CMS
NOTES
de la SMC

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2018

Journeying towards a 'thinking classroom'. . 11

IN THIS ISSUE / DANS CE NUMÉRO

CMS
SMC

Canadian Mathematical Society
Société mathématique du Canada

Vice-President's Notes / Notes de la Vice présidente

Sara Faridi *(Dalhousie University)*
Vice-President - Atlantic / Vice-présidente - Atlantique

Common Ground

What unites us as mathematicians is our love of mathematics. It goes beyond our personal, national or cultural identities. Some of us are attracted to the grand open problems, some to the glorious past, some to the intriguing puzzles, and some to the art of sharing it with enthusiastic students. In a way, mathematics is a culture of its own within which we mathematicians live. Unlike so many others, we are paid to inhabit a world we love.

A typical mathematical event is multicultural, but one can easily be blind to that because the math culture takes over, and people are often focused on a narrow field. The excitement of sharing our interests shrouds our differences.

This can change slightly if you visit a math department in a different country and immerse yourself in that department's culture. Each department does things a bit differently. However they have different lunch routines. However, there are many similarities: there are seminars, colloquia, students, exams and defenses, poster boards announcing upcoming conferences and interesting problems to think about. No matter how exotic the new land, the math department is an island offering the comforts of home. Just like Starbucks, you can trust it to be the same the world over!

At a recent meeting of Women in Mathematics in Nepal, I had a completely different experience. I had already been to a conference at Thribhuvan University in Kathmandu the previous year, and had enjoyed the conference, the people, and

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Terrain d'entente

Les mathématiciens sont unis par l'amour des mathématiques. C'est un lien plus fort que l'identité personnelle, culturelle ou nationale. Certains d'entre nous ont une prédilection pour les grands problèmes ouverts, d'aucuns pour le passé glorieux, d'autres encore par les casse-tête les plus mystérieux, et d'autres enfin sont attirés par l'art de partager leur passion avec des étudiants enthousiastes. D'une certaine façon, les mathématiques sont une culture en soi, celle où vivent les mathématiciens. À la différence de tant d'autres, nous sommes payés pour vivre dans un monde que nous adorons.

Une rencontre typique de mathématiciens est multiculturelle, mais il est facile de l'oublier parce qu'une culture prend le dessus : celle des mathématiques. Nous nous concentrons sur un champ plutôt étroit. La fièvre de l'échange voile ce qui nous distingue.

La situation sera légèrement différente si vous visitez un département de mathématiques dans un autre pays et que vous vous immergez dans sa culture propre. Chacun fait les choses un peu différemment, ne serait-ce que les habitudes qui entourent le repas de midi. Mais les similarités sont nombreuses : il y a des séminaires, des colloques, des étudiants, des examens et des soutenances de thèse, des affiches annonçant une conférence à venir et des problèmes intéressants qui appellent à la réflexion. Peu importe l'exotisme du lieu, le département de mathématiques est une île où vous retrouverez le confort de votre chez-soi. Tout comme Starbucks : identique partout au monde!

Or, j'ai participé récemment à une rencontre des femmes mathématiciennes au Népal, et j'y ai vécu une expérience complètement

Suite à la page 5

What Is A Textbook Worth?

Robert Dawson, *St. Mary's*
CMS Notes Editor-in-Chief

Last term I was teaching a differential equations course, the second of two. The first course had adopted a textbook I wasn't familiar with. I obtained a copy, and when I looked at it, I realized that while it wouldn't do for the course as described in the calendar, it came fairly close. It had a decent treatment of linear systems, and a very respectable treatment of partial differential equations. But there was no chapter for the four weeks on systems of nonlinear equations - though one short section several chapters earlier defined a few useful concepts such as "phase plane."

How did we get into such a mess? Well, quite a few years back my university had separate math courses for engineering students, and there are specialized engineering math texts that (like the fat calculus books) cover all those courses between one (rather widely spaced) pair of covers. Then we realized that, as the physicists' and mathematicians' rather similar needs were setting the pace in the other courses, we might as well combine the streams, and we dropped the Engineers' Big Book of Math. But some patterns of textbook selection had got frozen into the system.

I decided to use the text from the previous course, with handouts for the missing material. It was a fair amount of work, but it made sense for the students. Nobody should have to buy a $150 textbook for one seventy-page chapter! But it made me think again about what the strange economics of textbook marketing have done to prices. Set up a bookstore near a campus, and try to sell $150 novels, or cookbooks, or travel books. Should be easy, right? After all, the students pay that kind of price for textbooks all the time. And good luck...

How about faculty members? We may occasionally buy a professional treatise for that sort of money out of our own pockets: but we'll use grant money if we can, or try to persuade the library to buy it. And we know that those research books have fairly small print runs, whereas popular undergraduate textbooks enjoy sales that many fiction authors ($14.99 paperback, $24.99 hardback, at a bookstore near you) would envy. No, not every calculus textbook sells so well - but a company with an underperforming calculus textbook doesn't just lower the price till customers take an interest. Why not? Because the purchasers don't make the decision, their professors do.

A new development here is the B.C. Open Textbook Project, founded about five years ago. It has assembled a collection of high quality peer-reviewed textbooks in many subjects - including differential equations! - that are electronically available at no charge. BC campus provides curation and peer-reviewing; the various public-spirited authors make their work available. In many cases, students or instructors who have not outlived the love of books have the alternative of buying a commercially-printed copy from a third party publisher, at prices that generally seem reasonable. Even more intriguingly, at least textbooks are under Creative Commons licenses that allow instructors to adapt them for the need of the course, with TeX available.

Maybe this isn't the answer to everybody's problems, but it seems like an interesting new degree of freedom in an otherwise-overdetermined problem. Check it out at **https://open.bccampus.ca/**.

Que vaut un manuel de cours?

La session dernière, je donnais un cours sur les équations différentielles, le dernier d'une série de deux. Je ne connaissais pas le manuel de cours qui avait été utilisé pour la première partie. Je m'en suis procuré un exemplaire et lorsque je l'ai ouvert, j'ai vite constaté qu'il ne correspondait pas tout à fait au programme de mon cours tel qu'annoncé dans l'annuaire. Il abordait la plupart des notions, comme celles sur les systèmes linéaires, et il couvrait en profondeur les équations aux dérivées partielles, mais il ne contenait aucun chapitre sur les équations non linéaires, un sujet auquel nous devions consacrer quatre semaines. Il n'y avait à ce sujet qu'une courte section où l'on trouvait la définition de quelques concepts utiles, comme le « plan de phase ».

Comment s'était-on retrouvés dans cette fâcheuse situation? Je vous explique. Il y a plusieurs années, mon université offrait des cours de mathématiques distincts pour les étudiants en génie. Il existe des manuels de cours en mathématiques du génie qui (comme les gros manuels de calcul) englobent toute la matière enseignée entre deux couvertures généreusement espacées. Puis, on s'est aperçu que les besoins semblables des physiciens et mathématiciens donnaient le rythme à tous les autres cours et que les deux volets pouvaient être réunis. On a donc mis de côté le grand livre de mathématiques des ingénieurs. Mais certains mécanismes concernant le choix des manuels de cours sont demeurés figés dans le système.

J'ai décidé d'utiliser tout de même le manuel du cours précédent et de le compléter avec du matériel photocopié. C'était une tâche considérable, mais pleine de bon sens pour les étudiants. Personne ne devrait avoir à débourser 150 $ pour accéder à un seul chapitre de 70 pages d'un énorme manuel! Cette anecdote m'a fait réfléchir sur l'effet que les singulières techniques de commercialisation des manuels ont sur les prix. Ouvrez une librairie près d'une université et vendez-y des romans, des livres de recettes et des guides de voyage pour 150 $. Ça devrait fonctionner puisque les étudiants paient régulièrement ce prix pour des manuels, non? Mais en réalité, bonne chance!

Qu'en est-il du corps enseignant? À l'occasion, nous achèterons peut-être de notre poche un traité professionnel vendu à ce prix, mais nous utiliserons autant que possible l'argent d'une bourse. Sinon, nous tenterons de persuader la bibliothèque d'en faire l'acquisition. Nous savons de plus que ces ouvrages de recherche sont imprimés en peu d'exemplaires, alors que les manuels de cours de premier cycle enregistrent des ventes que bien des auteurs de fiction envieraient. (Vous trouverez leurs œuvres à 14,99 $ en format poche ou 24,99 $

en version cartonnée dans toute bonne librairie.) Même les manuels de calcul ne s'écoulent pas aussi bien – mais une entreprise qui publie un livre de calcul boudé par les clients ne baissera pas son prix jusqu'à ce qu'ils s'y intéressent. Pourquoi? Parce que ce ne sont pas les acheteurs qui décident : ce sont leurs professeurs.

Il y a cinq ans, une nouvelle initiative a vu le jour : le B.C. Open Textbook Project. Ce projet a permis de constituer une collection de manuels de cours évalués par les pairs, d'une grande qualité, portant sur de nombreux sujets – dont les équations différentielles! – et accessibles gratuitement en format électronique. De nombreux auteurs dévoués à l'intérêt public partagent librement leur travail dans cette grande banque de savoir, les services de conservation et d'évaluation par les pairs étant offerts par BCcampus. S'ils le souhaitent, les étudiants et les enseignants qui préfèrent encore les ouvrages papier peuvent dans bien des cas se procurer des exemplaires imprimés commercialement par un tiers éditeur à un prix qui, généralement, paraît raisonnable. Il est également fort intéressant de noter que les manuels sont régis par des licences Creative Commons permettant aux enseignants de les adapter à leur cours, notamment à l'aide du logiciel TeX.

Ce n'est peut-être pas la solution à tous les problèmes, mais c'est une idée qui apporte un nouveau degré de liberté dans un domaine autrement beaucoup trop rigide. Pour en savoir plus sur cette initiative, visitez le **https://open.bccampus.ca/**.

Letters to the Editors

The Editors of the NOTES welcome letters in English or French on any subject of mathematical interest but reserve the right to condense them. Those accepted for publication will appear in the language of submission. Readers may reach us at the Executive Office or at **notes-letters@cms.math.ca**

Lettres aux Rédacteurs

Les rédacteurs des NOTES acceptent les lettres en français ou en anglais portant sur n'importe quel sujet d'intérêt mathématique, mais ils se réservent le droit de les comprimer. Les lettres acceptées paraîtront dans la langue soumise. Les lecteurs peuvent nous joindre au bureau administratif de la SMC ou à l'adresse suivante : **notes-lettres@smc.math.ca**.

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Continued from cover

the country immensely. During that meeting, the local female mathematicians approached the female conference participants for help in professional development. The prospects for female Nepalese mathematicians, they said, were limited at best.

It seemed natural to want to help, so some colleagues and I decided to organize a conference in Kathmandu inviting women mathematicians from Southeast Asian countries (who are culturally similar to the Nepalese) to share their own experiences of mathematical life: obstacles they have had to overcome, motivation of students and outreach, where to look for opportunities, roads to success, and so on.

