The History of Mathematics, alive, alive-0 9

CMS NOTES de la SMC

October/ November octobre/ novembre 2019

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Canadian Mathematical Society
Société mathématique du Canada

Vice-President's Notes / Notes de la Vice-présidente

Monica Nevins (Ottawa)
Vice-President, Ontario / Vice-présidente - Ontario

Study Abroad

Internationalisation is all the buzz in academia today. While the impetus has largely been, and continues to be, the need for colleges and universities to generate more revenue by attracting fee-paying students from abroad, I warmly welcome what I see as a recent shift of focus: our schools, at all levels, touting their dedication to producing global citizens. As we get into the swing of our fall semester, I'm reflecting on what promoting "global citizenship" might mean to us, and to our professional society.

I've had a lot of reason to reflect on internationalisation, beyond the shifting demographics in my own classroom: at uOttawa, I am the Faculty of Science's Vice-Dean International Relations—a nebulous role that I am slowly defining for myself. What we've seen most tangibly is how our undergraduate and graduate programs benefit by attracting excellent students from around the world—not to mention how our research programs benefit from international collaborations, and from hiring globe-trotting postdoctoral fellows. But until I took on this role, I was unaware how intensely some countries promote global citizenship among their own students, and how comparatively far behind we are. For example, most programs in France have as a requirement that their students study at least one semester abroad—and their students are eager to seize this opportunity (due in part, no doubt, to the strong national funding behind the initiative).

What excites me particularly about internationalisation and global citizenship are

Continued on page 4

Étudier à l'étranger

Internationalisation est l'expression à la mode à l'université ces temps-ci. Quoique l'objectif principal des collèges et des universités soit d'augmenter le revenu en attirant un plus grand nombre d'étudiants payeurs des pays étrangers, je me réjouis de ce récent changement de perspective : nos universités se vantent de leurs initiatives pour produire les citoyen.ne.s mondial.e.s. Maintenant que la rentrée est derrière nous, je réfléchis à ce que cet élan de « citoyenneté mondiale » pourrait signifier pour notre société professionnelle.

Plusieurs facteurs ont déclenché mes réflexions, je vois, certes, une démographie changeante dans mes salles de classe, mais je suis aussi Vice-doyenne aux relations internationales à la Faculté des Sciences de l'Université d'Ottawa – un rôle nébuleux que je commence, quoique lentement, à redéfinir pour moi-même. Nous y avons vu, de façon tangible, les avantages que tirent les programmes du premier cycle et des cycles supérieurs d'attirer d'excellent.e.s étudiant.e.s de partout au monde– sans tenir en compte des bénéfices des collaborations internationales et de l'embauche des postdoctorant.e.s globe-trotters pour nos recherches. Avant d'assumer mes fonctions, j'ignorais l'intensité avec laquelle certains pays font la promotion de la citoyenneté mondiale auprès de leurs étudiant.e.s, et le retard dont nous souffrons par rapport à ces universités. À titre d'exemple, la plupart des programmes en France exigent que leurs étudiant.e.s passent au moins une session à l'étranger– et les étudiant.e.s saisissent cette occasion avec enthousiasme (grâce, en partie, au financement adéquat pour cette initiative).

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CMS Notes
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Beauty is Truth...

Robert Dawson, *St. Mary's*
CMS Notes Editor-in-Chief

...truth beauty. That, as Keats heard in the silent voice of the Grecian urn, is all we know on earth, and all we need to know.

From a mathematician's viewpoint, that rings true - as does Edna St. Vincent Millay's claim that "Euclid alone has looked on Beauty bare," and Hardy's " Beauty is the first test: there is no permanent place in the world for ugly mathematics." Mathematics deals with truths that may be abstract, but are also absolute, as true in their way as anything in the universe can be. And you and I know that we don't just do it for the huge paychecks, for the hordes of screaming fans, or for the million-dollar sponsorship contracts from the people who make those little white pencil erasers. We do it because it's beautiful.

Of course, mathematics is useful too. When I was a graduate student, one of my undergraduate mentors mailed me a copy of Clarence R. Wylie Jr.'s beautiful sonnet "Paradox." (Thanks, Ernie. I understood.) Look it up - I can't do justice to it with as short a fragment as copyright legislation would let me quote. It does as good a job of expressing the deep and strange relationship between mathematics and its applications in fourteen lines of iambic pentameter as most of us could do in a ten-page essay.

Mathematics is not all of poetry, architecture, music, or art, any more than it is all of engineering or chemistry. Yet it underlies all of these - or maybe they exist separately and mathematics explains them to us. Perhaps, seen properly, the two ideas are the same. Or, if not precisely the same, as intimately related as a pair of adjoint functors.

There's a significant literature of overtly mathematical poetry: one notable anthology is *Strange Attractors* edited by JoAnne Growney, who also keeps a blog on mathematical poems. Artists such as Escher, Ferguson, and Dalí have shown us snapshots of the visual beauty of our subject. And has any composer or architect ever managed to avoid bringing mathematics into their creations in one way or another?

My wish for you in this academic year: may your mathematics be beautiful, and may you succeed in showing that beauty to others.

Beauté et vérité

La vérité beauté, ce qu'entend Keats dans la voix silencieuse de l'urne grecque, est tout ce que nous savons de la terre, tout dont nous aurons besoin de savoir.

Cela paraît vrai au mathématicien – tout comme l'est l'affirmation d'Edna St. Vincent Millay qui déclare : « Euclid seul a regardé beauté nue », ou celle de Hardy qui dit « la beauté est la première épreuve : il n'y a pas de place durable dans le monde pour les mathématiques laides ». Les mathématiques traitent de la beauté parfois abstraite, mais tout aussi absolue; aussi vrai que l'univers et tout ce qu'il englobe. Rappelons-nous que nous ne sommes motivés ni par l'argent, ni par des hordes d'admirateurs et d'admiratrices en délire, ni par les contrats de commandite de millions de dollars avec les fabricant.e.s des petites gommes à effacer; en effet, nous y sommes parce que les mathématiques sont belles.

Certes, elles sont aussi utiles. Quand j'étais étudiant aux études supérieures, l'un de mes mentors au premier cycle m'a envoyé une copie du beau sonnet de Clarence R. Wylie Jr. qui s'intitule « Paradoxe » (merci, Ernie. Je l'ai compris.) Je vous laisse le trouver vous-même – des petits fragments que les lois sur le droit d'auteur me permettent n'y rendront pas justice. Ce sonnet exprime bien l'étrange et le profond lien entre les mathématiques et leurs applications, et ce, en seulement 14 vers en pentamètre iambique, ce que nous aurait pris un article de dix pages pour exprimer.

Les mathématiques ne peuvent pas à elles seules saisir la poésie, l'architecture, la musique et l'art, pas plus qu'elles rendent compte des domaines du génie ou de la chimie. Or, elles les sous-tendent– ou, peut-être, ces sujets sont dotés chacun d'une existence indépendante que les mathématiques mettent en lumière. Peut-être, à bien regarder, ces deux idées sont identiques. Ou, à tout le moins, elles sont intimement reliées comme une paire des foncteurs adjoints.

Il y a un grand corpus de la poésie mathématique : un exemple en est l'anthologie intitulée *Strange Attractors*, dirigée par JoAnne Growney qui publie aussi des poèmes mathématiques dans son blog. Les artistes tels que'Escher, Ferguson, et Dalí, ont saisi notre sujet et en ont présenté la beauté visuelle. Et quel.le compositeur, compositrice, ou architecte a pu éviter les mathématiques dans sa création?

Voici ce que je souhaite pour vous dans cette nouvelle année scolaire : que vos mathématiques soient belles, et que vous arriviez à présenter cette beauté aux autres.

2020 CMS MEMBERSHIP RENEWALS

The 2020 membership renewals have been sent! Please renew your membership online at **portal.cms.math.ca** by logging into your member account. Should you have any questions, please email us at **memberships@cms.math.ca**

RENOUVELLEMENTS 2020 À LA SMC

Le renouvellement pour l'an 2020 a été envoyé! S'il vous plaît renouveler votre adhésion en ligne à **portail.smc.math.ca** et en vous connectant à votre compte de membre. Si vous avez des questions, s'il vous plaît écrivez-nous à **adhesions@smc.math.ca**

Letters to the Editors

The Editors of the NOTES welcome letters in English or French on any subject of mathematical interest but reserve the right to condense them. Those accepted for publication will appear in the language of submission. Readers may reach us at the Executive Office or at **notes-letters@cms.math.ca**

Lettres aux Rédacteurs

Les rédacteurs et les rédactrices des NOTES acceptent les lettres en français ou en anglais portant sur n'importe quel sujet d'intérêt mathématique, mais ils se réservent le droit de les comprimer. Les lettres acceptées paraîtront dans la langue soumise. Le lectorat peut nous joindre au bureau administratif de la SMC ou à l'adresse suivante : **notes-lettres@smc.math.ca**.

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Continued from cover

the opportunities to promote outward mobility, at all levels. Going abroad is all about adding something global to our local knowledge. (Coincidentally or not, local-global relationships are the kind of mathematics I like, too.) What I've been learning about, and seeking to encourage, are the myriad of different interconnected ways students and academics go abroad. What really stands out for me is how almost uniquely well-positioned we are—as mathematicians, or students of mathematics—to take advantage of them.

Student exchanges had always struck me as quaint throwbacks, for simpler times. The appetite for specialised undergraduate programs, with long rigid lists of required courses, isn't all that compatible with exploration, let alone with studies at a university with a different take on the subject. Mathematics is certainly a nice exception to this rule: most universities are lucky to have a healthy core program that is well-respected, and yet enjoys enough flexibility to count courses from abroad towards their degree. I encourage my students to take advantage of the special and unique mathematics courses they discover at their host institution, for the chance to really experience the breadth and depth of mathematics.

Moreover, isn't mathematics among the simplest subjects to follow in a foreign language? (I've certainly argued this, successfully and not, trying to persuade students to try a math course in the other language at my bilingual University…) The point is rapidly becoming moot, though—the number of universities abroad offering international programs in English make an academic exchange easier to embrace than ever.

Fundamentally, I think that studying abroad, and having the chance to live and breathe in another culture (both socially and academically), truly enriches—maybe even defines—a university education. How many of us will attest to the relevance of graduate studies in another country to forging a successful career in academia? I certainly believe that doing my doctoral studies in the US had a profound impact on both my career and my world view.

The Canadian Mathematical Society had a long and proud tradition of promoting such student mobility. The NSERC-CMS Math in Moscow program, which ran from 2001 to 2014, funded between two and four Canadian students per year to study one semester at the Independent University of Moscow. Though Math in Moscow continues, our sponsorship ended with the withdrawal of our NSERC funding partner, and no equivalently highly prestigious program has taken its place. Perhaps it is madness to consider choosing a replacement; after all, why should there be one single best destination, from the dozens of outstanding mathematics institutions around the world? Nevertheless, I do think it was a singular opportunity to establish—with just three words!—that doing mathematics is a glorious and international activity.

Though I've focussed on academic exchanges, I would be remiss if I didn't at least mention some of the other options for internationalisation open to our students, and ourselves. For one, there is a growing market for students conducting research abroad. As universities around the world seek to raise their stature (and their rankings), we're seeing a steady growth in funded research internships for our undergraduate students, everywhere from Lyon to Shanghai. Our own Canadian Mitacs program facilitates and funds international undergraduate research internships, both incoming and outgoing. These are excellent opportunities, for our students, and for our research—and I, for one, welcome the competition for hiring and attracting the brightest students into mathematical research.

Thankfully, internationalisation and global citizenship doesn't have to end with graduation! Think of the opportunities that academics enjoy: to attend conferences in different countries, to collaborate with colleagues across the globe, and to enjoy research stays, including sabbaticals, abroad. Between videoconferencing technologies and generous Research in Pairs programs at mathematical institutes on different continents, becoming global mathematicians ourselves has never been easier.

