

Welcome to the March-April Issue of the CMS Notes

Issue Contents

March 2020 : Vol. 52, No. 2

Cover Article

[Mathematics and Reconciliation](#) — *Edward Doolittle*

Editorial

[The Shortest Path](#) — *Robert Dawson*

Book Reviews

[Dynamical Systems in Population Biology](#)

[Short Reviews](#)

Education Notes

[Prove It! Emphasizing Mathematical Communication in a Linear Algebra Class](#) — *Lauren DeDieu*

CSHPM Notes

[The Two Cultures of Mathematics](#) — *William Hackborn*

Calls for Nominations

[2020 CMS Fellows](#)

[2020 Adrien Pouliot Award](#)

[2020 Graham Wright Award for Distinguished Service](#)

[Crux Associate Editors](#)

[2021 CMS Research Prizes](#)

[2021 CJM/CMB Associate Editors](#)

CMS Meetings

[2020 CMS Summer Meeting](#)

[Call for Pictures and Anecdotes](#)

[2020 CMS Winter Meeting in Montreal - Call for Sessions](#)

Announcements

[Mathematical Congress of Americas - 2021](#)

[Master of Mathematics for Teachers \(MMT\) - Indigenous Scholarship](#)

Competitions

[Call for Marking Partners](#)

Editorial Team

[March-April 2020](#)

Edward Doolittle (First Nations University)



Residential Schools

Residential Schools were Canadian government-sponsored schools for Indigenous children, generally run by religious orders. The schools were designed to assimilate Indigenous children into Christian religion and the dominant Canadian culture. About 150,000 Indigenous children in Canada attended residential schools from the time the system was developed beginning in 1879 until the last school was closed in 1996.

Students in the residential school system, now known as Residential School Survivors, experienced an extraordinary level of mistreatment and abuse perpetrated by those in charge of the schools. The resulting legal action led to the [Indian Residential Schools Settlement Agreement \(IRSSA\)](#) in 2007, the largest class-action settlement in Canadian history, with \$1.9 billion in damages paid to all former students (the Common Experience Payment) and an additional \$3.1 billion paid for damages suffered beyond the norm. As part of the settlement, the IRSSA also established the Truth and Reconciliation Commission (TRC) to document and preserve the experiences of the survivors.

One of the most astonishing statistics uncovered by the TRC is that of the death rate of Indigenous students who attended residential school. Out of about 150,000 attendees, at least 6,000 died of various causes directly related to their attendance in residential school, a death rate of about 1 in 25 students. In comparison, the death rate of Canadian soldiers serving in World War II was lower, at about 1 in 26.

Many of the students' deaths were attributed to tuberculosis. While tuberculosis likely affected most of humanity before the discovery of antibiotics, the disease became deadly in the presence of malnutrition and poor living environments, conditions which were endemic in the under-funded residential schools system. In addition to malnutrition, violent physical and sexual abuse of children in the system was rampant. I personally know residential school survivors who have told me harrowing stories of malnutrition, physical abuse, and sexual abuse. Thousands more stories like theirs have been documented by the [TRC](#).

One might characterize the failures of residential schooling as malfunctions of a system that was badly conceived, badly funded, and badly executed. But we must remember that the primary goal of residential school was "to kill the Indian in the child," a policy of cultural genocide. While the residential school system generally failed in its duty to protect Indigenous children, it generally succeeded in its aim to disrupt the transmission of Indigenous cultures and languages from one generation to the next. Canadians have a moral duty to undo those effects of residential schools, and reestablish healthy, thriving Indigenous cultures and communities.

Truth and Reconciliation

To that end, the TRC issued a set of 94 Calls to Action directed at Canadian governments and Canadians in general. The complete set of [Calls to Action](#) may be found on the TRC web site. For example, Call to Action 62 reads, in part:

We call upon the federal, provincial, and territorial governments, in consultation and collaboration with Survivors, Aboriginal peoples, and educators, to: ... ii. Provide the necessary funding to post-secondary institutions to educate teachers on how to integrate Indigenous knowledge and teaching methods into classrooms.

Many of the Calls to Action are related to education. As Justice (now Senator) Murray Sinclair, chair of the TRC, has said, "Education has gotten us into this mess, and education will get us out."

Government Response

On December 15, 2015, the Canadian federal government committed to implementing the Calls to Action, which call for changes not only to the situation of Indigenous people in this country, but also to the relationship between Indigenous and non-Indigenous people.

As an example, in response to Call to Action 43, the Government of Canada has announced its full support of the United Nations Declaration on the Rights of Indigenous Peoples (UNDRIP). UNDRIP has the potential to cause far-reaching changes to Canadian legal and governmental frameworks, particularly those related to resource management, and ultimately changes to Canadian society. We all have a responsibility to learn about those changes so that we can prepare ourselves, our students, and our institutions.