We spent the next year planning, fundraising and writing to people. Most places we applied to were happy to support the event financially, and we managed to reach out to a large body of women: many from Nepal and neighbouring countries, and several more from Europe, the US, Japan, Korea and Canada.

The event was a success, we met our goal. We even made an appearance on National TV!

We talked about mathematics and being mathematicians, and once again, after much effort to get everyone together, we were in the familiar setting of a mathematics conference.

But this time things were different: the dominant culture was not that of mathematics, but that of that region, dictated by geography and by cultural norms. The women from the region shared their stories of what it took for them to do mathematics. While for me, mathematics was a choice of many fields I made in high school, for them it was a change of destiny and a breaking of all sorts of cultural barriers. Many successful female mathematicians from this area were wives, daughters-in-law, and mothers first, and mathematicians in their very limited spare time. Some picked their life-partners with a view of who would most be able and willing to support their mathematical career. Many women typically completed each degree after completing a family phase (children going to school, children getting married). There were women well into their forties who were contemplating their next academic degree.

The experience was humbling, and the obstacles faced by our colleagues seemed insurmountable.

As we planned the banquet dinner, we thought we should share some motivational stories with these women, about the journeys of female mathematicians in the previous century and their hardships, to smoothen their rocky road.

The Nepalese women disagreed. What we perceived as a rocky road was normal life for them, and the fact that we were there looking for ways to make things better warranted a celebration. It was a party after all, and there ought to be a talent contest! So we put all things in perspective, and spent the evening singing and dancing and celebrating our common love of mathematics in their way, and according to the cultures of the region.

2018 Graham Wright Award for Distinguished Service

In 1995, the Society established this award to recognize individuals who have made sustained and significant contributions to the Canadian mathematical community and, in particular, to the Canadian Mathematical Society. The award was renamed in 2008, in recognition of Graham Wright's 30 years of service to the Society as the Executive Director and Secretary.

Nominations should include a reasonably detailed rationale and be submitted by **March 31, 2018**.

All documentation should be submitted electronically, preferably in PDF format, by the appropriate deadline, to **gwaward@cms.math.ca**.

Suite de la couverture

différente. J'étais allée un an auparavant à un congrès organisé par l'Université Thribhuvan, à Katmandou, et j'avais beaucoup apprécié les présentations, les gens et le pays. À cette occasion, les mathématiciennes ont demandé aux participantes de les aider sur le plan du perfectionnement professionnel. Les possibilités offertes aux mathématiciennes népalaises sont pour le moins limitées, disaient-elles.

Il semblait tout naturel d'aider et c'est pourquoi quelques collègues et moi avons décidé d'organiser un congrès à Katmandou et d'inviter les mathématiciennes des pays d'Asie du Sud-Est (de culture semblable à celle du Népal) à relater leur propre expérience de leur vie en mathématiques, les obstacles qu'elles ont dû surmonter, la motivation des étudiants et le rayonnement, la recherche des possibilités, les voies de la réussite, etc.

Nous avons passé un an à planifier, à trouver du financement, à écrire à gauche et à droite. La plupart des organismes sollicités étaient heureux d'offrir leur aide financière, et nous avons réussi à toucher beaucoup de femmes, dont une bonne part au Népal et dans les pays voisins et plusieurs autres en Europe, aux États-Unis, au Japon, en Corée et au Canada.

L'activité a été un succès et nous avons atteint notre objectif. Nous avons même eu droit à un reportage à la télévision nationale.

Nous avons parlé de mathématiques et de la vie de mathématicienne, et une fois encore, au prix de valeureux efforts et après avoir réuni tout le monde, nous nous sommes retrouvées dans le contexte familier d'un congrès de mathématiques.

Cette fois, pourtant, les choses étaient différentes. La culture dominante n'était pas celle des mathématiques, mais celle de la région, dictée par la géographie et les normes culturelles. Les femmes de la région ont raconté ce qu'il leur a fallu faire pour arriver à leurs fins. Si pour moi les mathématiques résultent de nombreux choix que j'ai fait au secondaire, pour elles, c'est un changement de destinée et le renversement de barrières culturelles en tous genres. Dans cette région, nombre de mathématiciennes accomplies ont d'abord été épouses, belles-filles et mères, puis mathématiciennes à temps perdu (une denrée d'ailleurs rare). Certaines ont choisi leur conjoint en fonction de sa capacité et de sa volonté d'appuyer leur carrière en mathématiques. Beaucoup de ces femmes ont obtenu un grade après avoir franchi une étape de la vie familiale (l'entrée des enfants à l'école ou le mariage des plus vieux). Certaines, à plus de 40 ans, envisagent l'obtention d'un grade supérieur.

Belle leçon d'humilité. Comme ces obstacles nous ont semblé insurmontables!

Tandis que nous organisions le banquet, nous avons pensé échanger avec ces femmes quelques anecdotes de nature à nous motiver toutes sur le périple de mathématiciennes des siècles passés et leurs difficultés, afin d'aplanir un peu leur chemin difficile.

Nos collègues népalaises n'étaient pas d'accord. Ce qui nous semble un chemin difficile, c'est pour elle la vie, tout simplement, et le fait que nous ayons été là à chercher avec elles des moyens d'améliorer la situation méritait d'être célébré. Nous avons donc fait la fête et, bien sûr, le concours d'amateurs était de mise! Nous avons remis les choses en perspective, afin de jouir pleinement d'une soirée de chants et de danses, à célébrer notre passion commune pour les mathématiques, à la façon de nos collègues et suivant les cultures de la région.

Prix Graham-Wright pour service méritoire 2018

En 1995, la Société mathématique du Canada a créé un prix pour récompenser les personnes qui contribuent de façon importante et soutenue à la communauté mathématique canadienne et, notamment, à la SMC. Ce prix était renommé à compter de 2008 en hommage de Graham Wright pour ses 30 ans de service comme directeur administratif et secrétaire de la SMC.

Pour les mises en candidature prière de présenter des dossiers avec une argumentation convaincante et de les faire parvenir, **le 31 mars 2018** au plus tard.

Veuillez faire parvenir tous les documents par voie électronique, de préférence en format PDF, avant la date limite à **prixgw@smc.math.ca**.

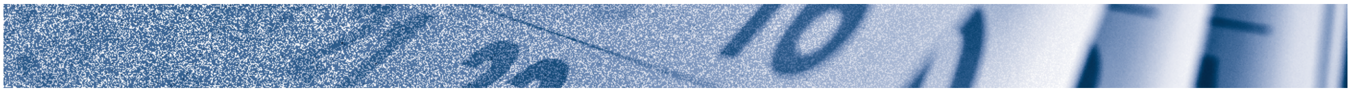
CMS Notes
Volume 50 No. 2, March-April 2018

The Calendar brings current and upcoming domestic and select international mathematical sciences and education events to the attention of the CMS readership. Comments, suggestions, and submissions are welcome.

Denise Charron, *Canadian Mathematical Society, (managing-editor@cms.math.ca)*

Le calendrier annonce aux lecteurs de la SMC les activités en cours et à venir, sur la scène pancanadienne et internationale, dans les domaines des mathématiques et de l'enseignement des mathématiques. Vos commentaires, suggestions et propositions sont le bienvenue.

Denise Charron, *Société mathématique du Canada (redacteur-gerant@smc.math.ca)*



MARCH 2018 MARS

5-9	Workshop on Human-Environment Systems: Feedback and Management, The Fields Institute, Toronto, Ont.
11-16	BIRS Workshop: Modular Forms and Quantum Knot Invariants, BIRS, Banff, Alta.
12-16	CRM Workshop: Workshop in Geometric Analysis, CRM, Montreal, Que.
14-16	1st Canadian Geometry and Topology Seminar, The Fields Institute, Toronto, Ont.
16-18	BIRS Workshop: Impact of Women Mathematicians on Research and Education in Mathematics, BIRS, Banff, Alta.
18-23	BIRS Workshop: New Developments in Open Dynamical Systems and Their Applications, BIRS, Banff, Alta.
19-22	Workshop on Algebraic Varieties, Hodge Theory and Motives, The Fields Institute, Toronto, Ont.
25-30	BIRS Workshop: Emerging Trends in Geometric Functional Analysis, BIRS, Banff, Alta.

APRIL 2018 AVRIL

1-6	BIRS Workshop: Physical, Geometrical and Analytical Aspects of Mean Field Systems of Liouville Type, BIRS, Banff, Alta.
8-13	BIRS Workshop: Entropies, the Geometry of Nonlinear Flows, and their Applications, BIRS, Banff, Alta.
9-13	Workshop on Recent Progress in Nonlinear Quantum Mechanics, Theory, Simulations and Experiment, The Fields Institute, Toronto, Ont.
15-20	BIRS Workshop: Workshop on Geometric Quantization, BIRS, Banff, Alta.
22-27	BIRS Worshop: Numerical Analysis and Approximation Theory meets Data Science, BIRS, Banff, Alta.
23-26	CRM Workshop: Workshop on Modern Challenges of Learning Theory, CRM, Montreal, Que.
27	2018 Math Horizons Day, University of Ottawa, Ottawa, Ont.
27-29	Conference First Year University Mathematics Across Canada: Facts, Community and Vision, The Fields Institute, Toronto, Ont.

MAY 2018 MAI

Apr 29-6	BIRS Workshop: Algebraic Structure of Cyclic Combinatorial Objects, BIRS, Banff, Alta.
11	Math on the move - 13e édition des 24 heures de science, CRM, Montréal, Qué.
13-18	BIRS Workshop: Asymptotically Hyperbolic Manifolds, BIRS, Banff, Alta.
14-18	School on Probability in Number Theory, CRM, Montreal, Que.
20-25	BIRS Workshop: Topics in the Calculus of Variations: Recent Advances and New Trends, BIRS, Banff, Alta.
21-Jun 1	CRM Workshop: Workshop on Probability in Number Theory, CRM, Montreal, Que.
22-25	Graph Complexes, Configuration Spaces and Manifold Calculus, University of British Columbia, B.C.
27-Jun 1	BIRS Workshop: Adaptive Numerical Methods for Partial Differential Equations with Applications, BIRS, Banff, Alta.
28-Jun 1	Shimura varieties and hyperbolicity of moduli spaces, UQAM, Montreal, Que.

JUNE 2018 JUIN

1-4	2018 CMS Summer Meeting / Réunion d'été de la SMC 2018, University of New Brunswick – Fredericton / Université du Nouveau-Brunswich – Frédericton, Fredericton, N.B.
1-5	CMESG (Canadian Mathematics Education Study Group) 2018 Conference, Quest University, Squamish, B.C.
3-8	BIRS Workshop: Hydraulic Fracturing: Modeling, Simulation, and Experiment, BIRS, Banff, Alta.
4-6	CSHPM 2018 Annual Meeting, Université du Québec à Montréal (UQAM), Montreal, Que.
4-7	CAIMS Annual Meeting 2018, Ryerson University, Toronto, Ont.
4-15	Séminaire de Mathématiques Supérieures, The Fields Institute, Toronto, Ont.
17-22	BIRS Workshop: Advanced Developments for Surface and Interface Dynamics - Analysis and Computation, BIRS, Banff, Alta.
24-29	BIRS Workshop: New Trends in Syzygies, BIRS, Banff, Alta.

