -module, where the Γ -action is induced by the shift operation on graded modules. $K_0^{gr}(A)$ is computed if A is a graded division ring, a graded local ring or a Leavitt path algebra. Several results about classical K_0 -groups are extended to the graded case, for example a construction in terms of idempotent matrices and a relative version with respect to a two-sided graded ideal. Chapter 4 studies the graded version of the Picard group of a graded ring. In Chapter 5, inspired by the Bratteli-Elliott Theorem, which says that the Grothendieck group, its positive cone and the position of identity classify ultramatricial algebras over a field, the author defines graded ultramatricial algebras over a graded field, and proves that they can be classified by using the graded Grothendieck group. In Chapter 6, Quillen's K -theory machinery is used to define the K -group $K_i^{gr}(A)$ for any $i \geq 0$. Several results connecting graded K -theory and ungraded K -theory are presented for certain classes of \mathbb{Z} -graded rings.

The presentation of the material is very well organized, with many examples and enlightening comments. These make the book an enjoyable one.

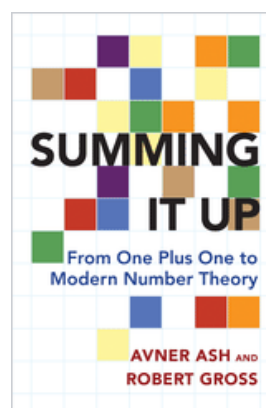
Summing it up

By Avner Ash and Robert Gross

Princeton University Press, 2016

ISBN 978-0691170190

Reviewed by Patrick Ingram, York University



Number theory is a natural setting for the exposition of mathematics, because everyone has experience with the natural numbers, but we can so quickly ask questions about them that are very hard to answer. *Which numbers are the sum of two primes?*, or *Which numbers are the sum of two squares?* Any naturally inquisitive layperson can become interested in these questions, perhaps more so when we reveal how much is unknown after centuries (in some cases millennia) of study.

In *Summing it up*, Ash and Gross use the problem of writing an integer as a sum of squares as the base camp for an excursion into the theory of modular forms. They begin with an overview of elementary number theory (primes and divisibility), and then introduce the reader to Waring's Problem, and natural variants. Waring's original question was whether or not there existed, for each $k \geq 1$, an N_k such that every natural number is the sum of at most N_k perfect k th powers; a positive answer was given by Hilbert. It is then natural to ask what the smallest N_k is, what it becomes if we allow finitely many exceptions, and in how many ways a natural number can be written as a sum of a certain number of k th powers. If $r_t(n)$ is the number of ways of writing n as a sum of t squares, counted in a particular fashion, one might write down the generating series

$$\theta^t(z) = \sum_{n=0}^{\infty} r_t(n) q^n, \quad q = e^{2\pi i z}.$$

It turns out that Θ^t satisfies a functional equation

$$\theta^t\left(\frac{az+b}{cz+d}\right) = (cz+d)^{t/2} \theta^t(z)$$

with respect to a certain group of Möbius transformations (at least for t even). In other words, it defines a modular form. Most of the book is devoted to laying the groundwork for this observation, developing some complex analysis so that we can properly consider generating functions, and exploring related constructs such as Dirichlet series. At the end, the authors illustrate the connections (which are many and deep) with their previous books *Fearless symmetry: Exposing the Hidden Patterns of Numbers* (2008, on Galois representations) and *Elliptic Tales: Curves, Counting, and Number Theory* (2012, on elliptic curves).

Expository and informal, but with numbered theorems and the occasional assigned exercise, this book is half-way between a textbook and the typical popularization of mathematics meant for the general public. It is probably best thought of as something to give an enthusiastic undergraduate who is disappointed that your department offers only a very introductory number theory course. It is also well suited to any mathematician outside of number theory, who wants to get a vague idea of what's happening in next week's colloquium on automorphic forms. By these measures the book delivers admirably, being well-written and detailed, while sweeping just the right details under the rug.

Call for Proposals: 2017 Endowment Grants Competition

The Canadian Mathematical Society is pleased to announce the *2017 Endowment Grants Competition*. The CMS Endowment Grants fund projects that contribute to the broader good of the mathematical community. Projects funded by the Endowment Grants must be consistent with the interests of the CMS: to promote the advancement, discovery, learning and application of mathematics.

An applicant may be involved in only one proposal per competition as a principal applicant. Proposals must come from CMS members, or, if joint, at least one principal applicant must be a CMS member.

The deadline for applications is **September 30, 2017**. Successful applicants will be informed in January 2018 and the grants issued in February 2018.

Further details about the endowment grants and the application process are available on the CMS website: www.cms.math.ca/Grants/EGC

The Endowment Grants Committee (EGC) administers the distribution of the grants and adjudicates proposals for projects. The EGC welcomes questions or suggestions you may have on the program. Please contact the Committee by e-mail at chair-egc@cms.math.ca.



Education Notes brings mathematical and educational ideas forth to the CMS readership in a manner that promotes discussion of relevant topics including research, activities, and noteworthy news items. Comments, suggestions, and submissions are welcome.

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Learn Leap Fly: Interview with two Canadian mathematicians taking on a Global Learning XPRIZE challenge

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While Amy Woodling was finishing up her PhD in McGill in number theory, she used to take the train back and forth from Ottawa to Montreal. One day, she was on the train listening to an interview of Peter Diamandis, a founder of the XPRIZE Foundation. He was introducing the upcoming competition for the Global Learning XPRIZE, putting out a challenge to use technology to teach basic literacy and numeracy skills to children who do not have access to teachers. The submitted proposals would be judged and the chosen finalists would then try their software in an 18-month field test with students in Africa. Now that, she thought, would be a really cool project to pursue. So she pitched the idea of participation to her husband Kjell later that day. He said yes immediately. Two and a half years later, on June 21, 2017, their team Learn Leap Fly was named as one of the Global Learning XPRIZE's 11 semi-finalists.

Now, Amy was not just an average person on the train thinking about educational technologies and Kjell was not just an average guy saying yes to developing them. Both Amy and Kjell Woodling have mixed interdisciplinary backgrounds that are, in many ways, perfect for a project like this. After completing her BSc in math, Amy attended Teacher's College before continuing on to do her PhD in arithmetic geometry. Kjell started as a computer engineer and then worked in industry before returning to academia to complete his math PhD degree. Furthermore, within their immediate families, they had a plethora of skills required for this project. Among these, Kjell's family members comprise of many educational specialists, while Amy's aunt runs a children's home in Kenya.

The intention here in this piece is to share more about the experience of the two mathematicians, Amy Woodling (AW) and Kjell Woodling (KW), with a focus on the aforementioned project. For instance, what challenges did they face and how did they address them? I sat down with the two of them to chat about their project's past, present and future. As the interviewer, my initials KG are used below with questions and prompts appearing in italics.

Notes pédagogiques présentent des sujets mathématiques et des articles sur l'éducation aux lecteurs de la SMC dans un format qui favorise les discussions sur différents thèmes, dont la recherche, les activités et les nouvelles d'intérêt. Vos commentaires, suggestions et propositions sont le bienvenue.

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KG: *So you didn't have to go very far to find all the necessary skills that one might need for a project like this, but this is still a big undertaking.*

KW: I think the thing that helped us the most is that we are both fundamentally researchers. We are people who tackle hard problems for a living. So when it came to this problem, it didn't seem unusual to try to do a thing that has never been done before. But we also came to it as a blank slate. We came to it not knowing what solution we wanted to see work, just that we wanted to see a solution. So we came to it very much with a sense of experimentation. We wanted to get in there and try things out, see what was working and what wasn't.

KG: *What was the development process like?*

KW: When we started this, there was an 18-month period before you had to submit your solution. So we took that 18-month period and we broke it up into three phases. We called our team *Learn Leap Fly* and we ended up doing that as our approach to the problem. First was the "Learn" phase, where we sat down with educational research to find out the well-known ways to teach literacy and numeracy. Then we started identifying products available already: what software is there now, what does it do well and what does it do poorly. By the end of this phase, we had a pretty good idea about where the challenges were. For the "Leap" phase, we chose the set of things we wanted to try and started trying them out. We started testing other people's software to see how children responded to it. We noticed several important things right away, for example that a child's hand is sometimes too small to do things on the screen the standard way. Some of these discoveries we were making are now obvious in hindsight, but impossible to know until you do those observations and experimentations. So this was our approach throughout the project: to try things out as soon as possible and see how the children are responding. The last phase, the "Fly" phase, was taking all of our best ideas, all the pieces that worked well, putting them together into a prototype and then would fly off to Kenya to try it out with the children there. Children, who in many cases, have never even seen a tablet before, so we got to see these first interactions with technology.