Universities' Responses

Canadian universities and other professional organizations have begun the long process of responding to the Calls to Action. Due to the comprehensive and far-reaching nature of the Calls to Action, universities' responses have been slow and tentative, and vary tremendously from one institution to another. However, there are some common themes emerging.

For example, Indigenization at various levels (the classroom; student services; the institution as a whole) is a common emerging theme of universities' responses to the Calls to Action. Indigenization is not yet well defined, but it includes the notion of incorporating Indigenous ideas, concepts, and practices into curricula, teaching methods, research, administration, and community service. Indigenization could conceivably impact every aspect of a university's operation, including its mathematics teaching and research.



Zapiro-1997

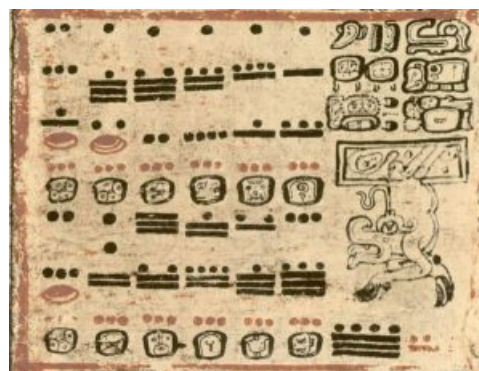
Reconciliation and Mathematics

As mathematicians, we are being asked by our institutions to Indigenize our practice as part of their overall effort. Yet we are among the least well-prepared for this request. Mathematics, with its dedication to truth (or at least validity), is sometimes valued for being beyond culture and politics. That is certainly one reason why I was first attracted to mathematics in my youth. Whether mathematics really can be so removed from the issues of the day is debatable, but there is no question that the associated attitude is common among mathematicians, and as a result, we may be poorly prepared when called upon to contribute to cultural, moral, and political endeavors.

Yet, I believe there is a compelling reason for mathematicians to contribute to the reconciliation effort, beyond the imperatives of our institutions and the moral imperative for all Canadians mentioned above. Consider what was taught in residential schools. My impression is that many students learned only four subjects in those schools: manual labour skills, generally farming for the boys and housekeeping for the girls, to establish the place of the Indigenous students in the economic order, and to help maintain the underfunded schools; religious studies, to colonize the students and break their connection to traditional cultures and spiritual beliefs; English or French, again to colonize the students and break their connection to traditional languages; and mathematics. "Mathematics is how they really got us," said one residential school survivor to me. The issue has not been studied at all, to my knowledge, but it seems likely to me that in residential school the power of mathematics was misused, as were the other residential school subjects, as a tool for colonization and repression. The study of that issue is within the domain of mathematics education, but all mathematicians and math educators should be aware of the potential of mathematics to do harm as well as good.

What can mathematicians do to contribute to the reconciliation effort? I would like to offer a modest collection of practical ideas, touching on the typical duties of mathematicians working in post-secondary institutions mentioned above, particularly curriculum, teaching methods, research, and community service. My suggestions are derived from my own personal experience and the local contexts in which I have worked. In my career, I have taught over one thousand Indigenous students in a variety of locations (reserves in Ontario and Saskatchewan; remote reserves ("fly-in communities") in northern Ontario; universities and colleges in Ontario and Saskatchewan) and for a variety of programs (generally math courses required as components of a non-science diploma or degree, but also math methods courses for K-12 preservice teachers, and even Indigenous Studies courses). Yet there is enormous variation among the hundreds of Indigenous communities and hundreds of thousands of Indigenous people across the country, of which my experience is but a narrow cross-section. I encourage anyone interested in these issues to seek out opportunities to gain their own different experiences with Indigenous cultures and to develop their own creative responses to the issues that face us. I continue to be astonished at the creativity and valuable contributions of colleagues who have taken up the challenge of Indigenousizing their practice.

I have had moderate success Indigenousizing curriculum in some courses, especially Introductory Finite Mathematics, which includes topics like arithmetic in other bases (like Mayan arithmetic and Chumash arithmetic) which has an interesting connection to the Unicode base 16 representation of Indigenous writing systems; modular arithmetic, which can be applied to time-keeping and calendars; and elementary number theory, which can be studied using bead work. Some examples I use are admittedly superficial; for example, choosing important dates in Indigenous history when showing students how to calculate the day of the week given the date, or using Indigenous data when studying the distribution of blood types with Venn diagrams; but I feel that anything is better than nothing, as long as a superficial example is not a reason to stop improving. In statistics, I use Indigenous games as examples in the study of probability, and there are many opportunities to use Indigenous data from Statistics Canada. I have not yet had much success Indigenousizing the calculus curriculum, but I believe there is an opportunity to draw examples from situations of interest to Indigenous people.



