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February / février 2018

Vice-President's Notes / Notes du Vice-président

Douglas Farenick (*University of Regina*)

Vice-President - West / Vice-président - Ouest

A Mathematical Visit to Africa



A number of years ago while at a conference in Amsterdam, I attended a lecture presented by a young mathematician from Kenya. I noticed right away that the lecturer made statements that were incorrect, but I didn't want to say anything publicly about it. And no one else from the rather small audience did either. But after the lecture, I spoke with the lecturer and showed him where he had some problems in his "theorems." It became apparent rather quickly that this young man had had very little contact with mathematicians in his field, and that he had fewer library resources than most of us at Canadian universities enjoy. When I returned home, I sent him some material related to the subject of his lecture, but I was left with the feeling that I could do more—not just for him, but for mathematicians like him: people who were talented in mathematics, but who lacked the intellectual environment, resources, and personal contacts for sustained research activity. It was about that time that the idea of teaching a graduate course on the African continent formed in my mind.

But one does not simply travel to another continent show up somewhere, and say "I'd like to teach a course on spectral theory." Lacking contacts in Africa who could facilitate my ambition to teach a graduate course there, I was not able to follow through with a concrete plan and, as the years passed, I began to accept that my idea was not likely to be anything more than wishful thinking.

Un mathématicien en Afrique

I y a de cela plusieurs années, dans un congrès à Amsterdam, j'ai assisté à une conférence donnée par un jeune mathématicien du Kenya. J'ai tout de suite remarqué que celui-ci faisait des affirmations inexакtes, mais je n'ai pas voulu les relever publiquement. Aucun autre membre de l'auditoire, assez peu nombreux, ne l'a fait non plus. Après la conférence, cependant, j'ai rencontré le jeune homme et lui ai expliqué ce qui n'allait pas dans ses « théorèmes ». J'ai vite compris qu'il avait eu très peu de contacts avec des mathématiciens de son champ de spécialisation et qu'il avait moins d'ouvrages de référence à sa disposition qu'on en trouve dans les bibliothèques des universités canadiennes. De retour au Canada, je lui ai envoyé quelques ressources documentaires liées au sujet de sa conférence, mais je restais avec le sentiment que j'aurais pu en faire plus – pour lui et pour d'autres mathématiciens comme lui, qui avaient un talent pour les mathématiques, mais à qui il manquait l'environnement intellectuel, les ressources et les contacts nécessaires pour mener une activité de recherche soutenue. C'est à cette époque qu'a germé dans mon esprit l'idée d'aller donner une formation de cycle supérieur sur le continent africain.

Mais voilà, on ne débarque pas comme ça sur un autre continent en disant au premier venu « je voudrais donner un cours sur la théorie spectrale ». Comme je ne connaissais personne en Afrique qui aurait pu m'aider à réaliser mon projet pédagogique, celui-ci restait obstinément abstrait et, les années passant, je commençais à accepter que mon idée ne soit jamais qu'un vœu pieux.



The Tools Of Our Trade

Robert Dawson, *St. Mary's CMS Notes Editor-in-Chief*



Earlier this month I wrote a short research note about a minor problem in number theory. Today I found myself thinking about the tools I used to do it, and how they have changed from my student days in the 1980s.

The first steps were carried out using paper and pencil. Nothing has changed there; there's still a wonderful flexibility in freeform writing and sketching on a two-dimensional surface, and a sharp pencil that produces a resolution that few displays can match. The horizontal surface is ergonomically ideal for most people. And you can spread out a dozen pages of earlier work on your desktop for a working area that few monitors can match. Will this technology stay cutting-edge for another thirty-five years? Who knows?

But when I needed some experimental data, suddenly the twenty-first century took over. Even the calculator application that comes with Windows 10 can display thirty digits (absurd for most people), and programs like MAPLE open depths of possibility that no one researcher will ever plumb. Geometers and statisticians have their own software, seven-league boots that open up vast lands of unexpected results. Even the utilitarian spreadsheet has a part to play.

As a student, I wrote my first paper by hand, and passed it (with proper authorization) to the departmental secretary, whose miraculous IBM Selectric typewriter could actually type mathematical characters - several dozen of them - and the whole Greek alphabet. It took two or three rounds of markup (in soft pencil) and correction (with whiteout fluid) to get it right.

Maybe eight years later, I learned how to use TeX. Now, with a modern computer, I reset an entire paper to change a comma - in a tenth of a second - and submit it electronically.

And I wonder: what changes will the new year bring to our research?

Nos outils de travail

Plus tôt ce mois-ci, j'ai écrit un bref article sur un problème mineur en théorie des nombres. Aujourd'hui, je me suis pris à réfléchir aux outils que j'avais utilisés pour le faire, et à leur évolution depuis mes années d'étudiant dans les années 1980.

Pour les premières étapes, j'ai utilisé du papier et un crayon. Rien de nouveau de ce côté-là; l'écriture spontanée offre toujours une merveilleuse souplesse, et la réalisation d'une esquisse sur une surface bidimensionnelle à l'aide d'un crayon bien aiguisé produit une résolution que peu d'affichages ne peuvent égaler. La surface horizontale est idéale pour la plupart des gens du point de vue ergonomique. Sans compter la possibilité d'étaler une douzaine de pages de travaux antérieurs sur son bureau, ce qui donne une surface de travail avec laquelle bien peu d'écrans peuvent rivaliser. Cette technologie sera-t-elle toujours de pointe dans 35 ans? Qui sait...

Toutefois, quand j'ai eu besoin de données expérimentales, le XXI^e siècle a pris le dessus. Même l'application de calcul fournie avec Windows 10 peut afficher 30 chiffres (ce qui est absurde pour la plupart des gens), et des logiciels comme MAPLE offrent des possibilités qu'aucun chercheur ne pourra jamais atteindre. Les géomètres et les statisticiens ont leurs propres logiciels, véritables bottes de sept lieues qui leur permettent de découvrir de vastes territoires de résultats inattendus. Même la feuille de calcul utilitaire a son rôle à jouer.

En tant qu'étudiant, j'ai rédigé mon premier article à la main et je l'ai remis (avec autorisation) à la secrétaire du département, dont la miraculeuse machine à écrire IBM Selectric pouvait produire plusieurs dizaines de caractères mathématiques et tout l'alphabet grec. Il a fallu deux ou trois rondes d'annotations (au crayon pâle) et de corrections (avec du liquide correcteur) pour obtenir le résultat souhaité.

Et environ huit ans plus tard, j'ai appris à utiliser le langage TeX. Maintenant, avec un ordinateur moderne, je reprends un document entier pour changer une virgule, en un dixième de seconde, et le soumettre électroniquement.

Et je me pose la question : quels changements la nouvelle année apportera-t-elle à notre domaine de recherche?



Letters to the Editors

The Editors of the NOTES welcome letters in English or French on any subject of mathematical interest but reserve the right to condense them. Those accepted for publication will appear in the language of submission. Readers may reach us at the Executive Office or at notes-letters@cms.math.ca

Lettres aux Rédacteurs

Les rédacteurs des NOTES acceptent les lettres en français ou en anglais portant sur n'importe quel sujet d'intérêt mathématique, mais ils se réservent le droit de les comprimer. Les lettres acceptées paraîtront dans la langue soumise. Les lecteurs peuvent nous joindre au bureau administratif de la SMC ou à l'adresse suivante : notes-lettres@smc.math.ca.

NOTES DE LA SMC

Les Notes de la SMC sont publiés par la Société mathématique du Canada (SMC) six fois par année (février, mars/avril, juin, septembre, octobre/novembre et décembre).

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Les Notes de la SMC, les rédacteurs et la SMC ne peuvent pas être tenus responsables des opinions exprimées par les auteurs.

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La Société mathématique du Canada appuie l'avancement, la découverte, l'apprentissage et l'application des mathématiques. L'exécutif de la SMC encourage les questions, commentaires et suggestions des membres de la SMC et de la communauté.

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The CMS promotes the advancement, discovery, learning and application of mathematics. The CMS Executive welcomes queries, comments and suggestions from CMS members and the community.

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Continued from cover

Fast forward to the present decade. In 2012, Neil Turok of the Perimeter Institute presented the CBC Massey Lectures, from which an intriguing book, *The Universe Within*, resulted. It was there that I first read about the African Institute for Mathematical Sciences – Next Einstein Initiative (AIMS-NEI). Turok argued that the African continent represents a huge pool of potential scientific talent, if only we could find a way to nurture and develop this vast untapped human resource. A starting point was the recognition that mathematics is at the foundation of science, and Turok spearheaded the creation of the AIMS-NEI network of graduate training centres, each of which focuses on masters-level education in mathematical science. Instruction at AIMS-NEI centres is done mainly by academics who come from abroad, using a three-week short course model, and the AIMS-NEI students come from all over the continent. While it remains to be seen whether “the next Einstein” will emerge as a direct or indirect result of this initiative, the creation of AIMS-NEI network provided me with an opportunity to realize my old desire to teach a graduate course in Africa.

Thus, it came to pass that I stepped off a plane one evening not long ago in Kigali, Rwanda, and the next morning faced 45 eager faces for the first of 15 two-hour lectures over three weeks at AIMS Rwanda. That first lecture went reasonably well, but I had to recalibrate my starting point a little. I quickly learned that many of the 45 students were not focusing on pure mathematics. Some students had backgrounds in physics, or computer science, or statistics, or the mathematics of finance. Thus, that first lecture was a learning experience for both the students and me. However, once I found the right starting point, the course progressed rather well.

Formally, the courses are offered in three-week blocks, and each block has three courses running, of which the students must select two. Concurrent with my linear algebra course, there were also courses on functional analysis and classical mechanics. The three-week block that followed mine had courses in quantum mechanics, differential geometry, and algebraic number theory, whereas the block that preceded mine had skill courses: Python programming, problem solving, and writing proofs. All the students were taught LaTeX beforehand, and they submitted their assignments (on Saturdays, by 11:59 p.m.) in typeset form. The very last day of the block is devoted to presentations, which are given by three students from each class, on a topic they prepare independently.

I had four tutors assigned to my course; they attended every lecture, kept the students in check (being late for a lecture is frowned upon by the tutors, for example), and they handled all the grading.