KG: *This is very different, because we now assume that everybody knows how to use a tablet.*

AW: Exactly, there are some things that are assumed that are not natural. Dragging is one of them. It's a learned thing of how to

interact with a mobile device, so now you need to do something design-wise to try to teach children to do that.

KW: Design was a huge challenge throughout for a number of reasons. One of them is when you are trying to appeal to children the world over. Obviously you have to avoid using a language because they can't read yet, but you also have to avoid metaphors, because they don't always work. For example, an icon to go back to the main menu is often a picture of a house. But that doesn't mean anything to these children since they have no context for it, so you have to find things which, whenever possible, communicate what you want to do directly. So we had to experiment to see what children would respond to, children who have never seen this technology before and wouldn't necessarily have any instruction on it.

KG: *What other things did the initial testing in Kenya reveal?*

KW: Here we have this interesting ideal of one device per child. When we did our alpha testing in Kenya, we had four tablets with us and a dozen children, so of course we would end up with multiple children per tablet. We noticed right away that we weren't just getting children in the age group we were aiming at, but we were getting everybody. At the same time.

AW: This was deliberate on our part, we wanted to test it out naturally. We didn't want to impose any constraints. We wanted to see what they would do and how they would work together, whether this is something we need to incorporate into our design, something we should focus on or not.

KW: It turned out to be very powerful. Having 3-4 children per tablet was actually a benefit, because it set up a social environment. Reading for example. Some of the children knew how to read and some didn't, but they surely knew how to listen and repeat. So you got this group activity of reading together and even those children who weren't attracted to the letters wanted to participate in the social part of this. We wanted to play on this social idea. So whenever possible, we started designing our software so it could be used not just by one child at a time, but by a group of children, all their hands on a tablet, working together.

KG: *So it has to be multi-touch because there will be a lot of little hands.*

KW: Exactly, there might be a dozen hands on the screen or as many as would fit, so your activities needed to accommodate all of those hands. Designing this kind of social software is not well known, well understood or well developed, so this meant rethinking a lot of the ways we typically design our activities.

AW: When we were faced with the problem of doing personalization to groups, we came up with the idea of "digital personalities", where each tablet has its own distinct personality: it may prefer certain types of activities or stories, whereas the tablet beside it will have different preferences. The first step is to give children the chance to self-select a tablet, but then we also have things that adapt underneath it.

KG: *What about the level and progression of activities? Since you cannot rely on the same child working with the same tablet every time, how does the software accommodate that?*



AW: There are three main threads: reading, writing and numbers. Some are mixed and matched in the sense that numbers may know what level of reading you are at, so they know whether they should ask you to write the numbers out or not and so on. But it is all driven by a set of stories and every set of activities adapts each time to reflect the story you just read. For vocabulary building activities, it will pull words from the story. For numbers, it may draw elements of the story and ask you to count them. So the stories control the certain amount of the leveling and allow us to control user preference in the sense that the software can always connect the activities to the elements from your favorite stories. Under the hood, we have a decision engine that keeps track of the user preferences, such as the kinds of stories you like, as well as which level you are on in various activities. To add a little bit of randomness and variation, there is always a distinct aspect of the digital personality present in these decisions. There is also always some notion of curriculum, as in where you are in terms on learning progression.

KW: This is where it's convenient that we happen to be mathematicians and data scientists. We can use things like machine learning to identify the level we think the children are at and adapt the activities accordingly. But again, we always have to be aware of the fact that we never know for sure if it is the same children coming back. So those digital personalities have to be able to recognize that behaviors have changed and change the way it is acting.

AW: If the software ever feels that the user has changed in a significant way, it goes through an annealing type process where it presents the child with a whole bunch of random options to quickly figure out their level and preferences. As behavior stabilizes over time between different groups of users on a particular tablet, it also uses clustering to break up different types of behaviors. So if it expects to see 10 different kinds of behaviors on this tablet, it just has to figure out which one it is currently dealing with.

KG: *How does it make sure that children practice various skills as opposed to just picking one type of activity they are already good at?*

AW: At the end of every activity, the main menu has 8 choices, so a child always has a choice, but it is how we fill those choices that matters. Some of the choices will be influenced by user preferences

and some by curriculum preferences. If the child only ever chooses user preference options, it will eventually be filled with all curriculum choices. So when you are trying to maximize engagement, you will present more user preference type choices; if the child is highly engaged, the software might decide it is a good time to learn something new and swing towards more curriculum type choices. It will oscillate back and forth to guide you in various directions.

KG: *Your software is driven by a set of stories and it pulls content from them. So if you decide to implement it in another language, you can just upload a new set of stories?*

KW: Exactly. One of the great appeals to us in this approach is that it allows us to easily engage with a new language or a new culture, say our Northern communities in Canada. We would first work with that community to develop the initial set of stories, so we get some cultural and linguistic familiarity. This aspect of being able to create stories which are culturally relevant to the group is a great way to be able to tailor the content. You start with a set of stories and do the work to separate the characters, objects, story elements and everything adapts.

KG: *What is the next step for you?*

AW: This fall, we will be in kindergarten classes in Ottawa and we will be introducing French content as well. Their curriculum here is 50-50 English and French time-wise, which is a recent change, so the teachers are looking to find more suitable resources for instruction. So we are working over the summer to get everything in French as well ready to go for the new school year in September.

KW: We were originally developing this software to be used essentially without schools and teachers. But we are also doing our testing in classes and kindergartens and after-school programs. Because it was designed for children at different levels and for children working in groups, it ended up working really nicely in classrooms, where you can set up stations with 4-5 devices for children to work together. It worked so well in the classroom environment that by the end of the school year we had teachers asking if they can use this in their classrooms on an ongoing basis, which was a pleasant surprise. So we figured, let's give it a shot. Starting September, we will be working with some classrooms here in Ottawa to align it better with their curriculum, to introduce activities that fit with how we do kindergarten here and to again bring it closer to home with French and English as a start.

KG: *Last question: what has been the most exciting part of this process?*

KW: In my opinion, all of the interesting stuff we do in mathematics occurs on the boundary where it meets something else in the world. We never could have predicted when we started out that we will be using machine learning or developing Bayesian inference systems to these digital personalities. But it's the combination, the ability to take all of these different areas together and put them into one solution that makes it really exciting.

AW: The ability to make a real difference in the world. To apply mathematics and research in a way that can change the lives of hundreds of millions of children. It's an incredible experience.

Note: Readers interested in more information on the work of Amy and Kjell Woodling may wish to visit learnleapfly.com

Appel de projets : Concours de bourses du fonds de dotation 2017

La Société mathématique du Canada (SMC) est heureuse d'annoncer la tenue du *Concours de bourses du fonds de dotation 2017*. Les bourses du fonds de dotation de la SMC financent des activités contribuant à l'essor global de la communauté mathématique. Les projets financés à partir des bourses du fonds de dotation doivent correspondre aux intérêts de la SMC : soit promouvoir et favoriser la découverte et l'apprentissage des mathématiques, et les applications qui en découlent.

Un demandeur ne peut présenter qu'un projet par concours en tant que demandeur principal. Les projets doivent venir de membres de la SMC. S'il s'agit d'un projet conjoint, au moins un des demandeurs principaux doit être membre de la SMC.

La date limite pour présenter sa demande est le **30 septembre 2017**. Les projets retenus seront annoncés en janvier 2018, et les bourses distribuées en février 2018.

Pour vous procurer un formulaire ou pour de plus amples renseignements sur l'appel de projets, passez sur le site de la SMC au : www.smc.math.ca/Grants/EGC/f

Le Comité d'attribution des bourses du fonds de dotation (CABFD) gère la répartition des bourses et évalue les projets. Pour toute question ou tout commentaire sur les bourses du fonds de dotation, veuillez communiquer par courriel avec le comité à pres-egc@smc.math.ca.



2018 David Borwein Distinguished Career Award

The **David Borwein Distinguished Career Award** recognizes mathematicians who have made exceptional, broad, and continued contributions to Canadian mathematics.

A complete nomination dossier consists of:

- A signed nomination statement from a present or past colleague, or collaborator (no more than three pages) having direct knowledge of the nominee's contribution;
- A short curriculum vitae, no than five pages;
- Two to four letters of support in addition to the nomination;
- Other supporting material may be submitted, no more than 10 pages.

A nomination can be updated and will remain active for three years.