Arithmetic in Mayan Base

In general, I find that applied mathematics is more interesting to Indigenous students than pure mathematics. I have had Indigenous students go from bored and disengaged to riveted when I introduce applications that resonate with them. I feel that we could do better to teach more applied mathematics throughout the curriculum, from elementary school through university. Our system now seems to be designed like an arrow pointing to multivariable calculus, even though only a miniscule proportion of students ever take that course; I feel that we would do better with Indigenous students, and perhaps with all students, if we thought of the math curriculum more as a study of interesting, practical problems, for which we can draw in tools as needed.

Coupled with modifications to curriculum, we need also to consider modifications to teaching methods, pedagogy and andragogy, to reach our Indigenous students more effectively. Indigenous students have all been affected by residential school, whether directly or indirectly through the experiences of their relatives and members of their communities. As a result, in my experience, Indigenous students mistrust formal education systems. As educators we must work to overcome that mistrust and by being absolutely trustworthy. Furthermore, despite all the attention that residential schooling has received, one of the major problems at the root of the system still persists: the underfunding of the federal school system responsible for teaching Indigenous students on reserve. [By some estimates](#), reserve schools are funded at a rate 30% less than provincial schools. Many reserve schools are also remote, making it difficult to stock and staff the schools. I have sat in classes in reserve schools without functioning science labs, in which teachers who have little math or science background just read from the textbook. The solution to those problems would be to improve funding for reserve schools, and I think we should all call on the federal government to do so, to eventually deal with the shortages that reserve schools experience. However, in the meantime, we need to be understanding about our university students' difficulties resulting from poor prior education. I encourage my students to use aids like calculators and help sheets in tests, for example. One of my students even brought in a multiplication table, which I encouraged her to use.

As university professors, I also feel we have much to learn about teaching methods from our colleagues in elementary math education, which will benefit all students, particularly Indigenous students. For example, I have successfully used "rich tasks" in which the class breaks into small groups of four students to solve a set of interesting and carefully designed problems in my finite mathematics classes. Universities can help by reducing class sizes, particularly for classes designed for Indigenous students, or at least providing more opportunities like tutorials for skilled teaching staff to interact with smaller groups of students.

There are many research opportunities for mathematicians in Indigenous contexts, particularly in applied mathematics addressing the many difficulties and challenges faced by Indigenous people in Canada. That type of research could also be viewed as community service. One of my current

research projects is to study the structure of word puzzles and games and to apply the results to assist in the construction of puzzles and games in Indigenous languages. In my project there is some mathematics (graph theory), some statistics, some computing (theory of effective and efficient computing; programming; perhaps machine learning), and some study of Indigenous languages. The mathematics is not particularly difficult or leading-edge, but there is potentially enormous value to Indigenous communities who are struggling to find ways to reverse the language loss which resulted from the residential school system. Water quality in Indigenous communities is another issue which may benefit from attention by researchers.

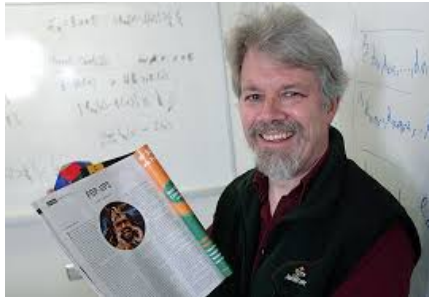
Beyond the current needs of Indigenous communities, there are research opportunities in cultural practices like games, arts, crafts, and other material culture. Those who are interested in pursuing research in those areas should be aware that ethics requirements are stronger for research in Indigenous communities, and should review the section on research in Indigenous communities the [Tri-Council Policy Statement TCPS2](#), and should also be aware of the [OCAP](#) (ownership, control, access, and possession) principles for Indigenous research data. It is important to have a true partnership with Indigenous researchers when doing Indigenous community-based research. Furthermore, researchers should realize that Indigenous cultures continue to use oral tradition, so researchers must consult elders and traditional teachers to obtain information which might be contained in books in other contexts.

We can overcome the shameful legacy of our past and build a better nation together. I hope that mathematicians and math educators can see the valuable role that they can play in doing so. I look forward to sharing success stories with you all in the future. Skennen (peace).

Copyright 2020 © Canadian Mathematical Society. All rights reserved.

Robert Dawson (Saint Mary's University)

Editor-in-Chief



...between two points may be a straight line. But that's not the shape of the usual path from a conjecture to a proof – whatever the writeup in our submitted paper may suggest. When we write a research paper, we leave out a lot. And that's usually a good thing.