JULY 2018 JUILLET

1-7	18th International Conference on Fibonacci Numbers and Their Applications, Dalhousie University, Halifax, N.S.
11-15	Canadian Undergraduate Mathematics Conference 2018, University of Saskatchewan, Saskatoon, Sask.
23-28	XIX International Congress on Mathematical Physics / XIXe Congrès international de la physique mathématique, Centre Mont-Royal, Montreal, Que.

2018 CMS Winter Meeting

December 7-10, 2018

Sheraton Vancouver Wall Centre, Vancouver, British Columbia

CALL FOR SESSIONS

Deadline: **March 31, 2018**

The Canadian Mathematical Society (CMS) welcomes and invites session proposals for the 2018 CMS Winter Meeting in Vancouver from December 7 - 10, 2018. Proposals should include a brief description of the focus and purpose of the session, the expected number of speakers, as well as the organizer's name, complete address, telephone number, e-mail address, etc. Sessions will be advertised in the CMS Notes, on the web site and in the AMS Notices. Speakers will be requested to submit abstracts, which will be published on the web site and in the meeting program. Those wishing to organize a session should send a proposal to the Scientific Directors.

Scientific Directors:

Franco Saliola (Université du Québec à Montréal)
saliola.franco@uqam.ca

Malabika Pramanik (University of British Columbia)
malabika@math.ubc.ca

Réunion d'hiver de la SMC 2018

7-10 décembre 2018

Sheraton Vancouver Wall Centre, Vancouver, Colombie Britannique

APPEL DE PROPOSITIONS DE SESSIONS

Date limite : **31 mars 2018**

La Société mathématique du Canada (SMC) vous invite à proposer des sessions pour la Réunion d'hiver de la SMC qui aura lieu à Vancouver du 7 au 10 décembre 2018. Les propositions doivent présenter une brève description de l'orientation et des objectifs de la session, le nombre de conférenciers prévu, de même que le nom, l'adresse complète, le numéro de téléphone et l'adresse électronique de l'organisateur. Toutes les sessions seront annoncées dans les Notes de la SMC, sur le site Web et dans les notices de l'AMS. Les conférenciers devront présenter un résumé, qui sera publié sur le site Web et dans le programme de la réunion. Toute personne qui souhaiterait organiser une session est priée de faire parvenir une proposition à un des directeurs scientifiques.

Directeur scientifique:

Franco Saliola (Université du Québec à Montréal)
saliola.franco@uqam.ca

Malabika Pramanik (University of British Columbia)
malabika@math.ubc.ca

Book Reviews brings interesting mathematical sciences and education publications drawn from across the entire spectrum of mathematics to the attention of the CMS readership. Comments, suggestions, and submissions are welcome.

Karl Dilcher, *Dalhousie University (notes-reviews@cms.math.ca)*

Les comptes-rendus de livres présentent aux lecteurs de la SMC des ouvrages intéressants sur les mathématiques et l'enseignement des mathématiques dans un large éventail de domaines et sous-domaines. Vos commentaires, suggestions et propositions sont les bienvenus.

Karl Dilcher, *Dalhousie University (notes-critiques@smc.math.ca)*

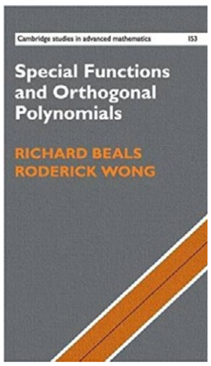
Special Functions and Orthogonal Polynomials

By Richard Beals and Roderick Wong

Cambridge University Press, 2016

ISBN: 978-1-107-10698-7

Reviewed by *Robert Milson*, Dalhousie University



The book under review is in equal measure a reference work on special functions and a textbook aimed at an advanced student working in a related area. Indeed, the core of the book comprises the authors' previous effort in this area, "Special Functions, a Graduate Text". The current version expands on the previous title by deepening existing sections and adding extra material, such as a chapter on Painlevé transcendants.

It may also be compared to "Special Functions" by Andrews, Askey, and Roy. While this latter work has some interesting specialist material, such as a section on q-identities, the present title wins my favour because it is both more readable and because it addresses a broader range of topics. The Beals and Wong book will certainly be of interest to individuals who attend meetings like the OPSFA series (International Symposium on Orthogonal Polynomials, Special Functions and Applications); but it will also serve as a useful reference to the broader community working in mathematical physics and integrable systems. This exceedingly well-written book is both comprehensive and accessible, exhibiting a remarkable balance between breadth and depth. The essential details are largely there, but the authors know when to draw the line so as to maintain the whole at a manageable size.

The book is constructed around a main theme, but contains a fair number of significant variations. As one may expect of a text on special functions, there is comprehensive coverage of classical orthogonal polynomials, hypergeometric functions, and related special functions that arise in applications. However, the authors also included a wealth of related topics: gamma and beta functions, asymptotic analysis, Riemann-Hilbert problems, general hypergeometric functions, elliptic functions, and a final chapter on Painlevé transcendants.

After an introductory chapter, the book opens with Chapter 2 on gamma, beta, and zeta functions. This material is essential for later application to asymptotic analysis of hypergeometric functions. For example, connection coefficients for hypergeometric solutions are expressed in terms of Γ, while B(x,y) is needed for the integral representation of the hypergeometric function.

Chapter 3 covers some essential elements of Sturm-Liouville theory, mostly specialized to Schrödinger-type operators. The book covers the bare minimum; this is not a reference on spectral theory. Here we find the comparison theorems, an introduction to symmetric operators, and the two types of basic transformation so essential for applications: changes of the independent variable and gauge transformations.

Chapter 4 follows as an introduction to the general theory of orthogonal polynomials: Hankel determinants, Padé approximants, measures, and some basic theorems on zeros. The coverage of topics is largely at an introductory level, but there are some strategic forays into more advanced and interesting material. For example, there is a proof of the existence part of Favard's theorem: 3-term relations satisfying a positivity condition correspond to a positive measure. However, the authors wisely avoid getting deeper into moment theory and do not tackle uniqueness. This is a characteristic feature of this book. It serves as a comprehensive introduction to everything, but the authors know where to draw the line, and provide pointers to more specialized literature.

Having covered the fundamentals of second-order linear differential equations and orthogonal polynomials, the book is now perfectly set up to treat classical orthogonal polynomials. The authors provide their own version of the Bochner-Lesky proof that the classical class is comprised of the celebrated families of Hermite, Laguerre, and Jacobi polynomials. This happens back in Chapter 3, as an aside on polynomial solutions. This is a bit curious as, in my opinion, logically this material belongs in Chapter 5, the chapter on classical O.P. There is also an enumeration of some well known subtypes of the Jacobi class, such as Legendre and Chebyshev polynomials, and some basic results on the asymptotics of the zeros. However, deeper analysis awaits the subsequent chapters devoted to techniques of asymptotic analysis. All in all, Chapter 5 reads largely as a handbook. To the authors' credit, they spice things up and conclude the chapter with some nice applications: completeness proofs of generalized expansions and a subsection on the electrostatic interpretation.

Chapter 6 follows with a brief diversion into the world of discrete orthogonal polynomials. After developing some foundational material regarding symmetric difference operators, the authors derive and enumerate the list of classical discrete polynomials:

Charlier, Meixner, Krawtchouk and Hahn — the discrete analogues of the Hermite, Laguerre, and the Jacobi families. Much of this section has a handbook feeling, but this is not unreasonable because the material is so essential and therefore serves a useful purpose in a work of reference. My only complaint is the choice of the title: semi-classical orthogonal polynomials. The latter is a well-known technical term introduced by Hendriksen and van Rossum and extended by Maroni in 1985 and, in common usage, does not refer to polynomials orthogonal with respect to a discrete measure. Thus, the "semi-classical" terminology is potentially confusing. My impressions is that most people would simply refer to the polynomials in this section as classical discrete polynomials.

Chapter 7 is an introduction to the Riemann-Hilbert method, which is presented as a tool for studying the asymptotic distribution of the zeros of classical OP. In recent times the RH method has become a leading tool of asymptotic analysis, and a chapter devoted to this important topic adds a lot of value to the book. Introductory lectures on the RH method crop up frequently in summer schools for graduate students, but up to now the choice of introductory reference material has been quite limited. The appearance of a modern book on special functions with an RH chapter is therefore quite welcome.

In Chapter 7, the authors introduce the Fokas-Its-Kitaev formulation of classical OP via a 2 x 2 Riemann-Hilbert problem, and then apply the Deift-Zhou method of non-linear steepest descent approach to derive both global and local asymptotics. The length of the exposition remains under control because the authors employ the didactic approach of working out one particular example, namely the asymptotics of the Hermite zeros, rather than attempt to develop a general theory. Another very nice feature of this chapter is that the RH method is contrasted to the more classical Plancherel-Rotach asymptotics. This chapter also exhibits and applies the important Sokhotski—Plemelj formula for scalar RH problems; the proof is left for an appendix.

Chaters 8-11 comprise a great deal of standard material on hypergeometric and related functions. Here we find the notions of contiguity, connection formulas, etc — a lot of useful and well known material that is readily available in many places. Perhaps the true value of these chapters are the exercises, which permit the interested student to go through the material in a systematic fashion. There are also some very interesting highlights, such as the derivation of the quadratic transformations for the hypergeometric functions.

The subsequent chapters are more eclectic, and represent a carefully chosen sampling of topics that connect naturally to special functions. Chapter 12 is a brief introduction to generalized hypergeometric functions. This is a large and active research area, and as per the general philosophy of the book, the authors content themselves with a carefully chosen introduction. Two highlights that caught my eye were subsections on the Meijer G-functions, and on the Mellin transform.

Chapter 13 is another foray into asymptotic analysis. This time the focus is on the WKB approximation, which is carefully derived as an application of the Liouville transformation. The second half of this section introduces the method of steepest descent. Asymptotic analysis is one of the great themes of the present book and these two topics certainly merit inclusion.

The final part of the book, Chapters 14 and 15, could almost be read independently of the rest of the book. The former deals with elliptic functions and the latter with Painlevé transcendants. On the surface, these are rather disconnected topics, but of course it is only reasonable to gather them under the umbrella of a book on special functions. The strategy is the same: a carefully chosen introduction of the topics, always stopping short of more intricate, and advanced developments. The chapter on elliptic function covers all the basics: elliptic integrals, the Jacobi inversion, classical theta functions, and finally Weirstrass's approach via series and the p-function. The Painlevé chapter focuses on PII, which is amply sufficient to introduce a number of key notions: rational and special function solutions, the isomonodromy formulation, and Bäcklund transformations. There is even room for a quick return to asymptotic analysis, which treats the asymptotic behaviour of the second transcendant for large X.