The complete nomination dossier must arrive at the CMS Executive Office **no later than November 15, 2017**.

All documentation should be submitted electronically, preferably in PDF format, by the appropriate deadline, to dbaward@cms.math.ca.

Prix David-Borwein de mathématicien émérite pour l'ensemble d'une carrière 2018

Le **prix David-Borwein de mathématicien émérite pour l'ensemble d'une carrière** rend hommage à un mathématicien qui a fait une contribution exceptionnelle et soutenue aux mathématiques canadiennes.

Le dossier de candidature comprendra les éléments suivants :

- une lettre de mise en candidature signée par un collègue ou un collaborateur actuel ou des années passées (trois pages maximum) qui connaît très bien les réalisations de la personne proposée;
- un bref curriculum vitae, maximum de cinq pages;
- de deux à quatre lettres d'appui, en plus de la mise en candidature;
- tout autre document pertinent, maximum de 10 pages.

Toute mise en candidature est modifiable et demeurera active pendant trois ans.

Le dossier complet doit parvenir au bureau administratif de la SMC **au plus tard le 15 novembre 2017**.

Veuillez faire parvenir tous les documents par voie électronique, de préférence en format PDF, avant la date limite à prixdb@smc.math.ca.

2018 CMS Excellence in Teaching Award

The CMS Excellence in Teaching Award Selection Committee invites nominations for the **2018 Excellence in Teaching Award**.

The Excellence in Teaching Award recognizes sustained and distinguished contributions in teaching at the post-secondary undergraduate level at a Canadian institution. The award focuses on the recipient's proven excellence as a teacher at the undergraduate level as exemplified by unusual effectiveness in the classroom and/or commitment and dedication to teaching and to students.

Nomination letters, *including at least three letters of reference*, should list the chosen referees and include a recent curriculum vitae for the nominee, if available.

Nominations and reference letters should be submitted electronically, preferably in PDF format, to: etaward@cms.math.ca no later than the deadline of **November 15, 2017**.

Prix d'excellence en enseignement de la SMC 2018

La Comité de sélection du Prix d'excellence en enseignement de la SMC invite un appel de mises en candidatures pour le **Prix d'excellence en enseignement 2018**.

Le Prix d'excellence en enseignement récompense des contributions exceptionnelles et soutenues en enseignement au collégial et au premier cycle universitaire dans un établissement canadien. Le Prix récompense l'excellence reconnue d'un enseignant ou d'une enseignante au niveau postsecondaire telle qu'illustrée par son efficacité exceptionnelle en classe ou son engagement et son dévouement envers l'enseignement et les étudiants.

Le dossier de candidature, *comportant au moins trois lettres de référence*, doit comprendre le nom des personnes données à titre de référence ainsi qu'un curriculum vitae récent du candidat ou de la candidate, dans la mesure du possible.

Veuillez faire parvenir les mises en candidature et lettres de référence par voie électronique, de préférence en format PDF, à : prixee@smc.math.ca avant la date limite du **15 novembre 2017**.

Research Notes brings mathematical research ideas forth to the CMS readership in a generally accessible manner that promotes discussion of relevant topics including research (both pure and applied), activities, and noteworthy news items. Comments, suggestions, and submissions are welcome.

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Les articles de recherche présentent des sujets mathématiques aux lecteurs de la SMC dans un format généralement accessible qui favorise les discussions sur divers sujets pertinents, dont la recherche (pure et appliquée), les activités et des nouvelles dignes de mention. Vos commentaires, suggestions et propositions sont le bienvenue.

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Unlikely intersections in arithmetic dynamics

Dragos Ghioca, Department of Mathematics, University of British Columbia

Combining ideas of Ihara-Serre-Tate, Lang [5] proved the following natural result. If a (complex, irreducible) plane curve $C \subset \mathbb{A}^2$ contains infinitely many points with both coordinates roots of unity, then C is the zero locus of an equation of the form $x^a y^b = \zeta$, where $a, b \in \mathbb{Z}$ and ζ is a root of unity. In other words, if $F \in \mathbb{C}[x, y]$ is an irreducible polynomial for which there exist infinitely many pairs (μ, ν) of roots of unity such that $F(\mu, \nu) = 0$, then (modulo multiplying by a constant) $F(x, y)$ is either a polynomial of the form $x^a - \zeta y^b$, or of the form $x^a y^b - \zeta$, where a and b are non-negative integers and ζ is a root of unity. In particular, Lang's result [5] provided the first instance when the Manin-Mumford Conjecture was proven: if a curve $C \subset \mathbb{G}_m^2$ contains a Zariski dense set of torsion points, then C is a torsion translate of a 1-dimensional torus. The proof of Ihara-Serre-Tate-Lang is a clever combination of various tools from mathematics (not only from number theory, but even basic complex analysis is used); for example, any graduate student in mathematics would benefit from reading their proof for the special case when the curve C is the graph of a polynomial.

The result of [5] is the first instance of the principle of *unlikely intersections* in arithmetic geometry, since Lang's result may be interpreted as follows: if an *unlikely* event (such as the existence on the plane curve C of a point with both coordinates roots of unity) occurs infinitely often, then this must be explained by a *global*, geometric condition satisfied by C (in this case, C is a translate of a 1-dimensional algebraic subgroup of \mathbb{G}_m^2 by a torsion point). Other famous conjectures in number theory, such as the Mordell-Lang, the Bogomolov, or the André-Oort conjectures may be coined in the same terminology of unlikely intersections, which is also paraphrased as "*special points and special subvarieties*" since the *special points* in this case are the ones with both coordinates roots of unity and the only subvarieties containing a Zariski dense set of special points are the *special subvarieties*, which are torsion translates of algebraic subgroups.

It is also natural to formulate a dynamical analogue of Lang's result [5] by interpreting the roots of unity as preperiodic points under the action of the squaring map $z \mapsto z^2$. We recall that given a self-map f on any set X , a point a is preperiodic if its orbit under the

action of $z \mapsto f(z)$ is finite, i.e., $f^m(a) = f^n(a)$ for some positive integers $m < n$ (where in dynamics, unless otherwise noted, f^ℓ always represents the ℓ -th compositional iterate of f for each positive integer ℓ). So, the dynamical reformulation of Lang's result yields the following: if a plane curve C contains infinitely many preperiodic points for the map $F : \mathbb{A}^2 \rightarrow \mathbb{A}^2$ given by $F(x, y) = (x^2, y^2)$, then C must be preperiodic under the action of F . Furthermore, one can replace the squaring map by an arbitrary polynomial (or more generally a rational function) and ask whether the same result holds; this was conjectured in early 90's by Zhang, with the conjecture being formally published in [11]. After a series of partial results obtained by several authors, Zhang's conjecture was proven in [3] for all plane curves.

The Dynamical Manin-Mumford theorem for split endomorphisms of $(\mathbb{P}^1)^2$. Let $f \in \mathbb{C}(x)$ be a rational function of degree $d > 1$ and let $F : (\mathbb{P}^1)^2 \rightarrow (\mathbb{P}^1)^2$ be the endomorphism given by $F(x, y) = (f(x), f(y))$. If the curve $C \subset (\mathbb{P}^1)^2$ contains infinitely many points preperiodic under the action of F , then C itself must be preperiodic.

Actually, in [3] we proved a more general result valid for the action of two *different* rational functions f_1 and f_2 (of same degree $d > 1$) on the two coordinate axes of $(\mathbb{P}^1)^2$; however, one needs to impose additional conditions when f_1 and f_2 are different Lattès maps of same degree corresponding to a CM elliptic curve (for more details, see [4]). Also, we note that the hypothesis that the two rational functions have the same degree (larger than 1) is absolutely necessary since otherwise there are obvious counterexamples, as seen by considering the diagonal line $\Delta \subset \mathbb{A}^2$ under the coordinatewise action of the two polynomials $z \mapsto z^2$ and respectively, $z \mapsto z^3$; indeed, Δ contains infinitely many points with both coordinates roots of unity, but Δ is not preperiodic under the action of $(x, y) \mapsto (x^2, y^3)$.