First, many of your readers will be busy researchers who can spare fifteen minutes to skim your papers for the highlights. Some will be driven by curiosity, some have just popped by to borrow a screwdriver to use in their own work, but in either case, the less they have to wade through, the happier they will be. Sure, they will want to know that the proof is there, but they'll trust you and the referee for the rest.

Some will be more interested, or will want to see if your proof provides some techniques that they can borrow. While they're going to read your proof, they'd still rather that you got to the point quickly. Proofs, after all, are heavy going.

So a research paper probably won't say much about the experiments you did to get your conjecture in the first place. You mention a result that somebody proved fifty years ago, but you leave out the afternoon you spent proving it another way, just to see if it could be done. You tried five different versions of your main theorem, all rather similar, and you only include the two most interesting (and the proof of one.) Your final definition is much more elegant than the one you began with, and so that's the definition that you keep.

And that's as it should be. The results are for your colleagues, for posterity. But the journey you took to get there? That was for you.

Request for Submissions!

Research Notes brings mathematical research ideas forth to the CMS readership in a generally accessible manner that promotes discussion of relevant topics including research (both pure and applied), activities, and noteworthy news items. Comments and suggestions are welcome.

Submissions are currently being sought for volume (51) of *CMS Notes*. To contribute contact Patrick Ingram, York University (pigram@yorku.ca)

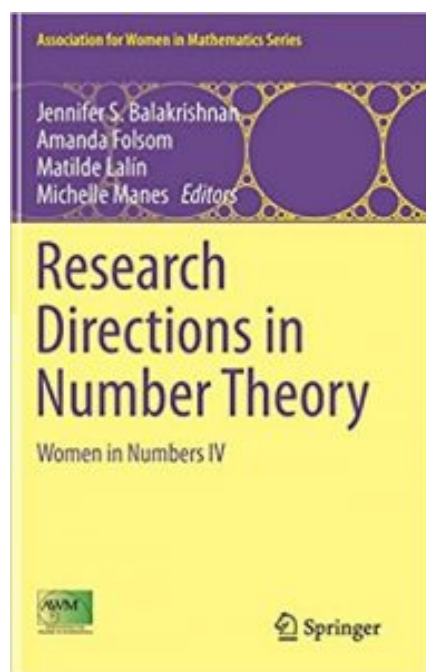
Demande de soumission!

Les Notes de recherche présentent des sujets mathématiques aux lecteurs de la SMC dans un format généralement accessible qui favorise les discussions sur divers sujets pertinents, dont la recherche (pure et appliquée), les activités et des nouvelles dignes de mention. Vos commentaires et suggestions sont les bienvenus.

Nous acceptons en ce moment des articles pour le volume 51 des *Notes de la SMC*. Pour contribuer à ce volume, veuillez contacter à Patrick Ingram, Université York (pigram@yorku.ca)

Book Reviews bring interesting mathematical sciences and education publications drawn from across the entire spectrum of mathematics to the attention of the CMS readership. Comments, suggestions, and submissions are welcome.

Karl Dilcher, Dalhousie University (notes-reviews@cms.math.ca)



Research Directions in Number Theory

Edited by Jennifer S. Balakrishnan, Amanda Folsom, Matilde Lalin, and Michelle Manes

AWM Series, Springer, 2019

ISBN: 978-3-030-19477-2

Reviewed by Karl Dilcher

There have been four conferences so far that were organized by the Women in Numbers (WIN) network. This volume originated from the 4th such conference, which took place at BIRS in Banff, Alberta, in August, 2017. As was the case with previous conferences, WIN4 was a working conference, with several hours each day devoted to research in project groups; the topics and members of these groups are listed in the volume under review.

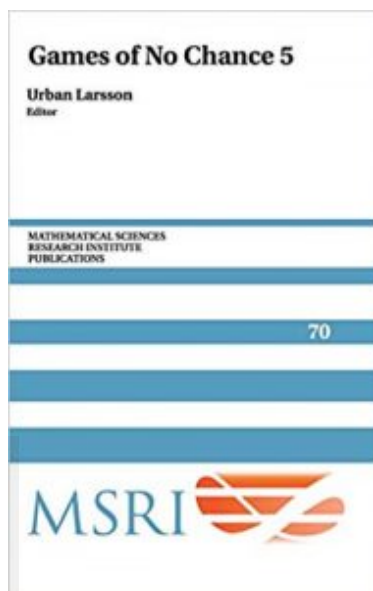
To quote from the Preface:

The editors solicited contributions from the working groups at the WIN4 workshop and sought additional articles through the Women in Numbers Network. [...] The articles collected here span algebraic, analytic, and computational areas of number theory, including topics such as elliptic and hyperelliptic curves, mock modular forms, arithmetic dynamics, and cryptographic applications. Several papers in this volume stem from collaborations between authors with different mathematical backgrounds, allowing the group to tackle a problem using multiple perspectives and tools.