From my perspective, teaching at AIMS-NEI Rwanda was an opportunity to work with highly motivated students. What they may have lacked in formal mathematics preparation (relative to Canadian universities), they made up for with their intelligence, work ethic, desire to understand, and desire to succeed. With students from across the African continent, it was a marvelous multicultural, multilingual setting. (During my first morning, I had the opportunity to chitchat in French while eating breakfast with some students from Cameroon and the Democratic Republic of Congo.) I held office hours during the afternoons, and sat in on the occasional tutorial that the students organized for themselves. I assisted students with their CVs and worked with students on their presentations. I prepared a set of course notes, which I updated daily to the course website. It was a lot of work, most certainly. But I went there with the specific aim of teaching, and so I didn't mind making the commitment of time and effort. I liked the students very much, and felt an obligation to give them my best.

After three weeks of working, living, and sharing meals with students and staff, I returned to Canada with many new friends and fond memories of my experience. Beyond the content of my course, my teaching in Kigali gave 45 young people access to a Canadian professor: someone who they can seek advice from or who can write a reference letter for them. For all involved, I believe it was a worthwhile experience—an experience that I hope to repeat in two or three years.

More information about AIMS-NEI can be found at <https://www.nexteinstein.org>; the Canadian office is in Toronto (<https://www.nexteinstein.org/canada/>).

2018 Graham Wright Award for Distinguished Service

In 1995, the Society established this award to recognize individuals who have made sustained and significant contributions to the Canadian mathematical community and, in particular, to the Canadian Mathematical Society. The award was renamed in 2008, in recognition of Graham Wright's 30 years of service to the Society as the Executive Director and Secretary.

Nominations should include a reasonably detailed rationale and be submitted by **March 31, 2018**.

All documentation should be submitted electronically, preferably in PDF format, by the appropriate deadline, to gward@cms.math.ca.

Suite de la couverture

Puis, en 2012, Neil Turok, directeur de l’Institut Périmètre de physique théorique, a présenté les CBC Massey Lectures – et, parallèlement, l’intrigant ouvrage *The Universe Within*. C’est dans cet ouvrage que j’ai découvert l’existence du projet « Next Einstein Initiative » (À la recherche du prochain Einstein) de l’Institut africain des sciences mathématiques (AIMS-NEI). Neil Turok y soutient que l’Afrique regorge de talents scientifiques prêts à se révéler, pour peu qu’on leur donne la possibilité de se développer. Il y eut un point de départ : la reconnaissance du fait que les mathématiques sont à la base de toute science. Et c’est sous l’impulsion de M. Turok qu’est né le réseau de centres AIMS-NEI, qui offrent tous des programmes de maîtrise en sciences mathématiques. L’enseignement dans ces centres est dispensé principalement par des universitaires étrangers, et les participants proviennent de partout sur le continent. L’avenir nous dira si cette initiative fera émerger, directement ou indirectement, « le prochain Einstein ». Grâce à elle, j’ai en tout cas réalisé mon vieux rêve d’enseigner aux niveaux supérieurs en Afrique.

C’est ainsi que me retrouvais récemment devant 45 visages enthousiastes, pour le premier de 15 cours magistraux de deux heures que j’allais donner en trois semaines au centre AIMS de Kigali, au Rwanda. Ce premier cours s’est raisonnablement bien déroulé. J’ai cependant dû rajuster un peu mon point de départ, car j’ai vite constaté qu’un grand nombre des participants n’avaient pas eu de formation antérieure formelle en mathématiques pures. Certains étaient formés en physique, d’autres, en informatique, en statistique ou en mathématiques des finances. Cette entrée en matière a donc été une occasion d’apprentissage pour moi comme pour les étudiants. Une fois le bon point de départ trouvé, cependant, les choses ont plutôt bien progressé.

La formation se donne par blocs de trois semaines, pour lesquelles les étudiants choisissent deux cours parmi trois proposés. Ainsi, parallèlement à mon cours d’algèbre linéaire se donnaient des cours d’analyse fonctionnelle et de mécanique classique. Le bloc qui suivait le mien proposait des cours de mécanique quantique, de géométrie différentielle et de théorie algébrique des nombres. Dans le bloc précédent, on enseignait des compétences techniques : la programmation en langage Python, la résolution de problèmes et la rédaction de démonstrations mathématiques. On apprend aux

étudiants à utiliser LaTeX pour soumettre leurs travaux (au plus tard le samedi à 23 h 59). La dernière journée de chaque bloc est consacrée à des présentations que donnent trois étudiants de chaque classe, sur un sujet de leur choix. Quatre tuteurs étaient affectés à mon cours; ils assistaient à chaque séance, surveillaient l’assiduité (réservant des froncements de sourcils aux retardataires, par exemple) et s’occupaient de la correction des travaux.

J’ai travaillé avec des étudiants hautement motivés au centre AIMS du Rwanda. Des étudiants dont l’intelligence, l’éthique de travail et le désir de comprendre et de réussir compensaient largement une préparation en mathématiques parfois lacunaire (par rapport à celle d’étudiants canadiens). Comme les participants venaient de partout en Afrique, j’ai aussi vécu une formidable expérience multiculturelle et multilingue – échangeant, par exemple, quelques mots de français avec des étudiants du Cameroun et de la République démocratique du Congo autour d’un déjeuner, dès le premier matin. J’étais disponible pour des rencontres à mon bureau en après-midi, et j’ai assisté à des séances de tutorat que des étudiants organisaient pour s’entraider. J’ai aidé des personnes à préparer leur curriculum vitae et des présentations. Et j’ai préparé un ensemble de notes de cours, que je mettais à jour quotidiennement sur le site du cours. Certes, ce fut beaucoup de travail. Mais j’étais là pour vivre une expérience d’enseignement, et le temps et les efforts à y consacrer ne me pesaient aucunement. J’aimais beaucoup mes étudiants et je me sentais une obligation de donner le meilleur de moi-même.

Au terme de ces trois semaines pendant lesquelles j’ai enseigné, mais aussi partagé la table et le quotidien des étudiants et du personnel du centre, je suis rentré au Canada riche de nouvelles amitiés et de précieux souvenirs. Quant aux 45 participants, au-delà de ce qu’ils ont appris, ils ont gagné l’accès à un professeur canadien – quelqu’un à qui ils peuvent demander conseil ou qui peut rédiger une lettre de recommandation pour eux. Une expérience enrichissante pour tous – que je répéterais volontiers dans deux ou trois ans.

Pour en savoir plus sur le projet AIMS-NEI : <https://www.nexteinsteinst.org> (en anglais); le bureau canadien est situé à Toronto (<https://www.nexteinsteinst.org/canada/> – en anglais).

Prix Graham-Wright pour service méritoire 2018

En 1995, la Société mathématique du Canada a créé un prix pour récompenser les personnes qui contribuent de façon importante et soutenue à la communauté mathématique canadienne et, notamment, à la SMC. Ce prix était renommé à compter de 2008 en hommage de Graham Wright pour ses 30 ans de service comme directeur administratif et secrétaire de la SMC.

Pour les mises en candidature prière de présenter des dossiers avec une argumentation convaincante et de les faire parvenir, le 31 mars 2018 au plus tard.

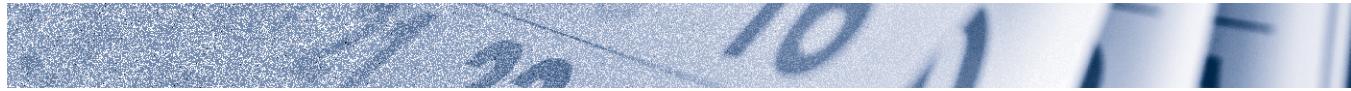
Veuillez faire parvenir tous les documents par voie électronique, de préférence en format PDF, avant la date limite à prixgw@smc.math.ca.

The Calendar brings current and upcoming domestic and select international mathematical sciences and education events to the attention of the CMS readership. Comments, suggestions, and submissions are welcome.

Denise Charron, Canadian Mathematical Society,
(managing-editor@cms.math.ca)

Le calendrier annonce aux lecteurs de la SMC les activités en cours et à venir, sur la scène pancanadienne et internationale, dans les domaines des mathématiques et de l'enseignement des mathématiques. Vos commentaires, suggestions et propositions sont le bienvenue.

Denise Charron, Société mathématique du Canada
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FEBRUARY 2018 FÉVRIER

- 4-9** BIRS Workshop: **Extremal Problems in Combinatorial Geometry**, BIRS, Banff, Alta.
- 11-16** BIRS Workshop: **Relativistic Fermions and Nodal Semimetals from Topology**, BIRS, Banff, Alta.
- 12-14** SUMM 2018 - Seminars in Undergraduate Mathematics in Montreal, Université Concordia, Montreal, Que.
- 13-14** CRM Workshop: **The 2018 Montreal-Toronto Workshop in Number Theory**, CRM, Montreal, Que.
- 18-23** BIRS Workshop: **Modelling Imbalance in the Atmosphere and Ocean**, BIRS, Banff, Alta.
- 26-28** Workshop on **Pollinators and Pollination Modeling**, The Fields Institute, Toronto, Ont.

MARCH 2018 MARS

- 5-9** Workshop on **Human-Environment Systems: Feedback and Management**, The Fields Institute, Toronto, Ont.
- 11-16** BIRS Workshop: **Modular Forms and Quantum Knot Invariants**, BIRS, Banff, Alta.
- 12-16** CRM Workshop: Workshop in Geometric Analysis, CRM, Montreal, Que.
- 14-16** 1st Canadian Geometry and Topology Seminar, The Fields Institute, Toronto, Ont.
- 16-18** BIRS Workshop: **Impact of Women Mathematicians on Research and Education in Mathematics**, BIRS, Banff, Alta.
- 18-23** BIRS Workshop: **New Developments in Open Dynamical Systems and Their Applications**, BIRS, Banff, Alta.

- 19-22** Workshop on **Algebraic Varieties, Hodge Theory and Motives**, The Fields Institute, Toronto, Ont.
- 25-30** BIRS Workshop: **Emerging Trends in Geometric Functional Analysis**, BIRS, Banff, Alta.