The Manin-Mumford conjecture (along with its further generalization, the Pink-Zilber conjecture) served as inspiration also for the following result of Masser-Zannier [7]. Given the Legendre family of elliptic curves: $y^2 = x(x-1)(x-t)$ parametrized by $t \in \mathbb{C} \setminus \{0, 1\}$, there exist at most finitely many $t_0 \in \mathbb{C}$ such that both the points $P_{t_0} := (2, \sqrt{2(2-t_0)})$ and $Q_{t_0} := (3, \sqrt{6(3-t_0)})$ are torsion for E_{t_0} . More generally, as shown in [8], given a curve C and an elliptic surface $\mathcal{E} \rightarrow C$ endowed with two sections $P, Q : C \rightarrow \mathcal{E}$, if there exist infinitely many $t \in C(\mathbb{C})$ such that both $P_t := P(t)$ and $Q_t := Q(t)$ are torsion points on the elliptic fiber E_t , then P

and Q are linearly dependent (globally) on the elliptic surface. In their proof, Masser and Zannier use the results of Pila-Wilkie [6] regarding the number of rational points on an analytic curve. Intrinsic to the method employed in [7, 8] is the fact that there exists a global analytic uniformization map for the family of elliptic curves.

The result of Masser-Zannier inspired Baker and DeMarco [1] to prove the following (first) result about unlikely intersections in arithmetic dynamics. Given an integer $d > 1$ and given complex numbers a and b , if there exist infinitely many $t \in \mathbb{C}$ such that both a and b are preperiodic under the action of $z \mapsto z^d + t$, then $a^d = b^d$. In other words, if the unlikely event that a and b behave similarly from a dynamics perspective (i.e., having finite orbit) with respect to the same map $f_{t_0}(z) := z^d + t_0$ occurs infinitely often, then a and b are mapped to the same point globally, by the entire family of maps f_t . Several papers followed, finally settling the generalization of the result from [1] in which $f_t(z)$ is allowed now to be any algebraic family of polynomials parametrized by \mathbb{P}^1 , while the starting points $a(t)$ and $b(t)$ are allowed to be polynomial in t (not necessarily constant); the conclusion is the same: the existence of infinitely many parameters $t_0 \in \mathbb{C}$ such that both $a(t_0)$ and $b(t_0)$ are preperiodic under the action of f_{t_0} yields that $a(t)$ and $b(t)$ satisfy an algebraic relation given by polynomials commuting (globally) with $f_t(z)$. If one attempts to generalize further the result from [1] to families $f_t(z)$ of rational functions parametrized by an arbitrary curve C (not necessarily \mathbb{P}^1), one encounters significant technical difficulties; see [2] for one of the very few known instances when such an extension was established. The strategy from [1, 2, 3] (along with all of the other papers on the new topic of unlikely intersections in arithmetic dynamics) employs powerful equidistribution theorems for points of small height with respect to metrized line bundles (see [10], for example) in order to prove that if there exist infinitely many parameters t_0 such that both $a(t_0)$ and $b(t_0)$ are preperiodic under the action of f_{t_0} (and also assuming neither $a(t)$ nor $b(t)$ is persistently preperiodic for the family $f_t(z)$), then for each parameter t , we have that $a(t)$ is preperiodic if and only if $b(t)$ is preperiodic. Finally, in order to go from the aforementioned *if and only if condition* to the *exact relation* between the starting points $a(t)$ and $b(t)$, one uses complex analysis (sometimes along with nontrivial real analysis) combined with the complete characterization of all invariant plane curves under a split polynomial action, as provided by Medvedev-Scanlon [9].

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An Uncertainty Principle For Roots

Felipe Gonçalves, *University of Alberta*

The uncertainty principle, from the quantum mechanics point of view, states that the standard deviation of the position and momentum of a particle cannot be measured simultaneously with arbitrarily small precision. Introduced first in 1927, by the German physicist Werner Heisenberg, its mathematical formulation takes the following form. Let

$$\widehat{f}(\mathbf{y}) = \int_{\mathbb{R}^d} f(\mathbf{x}) e^{-2\pi i \mathbf{x} \cdot \mathbf{y}} d\mathbf{x}$$

be the Fourier Transform of a given function $f : \mathbb{R}^d \rightarrow \mathbb{R}$ and let

$$\sigma(f) = \int_{\mathbb{R}^d} |\mathbf{x} f(\mathbf{x})|^2 d\mathbf{x}.$$

Then we have

$$\sigma(f)\sigma(\widehat{f}) \geq \frac{d^2}{16\pi^2},$$

if $\int_{\mathbb{R}^d} |f(\mathbf{x})|^2 d\mathbf{x} = 1$. After this discovery, a variety of mathematical uncertainty principles were found relating the behavior of a function f and its Fourier transform \widehat{f} (we recommend [1] for trip into the world of uncertainty).

A recent and interesting uncertainty principle found by Bourgain, Clozel and Kahane [2] is the following: If $f : \mathbb{R}^d \rightarrow \mathbb{R}$ is a radial function (a function which only depends on $|x|$) such that $f(0) \leq 0$ and $\widehat{f}(0) \leq 0$, then it is not possible for both f and \widehat{f} to be positive outside an arbitrarily small neighborhood of the origin. This suggests that the quantities

$$A(f) := \inf \{r > 0 : f(x) \geq 0 \text{ if } |x| > r\}$$

$$A(\widehat{f}) := \inf \{r > 0 : \widehat{f}(y) \geq 0 \text{ if } |y| > r\}$$

are strictly positive (possibly ∞) unless $f \equiv 0$. There is a dilation symmetry $x \rightarrow \lambda x$ having the reciprocal effect $y \rightarrow y/\lambda$ on the Fourier side and as a consequence, the product $A(f)A(\widehat{f})$ is invariant under this group action and becomes a natural quantity to consider (as the product $\sigma(f)\sigma(\widehat{f})$ was a natural quantity to consider in Heisenberg's uncertainty).

The purpose of the present expository article is to popularize the uncertainty principle established in [2] and to report the study done in [3], where we improve their results and establish new ones ([3] is a joint work with Diogo Oliveira e Silva from the Hausdorff Center for Mathematics and Stefan Steinerberger from Yale University).

Theorem 1. (Bourgain, Clozel & Kahane [2]).

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a nonzero, integrable, even function such that $f(0) \leq 0$, $\widehat{f} \in L^1(\mathbb{R})$ and $\widehat{f}(0) \leq 0$. Then

$$A(f)A(\widehat{f}) \geq 0.1687,$$

and 0.1687 cannot be replaced by 0.41.

The following is the first main result of [3], where we improve the bounds of the previous theorem.

Theorem 2. (Gonçalves, Oliveira e Silva and Steinerberger [3])

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a nonzero, integrable, even function such that $f(0) \leq 0$, $\widehat{f} \in L^1(\mathbb{R})$ and $\widehat{f}(0) \leq 0$. Then

$$A(f)A(\widehat{f}) \geq 0.2025,$$

and 0.2025 cannot be replaced by 0.353.

The proof of the lower bound in Theorem 2 relies on rearrangement inequalities of optimal transport flavor which do not admit a straightforward generalization to higher dimensions.

It is quite involved and cannot be improved much further: the third decimal place in the lower bound could be increased at the expense of some additional work, but a genuinely new idea seems needed for substantial further improvement. In contrast, we believe that the upper bound given by Theorem 2 might be very close to being optimal and that functions which almost realize the sharp constant look like the function depicted in Figure 1.

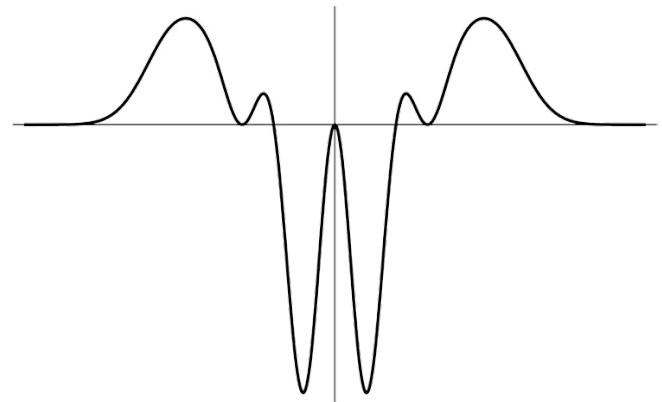


Figure 1. Plot of a function $f \in L^1(\mathbb{R})$ satisfying $\widehat{f} = f$ and $f(0) = 0$.

A version of Theorem 1 holds in higher dimensions.

Theorem 3. (Bourgain, Clozel & Kahane [2])

Let $d \geq 2$. Let $f \in L^1(\mathbb{R}^d)$ be a nonzero, real-valued, radial function such that $f(\mathbf{0}) \leq 0$, $\widehat{f} \in L^1(\mathbb{R}^d)$ and $\widehat{f}(\mathbf{0}) \leq 0$. Then

$$A(f)A(\widehat{f}) \geq \frac{1}{\pi} \left(\frac{1}{2} \Gamma\left(\frac{d}{2} + 1\right) \right)^{\frac{2}{d}},$$

and this lower bound cannot be replaced by $(d+2)/2\pi$.