Application deadlines for exchanges next September are fast approaching. Share your international experiences with your students, and encourage them to take advantage of some of these great opportunities!

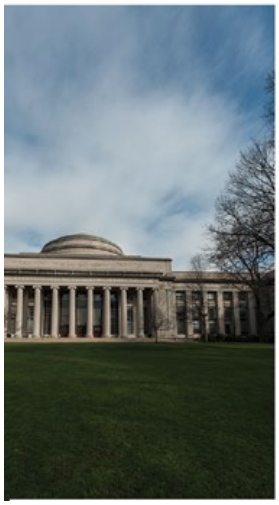



Photo credits: Ralph Nevins, Ralph Nevis Photography.

Suite de la couverture

Ce qui me passionne le plus à propos de cette internationalisation et de la citoyenneté mondiale, ce sont les occasions d'encourager une mobilité sortante dans tous les aspects de la vie universitaire. Voyager à l'étranger apporte quelque chose de mondial à notre savoir local. (Comme par hasard, le Principe local-global est mon sujet mathématique préféré.) Par toutes ces efforts de solliciter et d'encourager les échanges universitaires, je viens d'apprendre d'une myriade de différentes façons interreliées dont les étudiant.e.s et les universitaires voyagent à l'étranger; et ce qui me frappe le plus est le fait que nous, en tant que mathématicien.ne.s, sommes bien placé.e.s pour profiter de ces initiatives.

Les programmes d'échange étudiant m'ont toujours paru archaïques, peu pertinents à notre temps. Les exigences des études spécialisées du premier cycle, avec de longues et rigides listes de cours obligatoires, ne sont pas tout à fait compatibles avec l'exploration, et encore moins avec les différentes approches du sujet d'étude dans d'autres universités. Les mathématiques, toutefois, sont exemptes de cette règle : la plupart des universités sont dotées d'un programme de base solide, mais acceptent aussi les crédits pour des cours suivis par les étudiant.e.s dans d'autres universités. J'encourage mes étudiant.e.s à profiter des cours uniques et spéciaux en mathématiques qui se trouvent dans leur université hôte, et à se donner la chance de vivre l'étendue de l'expérience mathématique.

De surcroît, les mathématiques comptent parmi les sujets les plus faciles à suivre dans une langue étrangère. (Un fait sur lequel j'insiste pour encourager mes étudiant.e.s à suivre des cours de mathématiques dans l'autre langue dans mon Université bilingue…). Ce fait perd toutefois de sa pertinence, au moins pour nos étudiant.e.s anglophones, car le nombre d'universités étrangères qui offrent des programmes internationaux en anglais est en hausse. Cela qui rend l'échange universitaire d'autant plus facile.

Je crois fermement qu'étudier à l'étranger, et vivre d'autres cultures (autant au niveau culturel qu'universitaire) enrichit– ou peut-être même définit – l'éducation universitaire. Combien d'entre nous témoigneront de l'importance des études supérieures à l'étranger pour établir une carrière brillante à l'université? Je crois qu'avoir fait mes études doctorales aux États-Unis a eu un impact profond sur ma carrière, mais aussi sur ma vision du monde.

La Société mathématique du Canada a eu une longue tradition de promouvoir de telles mobilités étudiantes. Le programme "Math in Moscow", parrainé conjointement par le CRSNG et la SMC, qui s'est déroulé du 2001 à 2014 en est un exemple; il finançait le séjour des 2 à 4 étudiant.e.s canadien.ne.s par années pour la durée d'une session à l'Université indépendante de Moscou. Quoique « Math in Moscou » se continue à ce jour, mais la fin de la subvention du CRSNG a mis fin à la participation de la SMC et aucun autre programme de pareille envergure ne l'a remplacé. Il est peut-être insensé de songer à le remplacer, après tout, pourquoi envisager seulement une destination parmi des douzaines de meilleurs établissements mathématiques du monde? Cela dit, ledit programme était le seul à nous permettre de transmettre le message– et ce avec seulement trois mots!– qu'étudier les mathématiques est une activité internationale et glorieuse.

Bien que je me sois concentrée ici sur les échanges universitaires, je m'en voudrais de ne pas mentionner quelques autres options pour l'internationalisation, à la portée des étudiant.e.s et de nous. Premièrement, de plus en plus d'étudiant.e.s effectuent leurs recherches à l'étranger. Les universités désirent élever leur profil et leur classement; il y a donc, de Lyon à Shanghai, une croissance constante de stages payés offerts à nos étudiant.e.s de premier cycle. En plus, notre programme canadien Mitacs Globalink facilite et finance des stages internationaux de recherche de premier cycle, pour les entrant.e.s, tout comme pour les sortant.e.s. Ce sont d'excellentes occasions pour nos étudiant.e.s et pour la recherche– et je me réjouis personnellement de la compétition que cela suscite pour engager et attirer les étudiant.e.s les plus brillant.e.s en recherches mathématiques.

Heureusement, l'internationalisation et la citoyenneté mondiale ne cessent pas avec l'obtention du diplôme! Les universitaires en profitent pour assister aux colloques qui ont lieu dans les pays différents, collaborer avec les collègues à travers du monde et mener de recherches tout en résidant à l'étranger (surtout pendant leurs années sabbatiques). Grâce aux technologies de téléconférence et aux programmes « Research in Pairs » des différents instituts mathématiques, être un.e mathématicien.ne mondial.e n'a jamais été aussi facile.

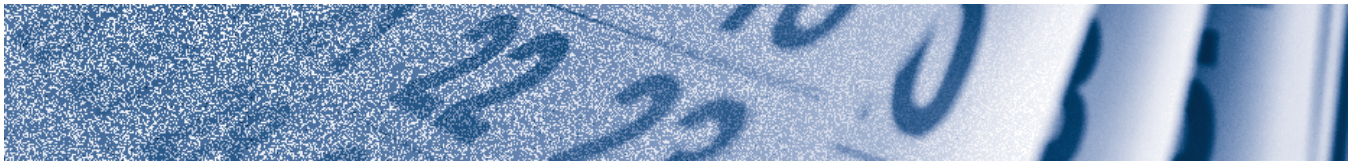
La date d'échéance pour les demandes d'échanges universitaires en septembre prochain approche à grands pas. Partagez vos expériences internationales avec vos étudiant.e.s et encouragez-les à profiter de ces belles occasions!

The Calendar brings current and upcoming domestic and select international mathematical sciences and education events to the attention of the CMS readership. Comments, suggestions, and submissions are welcome.

Denise Charron, *Canadian Mathematical Society, (mpagent@cms.math.ca)*

Le calendrier annonce au lectorat de la SMC les activités en cours et à venir, sur la scène pancanadienne et internationale, dans les domaines des mathématiques et de l'enseignement des mathématiques. Vos commentaires, suggestions et propositions sont les bienvenues.

Denise Charron, *Société mathématique du canada, (mpagent@smc.math.ca)*



OCTOBER 2019 OCTOBRE

S29-Oct 4	Classification Problems in von Neumann Algebras, BIRS, Banff, AB.
1-31	Mixed Integer Nonlinear Programming: Theory and Computation, CRM, Montreal, QC.
6-11	Herglotz-Nevanlinna Theory Applied to Passive, Causal and Active Systems, BIRS, Banff, AB.
9	Math Biology Seminar: David Holloway, University of British Columbia, Vancouver, BC.
13-18	Physics-Dynamics Coupling in Earth System Models, BIRS, Banff, AB.
13-18	Spaces of Embeddings: Connections and Applications, BIRS, Banff, AB.
14-18	Workshop on Automorphic p-adic L-functions and regulators, University of Lille, Laboratoire Paul Painlevé, Lille, France
15	Discrete Math Seminar: Chris Ryan, University of British Columbia, Vancouver, BC.
15-17	Math Matters - Maple in Mathematics Education and Research, Perimeter Institute and University of Waterloo, Waterloo, ON.
17-18	Mathematics of Vision Workshop, Fields Institute, Toronto, ON.
18-19	63e Congrès de l'AMQ, Cégep du Vieux Montréal, Montréal, QC.
20-25	Women in Commutative Algebra, BIRS, Banff, AB.
21-25	Borders in Public Health and Mathematical Epidemiology, Fields Institute, Toronto, ON.
27-Nov 1	Bridging the Gap between Kahler and non-Kahler Complex Geometry, BIRS, Banff, AB.

NOVEMBER 2019 NOVEMBRE

1-31	Mathematical Physiology – Better Health Through Mathematics, CRM, Montreal, QC.
3-8	Unifying 4-Dimensional Knot Theory, BIRS, Banff, AB.
4-7	2019 Fields Medal Symposium, Fields Institute, Toronto, ON.
10-15	Interactions between Brauer Groups, Derived Categories and Birational Geometry of Projective Varieties, BIRS, Banff, AB.
12	Workshop on Studying Mathematics Cognition in a Cybernetic Age, Fields Institute, Toronto, ON.
17-22	Dimers, Ising Model, and their Interactions, BIRS, Banff, AB.
18-22	Workshop on Higher Structures in Geometry and Physics, Fields Institute, Toronto, ON.
23	Math in Motion ... Girls in Gear!, University of Toronto Scarborough, Toronto, ON.
24-29	Theoretical Foundations of Relativistic Hydrodynamics, BIRS, Banff, AB.

DECEMBER 2019 DÉCEMBRE

1-6	Challenges in Mathematical and Computational Modeling of Complex Systems, BIRS, Banff, AB.
6-9	2019 CMS Winter Meeting / Réunion d'hiver de la SMC 2019, Chelsea Hotel, Toronto, ON.
8-13	Discrete Subgroups of Lie Groups, BIRS, Banff, AB.

JANUARY 2020 JANVIER

15-18	AMS/MAA Joint Mathematics Meeting 2020, Denver, CO, USA
24-26	2020 Combinatorial Algebra Meets Algebraic Combinatorics, Dalhousie University, Halifax, NS.

Book Reviews brings interesting mathematical sciences and education publications drawn from across the entire spectrum of mathematics to the attention of the CMS readership. Comments, suggestions, and submissions are welcome.

Karl Dilcher, *Dalhousie University (notes-reviews@cms.math.ca)*

Les comptes-rendus de livres présentent au lectorat de la SMC des ouvrages intéressants sur les mathématiques et l'enseignement des mathématiques dans un large éventail de domaines et sous-domaines. Vos commentaires, suggestions et propositions sont les bienvenues.

Karl Dilcher, *Dalhousie University (notes-critiques@smc.math.ca)*

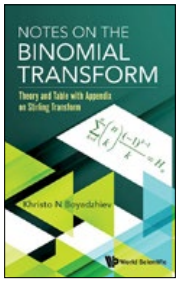
Editor's Note: *This issue contains another three short reviews, written by myself. As before, I quoted freely from prefaces or promotional texts, and in some cases from other published reviews. The publication of full reviews will resume in the next issue of the Notes. - Karl Dilcher.*

Notes on the Binomial Transform

by Khristo N. Boyadzhiev

World Scientific, 2018

ISBN 978-981-3234-97-0



Transforms of different kinds, especially integral transforms, play an important role in various areas of mathematics. Discrete transforms, on the other hand, transform a given sequence, which may or may not be an integer sequence, to another sequence. One of the most important examples of such a transform is the binomial transform, the topic of this book. The binomial transform can be seen as a higher-order extension of the basic (forward) difference operator. It is also related to the Euler transform and to exponential generating functions. The binomial transform has a beautiful inversion formula, which gives rise to a great deal of sctructure, and to connections with various important objects and sequences in combinatorics. Much of this is collected in a concise way in this fairly slim volume.