APRIL 2018 AVRIL

- 1-6** BIRS Workshop: **Physical, Geometrical and Analytical Aspects of Mean Field Systems of Liouville Type**, BIRS, Banff, Alta.
- 8-13** BIRS Workshop: **Entropies, the Geometry of Nonlinear Flows, and their Applications**, BIRS, Banff, Alta.
- 9-13** Workshop on **Recent Progress in Nonlinear Quantum Mechanics, Theory, Simulations and Experiment**, The Fields Institute, Toronto, Ont.
- 15-20** BIRS Workshop: **Workshop on Geometric Quantization**, BIRS, Banff, Alta.
- 22-27** BIRS Workshop: **Numerical Analysis and Approximation Theory meets Data Science**, BIRS, Banff, Alta.
- 27** 2018 Math Horizons Day, University of Ottawa, Ottawa, Ont.
- 27-29** Conference **First Year University Mathematics Across Canada: Facts, Community and Vision**, The Fields Institute, Toronto, Ont.

JUNE 2018 JUIN

- 1-4** 2018 CMS Summer Meeting / Réunion d'été de la SMC 2018, University of New Brunswick – Fredericton / Université du Nouveau-Brunswick – Frédericton, Fredericton, N.B.
- 4-6** CSHPM 2018 Annual Meeting, Université du Québec à Montréal (UQAM), Montreal, Que.



**La Société
mathématique du
Canada (SMC) est
fière d'annoncer ...**

A près 35 ans, le Congrès international de physique mathématique (CIPM) reviendra en Amérique du Nord en 2018 et se déroulera au Canada pour la première fois. Tenu tous les trois ans, le CIPM est l'événement le plus important de l'Association internationale de physique mathématique. Le XIXe CIPM aura lieu à Montréal en 2018 et, selon la nouvelle tradition, il sera précédé du Symposium des jeunes chercheurs. Ce Symposium se tiendra à l'Université McGill les 20 et 21 juillet, et le CIPM se déroulera au Centre Mont-Royal et à l'Université McGill du 23 au 28 juillet. Le Canada se réjouit à l'idée d'accueillir le monde de la physique mathématique en 2018!

Le CIPM 2018 sera organisé par la SMC en collaboration avec de nombreuses associations des domaines de la physique et des mathématiques, notamment : le CRM, l'Université McGill, le PIMS, l'Institut Fields, l'ISM, l'AARMA, le CANSSI, la SRIB, l'Institut Périmètre, l'Université de Montréal et l'UQAM.

<https://icmp2018.org/fr/inscription>



**The Canadian
Mathematical Society
(CMS) is pleased
to announce ...**

After 35 years, the International Congress on Mathematical Physics (ICMP) will return to North America in 2018, which will also mark the first time that Canada will host the congress. The ICMP, on its three year cycle, is the most important event of the International Association of Mathematical Physics. The XIXth ICMP will take place in Montreal, 2018, and, following recent tradition, it will be preceded by the Young Researchers Symposium (YRS). The YRS will be held at McGill University from July 20 to July 21 and the ICMP will be held at the Centre Mont-Royal and McGill University from July 23 to July 28. Canada is looking forward to welcoming the world of mathematical physics in 2018!

ICMP 2018 will be staged by the CMS in collaboration with many physics and mathematics organizations, including: CRM, McGill University, PIMS, FIELDS, ISM, AARMS, CANSSI, BIRS, Perimeter Institute, U. Montréal, and UQAM.

<https://icmp2018.org/en/registration>

LINEAR ALGEBRA WITH APPLICATIONS

BY W. KEITH NICHOLSON



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Book Reviews brings interesting mathematical sciences and education publications drawn from across the entire spectrum of mathematics to the attention of the CMS readership. Comments, suggestions, and submissions are welcome.

Karl Dilcher, Dalhousie University (notes-reviews@cms.math.ca)

Editor's Note

This issue of the Book Reviews is devoted to three short reviews, written by myself, of recent books published by the MAA Press. Although the books are very different from each other, a common theme is an attempt at dealing with the question, "how can we make the beginning undergraduate experience more interesting and engaging?". While this may be seen as more relevant for the Education Notes, I believe that the books presented here will also be of interest to the general readership of the Notes.

This column is also in recognition of the decades-long excellence of the MAA's book program, which has recently become an imprint of the AMS Book Program.

We look forward to further excellence in mathematics publishing.

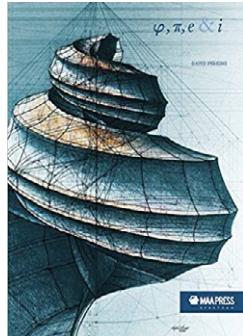
- Karl Dilcher

φ, π, e and i

by David Perkins

MAA Press, 2017

ISBN: 978-1-61444-525-8



Almost everybody would agree that π and e are the two most important constants in mathematics. Although not in the same category, i , the imaginary unit, is also of the greatest importance. The Euler constant γ should certainly be high on a list of special constants; however, a good case can be made that in a book aimed at beginning undergraduates, the formidable γ be replaced by the golden ratio φ .

In recent years, books have been published on each of these constants, but " φ, π, e and i " by David Perkins is rather unique in that it treats the four titular numbers together in one book, with each of the four chapters devoted to one of them. It is certainly reasonable to begin with φ ; of the four numbers it is the easiest, and it provides introductions to some concepts that will recur later in the book, such as irrationality and continued fractions. π and e are next, and the chapter on i , of course, uses these two numbers throughout.

To quote from the MAA website, "The book is written with the goal that an undergraduate student can read the book solo. With this goal in mind, the author provides endnotes throughout, in case

the reader is unable to work out some of the missing steps. Those endnotes appear in the last chapter, Extra Help."

It should also be mentioned that the book contains some historical notes, and each chapter ends with an interesting section on "Further Content". Examples of this are 4 different irrationality proofs for φ in Chapter 1, the BBP (Bailey-Borwein-Plouffe) formula in Chapter 2, an introduction to Euler's constant γ in Chapter 3 (so, it is mentioned after all!), and Hamilton's quaternions in Chapter 4.

I will keep this book in mind to recommend to first-year calculus students who wish to discover more and perhaps get a glimpse into the beginnings of complex variables. It may also serve as a resource for instructors who want to enrich their standard calculus courses.

Exercises in (Mathematical) Style

by John McCleary

MAA Press, 2017

ISBN: 978-0-88385-652-9



This is a fascinating and unusual book in many ways, beginning with the title, which was borrowed from Raymond Queneau. To quote from this book's Preface: "Queneau intended his *Exercises de style* to, in his words, 'act as a kind of rust remover to literature'. I share the same intentions with him for my exercises."

This book could therefore be seen as a 'rust remover to mathematical literature', although this would be a little unfair to some of the fresh and lively books that have recently appeared, especially in the field of enumerative combinatorics. And in fact, the book can be seen and read (and perhaps be used in a classroom or seminar setting) as an introduction to enumerative combinatorics, with some elements of combinatorial number theory. The material is presented in the form of 99 vignettes (again borrowing from Queneau) which are between one and three pages in length. All the important topics in the field are covered, or at least mentioned, including Stirling, Eulerian, Bernoulli and harmonic numbers, as well as q-analogues, partitions, the distribution of primes, and much more; all this while important methods of proof are introduced as well.

The vignettes can be read at random, although some neighbouring vignettes may be related. Many of them have no other prerequisites than high school mathematics, and those that require calculus and

more advanced background are marked in the table of contents. Some 30 pages of "Style notes" at the end give some additional remarks and references for further reading.

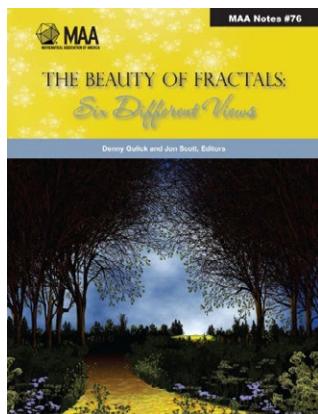
Let me finish by quoting from the book's web pages: "[...] the author investigates the world of that familiar set of numbers, the binomial coefficients. While the reader learns some of the properties, relations, and generalizations of the numbers of Pascal's triangle, each story explores a different mode of discourse – from arguing algebraically, combinatorially, geometrically, or by induction, contradiction, or recursion to discovering mathematical facts in poems, music, letters, and various styles of stories. [...] The ubiquitous nature of binomial coefficients leads the tour through combinatorics, number theory, algebra, analysis, and even topology. The book celebrates the joy of writing and the joy of mathematics, found by engaging the rich properties of this simple set of numbers." Indeed. Open this book at random, read the vignette you find, and you will not be disappointed.

The Beauty of Fractals: Six Different Views

by Denny Gulick and Jon Scott (Editors)

MAA Press, 2017

ISBN: 9780883851869



The Preface of this collection of essays begins with, "Fractals came onto the stage in the 1970s with the emergence of the Mandelbrot set, with its incredibly complicated and interesting boundary. During the 1980s a number of books appeared – including most especially those by Mandelbrot, Barnsley and Devaney – that gave a mathematical background for fractals and made fractals accessible to both students

and teachers. More recently, as computers and their users have become more sophisticated, the domain of fractals has broadened, from art to scientific application to mathematical analysis. In particular, students in high school as well as college are often introduced to fractals and fractal concepts. *The Beauty of Fractals: Six Different Views* includes six essays related to fractals, with perspectives different enough to give you a taste of the breadth of the subject.

"Each essay is self-contained and expository. Moreover, each of the essays is intended to be accessible to a broad audience that includes college teachers, high school teachers, advanced undergraduate students, and others who wish to learn or teach about topics in fractals that are not regularly in textbooks on fractals."

The titles and authors of the essays are: *Mathscapes—Fractal Scenery* by Anne M. Burns; *Chaos, Fractals, and Tom Stoppard's*

Arcadia by Robert L. Devaney; *Excursions Through a Forest of Golden Fractal Trees* by T. D. Taylor; *Exploring Fractal Dimension, Area, and Volume* by Mary Ann Connors; *Points in Sierpiński-like Fractals* by Sandra Fillebrown, Joseph Pizzica, Vincent Russo, and Scott Fillebrown; and *Fractals in the 3-Body Problem Via Symplectic Integration* by Daniel Hemeberger and James A. Walsh.