The individual articles are as follows:

- “Ramanujan Graphs in Cryptography”, by Anamaria Costache et al.;
- “Cycles in the Supersingular I -Isogeny Graph and Corresponding Endomorphisms”, by Efrat Bank et al.;
- “Chabauty–Coleman Experiments for Genus 3 Hyperelliptic Curves”, by Jennifer S. Balakrishnan et al.;
- “Weierstrass Equations for the Elliptic Fibrations of a $K3$ Surface”, by Odile Lecacheux;
- “Newton Polygons of Cyclic Covers of the Projective Line Branched at Three Points”, by Wanlin Li et al.;
- “Arboreal Representations for Rational Maps with Few Critical Points”, by Jamie Juul et al.;
- “Dessins D'enfants for Single-Cycle Belyi Maps”, by Michelle Manes et al.;
- “Multiplicative Order and Frobenius Symbol for the Reductions of Number Fields”, by Antonella Perucca;
- “Quantum Modular Forms and Singular Combinatorial Series with Distinct Roots of Unity”, by Amanda Folsom et al.

The next Women in Numbers Conference, WIN5, is scheduled to take place from November 15 to 20, 2020, again at BIRS.



Games of No Chance 5
 Edited by Urban Larsson
 Cambridge University Press, 2019
 ISBN: 978-1-108-48580-7
 Reviewed by Karl Dilcher

Although this book was published in the MSRI series (Volume 70) by Cambridge University Press, it has a very strong Canadian connection. The Editor, Urban Larsson, was a postdoctoral fellow at Dalhousie University for a few years, and at least 9 of the 23 papers have Canadian authors or co-authors. Furthermore, this volume was initiated at the Combinatorial Game Theory Workshop in January, 2011, at the Banff International Research Station.

For a brief review, this book is best described by quoting from the publisher's description:

This book surveys the state-of-the-art in the theory of combinatorial games, that is games not involving chance or hidden information. Enthusiasts will find a wide variety of exciting topics, from a trailblazing presentation of scoring to solutions of three piece ending positions of bidding chess. Theories and techniques in many subfields are covered,

such as universality, Wythoff Nim variations, misère play, partizan bidding (a.k.a. Richman games), loopy games, and the algebra of placement games. Also included are an updated list of unsolved problems, extremely efficient algorithms for taking and breaking games, a historical exposition of binary numbers and games by David Singmaster, chromatic Nim variations, renormalization for combinatorial games, and a survey of temperature theory by Elwyn Berlekamp, one of the founders of the field.

This substantial volume of almost 500 pages begins with an Introduction by the Editor, including a detailed overview of the contents. This is followed by seven survey articles:

- "Temperatures of games and coupons", by Elwyn Berlekamp;
- "Wythoff visions", by Eric Duchêne et al.;
- "Scoring games: the state of play", by Urban Larsson et al.;
- "Restricted developments in partizan misère game theory", by Rebecca Milley and Gabriel Renault;
- "Unsolved problems in combinatorial games", by Richard Nowakowski;
- "Misère games and misère quotients", by Aaron Siegel;
- "An historical tour of binary and tours", by David Singmaster.

The remaining 16 articles came out of workshop topics, or are other research papers. They are as follows:

- "A note on polynomial profiles of placement games", by J. I. Brown et al.;
- "A PSPACE-complete Graph Nim", by Kyle Burke and Olivia George;
- "A nontrivial surjective map onto the short Conway group", by Alda Carvalho and Carlos Pereira dos Santos;
- "Games and complexes I: transformation via ideals", by Sara Faridi et al.;
- "Games and complexes II: weight games and Kruskal-Katona type bounds", by Sara Faridi et al.;
- "Chromatic Nim finds a game for your solution", by Mike Fisher and Urban Larsson;
- "Take-away games on Beatty's theorem and the notion of k-invariance", by Aviezri Fraenkel and Urban Larsson;
- "Geometric analysis of a generalized Wythoff game", by Eric Friedman et al.;
- "Searching for periodicity in officers", by J. P. Grossman;
- "Good pass moves in no-draw HyperHex: two proverbs", by Ryan Hayward;
- "Conjoined games: Go-Cut and Sno-Go", by Melissa Huggan and Richard Nowakowski;
- "Impartial games whose rulesets produce continued fractions", by Urban Larsson and Mike Weimerskirch;
- "Endgames in bidding chess", by Urban Larsson and Johan Wastlund;
- "Phutball draws", by Sucharit Sarkar;
- "Scoring play combinatorial games", by Fraser Stewart;
- "Generalized misère play" by Mike Weimerskirch.