Overall, my impression of this book is overwhelmingly positive. Every chapter is both interesting and useful. The authors have included copious exercises; each chapter ends with a historical overview, and the bibliography is well researched. I would have no hesitation in recommending the book to PhD students in either mathematical physics or integrable systems — either for independent study or as part of a more formal course. Because of its mixed character, the book can also serve as a useful reference for established researchers: it is part handbook, part reference, part exposé of a certain number of key topics. For my part, I will keep this tome close at hand and imagine that it will acquire that well-worn look characteristic of the best books in one's collection.

Near The Horizon: An Invitation to Geometric Optics

By Henk W. Broer

MAA Press, 2017 (Carus Math. Monographs 33)

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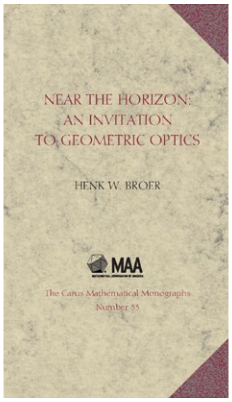

Imagine that you're on a westward-facing coast, watching the sun set. It's so low in the sky that you can look right at it, an orange blob slowly sinking into the sea. And then—just as the last sliver is about to disappear—a blob of light detaches itself from the top and turns green. That's the famous "green flash", caused by different wavelengths of light bending by different amounts as they pass through the nonhomogeneous atmosphere: have you ever seen it?

Or maybe you're on a ship at sea. The sextant and chronometer (or, today, the GPS) tell you that you're many days out from the nearest land. And yet, on the horizon, you see fantastic peaks and towers, wide platforms stacked impossibly on narrow columns: the fata morgana. Again, this is caused by light bending - in this case, bending enough that it follows the curvature of the Earth - and carrying the distorted image of land over the horizon to your eyes. (Similar things can be seen looking out over water from the shore, or even over dry land.)

There is one form of mirage that every landlubber has seen: the heat mirages on asphalt roads that look like silvery puddles ahead of you, but which "evaporate" as you draw near. Those mirages differ from the green flash and fata morgana, in that they are formed by light bent *upward* by hot air near the asphalt. If you're curious about any of these phenomena, Henk Broer's *Near The Horizon: An Invitation To Geometric Optics* should be of considerable interest to you.

The second part of the title is potentially misleading. To many people, the term "geometric optics" suggests primarily the study of lenses and mirrors. If you've picked up an old data projector at a yard sale and you want to know how to design a telescope using the lenses from your find, this is not the place to look! The book might be more accurately entitled "An Invitation to the Geometric Optics of the Atmosphere." It's a specialized treatise, and in its slim hundred and sixty pages, it does what it does very well indeed.

It begins with a description of common mirage types, and the physics involved - light travels along geodesics, locally minimizing the time of travel through a possibly inhomogeneous medium. (The density, hence refractive index, of air is affected both by height and by temperature. In the simplest models the density drops with increasing height - but for meteorological reasons that is not always the case.)

Phenomena such as total internal reflection are described and explained, and a first atmospheric model is derived - in which the Earth is flat and the atmosphere perfectly stratified like a *pousse-café* into discrete layers. This is already sufficient to explain some phenomena; but the author quickly generalizes it to the round Earth and the limit of continuously varying density. Many interesting phenomena are then explained.

The history of the subject permeates the book, but though some of these phenomena were studied before calculus was available, the author does not attempt to use "period" mathematics when better tools exist. There are interesting discussions of the brachistochrone problem and the geometry of the Poincaré upper halfplane, both of which are closely related.

The second part of the book goes more deeply into the theory, including a clear and interesting introduction to Hamilton's principle, geodesics on surfaces of revolution, and isometric embeddability.

All in all, it's a fascinating read. It would be accessible to an advanced undergraduate student in mathematics or physics, but most researchers in either discipline would find much new in it. It's affordable enough for a personal collection, and should certainly be in university libraries.

Notes de la SMC
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Education Notes brings mathematical and educational ideas forth to the CMS readership in a manner that promotes discussion of relevant topics including research, activities, and noteworthy news items. Comments, suggestions, and submissions are welcome.

John McLoughlin, *University of New Brunswick (johngm@unb.ca)*
Kseniya Garaschuk, *University of Fraser Valley (kseniya.garaschuk@ufv.ca)*

Les Notes pédagogiques présentent des sujets mathématiques et des articles sur l'éducation aux lecteurs de la SMC dans un format qui favorise les discussions sur différents thèmes, dont la recherche, les activités et les nouvelles d'intérêt. Vos commentaires, suggestions et propositions sont les bienvenus.

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Education issues (both challenges and successes) have been getting more and more attention recently. This was evident at the recent CMS meeting in Waterloo, where math education sessions attracted significant audiences of 50-some people. The topic of first-year university courses and our approach to them unveiled innovative projects and research done in various institutions across the country. We hope to continue the conversation throughout this year in Education Notes.

In the first piece of 2018, Wes Maciejewski shared some findings on the flexible teaching of procedures, which are ever-present in our first-year courses. This second issue features another take on pedagogy in the undergraduate classroom. Judy Larsen offers insight into the development of her own teaching while sharing on thinking classrooms and community building. Consider sending along your contribution to this focus on undergraduate mathematics, teaching, and related issues.

Journeying towards a 'thinking classroom': From covering content to community building

Judy Larsen, *University of the Fraser Valley (judy.larsen@ufv.ca)*

> If you cast the student in a passive role, then saying that he has "studied" your course may mean no more than saying of a cat that he has looked at a king. Mathematics is something that one does. (Moise, 1965)

Teaching as professing

My early experiences with teaching mathematics, or rather 'professing' it, in a university setting was that which many consider to be the normal experience. That is, you walk into a room, students are sitting in rows, books and technology armed around them as a form of protection and safety. Students position themselves carefully at the beginning of the term. Those in the back seem to yearn the least likelihood they will be asked to engage, and those near the door seem to be planning a stealthy escape. While this may be an uninspiring overgeneralization, I've had many experiences that seem to confirm these norms. What's more is that as the class proceeds, the curriculum, which is laid out in the textbook, logically unfolds on the board while students re-represent the ideas in the professor's interpretation. Control over the content rests in the hands of the professor.

While there are positive aspects to this form of 'covering' content, as a young and aspiring teacher, I quickly noticed its faults. Namely, no matter how slowly I proceeded, no matter how many questions I asked the class, there always seems to remain a trifurcation of the student body into those who couldn't keep up, those who found it easy and boring, and those who were tuned in. A small portion of those who were tuned in would answer my questions, giving me an inflated sense that I was doing a good job. However, even those who claimed to understand during class, would come to me afterwards and say that when they tried it at home, they were unable to use the ideas themselves. That is, even if they could 'follow' me in class, they couldn't independently recreate the thought process and reapply it. Although my student body typically consists of pre-undergraduate level adults trying to redeem themselves from their pre-calculus prerequisites, I sense this situation is familiar to many.

Offloading the Responsibility

In 2011, I came across the then-popular notion of the 'flipped classroom', coined by Bergman and Sams (2012), which is an instructional strategy that capitalizes on "the affordances provided by increasingly accessible technologies to deliver content asynchronously out of class time while dedicating class time for student learning" (Larsen, 2015, p. 51). I soon found myself creating instructional videos for all my course content and making it available online via YouTube for my students. I assigned videos for homework so that students could preview the content prior to engaging with it in class. I could then spend class time leading students through a variety of group activities and problems. In retrospect, the flipped classroom approach was a great way to step out of the idea that I needed to review all the material myself explicitly in class to satisfy my duties as an instructor.

Students also took well to the idea, and I found that those who chose to participate in all elements of the class, particularly the group activities, evidenced significant improvements in anxiety, self-efficacy, and orientation towards learning (Larsen, 2015). It also allowed students to self-pace through the content, and some, even with very poor attendance, were still able to succeed. However, the opposite also happened – those who participated in classroom activities still succeeded even without watching content videos. How were they learning if I wasn't 'telling' them the content and they weren't taking notes?

'Thinking' classrooms

It is hard to attribute all my successes to a flipped classroom because I made the class space very collaborative, which is not always typical for this approach, and many students indeed didn't watch the videos, and still succeeded. In fact, I eventually let go of emphasizing the necessity of previewing content with videos because it seemed to become an excuse that let students opt-out of participating in class activities by saying they needed to watch the videos instead. The main difference, then, was that my class time became a place for meaning making and collaboration around mathematical ideas, and my structures for leading such sessions had already been heavily influenced by the work of Peter Liljedahl, which has now become known as 'building thinking classrooms' (Liljedahl, 2016). I soon became more focused on how to build a thinking classroom within the class time I had and less focused on making sure what students did prior to my class. In fact, I now don't want students to preview the material as I find that it often spoils the aha effect.

Liljedahl (2016) defines a thinking classroom as:

> A classroom that is not only conducive to thinking but also occasions thinking, a space that is inhabited by thinking individuals as well as individuals thinking collectively, learning together and constructing knowledge and understanding through activity and discussion. It is a space wherein the teacher not only fosters thinking but also expects it, both implicitly and explicitly. (p. 364)

His work is rooted in research on the aha! experience and mathematical problem solving, and has developed into a set of tools that make such experiences possible. With over a decade of research on the mathematics classroom practices of teachers in various contexts, Liljedahl (2016) questioned the unquestioned elements of mathematics classrooms, often governed by institutional norms of school. Among other more recently emerging aspects, Liljedahl (2016) explored and identified ideals for:

1. the type of tasks used and when and how they are used
2. the way in which tasks are given to students
3. how groups are formed, both in general and when students work on tasks
4. student workspace while they work on tasks
5. room organization, both in general and when students work on tasks
6. how questions are answered when students are working on tasks
7. the ways in which hints and extensions are used, while students work on tasks
8. when and how a teacher levels their classroom during or after tasks
9. assessment, both in general and when students work on tasks (p. 365)

My intention here is not to summarize the large body of work on thinking classrooms, but rather to share my own experiences with implementing these ideas within a university context and to entice the reader to dig deeper into Liljedahl's work. However, to give a brief sense of the setting, I will outline a few key elements that most prominently characterize the desired 'thinking classroom' setting.

As one may expect, using rich tasks that engage students to collaboratively and evoke mathematical habits of mind is important, particularly at the beginning when the culture is being established. Near the beginning of a course, very unique and exciting tasks need to be used to spark interest and reset the focus towards processes and meaning making. However, once this culture is set, curricular tasks may be used if they are presented in a way that leaves students with something to think about. Giving the tasks orally with some supports of diagrams allows for students to dive into making meaning together rather than decoding instructions from text.

Students work in visibly random groups, which are randomized frequently enough that students don't feel overly committed or burdened by their group. Visibly randomizing groups has proven to be very effective at breaking down social barriers (Liljedahl, 2014) and contributing to a positive classroom culture. Further, students work on vertical non-permanent surfaces in their groups, usually in threes and twos, and share a pen to pursue the given problem collaboratively. Since they are writing on non-permanent surfaces, they may easily erase something they think is wrong (although they

rarely do), and since the surfaces are vertical, ideas may travel around the room and keep students engaged in the task.