The contents of this book are best described by quoting from the Preface: "The binomial transform leads to various combinatorial and analytical identities involving binomial coefficients. In particular, we present here new binomial identities for Bernoulli, Fibonacci, and harmonic numbers. Many interesting identities can be written as binomial transforms and vice versa.

"The volume consists of two parts. In the first part, we present the theory of the binomial transform for sequences with a sufficient prerequisite of classical numbers and polynomials. The first part provides theorems and tools which help to compute binomial transforms of different sequences and also to generate new binomial identities from the old. These theoretical tools (formulas and theorems) can also be used for summation of series and various numerical computations.

"In the second part, we have compiled a list of binomial transform formulas for easy reference. In the Appendix, we present the definition of the Stirling sequence transform and a short table of transformation formulas."

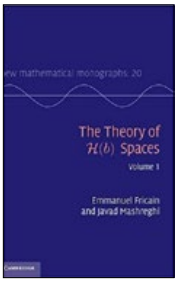
This book will be useful reference to researchers interested in enumerative combinatorics, special number and polynomial sequences, and combinatorial number theory.

The Theory of $\mathcal{H}(b)$ Spaces, Volumes 1 and 2

by Emmanuel Fricain and Javad Mashreghi

Cambridge University Press, 2016

ISBN 978-1-107-02777-0, 978-1-107-02778-7



To quote from the publisher's description of this massive 2-volume treatise, "An $\mathcal{H}(b)$ space is defined as a collection of analytic functions which are in the image of an operator. The theory of $\mathcal{H}(b)$ spaces bridges two classical subjects: complex analysis and operator theory, which makes it both appealing and demanding.

"The first volume of this comprehensive treatment is devoted to the preliminary subjects required to understand the foundation of $\mathcal{H}(b)$ spaces, such as Hardy spaces, Fourier analysis, integral representation theorems, Carleson measures, Toeplitz and Hankel operators, various types of shift operators, and Clark measures.

"The second volume focuses on the central theory. Both books are accessible to graduate students as well as researchers: each volume contains numerous exercises and hints, and figures are included throughout to illustrate the theory. Together, these two volumes provide everything the reader needs to understand and appreciate this beautiful branch of mathematics."

These two volumes have a total of 1300 pages. In addition to the exercises mentioned above, each of the 31 chapters ends with a few pages of notes. The lists of references contain 572, resp. 194 entries.

A long and very detailed review of these two volumes, written by Brett D. Wicks, recently appeared in the Bulletin of the AMS (Volume 56, No. 3, July 2019, Pages 535-542), or online at https://www.ams.org/journals/bull/2019-56-03/

I urge the interested reader to consult this review. To quote from the final section: "[The volumes] are of a monographic nature and are designed for a person who wants to learn the theory of these spaces and understand the state of the art in the area. All major results are included. In some situations the original proofs are provided, while in other cases they provide the 'better' proofs that have become

available since. The books are designed to be accessible to both experts and newcomers to the area. Comments at the end of each section are very helpful, and the numerous exercises were clearly chosen to help master some of the techniques and tools used. [The two volumes] are probably suitable for a year-long sequence in a topics course on complex analysis and operator theory. They are also appropriate for helping a novice learn the techniques and undertake research in the area."

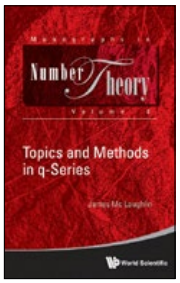
The reviewer ends by writing, "In sum, these are excellent books that are bound to become standard references for the theory of $\mathcal{H}(b)$ spaces."

Topics and Methods in q-Series

by James Mc Laughlin

World Scientific, 2018

ISBN 978-981-3223-36-3



As is the case with many other topics in pure and applied mathematics, the study of q-series goes back to Euler. While other early results were discovered by Gauss and Cauchy, the first systematic study is due to E. Heine in 1843, who considered q-series as natural analogues of classical hypergeometric series. The theory was then further developed in the early 20th century by the Rev. F.H. Jackson, G.N. Watson, W.N. Bailey, and Lucy Slater, among a few others.

The further development is best summarized by George E. Andrews who in the Foreword to the book under review wrote that "things blossomed in the late 1960s with the discovery that the world of partitions had its natural home in q-series. Seemingly esoteric objects like very well-poised q-hypergeometric series turned out to be the central tools for the exploration of partition identities.

"With the Discoveries of Ramanujan's Lost Notebook in the 1970s and Bailey Chains in the 1980s, q-series have subsequently become a center of intense research.

"James (Jimmy) Mc Laughlin has gathered together the components that fueled this resurgence of q-series and combined them into a natural and coherent text. The central theme revolves around the Bailey Chain, its extensons and implications. The book concludes with a nice development of related continued fractions and an introduction to Ramanujan's mock theta functions. There are sufficient exercises included to allow this book to be used as a text in a graduate course."

An informative review of this book, with references to the earlier liteature, appeared in MathSciNet under MR3752164. The reviewer, David M. Bressoud, ends by writing, "Mc Laughlin has produced an admirable book, clearly and knowledgeably written, upon which one could build a challenging undergraduate seminar as well as a graduate course designed to lead toward today's research questions."

The book ends with seven appendices, from "Frequently Used Theorems" to "Selected Summation Formulae", which are intended for quick reference. The book is available in hardcover, softcover, and as an e-book.

Education Notes brings mathematical and educational ideas forth to the CMS readership in a manner that promotes discussion of relevant topics including research, activities, and noteworthy news items. Comments, suggestions, and submissions are welcome.

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Les Notes pédagogiques présentent des sujets mathématiques et des articles sur l'éducation aux lecteurs de la SMC dans un format qui favorise les discussions sur différents thèmes, dont la recherche, les activités et les nouvelles d'intérêt. Vos commentaires, suggestions et propositions sont les bienvenues.

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Kseniya Garaschuk, *University of Fraser Valley (kseniya.garaschuk@ufv.ca)*

The History of Mathematics, alive, alive-O

Fiacre Ó Cairbre, *Maynooth University*

Introduction

> *wondering was it hebrew set to himmeltones or the quicksilversong of qwaternions; his troubles may be over but his doubles have still to come. — James Joyce in Finnegans Wake.*

In this article, I wish to illustrate how the history of mathematics is very much alive and vibrant in Ireland today. I will focus on how the history of mathematics is fresh and being commemorated in society at large in the sense that it is directly accessible to the general public. I see this freshness and creativity related to the history of mathematics mainly because of my work on Ireland's greatest mathematician, William Rowan Hamilton (1805–1865). Many people, mainly from outside mathematics, frequently contact me wanting to know about Hamilton and then many of them wish to create something related to Hamilton. I feel this creativity and public interest in Hamilton arises because the Hamilton story (in particular his creation of quaternions) is just a really good story. I also sometimes collaborate with artists on pieces related to Hamilton. From many years of experience, I feel the Hamilton story has played a significant role in mathematics education and outreach programmes here in Ireland. Below, I will discuss seven (of many) examples that show how the history of mathematics is alive and well in society at large in Ireland today.

The Hamilton walk

It all began on a bright Monday morning on October 16, 1843. Hamilton was walking along the banks of the Royal Canal in Dublin with his wife Helen, pondering a question in algebra that he had been working on for some time. As he passed Broombridge in Cabra, his mind gave birth to Quaternions in a flash of inspiration. In a nineteenth century act of graffiti, he scratched his quaternion formula on the bridge:

$$i^2 = j^2 = k^2 = ijk = -1$$

He named his new system of numbers Quaternions because each number had four components. Hamilton's act of mathematical vandalism had created a totally new structure in mathematics. The mathematical community was stunned at his audacity in creating a system of numbers that did not satisfy the usual commutative rule for multiplication in arithmetic ($ab = ba$). Today, students usually encounter noncommutativity when they first meet matrices; but it would be another decade before Sylvester and Cayley would introduce matrices. Hamilton has been called the Liberator of Algebra because his quaternions liberated algebra from arithmetic by shattering the previously accepted convention that any new useful algebraic number system should satisfy the rules of

ordinary arithmetic. His quaternions revealed a new mathematical world where mathematicians were now free to conceive new algebraic systems that were not constrained by the rules of ordinary arithmetic. See [1], [2], [3] for more on Hamilton's life and works.

Quaternions have many important and powerful applications. For example, they are crucial in space navigation and were fundamental in the Apollo 11 landing on the moon and the Curiosity landing on Mars. Quaternions also play a significant role in many aspects of computer animation, computer games and special effects in movies. See [4] for more applications of quaternions.

In 1990 Anthony G. O'Farrell initiated an annual walk on October 16 to commemorate Hamilton's eureka moment when he created these seemingly strange four dimensional numbers, quaternions. I organise the annual walk which now typically attracts about 200 participants and retraces Hamilton's steps from Dunsink Observatory along the banks of the Royal Canal to Broombridge. The walk takes about forty–five minutes. We now call October 16 Broomsday as a nod to James Joyce's Bloomsday which refers to the day on June 16 in 1904 when the events of his novel, Ulysses, happened. The participants come from a variety of backgrounds including staff and students from third level, second level and many from the general public. The walk is ideal for a mathematics outing for second level students and teachers have remarked that the walk and Hamilton story have had a very positive impact on students' perception of mathematics. Figure 1 shows a large group at the start of the 2018 walk. See [4] for more on the walk.

The large number of participants from the general public shows that there is a substantial public interest in Hamilton and the walk. Also, I receive many calls from the media (television, radio and newspaper) and other bodies and individuals every year expressing an interest in doing a piece on Hamilton and the walk. Consequently, Hamilton's story and the walk have appeared many times on television (including the main evening news on the national broadcast station), radio and in the newspaper and I have given many talks on Hamilton. The general public has an important role to play in mathematics education in second and third level because parents, decision makers and the media are all part of the general public and can have great influence on the attitude of young people and society at large towards mathematics.

Many famous people have come on the walk. For example, Andrew Wiles, Fields Medallists, Timothy Gowers and Efim Zelmanov and Nobel Prize winners in Physics, Murray Gellman, Steven Weinberg and Frank Wilczek have all participated in the walk in recent times.

Cabra Community Council (in the local Broombridge area) have made the end of the walk into a very festive event with a big banner about Hamilton draped across the bridge and stalls along the canal where they offer refreshments to the walkers. The following quote from Aodhan Perry of the Cabra Community Council in 2009 indicates the positive impact of the walk on the Cabra community:

The walk has had a huge impact on the local community. In fact it has gone beyond just being a walk because all the local school children and the community are extremely proud of Hamilton and their local connection with him. The walk really has touched the local community in a big way. The fact that famous mathematicians and Nobel Prize winners mingle with school children and the local community on the walk and at the bridge is great experience. Also, not one but two local artists have been commissioned in recent times to do portraits of Hamilton which are then publicly displayed at the bridge during the walk.

Here is another quote from local Cabra resident, Jack Gannon:

On account of the walk, Hamilton is in the folk consciousness of the local people.

Mick Kelly came on the walk in 2005 and wrote the following:

The Hamilton walk was my licence to explore and express myself around the subject of mathematics. By the age of nine I had decided I couldn't do mathematics, but I had also developed a strong interest in things technical and scientific and this created a conflict that simmered in the background of my educational and professional career for forty years. That is until I took part in the Hamilton walk in 2005. That walk had a profound effect on me. Hearing not only a Nobel laureate and a professor of mathematics sing Hamilton's praises, but also local poets, school children, balladeers and the Cabra community council, spurred me to turn my desire to celebrate Ireland's Science Heritage into action. That action turned out to be a family run business called Science Heritage Ireland selling placemats and coasters celebrating Hamilton. By the 2007 walk I could sense flaws developing in the glass wall I had built around learning mathematics and found it strange but very uplifting to be answering queries from people about quaternion algebra.