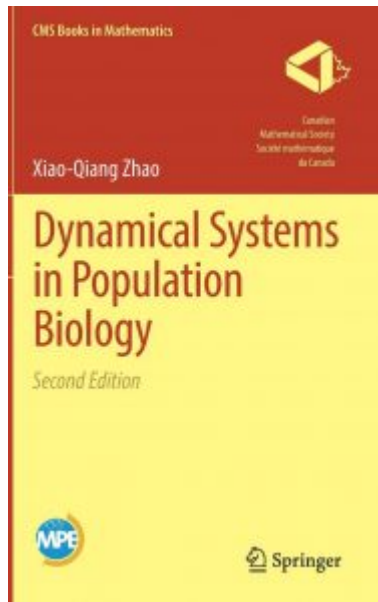
As the book's title indicates, this is Volume 5 in the "Games of No Chance" series; the first four volumes were also published by Cambridge in the MSRI

series between 1998 and 2015. Those were edited by Richard Nowakowski of Dalhousie University, with Volume 3 co-edited with Michael H. Albert of the University of Otago in New Zealand.

Copyright 2020 © Canadian Mathematical Society. All rights reserved.

Book Reviews bring interesting mathematical sciences and education publications drawn from across the entire spectrum of mathematics to the attention of the CMS readership. Comments, suggestions, and submissions are welcome.

Karl Dilcher, Dalhousie University (notes-reviews@cms.math.ca)



Dynamical Systems in Population Biology, 2nd Edition
by Xiao-Qiang Zhao
CMS Books in Mathematics, Springer, 2017
ISBN 978-3-319-56432-6

Reviewed by Frithjof Lutscher, University of Ottawa

It is by now widely accepted that many interesting problems in population biology can be formulated in the language of dynamical systems. The Lotka-Volterra equations for two interacting populations have found their ways into many textbooks on dynamical systems and serve as examples for phase-plane analysis and other dynamical systems techniques. The extent to which problems from population dynamics continue to inspire the development of highly sophisticated theories and analytical tools to study their behaviour is much less known. Zhao's book, *Dynamical Systems in Population Biology*, now in its second, substantially extended edition, documents the rich inspiration and challenging problems that population biology offers for the theory of finite- and infinite dimensional dynamical systems.

The particular biological aspect that is the basis for this book is the temporal variability that is present in so many biological systems, for example in the form of annual variation. From a dynamical system point of view, this leads to non-autonomous systems, potentially periodic, but not necessarily

so. The aim of the book then is to "provide an introduction to periodic semiflows on metric spaces and give applications to population dynamics." The preface already sets the tone as the author introduces the main ideas with the possibly simplest example and gives a very short, concise and elegant proof that every bounded solution of a planar, time-periodic competitive system converges to a periodic orbit. I strongly recommend taking time to read the preface. It clearly shows the mathematical emphasis and direction of the book. If it speaks to you, the book with its elegant and beautiful mathematical theory is for you. If you are more interested in the biological side of things, this book will likely not be your favourite.

The first three chapters are devoted to introducing the mathematical machinery required for the analysis in later chapters. While there is always some biological motivation, the focus is on the mathematical theory. The first chapter is about dissipative dynamical systems and considers attractors, chain transitivity, repellers, perturbations and related topics. The second chapter dives into the important concept of monotonicity that is also a recurring theme throughout the book. The third chapter discusses nonautonomous semiflows, periodic and asymptotically periodic semiflows and the connection to Poincaré maps and discrete dynamical systems.

Chapters 4–14 each cover a particular application in the form of a clearly defined population dynamic question. Most of these chapters can be read independently. They typically consist of material previously published in one or two research papers by the author and with a large variety of coauthors. But they are not simple reprints of the original papers. They contain more detailed explanations, they refer to the concepts and theorems introduced in chapters 1–3, and some contain new and alternative proofs of old results. Chapter 4 discusses a chemostat model of finitely many species. Rather than obtaining the discrete structure from periodicity, it starts with a discrete-time model directly. Chapters 5 and 6 consider periodic and almost periodic competitive systems of finitely many species. From Chapter 7 on, the statespace becomes infinite dimensional, either because of spatial structure or delay or both. Chapter 7 treats a three-species model with two competitors and one mutualist. Chapter 8 considers a bioreactor that is pulsed periodically. Chapter 9 looks at predator-prey interactions with delay and nonlocal interactions, and Chapter 10 treats travelling waves in the case of two locally stable equilibria.

