Research Directions in Number Theory
Edited by Jennifer S. Balakrishnan, Amanda Folsom, Matilde Lalín, and Michelle Manes
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There have been four conferences so far that were organized by the Women in Numbers (WIN) network. This volume originated from the 4th such conference, which took place at BIRS in Banff, Alberta, in August, 2017. As was the case with previous conferences, WIN4 was a working conference, with several hours each day devoted to research in project groups; the topics and members of these groups are listed in the volume under review.

To quote from the Preface:
The editors solicited contributions from the working groups at the WIN4 workshop and sought additional articles through the Women in Numbers Network. […] The articles collected here span algebraic, analytic, and computational areas of number theory, including topics such as elliptic and hyperelliptic curves, mock modular forms, arithmetic dynamics, and cryptographic applications. Several papers in this volume stem from collaborations between authors with different mathematical backgrounds, allowing the group to tackle a problem using multiple perspectives and tools.

The individual articles are as follows:

- « Ramanujan Graphs in Cryptography », by Anamaria Costache et al.;
- « Cycles in the Supersingular I-Isogeny Graph and Corresponding Endomorphisms », by Efrat Bank et al.;
- « Chabauty–Coleman Experiments for Genus 3 Hyperelliptic Curves », by Jennifer S. Balakrishnan et al.;
- « Weierstrass Equations for the Elliptic Fibrations of a K3 Surface », by Odile Lecacheux;
- « Newton Polygons of Cyclic Covers of the Projective Line Branched at Three Points », by Wanlin Li et al.;
- « Arboreal Representations for Rational Maps with Few Critical Points », by Jamie Juul et al.;
- « Dessins D’enfants for Single-Cycle Belyi Maps », by Michelle Manes et al.;
- « Multiplicative Order and Frobenius Symbol for the Reductions of Number Fields », by Antonella Perucca;
- « Quantum Modular Forms and Singular Combinatorial Series with Distinct Roots of Unity », by Amanda Folsom et al.

The next Women in Numbers Conference, WIN5, is scheduled to take place from November 15 to 20, 2020, again at BIRS.
Although this book was published in the MSRI series (Volume 70) by Cambridge University Press, it has a very strong Canadian connection. The Editor, Urban Larsson, was a postdoctoral fellow at Dalhousie University for a few years, and at least 9 of the 23 papers have Canadian authors or co-authors. Furthermore, this volume was initiated at the Combinatorial Game Theory Workshop in January, 2011, at the Banff International Research Station.

For a brief review, this book is best described by quoting from the publisher's description:

This book surveys the state-of-the-art in the theory of combinatorial games, that is games not involving chance or hidden information. Enthusiasts will find a wide variety of exciting topics, from a trailblazing presentation of scoring to solutions of three piece ending positions of bidding chess. Theories and techniques in many subfields are covered, such as universality, Wythoff Nim variations, misère play, partizan bidding (a.k.a. Richman games), loopy games, and the algebra of placement games. Also included are an updated list of unsolved problems, extremely efficient algorithms for taking and breaking games, a historical exposition of binary numbers and games by David Singmaster, chromatic Nim variations, renormalization for combinatorial games, and a survey of temperature theory by Elwyn Berlekamp, one of the founders of the field.

This substantial volume of almost 500 pages begins with an Introduction by the Editor, including a detailed overview of the contents. This is followed by seven survey articles:

- « Temperatures of games and coupons », by Elwyn Berlekamp;
- « Wythoff visions », by Eric Duchêne et al.;
- « Scoring games: the state of play », by Urban Larsson et al.;
- « Restricted developments in partizan misère game theory », by Rebecca Milley and Gabriel Renault;
- « Unsolved problems in combinatorial games », by Richard Nowakowski;
- « Misère games and misère quotients », by Aaron Siegel;
- « An historical tour of binary and tours », by David Singmaster.

The remaining 16 articles came out of workshop topics, or are other research papers. They are as follows:

- « A note on polynomial profiles of placement games », by J. I. Brown et al.;
- « A PSPACE-complete Graph Nim », by Kyle Burke and Olivia George;
- « A nontrivial surjective map onto the short Conway group », by Alda Carvalho and Carlos Pereira dos Santos;
- « Games and complexes I: transformation via ideals », by Sara Faridi et al.;
- « Games and complexes II: weight games and Kruskal-Katona type bounds », by Sara Faridi et al.;
- « Chromatic Nim finds a game for your solution », by Mike Fisher and Urban Larsson;
- « Take-away games on Beatty's theorem and the notion of k-invariance », by Aviezri Fraenkel and Urban Larsson;
- « Geometric analysis of a generalized Wythoff game », by Eric Friedman et al.;
- « Searching for periodicity in officers », by J. P. Grossman;
- « Good pass moves in no-draw HyperHex: two theorems », by Ryan Hayward;
- « Conjoined games: Go-Cut and Sno-Go », by Melissa Huggan and Richard Nowakowski;
- « Partizan games whose rulesets produce continued fractions », by Urban Larsson and Mike Weimerskirch;
- « Endgames in bidding chess », by Urban Larsson and Johan Wastlund;
- « Phutball draws », by Sucharit Sarkar;
- « Scoring play combinatorial games », by Fraser Stewart;
- « Generalized misère play » by Mike Weimerskirch.

As the book's title indicates, this is Volume 5 in the « Games of No Chance » series; the first four volumes were also published by Cambridge in the MSRI series.
series between 1998 and 2015. Those were edited by Richard Nowakowski of Dalhousie University, with Volume 3 co-edited with Michael H. Albert of the University of Otago in New Zealand.