Here are two examples of how the Hamilton story has spread through Irish society at large. In 2006, the group was so big at the bridge that it was necessary to have some policemen provide crowd/traffic control because of the dangerous road nearby. This may have been the first time for a police presence for crowd/traffic control at an outdoor mathematical event! I walked over to a policeman to thank him for helping out. He started to talk about Hamilton and said "Yeah, isn't Hamilton's maths used for space navigation nowadays". I replied "You're right". The second example involves a foreign mathematician who took a taxi from Dublin airport to a conference here in Maynooth. When the driver realised that his passenger was a mathematician, the driver spent the whole 30 minute journey talking about Hamilton. After arriving at Maynooth, the surprised mathematician asked "Are all Irish taxi drivers so knowledgeable about mathematics"?

If you are interested in coming on the walk, then please contact me at **fiacre.ocairbre@mu.ie**. I feel the walk is unique as a celebration of a mathematical creation because everything is still there in the sense that the bridge is still there, the canal path is still there and even Dunsink Observatory (where he started from on October 16) is still there. I don't think Archimedes' bathtub is still there!

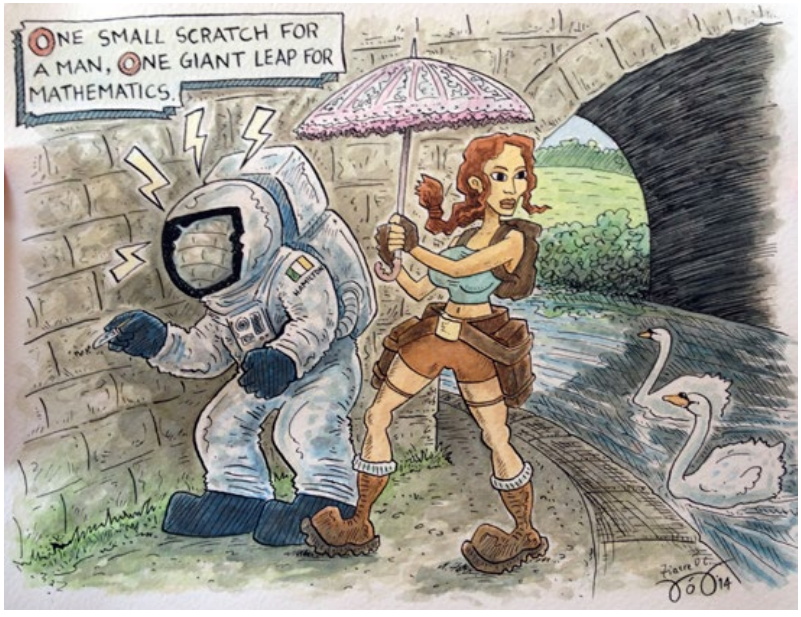
It's interesting to note that Hamilton's motivation for doing mathematics was the quest for beauty. Hamilton was also a poet and he wrote, "Mathematics is an aesthetic creation, akin to poetry, with its own mysteries and moments of profound revelation". Beauty

in mathematics typically corresponds to the beauty of an idea because mathematics essentially comprises an abundance of ideas. Also, Hamilton's creation of quaternions illustrates the freedom in mathematics because he was free to create the new idea of quaternions that liberated algebra from arithmetic and changed the world of mathematics forever. See [5], [6] for more on the beauty and freedom in mathematics.

In 2006 the walk was one of the main events on the first day of the inaugural Maths Week Ireland which was initiated by Eoin Gill and Sheila Donegan. There are now over 150 events across Ireland during Maths Week which occurs annually around the middle of October so that it includes October 16 and the walk. Maths Week aims to promote a positive attitude towards mathematics and is now the largest and most successful mathematics outreach programme/festival in the world and in 2018 it attracted participation from 400,000 people. See www.mathsweek.ie for more on Maths Week.

One small scratch for a man, one giant leap for mathematics

I was invited to submit a piece to an Art and Mathematics exhibition in Galway in 2014. I thought about trying to get a picture to reflect an expression I had coined many years ago about Hamilton's creation of quaternions. The expression was, 'One small scratch for a man, one giant leap for mathematics'. Figure 2 was a collaboration between the artist, Dómhnal Ó Bric and myself and it appeared in the exhibition. Figure 2 was accompanied by a text giving the background to the picture. Part of that text was: This is a picture of Hamilton, in a space suit, scratching his quaternion formula on the bridge. Lara Croft of Tombraider, is standing beside Hamilton. The space suit links in with Neil Armstrong's expression, 'One small step for man, one giant leap for mankind,' when he first set foot on the moon. The space suit also links in with the fact that quaternions are crucial in space navigation. Lara Croft appears because she was actually created using quaternions.



Saoirse sa Mhatamaitic (Freedom in Mathematics)

I had been aware for many years about the wide diversity of artworks related to the Hamilton story (see [7]). However, I was not aware of any such artworks involving the Irish language. So, I recently became interested in trying to produce a piece of art (in Irish) about Hamilton's eureka moment at Broombridge. Also, I wanted to try and get the artwork into the second level school, Coláiste Mhuire (St. Mary's College), which is just a few minutes walk from Broombridge. Coláiste Mhuire is a Gaelcholáiste which means that all subjects (except English) are taught through Irish. This led to a collaboration with Dómhnal Ó Bric again on another piece of art about Hamilton's creation of quaternions. We thought it would be a good idea to follow in Hamilton's footsteps by creating a piece of graffiti to commemorate his eureka moment. Figure 3 shows what the recently finished 8 foot by 4 foot artwork will look like before it is put on the wall beside the main front door of the school. The title of the piece is Saoirse sa Mhatamaitic (Freedom in Mathematics). We hope the piece will encourage freedom and creativity in all things, to those who pass through the main door of the school. Currently, there are at least three other artworks in progress related to the Hamilton story.

A ballad

In 2003, Jack Gannon, who lives close to Broombridge, came on the walk, and inspired by the walk, he wrote a song called "The Ballad of Rowan Hamilton" later in the year. The song was first performed at Broombridge at the end of the walk in 2004. Jack's song has been played many times on programmes about Hamilton and the walk on radio and television since. See [4] for the words of the song.

A plaque

In 1958, Taoiseach (Prime Minister) Éamon De Valera unveiled a plaque at Broombridge to commemorate Hamilton's creation of quaternions. The unveiling received substantial coverage in the newspapers. De Valera started his speech stating "I am glad as head of the Government, to be able to honour the memory of a great scientist and a great Irishman. It is a great personal satisfaction for me to be present, because it was well over fifty years ago since I first heard the story of the bridge and the birth of quaternions". De Valera also said "On many occasions since I first heard this story I have come to this place as a holy place". See [4] for the rest of what De Valera said. De Valera was a mathematician himself and lectured in mathematics here in Maynooth during 1912–1914. Also, in 1924, De Valera paid homage to Hamilton by

scratching the quaternion formula on the wall of his prison cell when he was in Kilmanham jail. There is no trace of De Valera's formula. However, in 1966 when he visited Kilmainham jail, he wrote the quaternion formula on an envelope. This envelope is now on display in the museum in Kilmainham jail.

A bog oak sculpture

In 2008 Joey Burns created a bog oak sculpture in Trim with some of Hamilton's equations on it. Hamilton had spent his youth in Trim living on the banks of the River Boyne across from the spectacular ruins of Trim castle. Trim town council had commissioned a piece of art for Castle Street. Joey told me he was inspired by a 2005 Hamilton television documentary to create something related to Hamilton. His sculpture shows the salmon of knowledge rising from the water and reaching for the hazelnuts. Hamilton's equations lie in the waves near the base of the sculpture. In Celtic Mythology, the salmon gains all the world's knowledge when it eats the hazelnuts from the trees surrounding the well at the source of the River Boyne. The well is in Cairbre (aka Carbury) in Co. Kildare and the Boyne flows through Trim near the sculpture.

Quicksilversong of qwaternions

I will now return to the quote from Finnegans Wake at the beginning of this article. In this (seemingly gobbledygook) quote, Joyce is conflating Hamilton with two other Hamiltons. James Hamilton was a Scottish clergyman who published a book of psalms (hence the himmeltones) and James Archibald Hamilton was the first astronomer at Armagh Observatory and he studied the transit of Mercury (hence the quicksilver). So, the lines of seemingly obscure gobbledygook actually have quite a bit going on in the background. The events in Finnegans Wake happen at night (compared with the daytime events in Joyce's other novel, Ulysses) and so Finnegans Wake is more mysterious and obscure. Joyce clearly liked to play around with words and language. However, he also said he could justify every word in Finnegans Wake and the conflation of the three Hamiltons above makes it at least plausible that he could joycetify every word. Joyce also said that Finnegans Wake should be read out loud because the music of the words is very important. I feel the expression "quicksilversong of qwaternions" has a beautiful sound to it that transcends any meaning. Joyce was a very good singer and had an ambition to be a professional tenor. He did end up as a tenner because his face appeared on the old Irish ten pound note! Hamilton liberated algebra from arithmetic by shattering the conventions of ordinary arithmetic. Can we say that Finnegans Wake liberated the novel by shattering the conventions of language? There is another reference to quaternions in Finnegans Wake and it follows to conclude the article. Quaternions are not explicitly mentioned and so see if you can figure out why the quote relates to quaternions (contact me if you like). I feel this quote(rnion) shows Joyce had a deeper knowledge of quaternions than one might think.

his sinister cyclopes after trigamies and spirals' wobbles pursuiting their rovinghamilton selves. — James Joyce in Finnegans Wake

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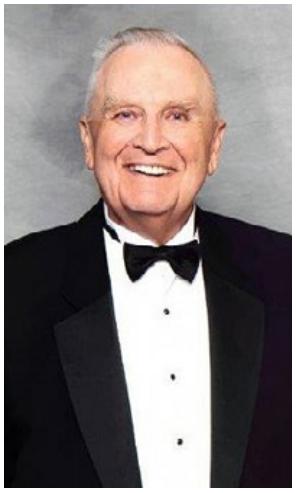
August 12, 2019

Laurence George Hoye (June 9, 1932—June 29, 2019)

Dennis Connolly, connolly@uleth.ca



Laurence George Hoye was born in rural Lloydminster, Alberta during the great depression of the 1930s. Life on a small quarter-section farm was tough, very tough. Laurence attended a one-room school for his first four grades. During WWII the family of four moved to Edmonton, living in a single room for the first two years.

In September 1951 Laurence enrolled in a four year Honours program of Mathematics at the University of Alberta; he was the first member of his family to earn a degree. In 1957, Laurence earned his MSc from the University of Alberta and transferred to the University of Washington in Seattle, where he completed his doctorate. In 1961 Laurence returned to Canada and took up a position teaching Math and Physics courses for the University of Alberta at the Lethbridge Community College.

In late 1966 Laurence transferred to the University of Lethbridge, which was scheduled to open July 1, 1967. Laurence was a member of the U of L's founding Board of Govenors, and recruited two faculty members to join him in the Math. Dept. Between the three of them they taught 18 courses in the first academic year. Incredibly, Laurence taught six of these courses despite his heavy load of admininstrative duties. Laurence remained Department Chair until 1977. It was important to him that his Department establish a strong, well respected reputation, and he was very strict about grade inflation: I remember him telling us to imagine someone from a well established university looking over your shoulder and approving the rigour and integrity of your grading.

After 10 years as Dept. Chair, Laurence became the Associate Dean of Arts and Science, a position which he held for 12 years. During this time, he continued to teach two courses each year in order to "stay in touch with what the whole enterprise is all about." During these intense administrative and teaching years, Laurence temporarily chaired at least four different Departments that were having difficulties, one of which required his leadership for two years. In addition to this work, Laurence also wrote the original computer programs (in Basic) for administrative use throughout the University.