This sort of space often naturally changes the dynamic of the room from being centered around the teacher's space to what Liljedahl refers to as 'defronting' the room. This means that a teacher may then watch students working around the room rather than be watched by them. The debrief after a task provides an opportunity to make important connections among concepts that all learners can access based on the work they have done. Although there are further tools that require more nuanced treatment, the above approaches have a significant enough effect to change the nature of the classroom and bring in a focus on *thinking*.

Teaching as culture building

Having had the incredible opportunity to experience Dr. Liljedahl's work in a variety of contexts (i.e. elementary schools, secondary schools, professional development workshops, undergraduate courses, graduate courses, etc.), I have found myself honing my own teaching practice around the principles developed out of the thinking classroom research. It is not possible to implement all the recommendations right away, but it is possible to feel a large enough change in student engagement quickly through some of the more obvious tools described (i.e., good problems, visibly random groups, vertical non-permanent surfaces).

Over the years, I have continued to make small tweaks to my implementation, and have experienced the profound effects they have had on my students. Namely, I've noticed the importance of keeping students in 'flow' (Csíkszentmihályi, 1990). One way to conceptualize this idea is that no one is ever finished, but rather, there is always more to learn. To engender this sort of attitude in the classroom, it is important to encourage students to act more autonomously. This encompasses efforts to learn how to challenge themselves or find another problem if they are done, to develop ways to convince themselves if they have achieved a plausible solution, etc. This takes a lot of work, and often takes a few weeks for me before my students realize they have ownership over their own learning, and that I am not there to provide them with answers. One way to maintain this stance is to not give in to answering questions that do not result in further thinking about the problem. Yes, this includes confirming for them that an answer is correct. Rather, I often ask them if they are convinced, and if so, why they think so. This communicates to them that my role is not content-giver, but thought-provoker.

Through my journey of transforming my teaching practice, I have learned that my primary duty as a teacher is to generate a culture in my room where students can engage in mathematical thinking in a safe space governed by respectful norms of collaboration and responsibility for not only one's own thinking, but the thinking of others in the room. One way I've been able to emphasize this within a university setting is to include a portfolio component to my grade in which students are expected to present to me their contributions, their aha! moments, and their moves towards making the classroom a better space for everyone. While I'm not entirely satisfied, which I may never be, assessment is the area that I am continuing to work on. Assessment is important because it communicates to my students what I value, and therefore, contributes to the culture of my classroom.

A new vision of teaching and learning

Although I have not gathered empirical evidence on my students' experiences in a thinking classroom, the anecdotal evidence I have encountered supports the findings of other empirical work on the topic. Most importantly, students enjoy coming to classes, they often lose track of time and choose to work on mathematics through their break times. They tell me about how much they've learned, and sometimes even reveal to me how much their attitude towards mathematics has changed. If only one student reveals a significant shift in attitude towards mathematics, I feel accomplished because I know I have initiated a spark that may one day come alive again.

I no longer plan my lessons around all the content I need to 'cover', but around the possible content we may likely encounter, which is driven by an ecological metaphor for learning. I still 'teach', but my interpretation of this word has changed. I 'teach' when the moment becomes 'teachable', but to make the space capable of occasioning such occurrences, a culture of thinking must be established. I know I have succeeded when I can stand in the centre of my room observing students engaging in mathematical thinking and discourse, but they don't see me. When I become invisible, the community in my room becomes visible.

References

[1] Bergmann, J., & Sams, A. (2012). *Flip your classroom: Reach every student in every class every day* [Kindle Edition]. Retrieved from Amazon.com

[2] Csíkszentmihályi, M. (1990). *Flow: The psychology of optimal experience.* New York: Harper & Row.

[3] Larsen, J. (2015). Adult students' experiences of a flipped mathematics classroom. *Adults Learning Mathematics: An International Journal*, 10(1), 50-67.

[4] Liljedahl, P. (2014). The affordances of using visually random groups in a mathematics classroom. In Y. Li, E. Silver, & S. Li (eds.), *Transforming Mathematics Instruction: Multiple Approaches and Practices.* New York, NY: Springer.

[5] Liljedahl, P. (2016). Building thinking classrooms: Conditions for problem solving. In P. Felmer, J. Kilpatrick, & E. Pekhonen (eds.), *Posing and Solving Mathematical Problems: Advances and New Perspectives.* (pp. 361-386). New York, NY: Springer.

[6] Moise, E. (1965). Activity and motivation in mathematics. *The American Mathematical Monthly*, 72(4), 407-412.

Research Notes brings mathematical research ideas forth to the CMS readership in a generally accessible manner that promotes discussion of relevant topics including research (both pure and applied), activities, and noteworthy news items. Comments, suggestions, and submissions are welcome.

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Les Notes de recherche présentent des sujets mathématiques aux lecteurs de la SMC dans un format généralement accessible qui favorise les discussions sur divers sujets pertinents, dont la recherche (pure et appliquée), les activités et des nouvelles dignes de mention. Vos commentaires, suggestions et propositions sont les bienvenus.

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On certain diophantine equations and the dynamical Mordell-Lang problem

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1. Some diophantine equations and the Mordell-Lang conjecture

We start with the simple diophantine equation $3^x + 4^y = 5^z$ in integers (x, y, z). One can quickly write down two solutions, namely $(0, 1, 1)$ and $(2, 2, 2)$. After a more careful consideration, it is not hard to show that those are the only solutions. While this equation only has finitely many solutions, a slightly more complicated one such as

$$3^x + 5^y = 7^z + 9^t \text{ for } x, y, z, t \in \mathbb{Z} \tag{1}$$

could have infinitely many such as the family of solutions $\{(2t, 0, 0, t) : \ t \in \mathbb{Z}\}$. Likewise, the "translated" version

$$3^x + 7 \times 5^y = 5 \times 7^z + 9^t \tag{2}$$

of equation (1) has the infinite family of solutions $\{(2t, 1, 1, t) : \ t \in \mathbb{Z}\}$.

From a geometric point of view, we can put $X = 3^x$, $Y = 5^y$, $Z = 7^z$, and $T = 9^t$. Then solving equation (1) is equivalent to describing the intersection between the hyperplane $X + Y = Z + T$ to the finitely generated subgroup $\Gamma = \{(3^a, 5^b, 7^c, 9^d) : \ a, b, c, d \in \mathbb{Z}\}$ of the group $(\mathbb{C}^*)^4$. The family $\{(2t, 0, 0, t)\}_{t\in\mathbb{Z}}$ "comes from" the subset of the hyperplane given by equations $Y = Z = 1$ and $X = T$; this is also an algebraic subgroup of $(\mathbb{C}^*)^4$. Similarly, with $X' = 3^x$, $Y' = 7 \times 5^y$, $Z' = 5 \times 7^z$, and $T' = 9^t$, the given family of solutions of (2) comes from the subset of the given hyperplane defined by $Y' = 7$, $Z' = 5$, and $X' = T$ which is actually the translate of the above algebraic subgroup by the point $(1, 7, 5, 1)$.

We now turn our attention to systems of equations. In the paper [2], Brassil and Reichstein reduce a problem considered by Hermite and Joubert in the 19th century to the study of the following system:

$$Z_1^3 + Z_2^3 + 9Z_3^3 = 0 \tag{3}$$

$$3^a Z_1 + 3^b Z_2 + Z_3 = 0 \tag{4}$$

with $a, b \in \mathbb{Z}$ and $[Z_1 : Z_2 : Z_3] \in \mathbb{P}^2(\mathbb{Q})$. The set of points $[Z_1 : Z_2 : Z_3] \in \mathbb{P}^2(\mathbb{C})$ satisfying the cubic equation (3) is an elliptic curve E after we choose a point O_E as the identity. For any fixed $(a, b) \in \mathbb{Z}^2$, we are looking at the intersection between E and the projective line $3^a Z_1 + 3^b Z_2 + Z_3 = 0$. By Bézout's theorem, this intersection has three points $[Z_1 : Z_2 : Z_3] \in \mathbb{P}^2(\mathbb{C})$. However, we are looking for *rational* points $[Z_1 : Z_2 : Z_3] \in \mathbb{P}^2(\mathbb{Q})$. As explained in [4], if K is a finitely generated field of characteristic 0 (such as $\mathbb{Q}$, $\mathbb{Q}(\sqrt{2})$, $\mathbb{Q}(t)$, etc.), then the above system has only finitely many solutions $(a, b, [Z_1 : Z_2 : Z_3]) \in \mathbb{Z}^2 \times \mathbb{P}^2(K)$ outside the infinite family

$$\{(m, m, [1 : -1 : 0]) : m \in \mathbb{Z}\}. \tag{5}$$

Put $X = 3^a$ and $Y = 3^b$, then solving for $(a, b, [Z_1, Z_2, Z_3]) \in \mathbb{Z}^2 \times E(K)$ in the above system is the same as describing the intersection between the subvariety V of $\mathbb{C}^* \times \mathbb{C}^* \times E$ given by $XZ_1 + YZ_2 + Z_3 = 0$ and the finitely generated subgroup $\Gamma = \{(3^a, 3^b) : \ a, b, \in \mathbb{Z}\} \times E(K)$. The family (5) comes from the translate of the algebraic subgroup $X = Y$ of $\mathbb{C}^* \times \mathbb{C}^* \times E$ by the element $(1, 1, [1 : -1 : 0])$.

These are examples of the much more profound Mordell-Lang conjecture which was proved by Faltings, Vojta, and McQuillan. Let X be a semiabelian variety (for example $(\mathbb{C}^*)^4$ or $(\mathbb{C}^*)^2 \times E$ as above) and let V be a subvariety of X. Let Γ be a finite rank subgroup of X which means that $\mathbb{Q} \otimes_{\mathbb{Z}} \Gamma$ is a finite-dimensional $\mathbb{Q}$-vector space. Roughly speaking, the Mordell-Lang conjecture states that the intersection $V \cap \Gamma$ is "induced" from the subvarieties of V which are translates of algebraic subgroups of X.

2. Dynamical analogue

A discrete dynamical system is a set X together with a map φ from X to itself and dynamics is the study of the family of iterates $\{\varphi, \varphi^2, \ldots\}$. After imposing structures on the pair (X, φ), we arrive at many highly interesting branches of dynamics. For example, in topological dynamics, X is a (compact metrizable) topological space and φ is continuous. In ergodic theory, X is a probability space and φ is a measure-preserving transformation. In arithmetic dynamics, X is an algebraic variety over a field K and φ is a morphism over K. An important source of problems in arithmetic dynamics comes from diophantine results on algebraic groups such as the following Dynamical Mordell-Lang conjecture:

Conjecture 2.1 *(Ghioca-Tucker) Let X be a quasi-projective variety and let φ be a morphism from X to itself defined over a field K*

of characteristic 0. *Let* V *be a subvariety of* X *and let* $P \in X$. *Then the set:*

$$\{n \in \mathbb{N}: \ \varphi^n(P) \in V\}$$

is the union of a finite set and finitely many arithmetic progressions.