The essays in this relatively brief volume of just under 100 pages are amply illustrated, and each has its own bibliography. The essays and references, some of which are online resources, could be used as a source for projects, both theoretical and computational, in a linear algebra, discrete mathematics, or analysis course. They could also serve as a source for enrichment material for such courses.

I'm happy to point to some Canadian content: The author of the third essay is Tara Taylor of St. Francis Xavier University; her initial research on the essay was done while she was a PhD student under Dorette Pronk at Dalhousie.

Finally: This book was initially published as an e-book, and it is available for free to all MAA members (along with other interesting and useful e-books). It can also be ordered on a print-on-demand basis.

Book Reviews in Crux Mathematicorum



Crux Mathematicorum (<https://cms.math.ca/crux/>), another CMS publication, also has a Book Reviews column, featuring reviews that might interest readers of the CMS Notes as well. On campuses with institutional membership (as is the case with almost all Canadian universities) there will be free electronic

access to Crux Mathematicorum. During the past year, the following books were reviewed:

Mathematics: Problem-Solving Challenges for Secondary School Students and Beyond by D. Linker and A. Sultan. World Scientific Press, 2016. (Vol. 43, #5).

Guesstimation 2.0 by L. Weinstein. Princeton University Press, 2012. (Vol. 43, #7).

The Calculus of Happiness by O.E. Fernandez. Princeton University Press, 2014. (Vol. 43, #8).

Education Notes brings mathematical and educational ideas forth to the CMS readership in a manner that promotes discussion of relevant topics including research, activities, and noteworthy news items. Comments, suggestions, and submissions are welcome.

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Les Notes pédagogiques présentent des sujets mathématiques et des articles sur l'éducation aux lecteurs de la SMC dans un format qui favorise les discussions sur différents thèmes, dont la recherche, les activités et les nouvelles d'intérêt. Vos commentaires, suggestions et propositions sont les bienvenus.

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On the Teaching of Procedures

Wes Maciejewski, San José State University,
wes@mathwes.ca

Procedures have a precarious status in mathematics education. They are seen as rote and the prevailing thought among primary and secondary educators and education researchers is that they should not be explicitly taught, at least not canonical forms of those procedures (think *division algorithm*). There are sound, research-based reasons for this that do not require a doctorate in education to understand: simply, there are better ways to learn something than being told about that thing. Ideas and concepts that are created or discovered by an individual and practiced extensively and effortfully are the ones that have the greatest sticking power. Moreover, procedures, by definition – at least, by traditional definitions – are context-bound, being tailored to their specific problem type. Concepts, on the other hand, reveal the nature of mathematics – understand those and procedures ought to follow.

The dichotomy above is a naïve characterisation of procedures and concepts which doesn't reflect the current state of understanding in the field. Both are vital to the development of mathematical knowledge. It is, though, a fair claim that the teaching and learning of procedures are currently de-emphasized, at least in primary and secondary schools and even then perhaps only "officially". At the same time, educators at all levels espouse the importance of teaching concepts and many claim that they are the focus of their teaching. Despite this proclaimed focus on the teaching of concepts, much of what comprises an education in mathematics remains as procedures, even at the university level where students are primarily assessed on implementing procedures (Maciejewski and Merchant, 2015).

So, there is a tension: we want to teach the deep, conceptual structure of mathematics but, perhaps for institutional reasons or assessment purposes or other factors, our teaching emphasizes procedures. How might we teach procedures so as to not be rote and to facilitate the growth of conceptual knowledge?

The Procedural/Conceptual Divide

Procedures have come to be associated with superficial knowledge and concepts with depth; the most widely used definitions of these terms have that association present (Hiebert and Lefevre, 1986).

Star (2005) suggests that this is a conflation of knowledge *type* and *quality*. Conceptual knowledge may be deep, but as our students surprise us with regularly, it may also be superficial. Therefore, is it also possible for procedures to be deep? What might be features of depth in procedures?

Deep Procedures?

Star (2005) suggests two features of deep procedures: *flexibility* – the ability to tailor the procedure to the particular task at hand, rearranging steps or skipping them entirely – and *innovation* – the creation of new action sequences in an established procedure to improve its applicability. The presence of either of these demonstrates that procedures *can* be something other than rote.

The genesis of deep procedural knowledge is still under study. What is clear is that practice alone is not always sufficient for the deepening of procedures. Experiences that reveal the inner workings of a procedure, or that call for flexibility, must be encountered. Can we provide such experiences for our students?

Teaching Procedures Deeply

The short answer to that question is: yes, and with only a modicum of extra effort. The key is in making flexibility a centrepiece of your instruction and imbibing the tasks and assessments you give your students with it. In your lectures, for example, when you pose a procedural question, first: don't solve it. Get your students to do that in their notebooks. Second, solicit responses from your students on how they solved it and reenact their solutions in front of the class – on the board, perhaps, or outright project some students' notebooks if you have a document camera. If you have two or more students, you are practically guaranteed to have two or more approaches. Emphasize that both are equally valid (if they yield the correct answer) – one may be longer or require more calculation, but both are fair game. Then poll your class quickly on which of the two they would each personally prefer to use. Draw attention to why one may be more appropriate for the given task, but avoid using language of one being better than another.

Keep in mind: no matter what you do or say in the lecture hall or classroom, your students adapt their study and overall mathematical behaviour to how they are assessed. If your assignment states

“differentiate the following using the quotient rule,” students will learn *the quotient rule*. That is, the task does not invite flexibility nor innovation nor any other potential for deep procedural knowledge. Rather, it promotes a rote view of procedures. Structure your tasks differently. Give a function that looks like it should be differentiated with the quotient rule and prompt your students to use the product rule. Or get your students to re-solve a given derivative task in as many ways they can think of. They will encounter less derivative tasks overall, but will learn more from the ones they do encounter.

I’ve shown in some recent work with Jon Star (Maciejewski and Star, 2016) that such simple approaches *do* promote flexibility in students’ procedures. Two sections of a first-year calculus course for finance and economics students participated in the study: one assigned to treatment, and the other control. At the beginning of the 2 week study, both groups wrote a pre-test on implementing the product and quotient rules. The instructor in the treatment section then emphasized flexibility in their instruction. First, they re-solved the pre-quiz in front of the class by soliciting student responses and displaying multiple solutions to each question. Second, the students were given an assignment in which they were required to differentiate functions each in two different ways: using the quotient or product rule, and then another method after simplifying. The page on this assignment was split vertically and the two solutions written side-by-side (Figure 1). At the bottom of the page the students were asked to comment on the solution they preferred.

Q1. Find the following derivative by using the quotient rule first and simplify your answer as much as possible. Find it again by simplifying first and then using an appropriate derivative rule.

$$\frac{d}{dx} \left[\frac{x^2 - 6x + 9}{x - 3} \right]$$

Quotient Rule First:



Simplify First:

Which of these two methods do you prefer? Why?

Figure 1: Sample from the assignment used in (Maciejewski and Star, 2016)

The control group saw solutions to the pre-quiz, but not multiple solutions. They were also given an assignment, but not prompted to re-solve the questions in another way, nor were they asked to reflect on a preferred solution.

At the end of the study frame, both sections wrote the same post-quiz. Flexibility of each section was measured as the aggregate proportion of solution methods used on the post-quiz. Perhaps not surprisingly, the treatment section did display greater procedural flexibility, and by a wide margin. What is surprising is that this was achieved with a small intervention and over a short time-frame.

There were good reasons to believe such an intervention design would promote flexibility. It incorporated a number of results from the

research literature as design features. The most important feature was the side-by-side comparison. Numerous studies, reviewed in Rittle-Johnson and Schneider (2015), have shown that comparison can develop procedural and conceptual knowledge and promote flexibility. The comparison need not be between two solutions generated by the student, though there is evidence that suggests having students work the problems themselves and become familiar with the procedure deepens learning gains. Comparison could also take place between worked examples of a procedure. In such a case, it may be surprising that students deepen their understandings of a procedure without actually using the procedure. Furthermore, comparison could take place between a correct and an incorrect solution, which, again, can deepen procedural and conceptual knowledge, but also reduce misconceptions.

Research also supports a number of other approaches that were not a main focus of the study design. One such example is the use of self-explanation, for example, in which students are asked to explain *why* a solution is correct or incorrect. Having students work problems *before* instruction on flexibility also appears to improve learning gains. Essentially, a task should have flexibility as a core concept and require students to engage with it in an effort- and mindful way.

There are other features of the study that have yet to be established in the research literature as effective, but seem as though they could be. Having students re-solve the same problem but in a different way seems to get at a form of comparison where the student is heavily involved. Emphasizing flexibility in instruction also seems promising, likely because it promotes a classroom culture of flexibility. The point here is that most, if not all, tasks and instruction can easily be made to include flexibility as a focus.

Conclusions

Procedures, when learned appropriately, can deepen a student’s overall mathematical knowledge. In a sense, there’s a ratcheting effect between conceptual and procedural knowledge: improvements in one can lead to improvements in the other and vice versa (Rittle-Johnson, Schneider, and Star, 2015). The key is that neither should be learned by rote. They ought to be learned deeply and flexibly to ensure transferability to different contexts and to create a richly connected web of mathematical knowledge. Results from the research literature are promising in that procedures can be taught in a way that facilitates such learning – see also (Maciejewski, Mgombelo, and Savard, 2011) for some Canadian perspectives.

Note that this essay is not an endorsement of various “Back to Basics” movements, promoting the re-introduction of “standard” procedures in arithmetic to school mathematics, that often pop

up in Canada and elsewhere in response to curricular changes. I've never found the suggestions of any such groups to be grounded in any rigorous research on how people learn. Moreover, such movements are regressive and work against our students and our discipline – the widespread distaste for mathematics is, in part, a result of the poor teaching of mathematics, the kind that emphasizes memorization and rote procedures. Why would we want to perpetuate poor teaching practices? Rather, I propose appealing to the most advanced contemporary research on the learning of mathematics and use it to inform practice. We can and ought to teach procedures alongside concepts in a way that enriches both and promotes an overall deeper understanding of mathematics.