In his last five years at the U of L, Laurence was the Associate Vice President. Laurence retired in 1993, the same year his beloved wife Fran died. Fran had been Laurence's companion and support for 38 years. They had two lovely daughters, Lorna and Valorie.Outside of Mathematics, Laurence had two passions: Trains and Nuclear Power. After retirement, Laurence spent a lot of time visiting and photographing trains (he had over 30,000 photos and slides) in Western US and Canada, and his home library had over 1,000 books on trains. Laurence was very concerned about global warming and canvassed hard to replace oil and coal generators with nuclear power, which he felt was much safer than it was often presented to be.

The U of L was so lucky to have such a powerhouse to establish their Mathematics Department and to provide leadership throughout the University in so many ways and over such an extended period.

We will all miss you Laurence:
You were one of the Greats.

Research Notes brings mathematical research ideas forth to the CMS readership in a generally accessible manner that promotes discussion of relevant topics including research (both pure and applied), activities, and noteworthy news items. Comments, suggestions, and submissions are welcome.

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Les Notes de recherche présentent des sujets mathématiques au lectorat de la SMC dans un format généralement accessible qui favorise les discussions sur divers sujets pertinents, dont la recherche (pure et appliquée), les activités et des nouvelles dignes de mention. Vos commentaires, suggestions et propositions sont les bienvenues.

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Chromatic numbers via commutative algebra

Adam Van Tuyl, *McMaster University*

Colouring graphs is a major subbranch of graph theory. We wish to illuminate an an overlap between this area and commutative algebra. Imagine a department where five committees must meet. Because some faculty members are on more than one committee, the meetings cannot all be scheduled at the same time (our imaginary faculty must attend their meetings!). Since faculty want to avoid meetings, we want to minimize the amount of time needed for meetings. Variations of this problem is a standard example of an application of the chromatic number of a graph. Specifically, represent each committee by a vertex, and join two vertices with an edge if there is someone who is on both committees. As an example, suppose in our fictional department, the corresponding graph is given as in Figure 1.

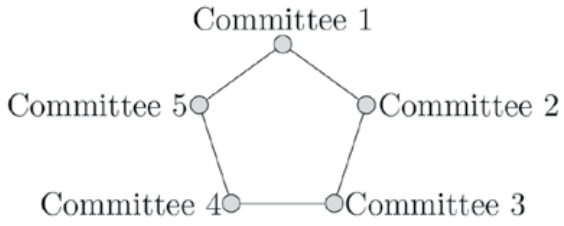

Figure 1. The graph representing committees and shared membership

We assign colours to each vertex so that vertices receive different colours if they are joined by an edge. We want the least number of colours needed for a valid colouring. Figure 1 can be minimally coloured with three colours (e.g., colour Committees 1 and 3 red, Committees 2 and 4 blue, and Committee 5 green). We can schedule our meetings in three hour long slots by scheduling all the committees with the same colour during the same hour slot.

More formally, a finite simple graph G is a pair $G = (V,E)$ where $V = \{x_1, \ldots, x_n\}$ are the vertices, and E consists of unordered pairs of distinct elements of V, called edges. Simple means that loops or multiple edges between vertices are not allowed. The **chromatic number** of G, denoted $\chi(G)$, is the least number of colours in a valid colouring of G. It can be quite difficult to compute $\chi(G)$ (in fact, it is an NP-complete problem).

The commutative algebra community, starting with Villarreal [7], has been interested in studying graphs algebraically. One associates with G two ideals in the polynomial ring $R = \mathbb{Q}[x_1, \ldots, x_n]$. The **edge ideal** of G is

$$I(G) = \langle x_i x_j \mid \{x_i, x_j\} \in E \rangle.$$

That is, the generators of the ideal $I(G)$ are in bijection with the edges of G. For example, the edge ideal of the graph of Figure 1 is

$$I(G) = \langle x_1x_2, x_2x_3, x_3x_4, x_4x_5, x_5x_1 \rangle.$$

The second ideal is the **cover ideal** of G:

$$J(G) = \bigcap_{\{x_i,x_j\} \in E} \langle x_i, x_j \rangle.$$

The nomenclature is due to a correspondence between the generators and the vertex covers of G (a subset $W \subseteq V$ such that $e \cap W \neq \emptyset$ for all $e \in E$). Within this framework, computing $\chi(G)$ can now be rephrased as an ideal membership problem, i.e., asking when a particular element belongs to an ideal. Below, $J(G)^d = \langle g_1 \cdots g_d \mid g_i \in J(G) \rangle$ is the d-th power of $J(G)$.

Theorem 1. ([3, Theorem 3.2]). *Let $G = (V, E)$ be a finite simple graph with cover ideal $J(G)$ and $|V| = n$. Then*

$$\chi(G) = \min \left\{ d \;\middle|\; (x_1 \cdots x_n)^{d-1} \in J(G)^d \right\}.$$

The proof of Theorem 1 exploits the fact that the set of vertices that do not receive a fixed colour form a vertex cover. What is remarkable is $\chi(G)$ can be computed without finding a colouring. Moreover, programs like *Macaulay2* [4] can compute $\chi(G)$ using the ideal membership property.

We can generalize $\chi(G)$ by assigning multiple colours to each vertex. For example, the 2-fold colouring of G, denoted $\chi_2(G)$, assigns a pair of colours to each vertex so that vertices joined by an edge have the property that their corresponding pairs are disjoint. Figure 2 gives a 2-fold colouring of our running example G, and in particular, $\chi_2(G) = 5$.

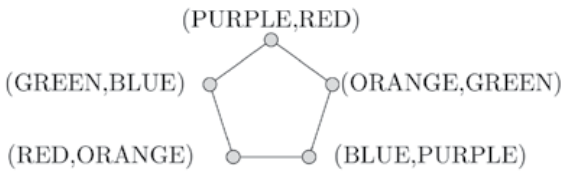

Figure 2. A minimal 2-fold colouring of our graph

For our scheduling problem, instead of one hour slots, we now use half-hour slots, and allow our meetings to take a break. For each slot, schedule all the committees where one of its two colours

match the colour of the slot. We can now schedule all of our meetings in 2.5 hours!

The b-fold chromatic number $\chi_b(G)$ is defined analogously. By normalizing, the **fractional chromatic number** of G is

$$\chi_f(G) = \lim_{b\to\infty} \frac{\chi_b(G)}{b},$$

an invariant of fractional graph theory [5]. For our example, $\chi_f(G) = \frac{5}{2}$.

The fractional chromatic number also has a commutative algebra interpretation. If G is a graph, then the s-th symbolic power of $I(G)$ is

$$I(G)^{(s)} = \bigcap_{W \text{ is a minimal vertex cover of } G} \langle x_i \mid x_i \in W\rangle^s.$$

For any homogeneous ideal K, we let $\alpha(K)$ be the smallest degree of a non-zero element in K. The Waldschmidt constant of $I(G)$ is then

$$\widehat{\alpha}(I(G)) = \lim_{s\to\infty} \frac{\alpha(I(G)^{(s)})}{s}.$$

The Waldschmidt constant has origins in complex analysis [8] and is related to the "ideal containment problem" [2]. Then $\widehat{\alpha}(I(G))$ and $\chi_f(G)$ are related:

Theorem 2. ([1, Theorem 4.6]) *Let $G = (V, E)$ be a finite simple graph with edge ideal $I(G)$. Then*

$$\chi_f(G) = \frac{\widehat{\alpha}(I(G))}{\widehat{\alpha}(I(G)) - 1}.$$

To prove Theorem 2, one uses the fact that both invariants can be described in terms of linear programs, which are then related to each other. Theorems 1 and 2 hold more generally for hypergraphs. There are additional connections between colourings and commutative algebra, including the irreducible decomposition of $J(G)^s$ [3] and secant ideals of $I(G)$ [6].

Acknowledgements. The author thanks C. Francisco, F. Galetto, and T. Hà for their feedback.

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CSHPM Notes brings scholarly work on the history and philosophy of mathematics to the broader mathematics community. Authors are members of the Canadian Society for History and Philosophy of Mathematics (CSHPM). Comments and suggestions are welcome; they may be directed to either of the column's co-editors:

Amy Ackerberg-Hastings, *Independent Scholar (aackerbe@verizon.net)*

or

Hardy Grant, *York University [retired] (hardygrant@yahoo.com)*

Les articles de la SCHPM présente des travaux de recherche en histoire et en philosophie des mathématiques à la communauté mathématique élargie. Les auteur.e.s sont membres de la Société canadienne d'histoire et de philosophie des mathématiques (SCHPM). Vos commentaires et suggestions sont les bienvenues et peuvent être adressées à:

Amy Ackerberg-Hastings, *Chercheuse indépendente (aackerbe@verizon.net)*

ou

Hardy Grant, *York University [retraité] (hardygrant@yahoo.com)*

Investigating the Structure of Inference in Scientific Practice with "Effective Logic"

Robert H. C. Moir, *Western University Department of Computer Science*

There is a tradition in the philosophy of science, spanning over a century, of using tools from mathematical logic to elucidate various aspects of scientific theories, concepts and methods. The idea is to leverage the conceptual precision and powerful formalism that logic provides in order to reconstruct products of science, resulting in clearer insight into their content and interpretation. This approach has had many successful implementations, underlying scores of influential philosophical accounts of the structure of theories, intertheoretic relations, confirmation, explanation, and so on.

More recently, and particularly in the last decade or so, philosophers of science have shown increased interest in a philosophical understanding of aspects of the practice of scientists and mathematicians. An example is the increasing interest in epistemological aspects of simulation and numerical methods. As the complexity and diversity of scientific practice have been increasingly appreciated, the focus on practice has contributed to a move away from general philosophical accounts of science toward precise accounts with more limited scientific scope.

Although, given the complexity of practice, this tendency toward specificity and localized accounts is correct and necessary to some extent, it leaves open the question of whether different methods could reveal general patterns or structures. I want to suggest it is possible to build general accounts of inference in scientific practice using a kind of modeling strategy where scientific inference itself is taken to be the phenomenon we investigate. The strategy I propose uses an informal generalized logic that I call "effective logic" to characterize structural patterns in scientific practice.

My research has revealed a surprisingly general pattern in scientific inference that appears in essentially any situation where a scientific method produces a problem to solve but does not provide the means to solve that problem directly given the constraints experienced in practice. The result is an inferential obstruction: we may have a precise specification of the problem and what its solution may look like, but no way of actually producing a solution. Since there is no path through the obstruction, we must go around it. The way around involves changing the framework within which the problem is posed to make solution feasible, and then mapping back to the original framework to reconstruct a solution to the original problem. In this way we overcome the inferential obstacle by strategically substituting the original unsolvable problem for a solvable one whose solution makes the original problem solvable.

I discovered this pattern in my dissertation research in philosophy, where I sought to investigate the structure of application of theories in scientific practice through a detailed case study of how impact probabilities of near-Earth objects are computed. The pattern shows up in two main places: the way that the orbit-determination problem is solved by connecting (phase space) positions in an inertial solar-system reference frame to image points on a telescope-mounted charge-coupled device (CCD) at an observatory; and the way that differential equations are solved using numerical methods and machine computing. Both involve a problem that is not directly solvable (solar-system orbit, nonlinear differential equation), but that becomes solvable by reducing the problem to a different problem that can be easily solved (by CCD measurements or CPU operations) and then mapped to a solution to the original problem (by solving least-squares problems or by data translation and interpolation).