In the previous section, when X is a semiabelian variety, the Mordell-Lang conjecture states that the intersection between a subvariety V and a finite rank subgroup Γ of X "comes" from translates of algebraic subgroups of X. In Conjecture 2.1, the orbit $\mathcal{O}_\varphi(P) := \{\varphi^n(P): \ n \in \mathbb{N}\}$ plays the role of the small subgroup Γ and we study the intersection $V \cap \mathcal{O}_\varphi(P)$. What should be an analogue of a translate of an algebraic subgroup and why do we need arithmetic progressions in Conjecture 2.1? Note that for any group G and $m \in \mathbb{N}$, we have the power map $x \mapsto x^m$ from G to itself and any subgroup of G is invariant under this map. Now supposed that an arithmetic progression of the form $(mk+\ell)_{k\in\mathbb{N}}$ belongs to the set in the conclusion of Conjecture 2.1, then we have that $\varphi^{mk+\ell}(P) \in V$ for every $k \in \mathbb{N}$. In other words, the appearance of the arithmetic progression $(mk+\ell)_{k\in\mathbb{N}}$ follows from the fact that the φ^m-invariant set $\{\varphi^{mk+\ell}(P): \ k \in \mathbb{N}\}$ belongs to V.

Another motivation for Conjecture 2.1 comes from the well-known Skolem-Mahler-Lech theorem which states that the zero set of a linear recurrence sequence is the union of a finite set and finitely many arithmetic progressions. Now consider the case when $X = \mathbb{C}^N$ and φ is the affine map $\varphi(\mathbf{x}) = A\mathbf{x} + \mathbf{v}$ where A is a given $N \times N$ matrix and $\mathbf{v}$ is a given vector. We can show that each of the N entries of $\{\varphi^n(P)\}_n$ forms a linear recurrence sequence. Let $f_1 = \ldots = f_M = 0$ denote the polynomial equations defining V, then $\varphi^n(P) \in V$ if and only if $f_i(\varphi^n(P)) = 0$ for $1 \le i \le M$. Hence we obtain the intersection of zero sets of finitely many recurrence sequences and Conjecture 2.1 in the current setting follows from the Skolem-Mahler-Lech theorem. For a comprehensive introduction including various partial results toward Conjecture 2.1, we refer the readers to [1].

We conclude this note with a brief discussion on a generalization of Conjecture 2.1 for $k \geq 2$ commuting morphisms $\varphi_1, \ldots, \varphi_k$ from X to itself. At first, it seems natural to expect that the set

$$S := \{(n_1, \ldots, n_k) \in \mathbb{N}^k: \ \varphi_1^{n_1} \circ \ldots \circ \varphi_k^{n_k}(P) \in V\}$$

is a finite union of translates of submonoids of $\mathbb{N}^k$. However, Scanlon and Yasufuku [5] show that the set S can be rather complicated even in the very simple situation when $X = (\mathbb{C}^*)^n$ and $\varphi_1, \ldots, \varphi_k$ are commuting algebraic endomorphisms. On the other hand, the more restricted "orbit intersection problem" when X is a linear space or a semiabelian variety has been solved in the paper [3].

References

[1] J.P. Bell, D. Ghioca, and T.J. Tucker, *The dynamical Mordell-Lang conjecture*, Mathematical Surveys and Monographs 210, American Mathematical Society (2016).

[2] M. Brassil and Z. Reichstein, *The Hermite-Joubert problem over p-fields*, in 2015 Clifford Lectures on Algebraic Groups: Structure and Actions, Contemporary Mathematics 94, American Mathematical Society (2017), 31–52.

[3] D. Ghioca and K.D. Nguyen, *The orbit intersection problem for linear spaces and semiabelian varieties*, to appear in Math. Res. Lett., arXiv:1604.02529

[4] K. D. Nguyen, *The Hermite-Joubert problem and a conjecture of Brassil-Reichstein*, preprint, arXiv:1709.06732

[5] T. Scanlond and Y. Yasufuku, *Exponential-polynomial equations and dynamical return sets*, Int. Math. Res. Not. IMRN 2014 (2014), 4357–4367.

CSHPM Notes brings scholarly work on the history and philosophy of mathematics to the broader mathematics community. Authors are members of the Canadian Society for History and Philosophy of Mathematics (CSHPM). Comments and suggestions are welcome; they may be directed to either of the column's co-editors:

Amy Ackerberg-Hastings, *Independent Scholar (aackerbe@verizon.net)*

Hardy Grant, *York University [retired] (hardygrant@yahoo.com)*

Les articles de la SCHPM présente des travaux de recherche en histoire et en philosophie des mathématiques à la communauté mathématique élargie. Les auteurs sont membres de la Société canadienne d'histoire et de philosophie des mathématiques (SCHPM). Vos commentaries et suggestions sont le bienvenue; ils peuvent être adressées à l'une des co-rédacteurs:

Amy Ackerberg-Hastings, *Chercheuse indépendente (aackerbe@verizon.net)*

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Mathematical Structures: behind and beyond

Jean-Pierre Marquis, *Université de Montréal*

Think about it: when you do mathematics, when you write about mathematics, or when you discuss mathematics, what are you actually talking *about*? At first glance this question seems completely silly. You just *know* what you are talking about! But *is* the answer that simple? Compare what you might say with what Euler might have said 250 years ago! Confused? This is a sure sign that you have now fallen into a philosophical territory, a territory that might feel like *Alice in Wonderland*. . . .

Or compare your answer to how a physicist might respond to the similar question "what is physics about?" Here the answer seems easy: physics is about properties of material bodies, of matter. If a general and satisfactory answer to what *matter* is might not come readily, at least our connections to these material bodies seem rather straightforward: we have a direct or indirect *sensory experience* of them. What would be the equivalent answer for mathematics? We don't *see* numbers, at least not literally or not in the same way as we see a tree or the Moon. We don't *feel* hyperbolic geometric space, at least not in the way that we feel heat. In the physical world perfect circles are nowhere to be found—only approximations. We have *representations* of numbers, circles and hyperbolic space, but these cannot *be* numbers, circles or hyperbolic space, for the *properties* of these representations are not the same as the properties of their archetypes.

In the 20th century, thanks to research in logic and the foundations of mathematics, the mathematical community found an answer that was simple, clear and satisfactory: mathematics is, ultimately, about *sets*. Although we cannot actually *see* sets, it is possible to show how mathematical language refers systematically and uniformly to sets, no matter what is talked about—integers, complex numbers, *p*-adic numbers, solutions to differential equations, groups, topological spaces, lattices, Hilbert spaces, etc. And the very notion of a set, together with its elements and the operations that can be performed on them, seems to be obvious. (Well, there are these paradoxes cropping up, but we basically know how to circumvent them . . . nothing to really worry about!)

There. Everyone is happy and can go back to doing mathematics with a peaceful mind. Philosophers have their answer.

Not so fast! At least, that is what Paul Benacerraf said in 1965. Say you are doing number theory. In vernacular language, you think about 0, 1, 2, 3, . . . and their properties. But now, since mathematics is really about sets, which set(s) are you referring to? The problem is that there are many different sets that can be the natural numbers. Here are the candidates put forward by Benacerraf himself. The first is obtained by starting with the empty set for zero, and then taking the set of the previous element, thus: $0, \{0\}, \left\{\{0\}\right\}, \left\{\left\{\{0\}\right\}\right\}$. In the second option, known as Von Neumann's ordinals, the successor of n is defined to be $n \cup \{n\}$; we obtain thus the usual ordinals: $0, \{0\}, \{0, \{0\}\}, \left\{0, \{0\}, \{0, \{0\}\}\right\}, \ldots$ Clearly, these are only the most obvious candidates; it would be easy to come up with infinitely many variations on the theme.

So the question arises again: which set is the number 3? Notice that the versions of 3 that emerge in different options are different as sets, and thus have different set-theoretical properties. Of course, if you are a number theorist, this sounds—again?—silly. A number theorist *never* gets bogged down on set-theoretical properties of a particular number, such as 3. The fact is: all the proposed collections of natural numbers are isomorphic from the number-theoretical point of view and there is a second-order axiomatic description of them. (There is also a first-order description, but we want our models to be isomorphic.) Benacerraf was well aware of that and, for that reason and others, suggested that what really interests number theorists are not particular, individual numbers, but the whole sequence of numbers, that is, the *structure* of the natural numbers and its properties. Thus, from this standpoint, mathematics is about *structures*. A structure can be described, informally, as a system of relations between objects. A *mathematical* structure is an *abstract* structure (you are supposed to understand what that means without any further explanation—I am exaggerating here, but not very much!).

Philosophers of mathematics picked up Benacerraf's cue and developed what is now called "mathematical structuralism." Numerous books have been written: Hellman, Resnik, Shapiro, Parsons, and Chihara, to mention but the most obvious, have all proposed variants. The challenges are many: one has to say what structures *are*, how we *know* them, how we *refer* to them, how what we say about them can be *true*, and whether or in what way they *exist*. And here we find basic substantive differences between the thinkers involved, together with subtle variations within basic positions. (For systematic discussions, see for instance Reck & Price or Cole.) Let me immediately point out that there is no agreed-upon classification of the various types of structuralism in the philosophical literature. Rather, depending on the book or article that a reader chooses, she will find different typologies of mathematical structuralism—such is the state of contemporary philosophy of mathematics.

Nonetheless, it is possible to highlight some of the bifurcation points among the variants. One major division is over the mode of existence of these structures. The first version of structuralism is probably the version mathematicians have in mind: structures arise from a given set-theoretical background and can be analyzed along model-theoretical lines. This position is sometimes called "set-theoretical structuralism" or "model-theoretical structuralism." Two obvious objections to this view are often made: first, the underlying universe is not coherent with the structuralist approach, for there one is dealing with particular objects and their properties instead of a structure; second, it is not clear that sets can deal with all kinds of mathematical structures, categories and homotopy types for instance.

The second version is often called "*ante rem*" or "universalist" structuralism. According to this view, all the systems that exemplify the natural numbers, for instance, have something in common, namely the natural number *structure*. It is *that* (abstract) structure, in itself, that is the object of study. This structure exists autonomously, that is, its existence does *not* depend on the existence of its instantiations. It is, in a sense, *prior* to its instantiations. Here one has to give an account of the mode of existence of these structures and of our knowledge of them.

A third option is to say that while talk about structures *as abstract objects* is, in some sense, heuristically useful, in principle such talk can be replaced by a discourse in which there is no reference to structures at all. There are various ways to go about this, but all of these variants belong to the family of "eliminative" structuralism. These are revisionist programs, that is, programs which, if they are successful, still have to explain why we are inclined to talk about structures and what, in fact, we are referring to, even if the reference is not explicit.

We can come back to our original question: what is pure mathematics about? From Gauss to Bourbaki, mathematicians progressively came to the conclusion that it was about *abstract structures*. For the working mathematician, that answer is usually accompanied by a clear methodological stance: mathematics has to be done up to isomorphism. As to what this exactly means and how it should be explained, it remains to philosophers to come up with a satisfactory and coherent answer.