References and Further Readings

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Prix Ribenboim en théorie des nombres



L'association canadienne en théorie des nombres (CNTA) lance un appel de candidatures pour le prix Ribenboim, lequel reconnaît des contributions importantes dans le domaine de la théorie des nombres réalisées par un mathématicien canadien ou un mathématicien ayant des liens étroits avec les mathématiques au Canada. Le prix est habituellement décerné à tous les deux ans lors des conférences CNTA. Le prochain prix Ribenboim sera décerné lors de la 15^{ème} conférence CNTA laquelle aura lieu du 9 au 13 juillet 2018 à l'Université Laval.

Le récipiendaire recevra un certificat et une médaille, et donnera un exposé lors de la conférence. Habituellement, le récipiendaire devrait avoir obtenu son doctorat au cours des 12 dernières années. Celui qui propose un candidat doit soumettre au comité de sélection, au plus tard le 28 février 2018, les documents suivants:

1. Une lettre de nomination (un individu ne peut se proposer lui-même) et une citation d'au plus 250 mots qui explique pourquoi le candidat devrait recevoir le prix.
2. Un curriculum vitae du candidat d'au plus 20 pages incluant une liste de publications.
3. Au moins 3 lettres de recommandation pour le candidat. Notez que le comité de sélection pourrait exiger des lettres de recommandation supplémentaires s'il le juge nécessaire.

Les proposants sont tenus de soumettre électroniquement les dossiers en format pdf à l'adresse courriel suivante:
hugo.chapdelaine@mat.ulaval.ca

Ribenboim Prize in Number Theory

The Canadian Number Theory Association (CNTA) solicits nominations for the Ribenboim Prize, for distinguished research in number theory by a mathematician who is Canadian or has connections to mathematics in Canada. This prize is normally awarded every 2 years in conjunction with a CNTA meeting. The intention is to award the next prize at the 15th CNTA meeting to be held from July 9–13, 2018 in Quebec City.

The prize winner will receive a certificate and medal and will give a plenary talk at the meeting. Normally the prize winner should have received his or her Ph.D. within the last 12 years. Nominators should submit to the selection committee, by February 28th 2018, the following documents:

1. A letter of nomination (an individual cannot nominate himself or herself), a citation of not more than 250 words, stating why the nominee should receive the award.
2. A curriculum vitae of the nominee of at most 20 pages, including a list of publications.
3. At least 3 supporting reference letters for the nominee.

Note that the selection committee can solicit additional reference letters if it deems it necessary. Electronic submission in pdf format to the following email address is required: hugo.chapdelaine@mat.ulaval.ca

CJM/CMB Associate Editors



The Publications Committee of the CMS solicits nominations for Associate Editors for the Canadian Journal of Mathematics (CJM) and the Canadian Mathematical Bulletin (CMB). The appointment will be for five years beginning January 1, 2019. The current members (with their end of term) are below.

For over fifty years, the Canadian Journal of Mathematics (CJM) and the Canadian Mathematical Bulletin (CMB) have been the flagship research journals of the Society, devoted to publishing original research works of high standard. The CJM publishes longer papers with six issues per year and the CMB publishes shorter papers

with four issues per year. CJM and CMB are supported by respective Editors-in-Chief and share a common Editorial Board.

Expressions of interest should include your curriculum vitae and your cover letter and sent electronically to: cjmcmbo-ednom-2018@cms.math.ca before September 15, 2018.

Current Members of CJM/CMB Editorial Board

Louigi Addario-Berry (McGill)	12/2021	Editor-in-Chief CJM
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Assaf Naor (Princeton)	12/2018	Associate Editor
Nilima Nigam (Simon Fraser)	12/2020	Associate Editor
Alistair Savage (Ottawa)	12/2021	Associate Editor
Juncheng Wei (UBC Vancouver)	12/2018	Associate Editor
Daniel Wise (McGill)	12/2018	Associate Editor

Rédacteur(trice) associé(e) pour le JCM et le BCM

Le Comité des publications de la SMC sollicite des mises en candidatures pour des rédacteurs associés pour le Journal canadien de mathématiques (JCM) et pour le Bulletin Canadien de mathématiques (BCM). Le mandat sera de cinq ans qui commencera le 1er janvier 2019. Les membres actuels (avec la fin de leur terme) sont ci-dessous.

Revues phares de la Société depuis plus de 50 ans, le Journal canadien de mathématiques (JCM) et le Bulletin canadien de mathématiques (BCM) présentent des travaux de recherche originaux de haute qualité. Le JCM publie des articles longs dans ses six numéros annuels, et le BCM publie des articles plus courts quatre fois l'an. Le JCM et le BCM ont chacun leur rédacteur en chef et partagent un même conseil de rédaction.

Les propositions de candidature doivent inclure votre curriculum vitae, votre lettre de présentation et doivent être envoyé par courriel électronique à : cjmcmbo-rednom-2018@smc.math.ca au plus tard le 15 septembre 2018.

Membres actuels du Conseil de rédaction scientifique pour le JCM et le BCM

Louigi Addario-Berry (McGill)	12/2021	Rédacteur en chef JCM
Eyal Goren (McGill)	12/2021	Rédacteur en chef JCM
Jie Xiao (Memorial)	12/2019	Rédacteur en chef BCM
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Fabrizio Andreatta (Università Studi di Milano)	12/2021	Rédacteur associé
Jason Bell (Waterloo)	12/2020	Rédacteur associé
Hans Boden (McMaster)	12/2020	Rédacteur associé
Alexander Brudnyi (Calgary)	12/2020	Rédacteur associé
Krzysztof Burdzy (University of Washington)	12/2021	Rédacteur associé
Guillaume Chapuy (CNRS, Paris)	12/2021	Rédacteur associé
Ilijas Farah (York)	12/2020	Rédacteur associé
Ailana Fraser (UBC Vancouver)	12/2020	Rédactrice associée
Alexander Furman (Illinois Chicago)	12/2021	Rédacteur associé
Wee Teck Gan (National University of Singapore)	12/2021	Rédacteur associé
Dragos Ghioca (UBC Vancouver)	12/2018	Rédacteur associé
Philippe Gille (CNRS & Université Claude Bernard)	12/2021	Rédacteur associé
Vojkan Jaksic (McGill)	12/2021	Rédacteur associé
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On arc-analytic functions and arc-symmetric sets

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In this note, we will mostly deal with semialgebraic geometry, that is, the study of real solutions of systems of polynomial equations and inequalities. A *semialgebraic set* E in \mathbb{R}^n is a finite union of sets of the form

$$\{x \in \mathbb{R}^n : f(x) = 0, g_1(x) > 0, \dots, g_s(x) > 0\},$$

where $s \in \mathbb{N}$ and f, g_1, \dots, g_s are polynomials in real variables $x = (x_1, \dots, x_n)$. A function $f : E \rightarrow \mathbb{R}$ is called semialgebraic if its graph Γ_f is a semialgebraic subset of $\mathbb{R}^n \times \mathbb{R}$. Given an open semialgebraic $U \subset \mathbb{R}^n$, a real analytic semialgebraic function $f : U \rightarrow \mathbb{R}$ is called *Nash*.

Our main object of interest here are the so called *arc-analytic* functions. A function $f : S \rightarrow \mathbb{R}$ on a set $S \subset \mathbb{R}^n$ is said to be arc-analytic when $f \circ \gamma$ is analytic for every real analytic arc $\gamma : (-\varepsilon, \varepsilon) \rightarrow S$.

Arc-analytic functions, although relatively unknown among non-specialists, play an important role in modern real algebraic and analytic geometry (see, e.g., [10] and the references therein). Indeed, Bierstone and Milman [3] proved that arc-analytic semialgebraic functions on a Nash manifold are precisely those that can be made Nash after composition with a finite sequence of blowings-up with smooth algebraic nowhere dense centres. In fact, this criterion is often the quickest way to determine arc-analyticity of a given function. Many classical examples in calculus are arc-analytic but not analytic.

Example 1. (a) The function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ defined as $f(x, y) = x^3/(x^2 + y^2)$ for $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$ is arc-analytic but not differentiable at the origin. Observe that f is made Nash after composition with a single blowing-up of the origin; for instance, $f(x, xy) = x/(1 + y^2)$. Note also that the graph Γ_f of f is not real analytic. In fact, the smallest real analytic subset of \mathbb{R}^3 containing Γ_f is the *Cartan umbrella* $\{(x, y, z) \in \mathbb{R}^3 : z(x^2 + y^2) = x^3\}$ (cf. [9]).

(b) The function $g : \mathbb{R}^2 \rightarrow \mathbb{R}$ defined as $g(x, y) = \sqrt{x^4 + y^4}$ is arc-analytic but not C^2 . The graph Γ_g of g is not real analytic. Indeed, the Zariski closure $\{(x, y, z) \in \mathbb{R}^3 : z^2 = x^4 + y^4\}$ of

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Γ_g has two C^1 sheets $z = \pm\sqrt{x^4 + y^4}$, but it is irreducible at the origin as a real analytic set (cf. [3]).

In general, the behaviour of arc-analytic functions may be surprising, if not pathological. For example, in [4] the authors construct an arc-analytic function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ which is not even continuous. However, in the semialgebraic setting, arc-analytic functions form a very nice family.

Arc-analytic functions were first considered by Kurdyka [9] on arc-symmetric semialgebraic sets. A set E in \mathbb{R}^n is called *arc-symmetric* when, for every analytic arc $\gamma : (-1, 1) \rightarrow \mathbb{R}^n$ with $\gamma((-1, 0)) \subset E$, one has $\gamma((-1, 1)) \subset E$. By a fundamental theorem [9], the arc-symmetric semialgebraic sets are precisely the closed sets of a certain noetherian topology on \mathbb{R}^n . (A topology is called *noetherian* when every descending sequence of its closed sets is stationary.) Following [9], we will call it the *\mathcal{AR} topology*, and the arc-symmetric semialgebraic sets will henceforth be called *\mathcal{AR} -closed sets*.

Given an \mathcal{AR} -closed set X in \mathbb{R}^n , we will denote by $\mathcal{A}(X)$ the ring of arc-analytic semialgebraic functions on X . By [9], the zero locus of every $f \in \mathcal{A}(X)$ is \mathcal{AR} -closed. Interestingly, despite noetherianity of the \mathcal{AR} topology, the ring $\mathcal{A}(\mathbb{R}^n)$ is not noetherian (see [9]).

