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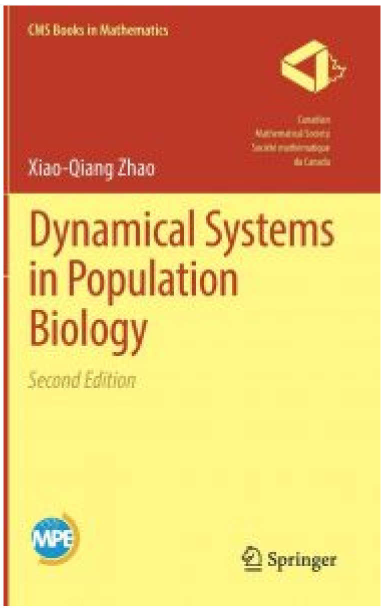

Dynamical Systems in Population Biology, 2nd Edition
by Xiao-Qiang Zhao
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It is by now widely accepted that many interesting problems in population biology can be formulated in the language of dynamical systems. The Lotka-Volterra equations for two interacting populations have found their ways into many textbooks on dynamical systems and serve as examples for phase-plane analysis and other dynamical systems techniques. The extent to which problems from population dynamics continue to inspire the development of highly sophisticated theories and analytical tools to study their behaviour is much less known. Zhao's book, *Dynamical Systems in Population Biology*, now in its second, substantially extended edition, documents the rich inspiration and challenging problems that population biology offers for the theory of finite- and infinite dimensional dynamical systems.

The particular biological aspect that is the basis for this book is the temporal variability that is present in so many biological systems, for example in the form of annual variation. From a dynamical system point of view, this leads to non-autonomous systems, potentially periodic, but not necessarily so. The aim of the book then is to "provide an introduction to periodic semiflows on metric spaces and give applications to population dynamics." The preface already sets the tone as the author introduces the main ideas with the possibly simplest example and gives a very short, concise and elegant proof that every bounded solution of a planar, time-periodic competitive system converges to a periodic orbit. I strongly recommend taking time to read the preface. It clearly shows the mathematical emphasis and direction of the book. If it speaks to you, the book with its elegant and beautiful mathematical theory is for you. If you are more interested in the biological side of things, this book will likely not be your favourite.

The first three chapters are devoted to introducing the mathematical machinery required for the analysis in later chapters. While there is always some biological motivation, the focus is on the mathematical theory. The first chapter is about dissipative dynamical systems and considers attractors, chain transitivity, repellors, perturbations and related topics. The second chapter dives into the important concept of monotonicity that is also a recurring theme throughout the book. The third chapter discusses nonautonomous semiflows, periodic and asymptotically periodic semiflows and the connection to Poincaré maps and discrete dynamical systems.

Chapters 4–14 each cover a particular application in the form of a clearly defined population dynamic question. Most of these chapters can be read independently. They typically consist of material previously published in one or two research papers by the author and with a large variety of coauthors. But they are not simple reprints of the original papers. They contain more detailed explanations, they refer to the concepts and theorems introduced in chapters 1–3, and some contain new and alternative proofs of old results. Chapter 4 discusses a chemostat model of finitely many species. Rather than obtaining the discrete structure from periodicity, it starts with a discrete-time model directly. Chapters 5 and 6 consider periodic and almost periodic competitive systems of finitely many species. From Chapter 7 on, the statespace becomes infinite dimensional, either because of spatial structure or delay or both. Chapter 7 treats a three-species model with two competitors and one mutualist. Chapter 8 considers a bioreactor that is pulsed periodically. Chapter 9 looks at predator-prey interactions with delay and nonlocal interactions, and Chapter 10 treats travelling waves in the case of two locally stable equilibria.

The tone changes slightly in Chapters 11–14, which were added in the second edition. Chapter 11 is devoted to a quantity of great interest in epidemiology: the basic reproduction ratio. This quantity is abstractly defined as the number of secondary infections that a single infective organism in a completely susceptible population will generate. Defining this quantity in models of great complexity, i.e. models that include spatial structure, delays, and interacting populations, is highly nontrivial. Proving that this number has the same properties as in the simple ODE models for which it was originally introduced, namely that it is the threshold between disease extinction and persistence, is very hard. It requires the tools and techniques introduced in the first chapters and several additional ideas. Chapters 12–14 then consider more applications of this basic reproduction ratio to populations with periodic delays, with spatial structure, and for the complicated dynamics of Lyme disease.

The author is a highly regarded specialist in dynamical systems theory, and the book gives a great introduction of the theory and comprehensive review of its many applications. In addition, I particularly enjoyed the notes at the end of each chapter that place the content into the wider mathematical literature and give some historical context. With its 450 references, the book is a treasure trove for graduate students as well as experienced researcher. It may not be the easiest introduction to analyzing population dynamic models, but it is an impressive compendium of the elegant and powerful mathematical theory required to analyze population dynamic models that contain the complexity required to make them meaningful.