As an immediate consequence, we have

$$(1) \quad \frac{d}{2\pi e} < \inf_f A(f)A(\hat{f}) < \frac{d+2}{2\pi},$$

where the infimum is taken over all functions f satisfying the assumptions of Theorem 3. The linear growth in terms of dimension given by inequalities (1) is expected in a wider class of related situations. The last chapter of the paper [2] shows that this problem and its solution are naturally related to the theory of zeta-functions in algebraic number fields. Arithmetic arguments show that the linear growth of the bounds with respect to dimension is natural in view of known properties of ramifications of these fields. We show, via a variation of the original argument employed in [2] to handle the one-dimensional case, that the lower bound can be improved in all higher dimensions.

Theorem 4. (Gonçalves, Oliveira e Silva and Steinerberger [3])

Let $d \geq 2$. Let $f \in L^1(\mathbb{R}^d)$ be a nonzero real-valued, radial function such that $f(\mathbf{0}) \leq 0$, $\hat{f} \in L^1(\mathbb{R}^d)$ and $\hat{f}(\mathbf{0}) \leq 0$. Then:

$$A(f)A(\hat{f}) \geq \frac{1}{\pi} \left(\frac{1}{1 + \lambda_d} \Gamma\left(\frac{d}{2} + 1\right) \right)^{\frac{2}{d}},$$

where the number λ_d is defined in terms of the Bessel function $J_{d/2}$ as

$$\lambda_d := -\inf_{u \in \mathbb{R}_+} \frac{\Gamma\left(\frac{d}{2} + 1\right) J_{d/2}(u)}{(u/2)^{d/2}}.$$

Moreover, $\lambda_d < \frac{1}{2}$ for every $d \geq 2$, and $\lambda_d \rightarrow 0$ as $d \rightarrow \infty$ exponentially fast.

Let \mathcal{A}_d denote the higher-dimensional version of the set of functions considered in Theorems 1 and 2. In other words, let $d \geq 1$, and say that a function $f : \mathbb{R}^d \rightarrow \mathbb{R}$ belongs to \mathcal{A}_d if it is nonzero, integrable with integrable Fourier transform, and such that $f(\mathbf{0}) \leq 0$ and $\hat{f}(\mathbf{0}) \leq 0$. Set

$$\mathbf{A}_d := \inf_{f \in \mathcal{A}_d} \sqrt{A(f)A(\hat{f})},$$

where $A(f)$ again denotes the smallest positive real number r such that $f(\mathbf{x}) \geq 0$, for every $|\mathbf{x}| > r$.

Our next result shows that the inequality

$$(2) \quad A(f)A(\hat{f}) \geq \mathbf{A}_d^2 \quad (f \in \mathcal{A}_d)$$

admits an extremizer.

Theorem 5. (Gonçalves, Oliveira e Silva and Steinerberger [3])

There exists a nonzero radial function $f \in \mathcal{A}_d$ such that $\hat{f} = f$, $f(\mathbf{0}) = \hat{f}(\mathbf{0}) = 0$, and $A(f) = \mathbf{A}_d$.

The next result shows that extremizers for inequality (2) exhibit an unexpected behavior if compared to extremizers for other uncertainty principles (recall, for instance, that Gaussians extremize the Heisenberg uncertainty inequality).

Theorem 6. (Gonçalves, Oliveira e Silva and Steinerberger [3])

Let $f : \mathbb{R}_+ \rightarrow \mathbb{R}$ be a function such that its radial extension, $\mathbf{x} \in \mathbb{R}^d \mapsto f(|\mathbf{x}|)$, belongs to the set \mathcal{A}_d and realizes equality in (2). Then f has infinitely many double roots in the interval $(A(f), \infty)$.

We approach Theorem 6 in two different ways, both of which follow a common general strategy: Assuming f to be an extremizer for inequality (2) with a finite number of double roots only, we identify a perturbation f_ε of f for which $A(f_\varepsilon)A(\hat{f}_\varepsilon) < A(f)A(\hat{f})$. The first argument works only if $d = 1$, but has the advantage that it relies on an explicit construction of the perturbation f_ε that seems generalizable to a number of related situations which we plan to address in future work. The second part of the proof of Theorem 6 works only in higher dimensions $d \geq 2$, and makes use of Laguerre expansions of radial functions.

Open Problems

(I) Can we identify the extremal function f of Theorem 5?

(II) Can we improve the upper and lower bounds at (1)? So far what we know only is that the \liminf and \limsup of $\frac{\mathbf{A}_d}{d}$ when $d \rightarrow \infty$ lies in the interval $[\frac{1}{2\pi e}, \frac{1}{2\pi}]$. Also, in an unpublished work, we were able to produce an exponentially small improvement on the upper bound

$$\mathbf{A}_d \leq \frac{d+2}{2\pi} - e^{-cd},$$

where $c \approx 10^{40}$.

(III) If we consider the sub-class \mathcal{A}_d^{Pol} of functions $f \in \mathcal{A}_d$ of the form $f(\mathbf{x}) = p(|\mathbf{x}|^2)e^{-\pi|\mathbf{x}|^2}$ where p is a polynomial, does the quantity

$$\mathbf{A}_d^{Pol} := \inf_{f \in \mathcal{A}_d^{Pol}} \sqrt{A(f)A(\hat{f})}$$

equals to \mathbf{A}_d ? If so, can we use computer programming to guess what is the value of \mathbf{A}_d for some small dimensions d and the shape of extremizers for Theorem 5?

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CSHPM Notes brings scholarly work on the history and philosophy of mathematics to the broader mathematics community. Authors are members of the Canadian Society for History and Philosophy of Mathematics (CSHPM). Comments and suggestions are welcome; they may be directed to either of the column's co-editors:

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Les articles de la SCHPM présente des travaux de recherche en histoire et en philosophie des mathématiques à la communauté mathématique élargie. Les auteurs sont membres de la Société canadienne d'histoire et de philosophie des mathématiques (SCHPM). Vos commentaires et suggestions sont le bienvenue; ils peuvent être adressées à l'une des co-rédacteurs:

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Constructions on a Spherical Blackboard

Robert Thomas, University of Manitoba

Sometimes historical work turns up mathematics of not merely historical interest. This column offers one example of that, namely constructions on a spherical blackboard. If you have a spherical blackboard, you can determine its diameter with compasses on the sphere and with compasses and a ruler on a planar blackboard. Three bits of theory are needed:

- 1) Any circle on the sphere has associated with it a centre and two poles, from the closer of which it can be drawn with compasses, and an axis, the line joining the poles through the centre of the circle and the centre of the sphere.
- 2) Any circle on the sphere is bisected perpendicularly by a great circle having the bisected circle's axis as a diameter.
- 3) The square inscribed in a great circle has as its diagonal the diameter of the sphere. The polar radius of great circles is the side of the square.

Determination of the sphere diameter, which allows setting the compasses to the polar radius of a great circle, can be done in a second set of three steps:

- 4) To draw on the plane the diameter of a circle through three points A, B, C , on the sphere.

Transfer the triangle ABC , using the compasses, to the plane. In the plane triangle abc with sides ab and ac extended, construct inward perpendiculars at vertices b and c to meet at d , the other end of the diameter from a in the circumcircle of abc , equal to the circle ABC on the sphere.

- 5) By transferring the triangle bcd back to BCD on the sphere, one determines the point D opposite A in the circle ABC whether the circle is present or missing.
- 6) To draw on the plane the diameter of the sphere. With any pole P , draw on the sphere a circle that is not a great circle. From three points A, B, C on the circle, determine the point D opposite A . Draw on the plane the diameter of the great circle APD bisecting circle $ABDC$.

This ingenious construction comes near the end of the first book of *Spherics* in the version attributed to Theodosios of Bithynia.

Although he lived approximately 100 BCE, the content of *Spherics* is much older, since it formed the mathematical basis of Euclid's astronomy book *Phenomena* in the fourth century BCE. No one knows what Theodosios is responsible for or who originally wrote the first book. It and the *Phenomena* are the main documents of the pre-Ptolemaic astronomy that was used as an introduction to that more sophisticated astronomy (using trigonometry) until the latter was retired. The Sadleirian professor of geometry at Oxford was expected to lecture on the *Spherics* as late as John Wallis in the seventeenth century. Book I begins by proving that a plane through three points on the sphere cuts the sphere in a circle. The above constructions allow the drawing of the great circle through any two points on the sphere and of the circle through any three points on the surface:

- 7) To draw a great circle through two points A and B . With poles A and B , draw great circles to intersect at two points C and D unless they are at ends of a diameter, in which case it is easier. With pole C or D , draw a great circle through A , which passes through B .
- 8) To find the pole of the circle through three points A, B , and C on the sphere. Find the points D and E diametrically opposite A and B using 5 above. Draw the great circles bisecting the circle (present or absent) at A and D and at B and E . They meet at the pole P .
- 9) If the circle is absent, it can be drawn with pole P and polar radius PA .