The usefulness of \mathcal{AR} topology comes from the fact that it contains and is strictly finer than the Zariski topology on \mathbb{R}^n . Moreover, it follows from the semialgebraic Curve Selection Lemma that \mathcal{AR} -closed sets are closed in the Euclidean topology in \mathbb{R}^n .

Noetherianity of the \mathcal{AR} topology allows one to make sense of the notions of irreducibility and components of a semialgebraic set much like in the algebraic case: An \mathcal{AR} -closed set X is called *\mathcal{AR} -irreducible* if it cannot be written as a union of two proper \mathcal{AR} -closed subsets. Every \mathcal{AR} -closed set admits a unique decomposition $X = X_1 \cup \dots \cup X_r$ into \mathcal{AR} -irreducible sets satisfying $X_i \not\subset \bigcup_{j \neq i} X_j$ for each $i = 1, \dots, r$. The sets X_1, \dots, X_r are called the *\mathcal{AR} -components* of X . The decomposition into \mathcal{AR} -components is finer than that into algebraic or Nash components and encodes more algebro-differential information (see [11]). In particular, by a beautiful characterisation of Kurdyka, there is a one-to-one correspondence between the \mathcal{AR} -components of X of maximal dimension and the connected components of a desingularization of the Zariski closure of X .

Desingularization arguments play a very important role in the study of arc-symmetry and arc-analyticity. Together with H. Seyedinejad [1], we used them recently to prove that every \mathcal{AR} -closed set X in \mathbb{R}^n is precisely the zero locus of a certain arc-analytic function $f \in \mathcal{A}(\mathbb{R}^n)$. It thus follows that the \mathcal{AR} topology coincides with the one defined by the vanishing of semialgebraic arc-analytic functions, which is not at all apparent from the intrinsic definition above.

Extending the techniques of [1], most recently we also proved in [2] an arc-analytic analogue of Efroymson's extension theorem [5]: Every arc-analytic semialgebraic function $f : X \rightarrow \mathbb{R}$ on an \mathcal{AR} -closed set $X \subset \mathbb{R}^n$ is, in fact, a restriction of an arc-analytic function $F \in \mathcal{A}(\mathbb{R}^n)$. Moreover, the function F may be chosen real analytic outside the Zariski closure of X .

This result is particularly interesting in the context of the so-called continuous rational functions, which form one of the most active research areas in contemporary real algebraic geometry (see, e.g., [7] and the references therein). A continuous function f is called *continuous rational* if it is generically of the form $\frac{p}{q}$, with p and q polynomial. Continuous rational functions on an \mathcal{AR} -closed set X form a subring of $\mathcal{A}(X)$, and the following example of Kollar-Nowak [8] shows that not every continuous rational function on an \mathcal{AR} -closed set admits an extension to the ambient space as a continuous rational function. Nonetheless, by [2], it does admit an extension as an arc-analytic one.

Example 2. The function $f(x, y, z) = \sqrt[3]{1 + z^2}$ is continuous rational on the real algebraic surface $S = \{(x, y, z) \in \mathbb{R}^3 : x^3 = (1 + z^2)y^3\}$, since $f|_S$ coincides with $\frac{x}{y}|_S$, but it has no continuous rational extension to \mathbb{R}^3 (see [8]). Note that f is Nash, and hence arc-analytic, on \mathbb{R}^3 .

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Betti numbers with a dash of representations

Federico Galetto

Consider an ideal I generated by homogeneous polynomials in the ring $\mathbb{C}[x_1, \dots, x_n]$. We associate to I a sequence of natural numbers $\beta_i(I)$ called the *Betti numbers* of I . Roughly speaking, Betti numbers count the minimal generators and relations of I . Although defined algebraically, Betti numbers carry significant geometric interest. In fact, they control such properties as dimension, degree and regularity of the algebraic variety defined by the common zeroes of polynomials in I [2].

We present an example which takes inspiration from representation theory to give a combinatorial description of Betti numbers (see [3] for more details). Let I be the ideal of $R = \mathbb{C}[x_1, x_2, x_3, x_4]$ generated by all square-free monomials of degree two:

$$x_1x_2, \quad x_1x_3, \quad x_2x_3, \quad x_1x_4, \quad x_2x_4, \quad x_3x_4. \quad (1)$$

The number $\beta_0(I)$ is the minimal number of generators of I . Clearly we have $\beta_0(I) = 6$, as none of the monomials above is an R -linear combination of the other ones. We represent a monomial $x_i x_j$ (with $i < j$) by a standard tableau $\begin{array}{|c|c|} \hline i & j \\ \hline k & l \\ \hline \end{array}$, so $\beta_0(I) = 6$ is the number of standard tableaux of shape $\begin{array}{|c|c|} \hline \square & \square \\ \hline \end{array}$ with entries from $\{1, 2, 3, 4\}$. Note how the symmetric group \mathfrak{S}_4 acts on $\mathbb{C}[x_1, x_2, x_3, x_4]$ by permuting the variables. This action permutes the monomial generators of I (or the corresponding tableaux), giving rise to a representation of \mathfrak{S}_4 .

The next Betti number $\beta_1(I)$ counts the minimal number of relations required to generate all R -linear relations among elements of I , the so-called *first syzygies* of I . For example, $x_3 \begin{array}{|c|c|} \hline 1 & 2 \\ \hline \end{array} - x_2 \begin{array}{|c|c|} \hline 1 & 3 \\ \hline \end{array}$ is a first syzygy of I , because replacing each tableau by the corresponding monomial yields zero. We can produce more relations, using the recipe:

$$\begin{array}{|c|c|} \hline i & j \\ \hline k & \\ \hline \end{array} \mapsto x_i \begin{array}{|c|c|} \hline k & j \\ \hline l & \\ \hline \end{array} - x_k \begin{array}{|c|c|} \hline i & j \\ \hline l & \\ \hline \end{array}, \quad (2)$$

where i, j, k are distinct elements of $\{1, 2, 3, 4\}$. Swapping i and k , simply changes the sign of the relation so, to avoid overcounting, we can make the identification

$$\begin{array}{|c|c|} \hline i & j \\ \hline k & \\ \hline \end{array} = - \begin{array}{|c|c|} \hline k & j \\ \hline i & \\ \hline \end{array}. \quad (3)$$

Moreover, we have an equality

$$\begin{array}{|c|c|} \hline i & j \\ \hline k & \\ \hline \end{array} = \begin{array}{|c|c|} \hline j & i \\ \hline k & \\ \hline \end{array} + \begin{array}{|c|c|} \hline i & k \\ \hline j & \\ \hline \end{array}, \quad (4)$$

which can easily be verified using (2). As before, \mathfrak{S}_4 acts on the tableaux of shape $\begin{array}{|c|c|} \hline \square & \square \\ \hline \end{array}$, giving rise to a representation. Equations (3) and (4) are well-known to hold for this kind of representation, allowing us to rewrite any tableau as a linear combination of the standard tableaux

$$\begin{array}{cccccccc} \begin{array}{|c|c|} \hline 1 & 2 \\ \hline 3 & \\ \hline \end{array} & \begin{array}{|c|c|} \hline 1 & 3 \\ \hline 2 & \\ \hline \end{array} & \begin{array}{|c|c|} \hline 1 & 2 \\ \hline 4 & \\ \hline \end{array} & \begin{array}{|c|c|} \hline 1 & 4 \\ \hline 2 & \\ \hline \end{array} & \begin{array}{|c|c|} \hline 1 & 3 \\ \hline 4 & \\ \hline \end{array} & \begin{array}{|c|c|} \hline 1 & 4 \\ \hline 3 & \\ \hline \end{array} & \begin{array}{|c|c|} \hline 2 & 3 \\ \hline 4 & \\ \hline \end{array} & \begin{array}{|c|c|} \hline 2 & 4 \\ \hline 3 & \\ \hline \end{array} \end{array},$$

namely the tableaux whose entries increase from left to right along rows and from top to bottom along columns. The relations among

generators of I obtained from these standard tableaux using (2) generate all the first syzygies of I . Thus we have $\beta_1(I) = 8$.

Now $\beta_2(I)$ counts the number of minimal generators for the *second syzygies* of I , i.e., the R -linear relations among the first syzygies of I . Similarly $\beta_3(I)$ counts the number of minimal generators for the *third syzygies* of I , and so on. The counting of $\beta_1(I)$ using tableaux generalizes to other Betti numbers as well. For example, the mapping

$$\begin{array}{|c|c|} \hline i & j \\ \hline k & \\ \hline l & \\ \hline \end{array} \mapsto x_i \begin{array}{|c|c|} \hline k & j \\ \hline l & \\ \hline \end{array} - x_k \begin{array}{|c|c|} \hline i & j \\ \hline l & \\ \hline \end{array} + x_l \begin{array}{|c|c|} \hline i & j \\ \hline k & \\ \hline \end{array} \quad (5)$$

produces second syzygies of I . Indeed, applying (2) to the tableaux on the right yields zero. Hence we get $\beta_2(I) = 3$, the number of standard tableaux of shape $\begin{array}{|c|c|} \hline \square & \square \\ \hline \end{array}$ with entries from $\{1, 2, 3, 4\}$. Adding one more box gives a tableau with five boxes; since we only have four entries, we deduce that $\beta_i(I) = 0$ for $i \geq 3$.

The famous Hilbert syzygy theorem implies that, for an arbitrary ideal, $\beta_i(I) = 0$ for all $i \geq n$ (where n is the number of variables) [2]. However, little more can be said in general about Betti numbers of arbitrary ideals. Instead, it is common to restrict to smaller families of ideals. For example, the Betti numbers of ideals generated by square-free monomials have interesting descriptions in terms of the (co)homology groups of certain simplicial complexes [4].

The interplay between representation theory and commutative algebra has lead to many fruitful applications. A spectacular example is the computation of Betti numbers of determinantal ideals by A. Lascoux [5]. Another significant instance is the proof by D. Eisenbud, G. Fløystad, and J. Weyman of one of the Boij-Soederberg conjectures on cones of tables of Betti numbers [1]. More recently, the theory of twisted commutative algebra by S. Sam and A. Snowden [6] pushes the interactions even further with a series of deep results on Betti numbers and other questions in commutative algebra, algebraic geometry, and representation theory.