The result of this investigation was a hypothesis, namely that this pattern would appear in scientific practice whenever a clear problem is encountered that cannot be solved directly given practical constraints. This hypothesis has turned out to be very well validated, not only in physical sciences, but also in computational, applied and even pure mathematics. The forms of the pattern that emerge in these contexts show how the pattern is used not only as a tool to make solution possible, but also as a tool to reduce either computational complexity or the difficulty of proof.

The case of computational mathematics is interesting, because two distinct forms of the pattern emerge as a result of computations being either exact or approximate. The latter case encompasses the majority of scientific computing, which relies on the fact that double-precision floating-point numbers are precise enough for most applications, meaning that computations with real numbers can be reduced to simple bit operations on 64-bit machine words. Together with numerical methods that allow for the discretization of continuous theoretical models, we then have a means of reducing insoluble nonlinear differential equations to easily computable

word operations on CPU. What is important about these numerical and machine methods is that they both introduce error, so that the solutions produced are only approximate solutions to the original problem. This is crucially important, because instability of the problem (ill-conditioning) or of the solution method (numerical instability) can cause the results of a computation to be meaningless. Thus, great care is needed to ensure that the solution method produces a reliable result (numerically stable methods where the problem is well-conditioned); hence the importance of numerical-stability proofs in numerical analysis.

The case of exact computation, on the other hand, avoids issues of instability but has limitations due to the size (space and time complexity) of computations. For these reasons, even if problems can be solved *algorithmically* in their original forms (which may not be the case), many problems cannot be *feasibly* solved in their original forms because the computations are infeasible (due to intermediate expression swell requiring too many CPU operations or too much memory), even on modern computing architectures. Accordingly, exact computation has developed many techniques to reduce complex computations to simple bit operations on 64-bit words. Rather than using floating-point numbers, though this is sometimes possible, the strategy is to reduce to finite fields modulo a small, i.e., 64-bit, prime and to then reconstruct the solution to the original problem using techniques from number theory and algebra (specifically, versions of the Chinese remainder theorem or Hensel's lemma). These so-called modular methods, then, are techniques among others in computer algebra that make many difficult problems, such as polynomial factorization, efficiently computable by strategically transforming the problem to a rapidly solvable one from which the exact solution to the original problem can be obtained.

The exact and approximate versions of the pattern also appear in pure and applied mathematics, depending on whether or not an exact or approximate result is provable. The approximate version is naturally common in many areas of applied mathematics, particularly those that rely on Taylor expansions, asymptotic methods and perturbation methods. For example, in the context of analytical fluid dynamics, it is common to reduce a single problem in partial differential equations (PDE) to many simpler such problems in asymptotic regimes (where different simplifying approximations or boundary conditions apply), and then these problems are further reduced to ordinary differential equations, which must then be solved by other methods. Matching or other methods can then be used to combine the solutions of the simpler PDE into an approximate solution to the original problem. But we can also see certain results in pure mathematics, such as the celebrated prime number theorem (particularly the non-elementary proof using Mellin transforms), as an example of the approximate-inference pattern where we gain approximate insight into the true distribution of primes by proving a feasible weaker result that gives an approximate distribution.

Given that the principal concern of mathematics is exact proof, we expect the exact version of the pattern to be ubiquitous in mathematics where a result cannot be proved directly. Indeed, we find examples of the exact pattern all over mathematics, from historical examples such as analytic geometry, which solves geometric problems by reducing them to feasible algebraic ones, to modern ones such as arithmetic geometry, where difficult problems of arithmetic are solved using a variety of techniques from other branches of mathematics, including p-adic analysis and algebraic geometry. Indeed, a strategy of transforming mathematical problems in order to simplify proofs, or to make proof possible, has become ubiquitous in mathematics following the rise of abstract algebra in the 20th century, leading to the many successes of modern algebraic geometry and algebraic topology. In these cases, the transformations between problems that make solution feasible take the form of precise mathematical transformations between categories (pairs of adjoint functors or categorical equivalences).

So what are we to make of the apparent ubiquity of this pattern of feasible problem-solving in scientific practice? Since there appears to be such a robust pattern in the methods used by practicing scientists across fields, the suspicion arises that there is a latent structure in scientific reasoning that has not been properly recognized. As a means of attempting to bring out this structure more clearly, I have indicated (see [1]) how we can understand this structure in terms of a kind of generalized logic. This logic attempts to describe the structure of reliable inference methods in terms of a generalized notion of valid inference: I define an inference to be "effectively valid" if nearly-true premises yield nearly-true conclusions. In distinction to traditional valid inferences, which preserve truth, effectively valid inferences preserve near-truth.

The feasible inference pattern we have been discussing is captured in this effective logic in terms of transformations of problems that enable the completion of effectively valid inferences. But the logic here is not like traditional formal logic, because it is not codified in terms of axioms and inference rules. Inference rules in effective logic are theorems or systematic methods developed by scientists that can be applied to make inferences feasible—they are discovered or developed, not given. Because effectively valid inferences preserve only near-truth we can account for the approximate version of the inference pattern, since an approximately true solution is a nearly-true one. But in the limit, where near-truth goes to exact truth, we recover deductive inference, meaning that the exact version of the inference pattern can be accounted for as well. Indeed, in perturbation-theoretic terms deductive inference is a singular limit of approximate inference, since the possibility for instability vanishes in the limit.

We may see, therefore, that an initial investigation into a specific case in scientific practice (impact-probability computation) has revealed a robust general pattern in inference methods in scientific practice, the structure of which can be captured in terms of an informal generalized logic. This project is still in very early stages, with the next steps involving a search for a more precise formalism,

a deeper probe of the structure of stability proofs where they are possible, and codification of the pattern in different areas of science. There are nonetheless intriguing hints of a latent logic of scientific practice that can be revealed by using modeling methods to elucidate extant patterns in how inference is accomplished by practicing scientists and mathematicians.

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[1] Moir, Robert H. C. (2019) Effective Validity: A Generalized Logic for Stable Approximate Inference. In Nicolas Fillion, Robert Corless and Ilias S. Kotsireas (eds), *Algorithms and Complexity in Mathematics, Epistemology, and Science*, 225–268. Fields Institute Communications vol. 82. New York: Fields Institute for Research in the Mathematical Sciences and Springer.

Robert H.C. Moir (**robert@moir.net**) *is a post-doctoral fellow in the department of computer science at Western University. He holds PhDs in both philosophy and applied mathematics. His interests in philosophy involve mapping scientific inference by constructing epistemic models of inference methods in scientific practice. His interests in computational science involve approximate symbolic-numeric and continuous exact methods for the integration problem in computer algebra. His current research focuses on developing an exact nonlinear system solver for the open-source bpas library* (**bpaslib.org**).

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CMS Notes
Volume 51, No. 5, October/November 2019

Canadian Mathematical Society
Société mathématique du Canada

DECEMBER 6-9 DÉCEMBRE, 2019

2019 CMS Winter Meeting
Réunion d'hiver de la SMC 2019

Toronto, ON

Plenary speakers | Conférences plénières

Maria Chudnovsky (Princeton)
Sarah Mayes Tang (Toronto)
Antonio Montalban (California)
Kirsten Morris (Waterloo)
Malabika Pramanik (British Columbia)
Lauren K. Williams (Harvard)

Public Lecture | Conférence publique

David Earn (McMaster)

Prizes | Prix

Doctoral Prize Lecture | Conférence de prix de doctorat
Dr. Mikhail Karpukhin (California)

Coxeter James Prize | Conférence de prix de Coxeter-James
Dr. Jacob Tsimerman (Toronto)

Adrien Pouliot Prize | Conférence de prix Adrien-Pouliot
Tiina Hohn (MacEwan)

Graham Wright Award for Distinguished Service | Prix Graham-Wright pour service méritoire
Karl Dilcher (Dalhousie)

G. de B. Robinson Award | Prix G. de B. Robinson
Lars Louder (University College London), Henry Wilton (Cambridge)

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Canadian Mathematical Society
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CMS WINTER MEETING | RÉUNION D'HIVER DE LA SMC

DECEMBER 6-9 DÉCEMBRE, 2019

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- Iwasawa Theory of Fine Selmer Groups | Théorie d'Iwasawa sur les groupes de Selmer finis
- Mathematical Finance | Finances mathématiques
- Mathematical Programming with Maple Workshop | Atelier de programmation mathématique Maple
- Modeling and Dynamics of Mosquito Population and Transmission of Mosquito-borne Diseases | Modélisation et dynamique d'une population de moustiques et
- Modeling Infestations using Bayesian Networks |modélisation des infestations avec les réseaux Bayésiens
- On the classification of simple nuclear C*-algebras | Sur la classification des C*-algèbres nucléaires simples
- Topological Data Analysis | Analyse topologique de données transmission des maladies émises par les moustiques
- Vertex Pursuit Games on Graphs | Jeux de poursuite vertex sur graphes

CHILDCARE | SERVICE DE GARDE D'ENFANTS

Parents are welcome to bring their children. CMS will be offering childcare from Saturday to Monday for registered participants during the 2019 Winter Meeting in Toronto. Register online. **If you are a CMS member this service will be complimentary**

Lors la Réunion d'hiver 2019 à Toronto, un service de garde sera offert sur place, de samedi à lundi, aux parents inscrit.e.s qui souhaitent amener leurs enfants. **Ce service sera gratuit pour les membres de la Société.**

Canadian Mathematical Society
Société mathématique du Canada

JUNE 5-8 JUIN, 2020

CMS 75th Anniversary Summer Meeting

Réunion d'été du 75e anniversaire de la SMC

Ottawa, ON

Plenary speakers | Conférences plénières:

Henri Darmon (McGill)
Moon Duchin (Tufts)
Matilde Marcolli (Toronto)
Aaron Naber (Northwestern)
Ian Putnam (Victoria)

Public Lecture | Conférence publique:

Anne Broadbent (Ottawa)

Scientific Directors | Directeurs scientifiques

Ailana Fraser (University of British Columbia)
Monica Nevins (University of Ottawa)
Mateja Šajna (University of Ottawa)

2020 CMS Math Competition Grants

The CMS is now accepting applications for the 2020 CMS Math Competition Grants program. The CMS supports activities that promote the learning of mathematics among Canadian youth. In addition to the Society's math competitions, the CMS offers math competition grants for activities at the elementary and secondary school levels.

The deadline for submissions is **November 15, 2019**. Successful applicants will be informed in January 2020 and the grants issued in February 2020.

Further details about the math competitions grants and the application process are available on the CMS website: **https://cms.math.ca/Competitions/grants**

The Committee on Grants for Provincial Competitions (CGPC) adjudicates proposals for support. Should you have further questions or comments, please contact the Committee by e-mail at **chair-grants-pc@cms.math.ca**

Applications should be submitted electronically, preferably in PDF format, **no later than November 15, 2019** to **mathgrants@cms.math.ca**.

Subventions pour les concours mathématiques de la SMC 2020

La SMC accepte maintenant des demandes de subventions pour le programme des concours de mathématiques de la SMC 2020. La SMC appuie des activités qui favorisent l'apprentissage des mathématiques chez les jeunes canadien.ne.s. En plus d'organiser ses propres concours de mathématiques, la SMC offre des subventions pour les concours de mathématiques scolaires au niveau primaire et secondaire.

La date limite pour présenter une demande est **le 15 novembre 2019**. Les projets retenus seront annoncés en janvier 2020, et les bourses seront distribuées en février 2020.

Pour vous procurer un formulaire ou pour plus de renseignements sur l'appel de projets, visitez le site de la SMC au : **https://cms.math.ca/Concours/grants**

Le comité du financement des concours provinciaux (CFCP) évalue les demandes de bourse. Pour toute question ou tout commentaire sur le financement des concours provinciaux, veuillez communiqué par courriel avec le comité à **pres-grants-pc@smc.math.ca**

Les demandes devraient être présentées par voie électronique, préférablement en format PDF, **au plus tard le 15 novembre 2019**, à l'adresse suivante : **subventionsmaths@smc.math.ca**

2020 Excellence in Teaching Award

The CMS Excellence in Teaching Award Selection Committee invites nominations for the **2020 Excellence in Teaching Award**.