This simple non-axiomatic geometry could almost be taught in high schools except for the lack of spherical blackboards. I became aware that such exist reading the *Notices of the AMS*, which last year published a photo of one at the Institute Mittag-Leffler in Djursholm, Sweden; see page three of www.ams.org/publications/journals/notices/201606/rnoti-p604.pdf. I have learned from an editor of this column that spheres coated to take chalk were used in teaching astronomy, geography, and spherical geometry and trigonometry in the nineteenth century. Those are now museum pieces (americanhistory.si.edu/collections/search/object/nmah_1064792), replaced by plastic (en.wikipedia.org/wiki/Lénárt_sphere).

It must be confessed that what I have outlined above is not all in Theodosios's book. In particular 4 assumes that the circle is present although it is not used and 5 is missing although something

unspecified is needed to find D in 6 and D and E in 8. The lack of 5 requires the circle to be present in 8, and so 9 is missing. I think that the geometry is more interesting as I have reconstructed it, but I do not claim that it is authentic. I just think that the original author, centuries before Theodosios, would have done what I have reconstructed, as he did all the work to get it. Construction 9 completes so neatly what the *Spherics*' first proposition (that the plane section of a sphere is a circle, not my 1 above) assures us exists, that it ought to be there. Furthermore, the original conclusion of Book I does seem to be missing. A fuller exposition of that book, intended to be readable by undergraduates, has been accepted by *Mathematics Magazine* for publication in 2018.

The *Spherics* was motivated by the ancient two-sphere model of the cosmos studied in the *Phenomena*. The Earth is a sphere (not flat) that is so small with respect to the cosmos that a circular disk tangent to the Earth bisects the sphere of the fixed stars. This disk, tangent at an observer, is the local horizon separating the 'visible' half of that sphere from the 'invisible'. Except for the pole star, stars rotate daily on parallel circles, those near the pole always in the visible half of the cosmos, then less and less in the visible half, and then those that are always below the horizon. What is of interest is the annual path of the Sun along the oblique great circle called the ecliptic or zodiac circle, the circle of the constellations composing the 'signs' of the zodiac band, conventionally equal twelfths of a circle. The Sun moves through these twelfths at a rate of approximately one degree a day. Two of the astronomical results are that the signs rise and set (cross the horizon circle) at different rates and that the ecliptic wobbles in the course of the year, accounting for how high the Sun is at noon through the year. These qualitative results were rendered passé by the invention of trigonometry, which allowed quantitative deductions and mathematical study of planetary motion. This mathematics and astronomy became the introduction to the more complex and up-to-date material.

While I am a founding member of the CSHPM, I have persisted in membership as audience, rather than as researcher, with one small exception. Since I taught myself Greek while doing my PhD, when Len Berggren mentioned at a meeting in Winnipeg that Euclid's astronomy book had not been translated into either French or English, I thought that this was something I should do. Len readily agreed to supply the necessary historian's craft, and together we published the *Phenomena* ten years later. The mathematical basis of that book is the *Spherics*, which another ten years later seemed a worthwhile project, if a little dull. I have now been working on the translation for another ten years for a study with Nathan Sidoli. To my surprise and pleasure, it has turned out to be more interesting than I could have hoped.

Robert Thomas (Robert.Thomas@umanitoba.ca), a former president of CSHPM, is retired from the University of Manitoba in Computer Science, Applied Mathematics (of which he was the last head), and Mathematics, where he studied bus routes, elastic waves in two-dimensional solids, and weaving respectively. He edits Philosophia Mathematica.



La Société mathématique du Canada (SMC) est fière d'annoncer...

Après 35 ans, le Congrès international de physique mathématique (CIPM) reviendra en Amérique du Nord en 2018 et se déroulera au Canada pour la première fois. Tenu tous les trois ans, le CIPM est l'événement le plus important de l'Association internationale de physique mathématique. Le XIXe CIPM aura lieu à Montréal en 2018 et, selon la nouvelle tradition, il sera précédé du Symposium des jeunes chercheurs. Ce Symposium se tiendra à l'Université McGill les 20 et 21 juillet, et le CIPM se déroulera au Centre Mont-Royal et à l'Université McGill du 23 au 28 juillet. Le Canada se réjouit à l'idée d'accueillir le monde de la physique mathématique en 2018!

Le CIPM 2018 sera organisé par la SMC en collaboration avec de nombreuses associations des domaines de la physique et des mathématiques, notamment : le CRM, l'Université McGill, le PIMS, l'Institut Fields, l'ISM, l'AARMA, le CANSSI, la SRIB, l'Institut Périmètre, l'Université de Montréal et l'UQAM.

<https://icmp2018.org/fr>

The Canadian Mathematical Society (CMS) is pleased to announce...

After 35 years, the International Congress on Mathematical Physics (ICMP) will return to North America in 2018, which will also mark the first time that Canada will host the congress. The ICMP, on its three year cycle, is the most important event of the International Association of Mathematical Physics. The XIXth ICMP will take place in Montreal, 2018, and, following recent tradition, it will be preceded by the Young Researchers Symposium (YRS). The YRS will be held at McGill University from July 20 to July 21 and the ICMP will be held at the Centre Mont-Royal and McGill University from July 23 to July 28. Canada is looking forward to welcoming the world of mathematical physics in 2018!

ICMP 2018 will be staged by the CMS in collaboration with many physics and mathematics organizations, including: CRM, McGill University, PIMS, FIELDS, ISM, AARMS, CANSSI, BIRS, Perimeter Institute, U. Montréal, and UQAM.

<https://icmp2018.org/en/welcome>

MATHEMATICAL CONGRESS OF THE AMERICAS 2017

Mathematical Congress of the Americas 2017 Recap

The second Mathematical Congress of the Americas (MCA) took place at Centre Mont-Royal and McGill University, July 24 to 28, 2017. Based largely on the success of the first congress which took place in Mexico in 2013, approximately 1100 mathematicians from North America, Central America, South America and the Caribbean attended.

The main goals of the MCA were to highlight the mathematical achievements of the Americas on an international level and to foster further collaboration between the continents' researchers, students, institutions and mathematical societies. The Canadian Mathematical Society (CMS) was honoured to play host to many invited speakers, award winners and students from the Americas.

The event opened with an opening ceremony that included the announcement of the recipients of the Americas Prize, the Solomon Lefschetz and the MCA Prizes. The prize winners and citations can be found at: <https://mca2107.org/program/prize-recipients>.

A welcome reception on Sunday night set the tone for a successful event and gave registrants the chance to meet and connect with colleagues before the week ahead.

The CMS Krieger-Nelson and Jeffery-Williams prize lectures preceded the welcome reception and various CMS activities took place as the MCA progressed.

A very rainy Monday morning did not dampen the spirits of attendees at the Université de Montréal, with plenary lectures following the Opening and Prize Ceremony.

MCA 2017 included two public lecturers, Erik Demaine (MIT) and Étienne Ghys (ENS Lyon); and five plenary speakers: Shafrira Goldwasser (MIT, USA); Yuval Peres (Microsoft Research, USA);



Stephanie van Willigenburg (UBC) receives CMS Krieger-Nelson Prize from CMS President Michael Bennett (UBC).



CMS Staff members receiving thank you baskets at MCA closing banquet from Jacques Hurtubise (McGill).

Manuel de Pino (Universidad de Chile); Peter Ozsvath (Princeton University, USA); and Kannan Souñdararajan (Stanford University, USA). In addition there were 20 other invited lecturers and more than 70 special sessions, seven prize lectures.

As part of the conference, participants were treated to a lively discussion on Gender and Mathematics in McGill's Trottier Atrium. The Panel included the Director of Research and Analysis for the Association for Women in Science (AWIS), Heather Metcalf, and Associate Professor of Psychology at New York University, Andrei Cimpian. Student mathematicians organized the Early Career Panel, 16 Student Research Presentations, 19 poster sessions, 16 contributed talks including a CV Writing Workshop, as well as a Fireworks Outing followed by a Social.