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Library Classification in Mathematics

Craig Fraser, University of Toronto

The classification of mathematical subjects occurred within the larger framework of library classification, a vast project which drew sustained attention between 1870 and 1920. The two American giants in the formative period were Melvil Dewey and Charles Cutter. In 1876 Dewey published the famous Dewey decimal system of classification, while Cutter's expansive scheme of 1885 would provide the basis for the Library of Congress (LC) system. The latter was established in 1905 by James Hanson and Charles Martel, both European immigrants to the United States.

Among all of the major systems of book classification, the Library of Congress scheme was the one that achieved dominance in university and research libraries. In 1870 the US Copyright Office was by legislation placed in the Library of Congress, and the Library received copies of all publications submitted for copyright. The holdings of the Library increased and became more complete than any elsewhere, including the collections of major university libraries and large public libraries. The importance of the LC system in the world of classification was apparent in the years following its establishment. While the major university libraries with their specialized collections containing many older and foreign-language books continued to maintain a patchwork

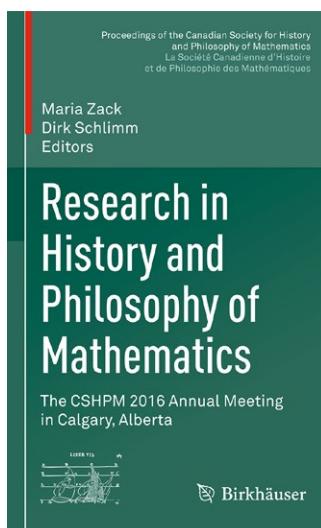
of local classification schemes, the LC's has made steady headway up to the present as the dominant and most widely used.

Unlike book classification, which was aimed at a very broad readership at various levels of engagement with the subject, the practices followed by journals reflected the outlook of advanced researchers in the field. The *Zeitschrift für Mathematik und Physik*, founded in 1856, was one of the first journals to explicitly divide its contents into subject categories. These were presented in this order: arithmetic and analysis, geometry, mechanics, optics, electricity and galvanism, and smaller and miscellaneous subjects. The grouping of analysis with arithmetic and its placement ahead of geometry reflected the prevailing view of advanced researchers, and indicated more generally the well-known "arithmetization of analysis" in mathematics in the nineteenth century.

At the turn of the century the Royal Society of London established the *International Catalogue of Scientific Literature* (1902), a major international bibliographic project that was intended to cover both periodical and book literature. Mathematics (which was also referred to as "pure mathematics") was divided into the following subject areas: fundamental concepts, algebra and number theory, analysis, and geometry. This ordering of subjects became canonical in the classification of twentieth-century mathematical literature, at least as this was followed by the LC and mathematical abstracting services.

The classification schedules for mathematical subjects in the original LC system of 1905 were compiled by J. David Thompson, chief of the

science section, under the direction of Martel, head of classification for the whole of LC. Thompson was a native of England who had studied mathematics at the University of Cambridge, graduating 16th Wrangler in 1895. In the preface to the volume on science (1905, 3) he states that he has relied notably on the schedules of the *International Catalogue of Scientific Literature* (ICSL). While the overall scheme of the LC system was patterned on the Cutter system of classification, the organization of scientific subjects followed the ICSL. The 1905 edition



CSHPM Proceedings

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of the LC science schedules was republished in multiple later editions, each modifying and extending the original scheme.

In the LC classification system books on science are classified under Q, and those on mathematics are classified under QA. In 1905 some parts of mathematics hardly existed yet as recognized subject areas. In the ICSL under arithmetic there was a subject entry on “aggregates”, what would later be called the theory of sets, but there was no entry at all for this subject in the LC. When Abraham Fraenkel’s *Einleitung in die Mengenlehre* appeared in 1919 it was classified in the LC under foundations of arithmetic (QA248) in the algebra section, and that became the standard LC subject classification for books on set theory. A part of mathematics that was very well established in 1905 was analysis, and books on this subject received call numbers in the range from QA300 to QA400. The theory of functions was designated QA331 and was made up of books we would regard today as belonging to complex analysis. The theory of functions of a real variable came to be designated QA331.5, being regarded as a branch or offshoot of the theory of functions. The classification scheme is evident in the following two books on analysis from the early years of the century:

- QA331 Heinrich Burkhard, *Theory of Functions of a Complex Variable* (1913)
- QA331.5 James Pierpont, *Lectures on the Theory of Functions of Real Variables* (1905–12)

When Lars Ahlfors’s *Complex Analysis* was published in 1953 it was given the LC subject designation QA331. In the 1960s “complex analysis” replaced “the theory of functions” as the standard subject name for the theory of functions of a complex variable. One also began to see the publication of books with the term “real analysis” in the title. H.L. Royden’s *Real Analysis* appeared in 1963 and was given the subject designation QA331.5. Thus real analysis was envisaged in this classification scheme as an offshoot of complex analysis. The earlier subject classifications QA331 (theory of functions, implicitly functions of a complex variable) and QA331.5 (theory of functions of a real variable) mapped onto the new subject names “complex analysis” (QA331) and “real analysis” (QA331.5).

In the LC books on analysis with the classification QA300 are devoted to the more general parts of analysis and the foundations of the subject. A widely used primer on analysis for senior undergraduate and graduate students from the 1950s and 1960s was Walter Rudin’s *Principles of Analysis* (1953 and later editions). Rudin’s book was classified under QA300. We have the classification sequence:

- Q Science
- QA Mathematics
- QA300 Rudin *Principles of Analysis*
- QA331 Ahlfors *Complex Analysis*
- QA331.5 Royden *Real Analysis*

By the 1970s some books on real analysis were assigned the designation QA300, and thus were understood to belong to more general parts of analysis, prior in the classification scheme to complex analysis. Other books on real analysis continued to receive the traditional designation QA331.5. There was an overhaul of LC mathematical analysis subject designations in the 1980s, a change

that was completed by around 1990. Here is how the breakdown for subjects in analysis is now given:

- QA300 *Analysis*
- QA300 *General works, treatises, and textbooks*
- QA331 *Theory of functions*
- QA331 *General works, treatises, and advanced textbooks*
- QA331.3 *Elementary textbooks*
- QA331.5 *Functions of real variables*
- QA331.7 *Functions of complex variables; Riemann surfaces including multiform, uniform functions*

Evidently the QA331 section dealing with the theory of functions has been reorganized to reflect the standard order of subject presentation: first general works, followed by elementary presentations, and then, according to some presumably natural principle, an ordered list of the subject areas that fall under the theory of functions. An old principle of book classification followed by the LC that is very useful to the historian is that books are not reclassified when a revision, either major or minor, of the classification system takes place. This seems to be partly for practical reasons—it would be difficult for libraries to be continually reclassifying the materials in their collections. But it should be noted that although the *classification* of a book is not changed, in the LC a later edition of a given book may have a different *call number*. For example, Stanley G. Krantz’s *Function Theory of Several Complex Variables* was classified as QA331 when it appeared in 1982, a designation that remains unchanged to this day, while the second edition of this book in 1992 received the call number QA331.7.

For more information on the subject of the present essay see Fraser (2018).

References

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- [5] Library of Congress. (1905) *Library of Congress Classification Class Q Science*. Washington, D.C.: Government Printing Office.
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CMS 2017 Winter Meeting Recap

Patricia Dack, *Fundraising and Communications Officer, CMS*

More than 350 mathematicians were welcomed to the University of Waterloo for the 2017 CMS Winter meeting, December 8 to 11th. Participants attended 23 Regular Sessions; five Plenary Lectures; four Prize Lectures and one Public Lecture over the course of the meeting.

The meeting opened with an hilarious Public Lecture given by Professor Edward Burger (Southwestern) on "How Always to Win at Limbo" which was enjoyed by a packed house and included an hilarious 'limbo demo' which you can find on the CMS Facebook page!

On Sunday December 10th the CMS Awards Banquet recognized the 2017 CMS Award winners: They are: Professor Richard Hoshino (Quest) recipient of the Adrien Pouliot Award; Bernard Hodgson (Laval) recipient of the Excellence in Teaching Award; and Professor Sabin Cautis (UBC) recipient of the Coxeter-James Prize; Konstantin Tikhomirov (Princeton) recipient of the Doctoral Prize; and Alan Beardon (Cambridge) and Joseph Khouri (Ottawa) recipients of the G de B Robinson Award and the Award for Distinguished Service respectively.

Professor Jeremy Quastel of the University of Toronto was named the recipient of the 2018 Centre de recherche mathématiques - Fields - PIMS Prize Winner.

The Student Committee Awards were also presented at the banquet:

AARMS Prize: Farinaz Forouzannia (Waterloo); CMS President's Prize: Behnoosh Zamanlooy (Concordia); and CMS Student Committee Prize: François Larivière (Montréal).

The CMS would like to acknowledge the financial and administrative support of University of Waterloo, particularly Kenneth Davidson and Cameron Stewart co-Scientific Directors, and a huge thank you to Kathryn Hare for chairing the meeting and to the staff at the University of Waterloo.

Finally we would like to acknowledge the 23 session organizers for their part in making the 2017 CMS Winter Meeting a success.



Kathryn Hare
(Waterloo) receives
a gift from CMS
President Michael
Bennett (UBC)



Cameron Stewart
(Waterloo) receives
a thank you from
CMS President
Michael Benett
(UBC)



Kenneth Davidson
(Waterloo) receives
gift from CMS
President Michael
Bennett (UBC)



Prof. Jeremy Quastel
(Toronto) receives
the CRM - Fields
- PIMS Prize from
CRM director Luc
Vinet (Montréal)

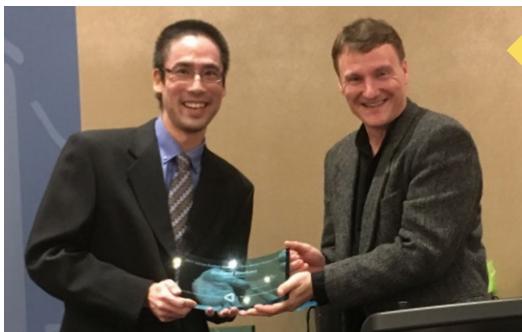


Alexis Langlois-
Rémillard (Montréal)
presents AARMS
Prize to Farinaz
Forouzannia
(Waterloo)



RÉSUMÉ DE LA RÉUNION D'HIVER 2017 DE LA SMC

Behnoosh Zamanlooy (Concordia) reçoit le Prix de la SMC pour la session de présentations par affiches d'Alexis Langlois-Rémillard (Montréal).