The Excellence in Teaching Award focuses on the recipient's proven excellence as a teacher at the undergraduate level, including at universities, colleges and cégeps, as exemplified by unusual effectiveness in the classroom and/or commitment and dedication to teaching and to students. The dossier should provide evidence of the effectiveness and impact of the nominee's teaching. The prize recognizes sustained and distinguished contributions in teaching at the post-secondary undergraduate level at a Canadian institution. Only full-time teachers or professors who have been at their institution for at least five years will be considered. The nomination will remain active for three years, with a possibility to update.

Nomination letters, *including at least three letters of reference*, should list the chosen referees and include a recent curriculum vitae for the nominee, if available.

CMS aims to promote and celebrate diversity in the broadest sense. We strongly encourage department chairs and nominating committees to put forward nominations for outstanding colleagues regardless of race, gender, ethnicity or sexual orientation.

Nominations and reference letters should be submitted electronically, preferably in PDF format, to: etaward@cms.math.ca no later than the deadline of **November 15, 2019**.

Prix d'excellence en enseignement 2020

Le comité de sélection du Prix d'excellence en enseignement de la SMC sollicite des mises en candidature pour le **Prix d'excellence en enseignement 2020**.

Le Prix d'excellence en enseignement de la SMC récompense l'excellence reconnue d'un.e enseignant.e ou d'un.e professeur.e de niveau postsecondaire (universités, collèges et cégeps), telle qu'illustrée par son efficacité exceptionnelle en classe et/ou son engagement et son dévouement envers l'enseignement et les étudiant.e.s. Le dossier de candidature doit montrer l'efficacité et les effets de l'enseignement du candidat ou de la candidate. Ce prix récompense des contributions exceptionnelles et soutenues en enseignement collégial et de premier cycle universitaire dans un établissement canadien. Seules les candidatures d'enseignant.e.s et de professeur.e.s à temps plein qui travaillent dans le même établissement depuis au moins cinq ans seront retenues. Une candidature peut être mise à jour et demeure active pendant 3 ans.

Le dossier de candidature, *comprenant au moins trois lettres de référence*, doit comprendre le nom des personnes données à titre de référence ainsi qu'un curriculum vitae récent du candidat ou de la candidate, dans la mesure du possible.

La SMC a pour but de promouvoir et de célébrer la diversité au sens le plus large. Nous encourageons fortement les directeurs et directrices de département et les comités de mise en candidature à proposer des collègues exceptionnel.le.s sans distinction de race, de genre, d'appartenance ethnique ou d'orientation sexuelle.

Veuillez faire parvenir les mises en candidature et lettres de référence par voie électronique, de préférence en format PDF, à : prixee@smc.math.ca avant la date limite du **15 novembre 2019**.

2020 Doctoral Prize

The **CMS Doctoral Prize** recognizes outstanding performance by a doctoral student. The prize is awarded to one or two recipients of a Ph.D. from a Canadian university whose overall performance in graduate school is judged to be the most outstanding. Although the dissertation will be the most important criterion (the impact of the results, the creativity of the work, the quality of exposition, etc.) it will not be the only one. Other publications, activities in support of students and other accomplishments will also be considered.

Nominations that were not successful in the first competition will be kept active for a further year (with no possibility of updating the file) and will be considered by the Doctoral Prize Selection Committee in the following year's competition.

The CMS Doctoral Prize will consist of an award of $500, a two-year complimentary membership in the CMS, a framed Doctoral Prize certificate and a stipend for travel expenses to attend the CMS meeting to receive the award and present a plenary lecture.

Nominations

Candidates must be nominated by their university and the nominator is responsible for preparing the documentation described below, and submitting the nomination to the address below. CMS aims to promote and celebrate diversity in the broadest sense. We strongly encourage department chairs and nominating committees to put forward nominations for outstanding colleagues regardless of race, gender, ethnicity or sexual orientation. The deadline for the receipt of nominations is **January 31, 2020**.

The documentation shall consist of:

- A curriculum vitae prepared by the student.
- A resumé of the student's work written by the student and which must not exceed ten pages. The resumé should include a brief description of the thesis and why it is important, as well as of any other contributions made by the student while a doctoral student.
- Three letters of recommendation of which one should be from the thesis advisor and one from an external reviewer. A copy of the external examiner's report may be substituted for the latter. More than three letters of recommendation are not accepted.

All documentation, including letters of recommendation, should be submitted electronically, preferably in PDF format, no later than January 31, 2020, to **docprize@cms.math.ca**.

Prix de doctorat 2020

La SMC a créé ce **Prix de doctorat** pour récompenser le travail exceptionnel d'un.e étudiant.e au doctorat. Le prix sera décerné à une personne qui aura reçu son diplôme de troisième cycle d'une université canadienne l'année précédente (entre le 1er janvier et le 31 décembre) et dont les résultats pour l'ensemble des études supérieures seront jugés les meilleurs. La dissertation constituera le principal critère de sélection (impact des résultats, créativité, qualité de l'exposition, etc.), mais ne sera pas le seul aspect évalué. On tiendra également compte des publications de l'étudiant.e, de son engagement dans la vie étudiante et de ses autres réalisations.

Les mises en candidature qui ne seront pas choisies dans leur première compétition seront considérées pour l'année suivante (sans possibilité de mise à jour du dossier), et seront révisées par le comité de sélection du Prix de doctorat l'an prochain.

Le lauréat ou la lauréate du Prix de doctorat de la SMC aura droit à une bourse de 500 $. De plus, la SMC lui offrira l'adhésion gratuite à la Société pendant deux ans et lui remettra un certificat encadré et une subvention pour frais de déplacements lui permettant d'assister à la réunion de la SMC où il recevra son prix et présentera une conférence.

Candidatures

Les candidat.e.s doivent être nommés par leur université; la personne qui propose un.e candidat.e doit se charger de regrouper les documents décrits aux paragraphes suivants et de faire parvenir la candidature à l'adresse ci-dessous. La SMC a pour but de promouvoir et de célébrer la diversité au sens le plus large. Nous encourageons fortement les directeurs et directrices de département et les comités de mise en candidature à proposer des collègues exceptionnels sans distinction de race, de genre, d'appartenance ethnique ou d'orientation sexuelle. Les candidatures doivent parvenir à la SMC au plus tard le **31 janvier 2020**.

Le dossier sera constitué des documents suivants :

- Un curriculum vitae rédigé par l'étudiant.e.
- Un résumé du travail du candidat ou de la candidate d'au plus dix pages, rédigé par l'étudiant.e, où il ou elle décrira brièvement sa thèse et en expliquera l'importance, et énumérera toutes ses autres réalisations pendant ses études de doctorat.
- Trois lettres de recommandation, dont une du directeur ou d'une directrice de thèse et une d'un.e examinateur.e de l'extérieur (une copie de son rapport serait aussi acceptable). Le comité n'acceptera pas plus de trois lettres de recommandation.

Veuillez faire parvenir tous les documents par voie électronique, de préférence en format PDF, au plus tard le 31 janvier 2020 à **prixdoc@smc.math.ca**.

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Volume 51, No. 5, October/November 2019

Call for CMS Meeting University Hosts

Deadline: December 16, 2019

The Canadian Mathematical Society (CMS) welcomes and invites host proposals from Canadian Universities for the 2021 CMS Winter Meeting, the 2022 CMS Winter Meeting and the 2023 CMS Summer Meeting.

CMS will provide all logistical support and contract negotiation with local venues. CMS is looking for Canadian Universities who are willing and able to showcase their department and University to students and faculty from across Canada. It is asked that proposals include the following information:

1. LOCATION

   How would people get from the airport to the venue? What are the reasons your city may be of interest to Canadian Mathematicians?

2. SITE

   (For summer meetings) Describe your University where the meeting would be held. Which building would the meeting be in and how many rooms are available for meeting sessions and plenaries? What technological support is available in session rooms? Will these rooms be available during the proposed dates?

   (For winter meetings) Do you have a venue in mind for the meeting, is your University available to host the meeting onsite? If not, CMS will find a property outside the university.

3. LODGING

   Is your university able to offer any residence lodging during the conference dates? CMS will take care of contracting and negotiating with hotels.

4. HOST UNIVERSITY

   Please describe briefly your institution and department. What funding support will the Host University have for the CMS Meeting? Is the University available for regular calls and updates on the meeting progress? Is the Host University able to commit and provide at least one scientific director for the meeting? What level of participation do you think there might be from academics at your institution?

The CMS Meetings typically run from Friday to Monday on the first weekend in June and December but we are open to other possibilities. Summer meetings typically have 250-350 registrants and winter meetings are typically 400-600 in larger cities.

Please admit your submissions to Sarah Watson (**meetings@cms.math.ca**) before December 16th, 2019.

Appel de mise en nomination pour l'accueil de la Réunion de la SMC

Date d'échéance : Le 16 décembre 2019

La Société mathématique du Canada (SMC) sollicite les universités canadiennes souhaitant accueillir les Réunions d'hiver 2021 et 2022 et la Réunion d'été 2023 de la SMC d'envoyer leurs propositions.

La SMC se charge du soutien logistique et de toute négociation de contrats auprès des fournisseurs locaux. La SMC est à la recherche d'Universités canadiennes disposées à mettre en valeur leur département et leur université auprès des étudiant.e.s et des professeur.e.s partout au Canada. Les propositions doivent contenir les informations suivantes :

1. LOCALISATION

   Comment les personnes participant à la Réunion pourront-elles se rendre au lieu depuis l'aéroport? Pourquoi votre ville intéressera-t-elle les mathématicien.ne.s canadien.ne.s?

2. SITE

   (Pour la Réunion d'été) Une description de votre Université où aura lieu la réunion.

   Les immeubles où sera tenue la réunion et le nombre de salles disponibles pour les sessions plénières et parallèles. Les technologies disponibles dans les salles. La disponibilité des salles pendant les dates proposées.

   (Pour les Réunions d'hiver) Le lieu de la Réunion : Votre Université est-elle en mesure d'accueillir la réunion sur place? Sinon, la SMC se charge de trouver un lieu en dehors de l'Université.

3. LOGEMENT

   Votre Université sera-t-elle en mesure d'offrir des logements pendant les dates de la réunion? La SMC se charge de toutes les négociations auprès des hôtels.

4. UNIVERSITÉ HÔTE

   Veuillez décrire votre établissement et votre département. Quels sont les soutiens financiers offerts par l'Université à la SMC? Votre Université est-elle en mesure d'être régulièrement en contact avec la Société pour avancer l'organisation de la Réunion? Y a-t-il au moins un directeur ou une directrice scientifique qui commettra son temps à la Réunion durant les dates proposées? Quel sera le taux de participation des membres de votre établissement?

Les Réunions de la SMC ont normalement lieu du vendredi au lundi de la première fin de semaine de juin et de décembre, mais nous sommes ouvert.e.s à d'autres possibilités. Les Réunions d'été reçoivent typiquement entre 250 et 350 participant.e.s et les Réunions d'hiver entre 400 et 600 participant.e.s quand elles ont lieu dans de grandes villes.

Veuillez envoyer vos propositions à Sarah Watson (**reunions@smc.math.ca**) avant le 16 décembre 2019.

YORK UNIVERSITÉ UNIVERSITY

Department of Mathematics and Statistics, Faculty of Science

Statistics, Assistant Professor

The Department of Mathematics and Statistics, Faculty of Science at York University invites applications for a tenure-track professorial stream appointment in Statistics at the Assistant Professor level, to commence July 1, 2020. Salary will be commensurate with qualifications and experience. All York University positions are subject to budgetary approval.