The tone changes slightly in Chapters 11–14, which were added in the second edition. Chapter 11 is devoted to a quantity of great interest in epidemiology: the basic reproduction ratio. This quantity is abstractly defined as the number of secondary infections that a single infective organism in a completely susceptible population will generate. Defining this quantity in models of great complexity, i.e. models that include spatial structure, delays, and interacting populations, is highly nontrivial. Proving that this number has the same properties as in the simple ODE models for which it was originally introduced, namely that it is the threshold between disease extinction and persistence, is very hard. It requires the tools and techniques

introduced in the first chapters and several additional ideas. Chapters 12–14 then consider more applications of this basic reproduction ratio to populations with periodic delays, with spatial structure, and for the complicated dynamics of Lyme disease.

The author is a highly regarded specialist in dynamical systems theory, and the book gives a great introduction of the theory and comprehensive review of its many applications. In addition, I particularly enjoyed the notes at the end of each chapter that place the content into the wider mathematical literature and give some historical context. With its 450 references, the book is a treasure trove for graduate students as well as experienced researcher. It may not be the easiest introduction to analyzing population dynamic models, but it is an impressive compendium of the elegant and powerful mathematical theory required to analyze population dynamic models that contain the complexity required to make them meaningful.

Copyright 2020 © Canadian Mathematical Society. All rights reserved.

Lauren DeDieu (University of Calgary)

Education Notes bring mathematical and educational ideas forth to the CMS readership in a manner that promotes discussion of relevant topics including research, activities, issues, and noteworthy news items. Comments, suggestions, and submissions are welcome.

John McLoughlin, University of New Brunswick (johnm@unb.ca)
 Kseniya Garaschuk, University of Fraser Valley (kseniya.garaschuk@ufv.ca)

“Writing in a math class?!” Many of our students tend to view communication and mathematics as two distinct entities. They enter our classroom with a preconceived notion of what a math class is and how they will be assessed. However, many of us decide to push back against this norm in our classrooms. Three factors which have motivated me to emphasize mathematical communication in my classes are: authenticity, student learning, and long-term student success.

A crucial part of a mathematician's job is to find the best ways to communicate their results both verbally and in writing. Incorporating mathematical communication into our assessments can help make our classrooms more authentic. Communication-based assessments can also accelerate student learning in a variety of ways. Writing provides a natural venue for enabling students to combine and unify course concepts. Reflective writing assignments can help boost student motivation and refine metacognitive skills. Moreover, research suggests that oral communication activities in a group setting boost achievement, persistence, and attitudes among undergraduate math students (Springer, Stanne, & Donovan, 1999). These benefits have been shown to be especially effective for female students (Herreid, 1998) and groups underrepresented in STEM (Treisman, 1992). Finally, some of us may be compelled to help students cultivate skills they will need in the workplace, which can lead to long-term student success. Mathematical communication exercises can help train our students to become logical thinkers, construct rigorous arguments, and effectively communicate complex ideas to non-experts.

In 2018/2019, I taught a second-year linear algebra course three semesters in a row which introduced the theory of abstract vector spaces. I took this opportunity to develop communication-based course components. I packaged these components into a Communication Score worth 25% of the course grade. For context, the other course components were: online Lyryx assignments (10%), tests (30%), and a final exam (35%). Each component of the Communication Score is outlined below.

Essay/Written Tasks (5%). Halfway through the course, students submitted a 500-word “Applications of Linear Algebra” essay. Students were encouraged to choose an application applicable to their major. They were instructed to avoid field-specific jargon; their audience was any person who had taken this course. This essay gave students the opportunity to practice communicating complex idea to non-experts. It was also meant to help prepare students for their upcoming poster presentation.

Essays were graded according to the following rubric:

Essay Rubric		Possible Points
A	Appropriate Mathematical Depth/ Contains an Explicit Example	40
B	Clarity	40
C	Narrative/Organization	10
D	Grammar/Spelling/Punctuation/Legibility	10

Note: Your B score cannot exceed your A score. Moreover, your scores in categories C/D cannot exceed your A score divided by 2. (e.g. If you receive 10/40 in category A, then you can receive at most 10/40 in B, 5/10 in C, and 5/10 in D.)

I designed this rubric in response to the common mistakes I saw the first time around. The weakest essays were superficial in nature and said something along the lines of, “Linear algebra is awesome. It's used in quantum mechanics, robotics, and signal processing. The end.” To help prevent essays like this, clarify expectations, and streamline grading, I anchored the rubric so that one needs appropriate mathematical depth to do well.

Another common critique was that some students were too ambitious, which made their essay difficult to understand; explaining a concept in complete generality in only 500 words so that an average classmate can understand can be extremely difficult! I found that essays which led with an example were a

lot clearer and helped to demonstrate that students understood their topic. As such, I included “Contains an Explicit Example” in the description of the anchoring category “A” of the rubric.