References

[1] Benacerraf, P. (1965) What Numbers could not be. *Philosophical Review*, 74(1), 47–73.

[2] Chihara, C. (2004) *A Structural Account of Mathematics*. Oxford University Press.

[3] Cole, J. (2010) Mathematical Structuralism Today. *Philosophy Compass*, 5/8, 689–699.

[4] Hellmann, G. (1989) *Mathematics without Numbers. Towards a Modal-Structural Interpretation*. New York: The Clarendon Press.

[5] Parsons, C. (1990) The Structuralist View of Mathematical Objects. *Synthese*, 84(3), 303–346.

[6] Reck, E. & Price, M. (2000) Structures and Structuralism in Contemporary Philosophy of Mathematics. *Synthese*, 125(3), 341–383.

[7] Resnik, M. (1997) *Mathematics as a Science of Patterns*. Oxford University Press.

[8] Shapiro, S. (1997) *Philosophy of Mathematics: Structure and Ontology*. Oxford University Press.

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CMS Notes
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2018 CMS Summer Meeting

June 1 – 4, 2018

University of New Brunswick, New Brunswick, Fredericton

Scientific Directors:

Colin Ingalls (University of New Brunswick)
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Alexandre Girouard (Université Laval)
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Réunion d'été de la SMC 2018

1 – 4 juin, 2018

l'Université du Nouveau Brunswick, Nouveau-Brunswick, Fredericton

Directeur scientifique :

Colin Ingalls (l'Université du Nouveau Brunswick)
cingall@unb.ca

Alexandre Girouard (Université Laval)
alexandre.girouard@mat.ulaval.ca

CMS Summer Meeting Scientific Sessions

All scientific sessions will take place at the University of New Brunswick.

Advances in Harmonic Analysis and PDEs

Org: **David Cruz-Uribe** (University of Alabama), **Galia Dafni** (Concordia University) and **Scott Rodney** (Cape Breton University)

Algebraic groups and related topics

Org: **Eddy Campbell** (University of New Brunswick) and **David Wehlau** (Royal Military College of Canada)

Categories and Topology

Org: **Darien DeWolf** (St. Francis Xavier University), **Pieter Hofstra** (University of Ottawa) and **Dorette Pronk** (Dalhousie University)

Cohomology – a link between numbers and geometry

Org: **Stefan Gille** (University of Alberta), **Jan Minac** (Western University) and **Kirill Zainoulline** (Univeristyof Ottawa)

Combinatorial Game Theory

Org: **Svenja Huntemann** (Dalhousie University), **Neil McKay** (University of New Brunswick, St. John) and **Rebecca Milley** (Memorial University)

Computational and Diophantine Number Theory

Org: **Michael Bennett** (University of British Columbia), **Keith Johnson** (Dalhousie University) and **Gary Walsh** (University of Ottawa)

Contributed Papers

Dynamical systems and spatial models in ecology

Org: **Amy Hurford** (memorial University)

Ergodic Theory, Dynamical systems, Fractals and Applications

Org: **Shafiqul Islam** (University of Prince Edward island) and **Franklin Mendivil** (Acadia University)

Geometric Potential Theory

Org: **Jie Xiao** (Memorial University)

Graph Searching & Pursuit-Evasion Games on Graphs

Org: **Dr. Danielle Cox** (Mt. St. Vincent University) and **Dr. Christopher Duffy** (University of Saskatchewan)

Mathematical Aspects of Quantum Information Theory

Org: **Nathaniel Johnston** (Mount Allison University) and **Sarah Plosker** (Brandon University)

Mathematical Epidemiology

Org: **Lin Wang** and **James Watmough** (University of New Brunswick)

Noncommutative Geometry and Topology

Org: Branimir Ćaćić (University of New Brunswick), **Dan Kucerovsky** (University of New Brunswick) and **Martin Mathieu** (Queen's University Belfast)

Number Theory

Org: **Alia Hamieh** (University of Northern British Columbia) and **Matilde Lalin** (University of Montreal)

Partial Differential Equations and Variational Problems

Org: Mohammad El Smaily (University of New Brunswick)

Prize Lectures

Reesentation theory of algebras and related topics

Org: **Colin Ingalls** (University of New Brunswick) and **Charles Paquette** (Royal Military College of Canada)

Singularities and Phase transitions in Nonlinear PDE's

Org: **L. Bronsard** (mcMaster University), **T. Giorgi** (New Mexico State University) and **I. Topaloglu** (Virginia Commonwealth University)

Schedule to be determined

Thomas Bellsky (University of Maine)
Rohit Jain (McGill University)
Maxime Laborde (McGill University)
Xin Yang Lu (Lakehead University)
Guanying Peng (The University of Arizona)
Scott Rodney (Cape Breton University)

Studc Student Research Session

Org: **Aram Demenjian** (UQAM) and **Jean Lagacé** (University of Montreal)

2018 Adrien Pouliot Award

Nominations of individuals or teams of individuals who have made significant and sustained contributions to mathematics education in Canada are solicited. Such contributions are to be interpreted in the broadest possible sense and might include: community outreach programs, the development of a new program in either an academic or industrial setting, publicizing mathematics so as to make mathematics accessible to the general public, developing mathematics displays, establishing and supporting mathematics conferences and competitions for students, etc.

Nominations must be received by the CMS Office **no later than April 30, 2018.**

Please submit your nomination electronically, preferably in PDF format, to **apaward@cms.math.ca**.

Nomination requirements

- Include contact information for both nominee and nominator.
- Describe the nominated individual's or team's sustained contributions to mathematics education. This description should provide some indication of the time period over which these activities have been undertaken and some evidence of the success of these contributions. This information must not exceed four pages.
- Two letters of support from individuals other than the nominator should be included with the nomination.
- Curricula vitae should not be submitted since the information from them relevant to contributions to mathematics education should be included in the nomination form and the other documents mentioned above.
- If nomination was made in the previous year, please indicate this.
- Members of the CMS Education Committee will not be considered for the award during their tenure on the committee.

Renewals

Individuals who made a nomination last year can renew this nomination by simply indicating their wish to do so by the deadline date. In this case, only updating materials need be provided as the original has been retained.

Prix Adrien Pouliot 2018

Nous sollicitons la candidature de personne ou de groupe de personnes ayant contribué d'une façon importante et soutenue à des activités mathématiques éducatives au Canada. Le terme « contributions » s'emploie ici au sens large; les candidats pourront être associés à une activité de sensibilisation, un nouveau programme adapté au milieu scolaire ou à l'industrie, des activités promotionnelles de vulgarisation des mathématiques, des initiatives spéciales, des conférences ou des concours à l'intention des étudiants, etc.

Les mises en candidature doivent parvenir au bureau de la SMC **avant le 30 avril 2018**.

Veuillez faire parvenir votre mise en candidature par voie électronique, de préférence en format PDF, à **prixap@smc.math.ca**.

Conditions de candidature

- Inclure les coordonnées du/des candidat(s) ainsi que du/des présentateur(s).
- Décrire en quoi la personne ou le groupe mis en candidature a contribué de façon soutenue à des activités mathématiques. Donner un aperçu de la période couverte par les activités visées et du succès obtenu. La description ne doit pas être supérieure à quatre pages.
- Le dossier de candidature comportera deux lettres d'appui signées par des personnes autres que le présentateur.
- Il est inutile d'inclure des curriculums vitae, car les renseignements qui s'y trouvent et qui se rapportent aux activités éducatives visées devraient figurer sur le formulaire de mise en candidature et dans les autres documents énumérés ci- dessus.
- Si la candidature a été soumise l'année précédente, veuillez l'indiquer.
- Les membres du Comité d'éducation de la SMC ne pourront être mis en candidature pour l'obtention d'un prix pendant la durée de leur mandat au Comité.

Renouveler une mise en candidature

Il est possible de renouveler une mise en candidature présentée l'année précédente, pourvu que l'on en manifeste le désir avant la date limite. Dans ce cas, le présentateur n'a qu'à soumettre des documents de mise à jour puisque le dossier original a été conservé.

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The CMS is part of the **CMS/Air Canada Charitable Pooling program for non-profit organizations**.

Charitable Pooling is a mechanism by which Air Canada passengers can donate some or all of their earned frequent flyer miles to a non-profit organization. The organization can then use those miles to pay airfares for travel that aligns with the organization's mission.

Every year, the CMS uses frequent flyer miles that have been donated to our Charitable Pooling account for IMO team travel and travel for graduate students/post-docs to attend CMS meetings.

Since anyone can donate their Air Canada frequent flyer miles, please spread the word to your colleagues, family, and friends. The general public is also invited to donate their miles.

To donate your frequent flyer miles to the CMS, please visit the following link: **https://beyondmiles.aeroplan.com/eng/charity/941**

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Comme tout le monde peut faire don de ses milles Aéroplan d'Air Canada, passez le mot à vos collègues, parents et amis! Le grand public est également invité à donner des milles.

Pour faire don de milles Aéroplan à la SMC, suivez le lien suivant : **https://beyondmiles.aeroplan.com/fra/charity/941**

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that CMS Membership has several benefits including discounts?

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- Jusqu'à 50% réduction sur les publications
- Inclus notre bulletin – Notes de la SMC

CMS Notes
Volume 50 No. 2, March-April 2018

Florin Diacu (1959–2018)

Florin Diacu passed away on February 13, 2018. Florin was an award-winning mathematician and author, and a former Vice President of the Canadian Mathematical Society (CMS). He authored 6 books, over 60 research papers, and several popular science, newspaper and encyclopedia articles.

Florin Diacu was born in Sibiu, Romania, on April 24, 1959. He graduated with a Diploma in Mathematics from the University of Bucharest in 1983. After graduation he worked as a high-school teacher in Mediaş, a small town in Transylvania, doing research in his spare time that would later become his PhD thesis. Defecting from communist Romania in 1988, he earned a Ph.D. in Mathematics at the University of Heidelberg in 1989. After his PhD he held a visiting position at the University of Dortmund, and then became a post-doctoral fellow at the Centre de Recherches Mathématiques (CRM) in Montreal.

Florin began his academic and teaching career in 1991 at the University of Victoria in British Columbia (UVic), where he remained for 26 years.

He also held short-term visiting positions at the Victoria University of Wellington, New Zealand (1993), University of Bucharest (1998), University of Pernambuco in Recife, Brazil (1999), The Bernoulli Centre of the EPFL in Lausanne, Switzerland (2004),

the University of Chongqing (2012) and the University of Chendu (2016 -17). In 2017, he went on leave from UVic to take up a position at the Yale-NUS College in Singapore where he was a Professor and Head of Studies of Mathematical, Computational & Statistical Sciences, until his death. During his career at UVic, Florin supervised 8 Masters students and 4 PhD students, including the authors of this obituary.