Le professeur Richard Hoshino (Quest) reçoit le prix Adrien-Pouliot du président de la SMC, Michael Bennett (UBC).

Le professeur Bernard Hodgson (Laval) reçoit le Prix d'excellence en enseignement du président de la SMC, Michael Bennett (UBC).



Le professeur Edward Berger (Southwestern) nous montre « Comment toujours gagner au limbo ».

Le professeur Sabin Cautis (UBC) prend la pose avec le président de la SMC, Michael Bennett (UBC), après la conférence du prix Coxeter-James.



Résumé de la Réunion d'hiver 2017 de la SMC

Patricia Dack, agente de la collecte de fonds et des communications, SMC

L'Université de Waterloo a accueilli plus de 350 mathématiciens pour la Réunion d'hiver 2017 de la SMC, du 8 au 11 décembre. Les participants ont assisté à 23 sessions régulières, cinq conférences plénières, quatre conférences de lauréats et une conférence publique dans le cadre de la Réunion.

Le programme s'est ouvert sur une conférence publique hilarante du professeur Edward Burger (Southwestern) sur l'art de toujours gagner au limbo, devant une salle comble ravie, et comprenant une désopilante démonstration de limbo (le tout sur la page Facebook de la SMC!).

Le dimanche 10 décembre, le banquet des prix de la SMC a récompensé les lauréats de 2017 : le professeur Richard Hoshino (Quest), prix Adrien-Pouliot; le professeur Bernard Hodgson (Laval), Prix d'excellence en enseignement; le professeur Sabin Cautis (UBC), prix Coxeter-James; Konstantin Tikhomirov (Princeton), Prix de doctorat; Alan Beardon (Cambridge), prix G. de B. Robinson et Joseph Khoury (Ottawa), Prix pour service méritoire.

Le professeur Jeremy Quastel de l'Université de Toronto a reçu quant à lui le Prix Centre de recherches mathématiques-Fields-PIMS 2018.

Les prix pour étudiants ont également été remis au banquet :

Prix AARMS, Farinaz Forouzannia (Waterloo); prix du président de la SMC, Behnoosh Zamanlooy (Concordia); prix du Comité des étudiants de la SMC, François Larivière (Montréal).

La SMC aimerait remercier l'Université de Waterloo de son soutien administratif et financier, et en particulier les codirecteurs scientifiques Kenneth Davidson et Cameron Stewart. Un grand merci également à Kathryn Hare, présidente de la Réunion, et au personnel de l'Université de Waterloo.

Enfin, nous aimerions remercier les 23 organisateurs de sessions d'avoir contribué à faire de la Réunion d'hiver 2017 un franc succès.

2018 Adrien Pouliot Award

Nominations of individuals or teams of individuals who have made significant and sustained contributions to mathematics education in Canada are solicited. Such contributions are to be interpreted in the broadest possible sense and might include: community outreach programs, the development of a new program in either an academic or industrial setting, publicizing mathematics so as to make mathematics accessible to the general public, developing mathematics displays, establishing and supporting mathematics conferences and competitions for students, etc.

Nominations must be received by the CMS Office **no later than April 30, 2018**.

Please submit your nomination electronically, preferably in PDF format, to apaward@cms.math.ca.

Nomination requirements

- Include contact information for both nominee and nominator.
- Describe the nominated individual's or team's sustained contributions to mathematics education. This description should provide some indication of the time period over which these activities have been undertaken and some evidence of the success of these contributions. This information must not exceed four pages.
- Two letters of support from individuals other than the nominator should be included with the nomination.
- Curricula vitae should not be submitted since the information from them relevant to contributions to mathematics education should be included in the nomination form and the other documents mentioned above.
- If nomination was made in the previous year, please indicate this.
- Members of the CMS Education Committee will not be considered for the award during their tenure on the committee.

Renewals

Individuals who made a nomination last year can renew this nomination by simply indicating their wish to do so by the deadline date. In this case, only updating materials need be provided as the original has been retained.

Prix Adrien Pouliot 2018

Nous sollicitons la candidature de personne ou de groupe de personnes ayant contribué d'une façon importante et soutenue à des activités mathématiques éducatives au Canada. Le terme « contributions » s'emploie ici au sens large; les candidats pourront être associés à une activité de sensibilisation, un nouveau programme adapté au milieu scolaire ou à l'industrie, des activités promotionnelles de vulgarisation des mathématiques, des initiatives spéciales, des conférences ou des concours à l'intention des étudiants, etc.

Les mises en candidature doivent parvenir au bureau de la SMC **avant le 30 avril 2018**.

Veuillez faire parvenir votre mise en candidature par voie électronique, de préférence en format PDF, à prixap@smc.math.ca.

Conditions de candidature

- Inclure les coordonnées du/des candidat(s) ainsi que du/des présentateur(s).
- Décrire en quoi la personne ou le groupe mis en candidature a contribué de façon soutenue à des activités mathématiques. Donner un aperçu de la période couverte par les activités visées et du succès obtenu. La description ne doit pas être supérieure à quatre pages.
- Le dossier de candidature comportera deux lettres d'appui signées par des personnes autres que le présentateur.
- Il est inutile d'inclure des curriculums vitae, car les renseignements qui s'y trouvent et qui se rapportent aux activités éducatives visées devraient figurer sur le formulaire de mise en candidature et dans les autres documents énumérés ci-dessus.
- Si la candidature a été soumise l'année précédente, veuillez l'indiquer.
- Les membres du Comité d'éducation de la SMC ne pourront être mis en candidature pour l'obtention d'un prix pendant la durée de leur mandat au Comité.

Renouveler une mise en candidature

Il est possible de renouveler une mise en candidature présentée l'année précédente, pourvu que l'on en manifeste le désir avant la date limite. Dans ce cas, le présentateur n'a qu'à soumettre des documents de mise à jour puisque le dossier original a été conservé.

2018 CMS Winter Meeting

December 7-10, 2018

Sheraton Vancouver Wall Centre, Vancouver, British Columbia

CALL FOR SESSIONS

Deadline: **March 31, 2018**

The Canadian Mathematical Society (CMS) welcomes and invites session proposals for the 2018 CMS Winter Meeting in Vancouver from December 7 - 10, 2018. Proposals should include a brief description of the focus and purpose of the session, the expected number of speakers, as well as the organizer's name, complete address, telephone number, e-mail address, etc. Sessions will be advertised in the CMS Notes, on the web site and in the AMS Notices. Speakers will be requested to submit abstracts, which will be published on the web site and in the meeting program. Those wishing to organize a session should send a proposal to the Scientific Directors.

Scientific Directors:

Franco Saliola (Université du Québec à Montréal)
saliola.franco@uqam.ca

Malabika Pramanik (University of British Columbia)
malabika@math.ubc.ca

Réunion d'hiver de la SMC 2018

7-10 décembre 2018

Sheraton Vancouver Wall Centre, Vancouver, Colombie Britannique

APPEL DE PROPOSITIONS DE SESSIONS

Date limite : **31 mars 2018**

La Société mathématique du Canada (SMC) vous invite à proposer des sessions pour la Réunion d'hiver de la SMC qui aura lieu à Vancouver du 7 au 10 décembre 2018. Les propositions doivent présenter une brève description de l'orientation et des objectifs de la session, le nombre de conférenciers prévu, de même que le nom, l'adresse complète, le numéro de téléphone et l'adresse électronique de l'organisateur. Toutes les sessions seront annoncées dans les Notes de la SMC, sur le site Web et dans les notices de l'AMS. Les conférenciers devront présenter un résumé, qui sera publié sur le site Web et dans le programme de la réunion. Toute personne qui souhaiterait organiser une session est priée de faire parvenir une proposition à un des directeurs scientifiques.

Directeur scientifique:

Franco Saliola (Université du Québec à Montréal)
saliola.franco@uqam.ca

Malabika Pramanik (University of British Columbia)
malabika@math.ubc.ca



2018 CMS Summer Meeting

June 1 – 4, 2018

University of New Brunswick, New Brunswick, Fredericton

The Canadian Mathematical Society (CMS) welcomes and invites session proposals for the 2018 CMS Summer Meeting in Fredericton from June 1 to 4, 2018. Proposals should include a brief description of the focus and purpose of the session, the expected number of speakers, as well as the organizer's name, complete address, telephone number, e-mail address, etc. Sessions will be advertised in the *CMS Notes*, on the web site and in the AMS Notices. Speakers will be requested to submit abstracts, which will be published on the web site and in the meeting program. Those wishing to organize a session should send a proposal to the Scientific Directors.

Scientific Directors:

Colin Ingalls (University of New Brunswick)

cingall@unb.ca

Alexandre Girouard (Université Laval)

alexandre.girouard@mat.ulaval.ca



Réunion d'été de la SMC 2018

1 – 4 juin, 2018

l'Université du Nouveau Brunswick, Nouveau-Brunswick,
Fredericton

La Société mathématique du Canada (SMC) vous invite à proposer des sessions pour la Réunion d'été de la SMC qui aura lieu à Fredericton du 1 au 4 juin 2018. Ces propositions doivent présenter une brève description de l'orientation et des objectifs de la session, le nombre de conférenciers prévu, de même que le nom, l'adresse complète, le numéro de téléphone et l'adresse électronique de l'organisateur. Toutes les sessions seront annoncées dans les Notes de la SMC, sur le site Web et dans les notices de l'AMS. Les conférenciers devront présenter un résumé, qui sera publié sur le site Web et dans le programme de la réunion. Toute personne qui souhaiterait organiser une session est priée de faire parvenir une proposition à un des directeurs scientifiques.

Directeur scientifique :

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