The MCA Banquet was well attended and was a wonderful culmination to the week. The Canadian Mathematical Society wishes to thank the members of the organizing committee, the program committee, the steering committee, the awards subcommittee and the Council for MCA for helping the CMS bring the event together. The Society is very grateful for the support from McGill University, Université de Montréal, the AMS and the other sponsors...

For a full list of MCA activities visit <https://mca2017.org/program/mca-schedule> as well as #mca2017.

The next MCA is scheduled for 2021 in Buenos Aires.



CMS President Michael Bennett (UBC) thanks Jacques Hurtubise (McGill) at MCA closing banquet.

CONGRÈS MATHÉMATIQUE DES AMÉRIQUES 2017

Retour sur le Congrès mathématique des Amériques 2017

Le deuxième Congrès mathématique des Amériques (CMA) a eu lieu au Centre Mont-Royal et à l'Université McGill du 24 au 28 juillet 2017. Reposant en grande partie sur le succès du premier congrès, tenu au Mexique en 2013, le CMA 2017 a attiré quelque 1100 mathématiciens d'Amérique du Nord, d'Amérique centrale, d'Amérique du Sud et des Caraïbes.

Le CMA avait comme principaux objectifs de mettre en évidence les réalisations mathématiques des Amériques sur la scène internationale et de renforcer la collaboration entre les chercheurs, les étudiants, les établissements d'enseignement et les sociétés mathématiques des Amériques. La Société mathématique du Canada (SMC) a eu l'honneur d'accueillir de nombreux conférenciers, lauréats et étudiants des Amériques.

L'événement a commencé par une cérémonie d'ouverture où ont été annoncés les lauréats du Prix des Amériques, de la Médaille Salomon-Lefshetz et du Prix CMA. La liste des lauréats et leur citation se trouvent à la page : <https://mca2017.org/fr/programme/recipiendaires-de-prix>.

Le dimanche soir, une réception d'accueil a donné le ton à un événement réussi et a permis aux participants de se rencontrer et de retrouver leurs collègues avant d'entreprendre la semaine.



Robert McCann (Toronto) receives CMS Jeffrey-Williams Prize from CMS Past President Lia Bronsard (McMaster)

Les conférences des prix Krieger-Nelson et Jeffery-Williams de la SMC ont précédé la réception d'accueil, et diverses activités de la SMC se sont déroulées pendant le CMA.

Un lundi matin très pluvieux n'a pas refroidi les participants qui se sont rendus à l'Université de Montréal pour les conférences plénières, qui ont suivi la cérémonie d'ouverture et de remise des prix.

Deux conférences publiques étaient au programme du CMA 2017, prononcées par Erik Demaine (MIT) et Étienne Ghys (ENS Lyon), de même que cinq conférenciers pléniers : Shafrira Goldwasser (MIT, É.-U.); Yuval Peres (Microsoft Research, É.-U.), Manuel de Pino (Université du Chili); Peter Ozsvath (Université de Princeton, É.-U.); et Kannan Soundararajan (Université Stanford, É.-U.). Quelque 20 autres conférenciers invités, plus de 70 sessions spéciales et 7 conférences de lauréats étaient aussi au programme.

Dans le cadre du congrès, les participants ont également eu droit à une discussion animée sur le déséquilibre des sexes en mathématiques au pavillon Trottier de l'Université McGill. Participaient à cette table ronde Heather Metcalf, directrice de la recherche et de l'analyse à l'Association for Women in Science (AWIS), et Andrei Cimpian, professeur agrégé de psychologie à l'Université de New York. Les étudiants en mathématiques ont quant à eux organisé une Table ronde sur les carrières, 16 présentations de recherche étudiante, 19 présentations par affiches, 16 communications, dont un atelier de rédaction de CV, ainsi qu'une sortie aux Feux d'artifice suivie d'une activité sociale.

Le banquet du CMA a accueilli un grand nombre de convives et a clôturé la semaine en beauté. La SMC tient à remercier les membres du comité organisateur, du comité du programme, du comité consultatif, du sous-comité des prix et du conseil du CMA de leur aide pour l'organisation de ce congrès. La Société remercie sincèrement l'Université McGill, l'Université de Montréal, l'AMS et les autres commanditaires de leur soutien.

Vous trouverez la liste complète des activités du CMA à l'adresse <https://mca2017.org/fr/program/mca-schedule> ou en cherchant le mot-clic #mca2017.

Le prochain CMA aura lieu en 2021 à Buenos Aires.



2017 International Mathematical Olympiad - Leader's Report

The International Mathematical Olympiad (IMO) is an annual competition for the brightest math students from around the world. This year's competition in Rio de Janeiro attracted 615 students from 111 countries. The Canadian team was selected based largely on the results of three competitions: the Canadian Mathematical Olympiad, the Asian Pacific Mathematical Olympiad, and the USA Mathematical Olympiad. This year, the team consisted of Thomas Guo, Qi Qi, Victor Rong, Ruiming (Max) Xiong, Ruizhou (Steven) Yang, and William Zhao. The team was introduced at the 2017 IMO Send-Off in Toronto. The Send-Off was well attended, including many former contestants, leaders, family members of the team, and representatives from the Mexican and Brazilian consulates.

I was joined on the leadership team by Sarah Sun (Toronto Math Circles), and Matthew Brennan (MIT). It was our task to train the students, accompany them to the competition, and to make cases to the coordinators for how many marks they deserve. While we normally hold the training at the BIRS site in Banff, this year the training was done joint with the Mexican team at the BIRS site in Oaxaca.

Over the two week training camp, the Mexicans provided a total of seven different trainers to help out, and they were fantastic! The training went very well, and we also had a couple of excursions to experience the local culture. The second excursion brought us to Monte Albán, an amazing archaeological site, first inhabited around 500 BCE.

I left for Rio de Janeiro three days before the team; I, along with the other 108 team leaders, set the two contest papers of three questions, from a shortlisted set of 32 problems. While we chose the nicest contest possible, there was an unfortunate side effect. The two easier problems were too easy, and the two medium problems were too hard. This caused a huge pileup of scores, with many weaker students being able to solve two problems, and many strong students being unable to solve more than two problems!

Rio de Janeiro is a fabulous city with mild temperature, friendly people, and an endless coast surrounding the region. We were lucky enough to stay at the Hotel Windsor Oceania, a four-star hotel just beside the beach. After the nine-hour Olympiad exam, the team visited the 2014 World Cup

Stadium, Christ the Redeemer Cathedral, and the Sugar loaf Mountain, the most famous tourist attractions in Rio de Janeiro.

The Canadian team did very well, obtaining five medals. William Zhao received a gold medal, and was one of 14 people to solve question 6. Attending their first IMO, Thomas Guo and Victor Rong picked up silver medals. Qi Qi and Max Xiong received bronze medals, and Steven Yang received an honourable mention. With this result, Canada has received at least one gold medal for nine consecutive years, an impressive feat!

This year was also the debut of a women's award, awarded to 5 female participants from around the world, including Canada's Qi Qi. It was later suggested that the award be named in honour of Maryam Mirzakhani, the first female Fields medalist, who passed away just a week prior. She was a two time IMO gold medalist, including a perfect score in 1995.

I will leave you with the very interesting problem 3. Despite having a very elementary and nice solution, this turned out to be the hardest IMO problem ever, with only 7 students receiving a non-zero score, and only 2 full solutions. However, don't let this deter you from attempting it; it is a lot of fun to play around with, and unlike the contestants, you don't have a strict time limit for solving it!

Problem 3. A hunter and an invisible rabbit play a game in the Euclidean plane. The rabbit's starting point, A_0 , and the hunter's starting point, B_0 , are the same. After $n - 1$ rounds of the game, the rabbit is at point A_{n-1} and the hunter is at point B_{n-1} . In the n^{th} round of the game, three things occur in order.

- 1) The rabbit moves invisibly to a point A_n such that the distance between A_{n-1} and A_n is exactly 1.
- 2) A tracking device reports a point P_n to the hunter. The only guarantee provided by the tracking device to the hunter is that the distance between P_n and A_n is at most 1.
- 3) The hunter moves visibly to a point B_n such that the distance between B_{n-1} and B_n is exactly 1.

Is it always possible, no matter how the rabbit moves, and no matter what points are reported by the tracking device, for the hunter to choose her moves so that after 10^9 rounds she can ensure that the distance between her and the rabbit is at most 100?