The successful candidate will have a PhD in Statistics or a related discipline and an outstanding research record (appropriate to their career stage) including sustained and recent publications in high quality peer reviewed journals. Outstanding candidates in any area of statistics will be considered with particular emphasis on machine learning, high dimensional data analysis, data science and artificial intelligence. The successful candidate will be expected to develop an excellent and innovative research program building on the current strengths of the Department, and secure and maintain external peer-reviewed research funding.

The successful candidate must be suitable for prompt appointment to the Faculty of Graduate Studies. The position will involve graduate teaching and supervision, as well as undergraduate teaching. Pedagogical innovation in high priority areas such as experiential education and technology enhanced learning is preferred.

York University has a policy on **Accommodation in Employment for Persons with Disabilities** and is committed to working towards a barrier-free workplace and to expanding the accessibility of the workplace to persons with disabilities. Candidates who require accommodation during the selection process are invited to contact Paul Szeptycki, Chair of the Department, at **szeptyck@yorku.ca**.

York University is an Affirmative Action (AA) employer and strongly values diversity, including gender and sexual diversity, within its community. The AA Program, which applies to women, members of visible minorities (racialized groups), Aboriginal (Indigenous) people and persons with disabilities, can be found at **www.yorku.ca/acadjobs** or by calling the AA line at 416-736-5713. Applicants wishing to self-identify as part of York University's Affirmative Action program can do so by downloading, completing and submitting the form found at: **http://acadjobs.info.yorku.ca/affirmative-action/self-identification-form**. All qualified candidates are encouraged to apply; however, Canadian citizens, permanent residents and Indigenous peoples in Canada will be given priority. No application will be considered without a completed mandatory Work Status Declaration form which can be found at: **http://acadjobs.info.yorku.ca/affirmative-action/work-authorization-form**.

Applications must be received by **November 1, 2019**. Only applications received through the AMS MathJobs website, **www.mathjobs.org**, will be considered. Applicants are required to provide a CV, three signed letters of reference, a statement on teaching, a statement on research, the Work Status Declaration (Citizenship) form, and a covering letter. Applicants wishing to self-identify may include the self-identification form which can be found at the above mentioned link. Once this form has been signed it can be uploaded to MathJobs.

Pure Mathematics, Assistant Professor

The Department of Mathematics & Statistics, Faculty of Science at York University invites applications for a professorial-stream tenure-track appointment in any field of Pure Mathematics at the Assistant Professor level, to commence July 1, 2020. Salary will be commensurate with qualifications and experience. All York University positions are subject to budgetary approval.

The successful candidate will have a PhD in Mathematics or a related discipline, postdoctoral experience in relevant areas, an outstanding research record, (appropriate to their career stage), including sustained and recent publications in high quality peer reviewed journals. Applicants with an excellent program of research specializing in any area of pure mathematics will be considered. Candidates must have a proven record of independent research excellence, be expected to develop an excellent and innovative research program, secure and maintain external peer-reviewed research funding and show evidence of potential for superior teaching and mentoring of students and postdoctoral fellows. All candidates are encouraged to apply including those who reinforce existing strengths and those who fill gaps in important areas of pure mathematics that are currently underrepresented in the department.

The successful candidate must be suitable for prompt appointment to the Faculty of Graduate Studies. The position will involve graduate teaching and supervision, as well as undergraduate teaching. A record of teaching excellence and pedagogical innovation in areas such as experiential education and technology enhanced learning is preferred.

York University has a policy on **Accommodation in Employment for Persons with Disabilities** and is committed to working towards a barrier-free workplace and to expanding the accessibility of the workplace to persons with disabilities. Candidates who require accommodation during the selection process are invited to contact Paul Szeptycki, Chair of the Department, at **szeptyck@yorku.ca**.

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Applications must be received by **November 1, 2019**. Only applications received through the AMS MathJobs website, **www.mathjobs.org**, will be considered. Applicants are required to provide a CV, three signed letters of reference, a statement on teaching, a statement on research, the Work Status Declaration (Citizenship) form, and a covering letter. Applicants wishing to self-identify may include the self-identification form which can be found at above mentioned link. Once this form has been signed it can be uploaded to MathJobs.

Actuarial Science, Assistant Professor and Associate Professor (2 positions)

The Department of Mathematics and Statistics, Faculty of Science at York University invites applications for two (2) professorial stream tenure-track appointments in Actuarial Science. One of these appointments is at the Assistant Professor level, the other one is at the Associate Professor level. Both positions are to commence July 1, 2020. Salary will be commensurate with qualifications and experience. All York University positions are subject to budgetary approval.

For the appointment at the Assistant Professor level, successful applicants are expected to have a Ph.D. in Actuarial Science, or Mathematics, or Statistics (or a dissertation thesis in a related area). In addition, successful applicants should have a professional designation of Fellow from one of the SoA / CAS / CIA / IFoA / IAA (or equivalent) by the date of appointment. That said, exceptional applicants that have the designation of Associate from one of the just-mentioned organizations and are actively pursuing fellowship will be considered. In addition, successful applicants must demonstrate potential for excellence in scholarly research, teaching and service.

Excellence in scholarly research involves an ongoing program of research in which the output appears in top Actuarial Science journals, e.g., *Insurance: Mathematics and Economics; North American Actuarial Journal; ASTIN Bulletin: The Journal of the International Actuarial Association; or the Scandinavian Actuarial Journal.* Applicants that specialize in any one of the following areas: risk measurement and management, insurance pricing, dependence modelling, retirement income modeling, are particularly welcome to apply and will be given priority, although exceptional candidates from all areas of Actuarial Science will be considered.

Successful applicants must be suitable for prompt appointment to the Faculty of Graduate Studies as the positions will involve graduate teaching and supervision, as well as undergraduate teaching. Pedagogical innovation in high priority areas such as experiential education, technology enhanced learning and integrated Actuarial Science and Data Science / Business / Economics / Accounting / Computer Science curricula is considered a strong asset.

A record of professional experience in industry will be given additional weight when comparing candidates with similar research credentials.

For the appointment at the Associate Professor level, successful applicants are expected to satisfy all the criteria required for the appointment at the Assistant Professor level, listed above. In addition, these applicants should demonstrate excellence in scholarly research, exceptional record of recent scholarly funding, a record of supervision of undergraduate and graduate students, as well as substantial and ongoing involvement within professional and scientific communities. Examples of the former are: serving on committees with one of the SoA / CAS / CIA / IFoA / IAA (or equivalent), and examples of the latter are: serving on the editorial boards of international scholarly journals, organizing international conferences. A record of experience leading a large Actuarial Science program will be given additional weight when comparing candidates with similar research and teaching credentials.

Note that York University has a policy on **Accommodation in Employment for Persons with Disabilities** and is committed to working towards a barrier-free workplace and to expanding the accessibility of the workplace to persons with disabilities. Candidates who require accommodation during the selection process are invited to contact Paul Szeptycki, Chair of the Department, at **szeptyck@yorku.ca**.

York University is an Affirmative Action (AA) employer and strongly values diversity, including gender and sexual diversity, within its community. The AA Program, which applies to women, members of visible minorities (racialized groups), Aboriginal (Indigenous) people and persons with disabilities, can be found at www.yorku.ca/acadjobs or by calling the AA line at 416-736-5713. Applicants wishing to self-identify as part of York University's Affirmative Action program can do so by downloading, completing and submitting the form found at: **http://acadjobs.info.yorku.ca/affirmative-action/self-identification-form**. All qualified candidates are encouraged to apply; however, Canadian citizens, permanent residents and Indigenous peoples in Canada will be given priority. No application will be considered without a completed mandatory Work Status Declaration form which can be found at: **http://acadjobs.info.yorku.ca/affirmative-action/work-authorization-form**.

Applications must be received by **October 21, 2019**. Only applications received through the AMS MathJobs website, **www.mathjobs.org**, will be considered. Applicants are required to provide three signed letters of reference, CV, a statement on teaching, a statement on research, the Work Status Declaration (Citizenship) form, and a covering letter. Applicants wishing to self-identify may also include the self-identification form which can be found at above mentioned link. Once this form has been signed it can be uploaded to MathJobs.

CMS Notes
Volume 51, No. 5, October/November 2019

Annual Fundraising Campaign

Last year, members of the CMS Board of Directors, staff members and friends of the CMS took time to contact current and past members of the Society (either by email or phone) and thank them for their participation in CMS activities and to encourage them to renew their CMS membership (if they hadn't already done so). This friend-raising and fundraising campaign was very successful and gave members a chance to comment upon what they enjoyed about belonging to the Society, as well as constructive advice on how the Society could be improved.

Once again, at the close of the year, the CMS plans to contact and thank its community again, as well as to encourage further membership in the Society and to invite its members to give as generously as possible to the CMS.

With your regular donations, as well as those from foundations, corporations, governments and institutions, the CMS programs and activities will be expanded as will our ability to truly represent the full community of mathematicians in Canada.

The CMS encourages you to consider donations through **Planned Giving**.

You can also help by:

- If you have contacts in private industry or with foundations, with whom the CMS can form potential partnerships, please communicate this information to Zishad Lak, CMS Fundraising and Communications Officer.
- If you are interested in volunteering for a CMS committee, please contact the Chair of the Nominating Committee, David Pike (Memorial) - **chair-nomc@cms.math.ca**
- Encourage colleagues to become members of the CMS!

Working together, we can continue to promote the advancement, discovery, learning, and application of mathematics. If you have questions or want more information, please contact Zishad Lak at **zlak@cms.math.ca**.

Campagne annuelle de financement

L'année dernière, les membres du conseil d'administration, le personnel, et les ami.e.s de la SMC, ont pris le temps de contacter les membres actuel.le.s et les ancien.ne.s membres de la Société (par courriel ou téléphone), pour les remercier de leur participation aux activités de la SMC et les encourager à renouveler leur adhésion (s'ils ou elles ne l'avaient pas déjà fait). Cette campagne de recrutement et de collecte de fonds a été couronnée de succès et a permis aux membres de nous dire ce qu'ils ou elles aimaient du fait d'être membres de la Société et d'offrir des conseils constructifs sur ce que la Société pourrait faire de mieux.

Une fois de plus, à la fin de l'année, la SMC prévoit contacter et remercier sa communauté, encourager d'autres personnes à devenir membres et inviter ses membres à donner le plus généreusement possible à la Société.

Grâce à vos dons constants, ainsi qu'aux contributions des fondations, des sociétés, des gouvernements et des établissements d'enseignement, les programmes et les activités de la SMC seront améliorées, tout comme notre capacité de représenter véritablement l'ensemble de la communauté mathématique canadienne.

La SMC vous invite à réfléchir à la possibilité de faire un **don planifié**.

Ce que vous pouvez aussi faire pour aider la SMC :

- Si vous avez des contacts dans le secteur privé ou les fondations avec qui la SMC peut discuter d'éventuels partenariats, parlez-en à Zishad Lak, agente de la collecte de fonds et des communications à la SMC.
- Si vous souhaitez devenir membre d'un comité de la SMC, veuillez contacter le président du Comité des mises en candidature, David Pike (Memorial) – **chairnomc@smc.math.ca**
- Encouragez vos collègues à devenir membres de la SMC!

En travaillant ensemble, nous pouvons continuer à promouvoir l'avancement, la découverte, l'apprentissage et l'application des mathématiques. Si vous avez des questions ou pour obtenir plus d'informations, veuillez contacter Zishad Lak à l'adresse **zlak@smc.math.ca**.

If undelivered, please return to:
Si NON-LIVRÉ, veuillez retourner à :

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209 - 1725 St. Laurent Blvd Ottawa, ON K1G 3V4 Canada