Florin's research focused on qualitative aspects of the n-body problem of celestial mechanics. From the early 1990s until the early 2000s his work consisted in studying modifications of the classical Newtonian n-body problem to problems with quasihomogeneous potentials. Florin's work on these topics stimulated a sizable number of papers by many different authors. In the mid 2000s his work was inspired by a conjecture of Don Saari, which states that every solution of the n-body problem with constant moment of inertia is a relative equilibrium. From the second half of the 2000s he concentrated on the n-body problem in spaces of constant curvature. He was awarded the prestigious SIAM Crawford Prize for his work in this direction, the prize citation praising his "novel approach to the n-body problem in curved space, blending dynamical systems, differential geometry, and geometric and celestial mechanics in a lucid, inspirational manner".

Florin was also the author of several books. He wrote two monographs, one on celestial mechanics and one on the n-body problem in spaces of constant curvature, as well as a differential equations textbook. An energetic and passionate writer, he had a gift for turning complex mathematics into simple-seeming and fascinating stories. He authored three popular science books: "Celestial Encounters: The Origins of Chaos and Stability" (coauthored with P. Holmes, this was one of Choice Magazine's Outstanding Academic Titles for 1997); "The Lost Millennium: History's Timetables Under Siege"; and "Megadisasters: The Science of Predicting the Next Catastrophe" (one of Choice's Outstanding Academic Titles for 2010).

In addition to his research and writing activities, Florin served the mathematical profession in many ways, especially through the Canadian scientific societies. He was the UVic site director of the Pacific Institute for the Mathematical Sciences between 1999 and 2003, and an editor of "π in the Sky" for many years. Between 2012 and 2015 he served the CMS as the Research Notes Editor to the Notes, from 2015 to 2017 he was Vice President of the CMS, and between 2013 and 2017 he was an associate editor of the Canadian Journal of Mathematics and the Canadian Mathematical Bulletin. He also served as a member of the NSERC Mathematics Scholarships and Fellowships Committee and the NSERC Mathematics and Statistics Committee and Evaluation Group. He was an editor for Libertas Mathematica and the Romanian Astronomical Journal.

Florin Diacu is survived by his wife, Marina; his son, Razvan; and his mother, Elena. Florin has had a lasting impact on the lives of many, and on mathematics. His determined and consistent efforts have blazed new trails in celestial mechanics and invited others to walk them. He will be greatly missed.

Seeking an Executive Director for the Canadian Mathematical Society

The Canadian Mathematical Society is currently seeking an individual with enthusiasm, initiative, and a strong interest in supporting the development of the Canadian Mathematical community, to be its next Executive Director.

The mission of the CMS is to promote the discovery, learning and application of mathematics in Canada. The CMS enhances the practice of mathematics in Canada through national conferences and publishing research journals, books, and newsletters in both print and electronic formats. The CMS supports efforts that identify and develop young mathematicians through its math competitions, math camps and other educational activities. The work of the CMS is carried out by a large number of dedicated and enthusiastic volunteers, together with a small experienced staff at the CMS Executive Office in Ottawa.

The role of the Executive Director is to support the CMS President through the CMS Executive Committee, Board of Directors, and the chairs of CMS standing committees. The Executive Director is responsible for the planning and implementation of CMS policy decisions and establishing the priorities of the Executive Office staff.

The Executive Director position requires an individual with proven administrative experience and a broad knowledge of the Canadian academic community. Excellent organizational, interpersonal, and problem solving skills are required. A graduate degree (preferably doctoral), fluency in English and French, and fundraising experience are assets.

This position offers is a unique opportunity to support the Canadian Mathematical Society, and to enhance the Canadian mathematical community. The position is full time and based in Ottawa and requires both vision and administrative talent.

The CMS is committed to an equitable, diverse, and inclusive workforce. We welcome applications from all qualified persons. We encourage women; First Nations, Métis and Inuit persons; members of visible minority groups; persons with disabilities*; persons of any sexual orientation or gender identity and expression; and all those who may contribute to the further diversification of ideas and perspectives at the CMS to apply.

Applications, including a covering letter, curriculum vitae and references, should be sent to:

ed-search@cms.math.ca. **Please contact the CMS Executive Secretary, Graham Wright, at** gpwright@cms.math.ca **if you have any questions about the position.**

We will start reviewing applications on April 15, 2018. The deadline for applications is April 15, 2018 or until the position is filled.

**building not wheelchair accessible.*

La Société mathématique du Canada recherche un directeur administratif

La Société mathématique du Canada (SMC) est à la recherche d'une personne enthousiaste, dotée du sens de l'initiative et ayant un vif intérêt pour le développement du milieu des mathématiques au Canada pour succéder à son actuel directeur administratif.

La SMC a pour mission de promouvoir la découverte, l'apprentissage et l'application des mathématiques au Canada. La Société renforce la pratique des mathématiques au Canada par ses congrès nationaux et par la publication de revues scientifiques, d'ouvrages et de bulletins imprimés et électroniques. Elle contribue à la découverte et à la formation de jeunes mathématiciens par ses concours et ses camps de mathématiques et par d'autres activités éducatives. Ce travail est accompli par de nombreux bénévoles dévoués et dynamiques et par le petit effectif expérimenté du bureau administratif de la SMC, à Ottawa.

Le directeur administratif de la SMC a pour rôle d'appuyer le président de la Société, par l'intermédiaire du comité exécutif et du conseil d'administration, et d'appuyer les présidents des comités permanents de la SMC. Il est responsable de planifier et de mettre en application les décisions stratégiques de la SMC et d'établir les priorités du personnel du bureau administratif.

Le poste de directeur administratif demande de l'expérience en administration et une connaissance étendue du milieu de l'enseignement et de la recherche au Canada. Le travail nécessite d'excellentes aptitudes organisationnelles, beaucoup d'entregent et une grande capacité de résoudre des problèmes. Un diplôme d'études supérieures (de préférence, un doctorat), la maîtrise du français et de l'anglais et l'expérience de campagnes de financement seraient des atouts.

Ce poste offre une occasion unique d'appuyer le travail de la SMC et de contribuer à la vitalité du milieu des mathématiques au Canada. Il s'adresse à une personne à la fois visionnaire et douée pour l'administration. Il s'agit d'un poste à temps plein à Ottawa.

La SMC a à cœur d'offrir un milieu de travail équitable, diversifié et inclusif. Toutes les personnes qualifiées sont invitées à postuler. Nous encourageons à poser leur candidature les femmes, les membres des Premières Nations, les Métis et les Inuits, les membres de minorités visibles, les personnes handicapées*, les personnes de toutes orientations sexuelles et de toutes identités ou expressions de genre et toutes les personnes pouvant contribuer à une diversification accrue des idées et des perspectives à la SMC.

Les candidatures, constituées d'une lettre de présentation, d'un curriculum vitae et de références, doivent être soumises à l'adresse da-recherche@smc.math.ca. **Pour toute question au sujet du poste, veuillez communiquer avec Graham Wright, secrétaire exécutif de la SMC, à l'adresse** gpwright@smc.math.ca.

L'examen des candidatures commencera le 15 avril 2018. Nous accepterons des candidatures jusqu'au 15 avril 2018 ou jusqu'à ce que le poste soit pourvu.

**immeuble non accessible en fauteuil roulant*

CMS Research Prizes

The CMS Research Committee is inviting nominations for three prize lectureships. These prize lectureships are intended to recognize members of the Canadian mathematical community.

The **Coxeter-James Prize** Lectureship recognizes young mathematicians who have made outstanding contributions to mathematical research. The recipient shall be a member of the Canadian mathematical community. Nominations may be made up to ten years from the candidate's Ph.D. A nomination can be updated and will remain active for a second year unless the original nomination is made in the tenth year from the candidate's Ph.D. For more information, visit: **https://cms.math.ca/Prizes/cj-nom**

The **Jeffery-Williams Prize** Lectureship recognizes mathematicians who have made outstanding contributions to mathematical research. The recipient shall be a member of the Canadian mathematical community. A nomination can be updated and will remain active for three years. For more information: **https://cms.math.ca/Prizes/jw-nom**

The **Krieger-Nelson Prize** Lectureship recognizes outstanding research by a female mathematician. The recipient shall be a member of the Canadian mathematical community. A nomination can be updated and will remain active for two years. For more information: **https://cms.math.ca/Prizes/kn-nom**

The deadline for nominations, including at least three letters of reference, is **September 30, 2018**. Nomination letters should list the chosen referees and include a recent curriculum vitae for the nominee. Some arms-length referees are strongly encouraged. Nominations and the reference letters from the chosen referees should be submitted electronically, preferably in PDF format, to the corresponding email address and **no later than September 30, 2018**:

Coxeter-James: **cjprize@cms.math.ca**
Jeffery-Williams: **jwprize@cms.math.ca**
Krieger-Nelson: **knprize@cms.math.ca**

Prix de recherche de la SMC

Le Comité de recherche de la SMC lance un appel de mises en candidatures pour trois de ses prix de conférence. Ces prix ont tous pour objectif de souligner l'excellence de membres de la communauté mathématique canadienne.

Le **Prix Coxeter-James** rend hommage aux jeunes mathématiciens qui se sont distingués par l'excellence de leur contribution à la recherche mathématique. Cette personne doit etre membre de la communauté mathématique canadienne. Les candidats sont admissibles jusqu'à dix ans après l'obtention de leur doctorat. Toute mise en candidature est modifiable et demeurera active l'année suivante, à moins que la mise en candidature originale ait été faite la 10e année suivant l'obtention du doctorat. Pour les renseignements, voir : **https://cms.math.ca/Prix/cj-nom**

Le **Prix Jeffery-Williams** rend hommage aux mathématiciens ayant fait une contribution exceptionnelle à la recherche mathématique. Cette personne doit être membre de la communauté mathématique canadienne. Toute mise en candidature est modifiable et demeurera active pendant trois ans. Pour les renseignements, voir : **https://cms.math.ca/Prix/jw-nom**

Le **Prix Krieger-Nelson** rend hommage aux mathématiciennes qui se sont distinguées par l'excellence de leur contribution à la recherche mathématique. La laureate doit etre membre de la communauté mathématique canadienne. Toute mise en candidature est modifiable et demeurera active pendant deux ans. Pour les renseignements, voir : **https://cms.math.ca/Prix/info/kn**

La date limite pour déposer une candidature, qui comprendra au moins trois lettres de référence, est **le 30 septembre 2018**. Le dossier de candidature doit comprendre le nom des personnes données à titre de référence ainsi qu'un curriculum vitae récent du candidat ou de la candidate. Veuillez faire parvenir les mises en candidature et lettres de référence par voie électronique, de préférence en format PDF, avant la date limite, à l'adresse électronique correspondante et **au plus tard le 30 septembre 2018 :**

Coxeter-James : **prixcj@smc.math.ca**
Jeffery-Williams : **prixjw@smc.math.ca**
Krieger-Nelson : **prixkn@smc.math.ca**

If undelivered, please return to:
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209 - 1725 St. Laurent Blvd Ottawa, ON K1G 3V4 Canada