James Rickards (McGill)



**December 8-11, 2017**

University of Waterloo – Waterloo, Ontario

cms.math.ca/Events/winter17**Prizes | Prix**2017 Excellence in Teaching Award | Prix
d'excellence en enseignement

Bernard Hodgson (Laval)

2017 Adrien Pouliot Award | Prix Adrien-Pouliot

Richard Hoshino (Quest)

2017 Coxeter-Jame Prize | Prix Coxeter-James et conférence

Sabin Cautis (UBC)

2017 Doctoral Prize | Prix de doctorat

Konstantin Tikhomirov (Princeton)

2017 Graham Wright Award for Distinguished Service |
Prix Graham Wright pour service méritoire

Recipient to be announced | Lauréat à confirmer

2017 G. de B. Robinson Award | Prix G. de B. Robinson

Alan Beardon (formerly Cambridge)

Scientific Directors | Directeurs scientifique

Kenneth Davidson, University of Waterloo

Cameron Stewart, University of Waterloo

8-11 décembre 2017

Université de Waterloo – Waterloo, Ontario

cms.math.ca/Reunions/hiver17**Plenary Lectures | Conférences plénières**

Bill Cook, University of Waterloo

Ilijas Farah, York University

Joel Kamnitzer, University of Toronto

Niky Kamran, McGill University

Natalia Komarova, UC-Irvine

Public Lecture | Conférence publique

Edward Burger, Southwestern University

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Regular Sessions | Sessions générales

Algebraic Graph Theory | Théorie algébrique des graphes

Org: Chris Godsil (University of Waterloo)

Analytic Number Theory | Théorie analytique des nombres

Org: Kevin Hare, Wentang Kuo and Yu-Ru Liu (University of Waterloo)

Application of Mathematics to Medicine & Biology | Application des mathématiques à la médecine et à la biologie

Org: Sivabal Sivaloganathan (University of Waterloo)

Applications of Combinatorial Topology in Commutative Algebra | Applications de la topologie combinatoire en algèbre commutative

Org: Sara Faridi (Dalhousie University) and Adam Van Tuyl (McMaster University)

Arithmetic Dynamics | Dynamique arithmétique

Org: Jason Bell (University of Waterloo) and Patrick Ingram (York University)

Contributed Papers | Communications libres

Org: to be announced | Org : à venir

Cyclic homology and noncommutative geometry | Homologie cyclique et géométrie non commutative

Org: Masoud Khalkhali (Western University) and Ilya Shapiro (University of Windsor)

Design Theory | Théorie de la conception

Org: Hadi Kharaghani (University of Lethbridge) and Doug Stinson (University of Waterloo)

Dynamics of Microbial Systems | Dynamique des systèmes microbiens

Org: Gail Wolkowicz (McMaster University)

Environmental and Geophysical Fluid Dynamics | Dynamique des fluides en géophysique et en science de l'environnement

Org: Kevin Lamb, Francis Poulin and Marek Stastna (University of Waterloo)

Explicit finiteness of integral points on hyperbolic curves | Finitude explicite des points entiers sur les courbes hyperboliques

Org: David McKinnon (University of Waterloo) and Jerry Wang

Geometric Analysis | Analyse géométrique

Org: Benoit Charbonneau and Spiro Karigiannis (University of Waterloo)

History of Mathematics | Histoire des mathématiques

Org: Maritza M. Branker (Niagara University)

Logic and Operator Algebras | Logique et algèbres des opérateurs

Org: Ilijas Farah (York University) and Marcin Sabok (McGill University)

Low dimensional topology and geometric group theory | Topologie en basses dimensions et théorie des groupes géométriques

Org: Adam Clay (University of Manitoba) and Tyrone Ghaswala (University of Waterloo)

Mathematical aspects of quantum information | Aspects mathématiques de l'information quantique

Org: David Kribs, Rajesh Pereira and Bei Zeng (University of Guelph)

Model Theory | Théorie des modèles

Org: Rahim Moosa (University of Waterloo) and Sergei Starchenko (University of Notre Dame)

Operator algebras | Algèbres des opérateurs

Org: Matthew Kennedy (University of Waterloo) and Paul Skoufranis (York University)

Symmetric functions and generalizations | Fonctions symétriques et généralisations

Org: Angele Hamel (Wilfrid Laurier University) and Stephanie van Willigenburg (University of British Columbia)

Toric geometry | Géométrie torique

Org: Matthew Satriano (University of Waterloo) and Greg Smith (Queen's University)

Variational Analysis and Monotone Operator Theory | Analyse variationnelle et théorie des opérateurs monotones

Org: Heinz Bauschke and Xianfu Wang (University of British Columbia Kelowna)

Graduate Student Poster Session

Présentations par affiches pour étudiants

Org: to be announced | Org : à venir

Using Digital Assets in Mathematics Education and Outreach | Utiliser les outils numériques en éducation et en sensibilisation aux mathématiques

Brian and Barbara Forrest (Waterloo)

CMS Research Prizes

The CMS Research Committee is inviting nominations for three prize lectureships. These prize lectureships are intended to recognize members of the Canadian mathematical community.

The **Coxeter-James Prize** Lectureship recognizes young mathematicians who have made outstanding contributions to mathematical research. The recipient shall be a member of the Canadian mathematical community. Nominations may be made up to ten years from the candidate's Ph.D. A nomination can be updated and will remain active for a second year unless the original nomination is made in the tenth year from the candidate's Ph.D. For more information, visit: <https://cms.math.ca/Prizes/cj-nom>

The **Jeffery-Williams Prize** Lectureship recognizes mathematicians who have made outstanding contributions to mathematical research. The recipient shall be a member of the Canadian mathematical community. A nomination can be updated and will remain active for three years. For more information: <https://cms.math.ca/Prizes/jw-nom>

The **Krieger-Nelson Prize** Lectureship recognizes outstanding research by a female mathematician. The recipient shall be a member of the Canadian mathematical community. A nomination can be updated and will remain active for two years. For more information: <https://cms.math.ca/Prizes/kn-nom>

The deadline for nominations, including at least three letters of reference, is **September 30, 2017**. Nomination letters should list the chosen referees and include a recent curriculum vitae for the nominee. Some arms-length referees are strongly encouraged. Nominations and the reference letters from the chosen referees should be submitted electronically, preferably in PDF format, to the corresponding email address and **no later than September 30, 2017**:

Coxeter-James: cjprize@cms.math.ca
Jeffery-Williams: jwprize@cms.math.ca
Krieger-Nelson: knprize@cms.math.ca

Prix de recherche de la SMC

Le Comité de recherche de la SMC lance un appel de mises en candidatures pour trois de ses prix de conférence. Ces prix ont tous pour objectif de souligner l'excellence de membres de la communauté mathématique canadienne.

Le **Prix Coxeter-James** rend hommage aux jeunes mathématiciens qui se sont distingués par l'excellence de leur contribution à la recherche mathématique. Cette personne doit être membre de la communauté mathématique canadienne. Les candidats sont admissibles jusqu'à dix ans après l'obtention de leur doctorat. Toute mise en candidature est modifiable et demeurera active l'année suivante, à moins que la mise en candidature originale ait été faite la 10^e année suivant l'obtention du doctorat. Pour les renseignements, voir : <https://cms.math.ca/Prix/cj-nom>

Le **Prix Jeffery-Williams** rend hommage aux mathématiciens ayant fait une contribution exceptionnelle à la recherche mathématique. Cette personne doit être membre de la communauté mathématique canadienne. Toute mise en candidature est modifiable et demeurera active pendant trois ans. Pour les renseignements, voir : <https://cms.math.ca/Prix/jw-nom>

Le **Prix Krieger-Nelson** rend hommage aux mathématiciennes qui se sont distinguées par l'excellence de leur contribution à la recherche mathématique. La lauréate doit être membre de la communauté mathématique canadienne. Toute mise en candidature est modifiable et demeurera active pendant deux ans. Pour les renseignements, voir : <https://cms.math.ca/Prix/info/kn>

La date limite pour déposer une candidature, qui comprendra au moins trois lettres de référence, est **le 30 septembre 2017**. Le dossier de candidature doit comprendre le nom des personnes données à titre de référence ainsi qu'un curriculum vitae récent du candidat ou de la candidate. Veuillez faire parvenir les mises en candidature et lettres de référence par voie électronique, de préférence en format PDF, avant la date limite, à l'adresse électronique correspondante et **au plus tard le 30 septembre 2017** :

Coxeter-James : prixcj@smc.math.ca
Jeffery-Williams : prixjw@smc.math.ca
Krieger-Nelson : prixkn@smc.math.ca

If undelivered, please return to:
Si NON-LIVRÉ, veuillez retourner à :

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