Presentation (5%). The Presentation component was a group poster presentation based on the “Applications of Linear Algebra” essay. Students worked in groups of four. Most groups chose to report on one or two of their group members’ essay topics. The class had two 50-minute tutorial sections consisting of ~60 students. The last two tutorials were dedicated to poster presentations.

Eight groups presented at each tutorial. They were instructed to prepare a three-minute presentation and to allow two minutes for questions. Posters were set up around the perimeter of the classroom. The eight non-presenting groups were assigned a starting position. They spent five minutes with each poster, then moved clockwise (so each group gave their presentation eight times). Non-presenting groups kept track of the questions they asked and handed their questions in at the end, as part of their Tutorial Quiz score. This format was meant to mirror a conference poster session and gave me the opportunity to view each presentation. Posters were graded based on content, organization, visual design, and oral presentation.

Tutorial Tasks (10%). Tutorials were designed to cultivate students’ ability to communicate mathematics orally and to gain practice in proof writing. The first five minutes were dedicated to a short quiz concerning the previous week’s material. Students brought their own paper to complete the quiz. The quiz was graded out of one point; students received 0.25 points for completion and 0.75 points for correctness. These quizzes were included to encourage students to review their notes before coming to tutorial so that they could productively engage in group work activities.

The next 10 minutes of tutorial was dedicated to proof writing. Some weeks students were given several possibly faulty proofs to evaluate according to a rubric; in tutorial, students would discuss the score they gave each proof and the TA would reveal their score. Other weeks, students were given a prompt ahead of time and asked to write a proof; in tutorial, the TA would go through the proof at the board.

The remaining 35 minutes was dedicated to group work activities. Students moved their desks into groups of four and worked on worksheets. The TA and I floated around to answer questions. Students received an Oral Communication Score based on their participation. To provide incentive to complete the worksheets, a quiz was given in class at the end of the week worth bonus points; students received one bonus point towards their test score if they got a 5/5 on this quiz.

One interesting note about the worksheets is that I found that the way I packaged these materials mattered. When I called these exercises “Worksheets”, many students complained that the exercises were too long and that they did not have enough time to complete them in tutorial – even though I had emphasized in class that they were not expected to finish the worksheets in tutorial. The second semester, I renamed the worksheets “Homework” and I did not receive a single criticism about the exercises being too long.

The Tutorial Quizzes and Oral Communication Score were worth 8% and 2%, respectively. Although, since students receive 25% on each quiz for completion, I really considered them each to be worth 6% and 4%, respectively. The quizzes provided a means to take attendance. Students received full points on their Oral Communication Score if they actively worked in tutorial, which meant I only needed to make note of students who were not working (which almost never happened). Two tutorial absences were excused.

Final Proof (5%). This course served as an introduction to proofs for many students. Proof writing was emphasized throughout the semester and was evaluated on four term tests (best 3 of 4 were counted). The Final Proof appeared on the last page of the final exam. The marking scheme and difficulty mirrored the term-test proofs.

I chose to package this final proof as part of the Communication Score (instead of making the final exam worth 40%) in order to underline the critical role proof writing would play in the course. Putting the Final Proof on the Course Outline helped to give the course direction and gave students a goal to work towards.

Written Tasks (bonus). There were two optional bonus reflections which could each bump students’ 25-point Communication Score by 0.25 points, so long as they appeared to be genuine and well thought out. These reflections were originally mandatory, but I decided to make them optional based on student feedback that the course had too many due dates.

The first reflection was designed to help students find value in proof writing:

In this course, we have been gaining experience in formal proof-writing. With your future career goals in mind, reflect upon if/how engaging in this formal proof-writing may help or influence you in your post-university life/career.

The second reflection was designed to push students to reflect on their learning. It had the added benefit of providing insights to me about what students felt like they were taking from the course:

What were 3 of the most important discoveries or realizations you made in this class? In other words, what are you taking away from this class that you think might stick with you and/or influence you in the future? These can include things you had not realized about

mathematics, specific homework problems or theorems, discussions with other students or the instructor, connections to other courses, etc. Explain why these three discoveries or realizations are important to you.

What did students think about the Communication Score? The most memorable component appeared to be the poster presentation; the first time I read students' "most important discoveries or realizations" in the reflective prompt above, I was surprised to see that the majority of students commented on the widespread applications of linear algebra. We did not do any applications in class, so this takeaway is most likely due to the poster presentations they viewed – or possibly the research they did for their essay. I was also delighted to hear students report in this reflection and in unsolicited feedback – sometimes months later – their appreciation for the emphasis this course placed on group work and writing, because it led to deeper understanding of course material.

As can be expected, there were many students who were not fans of the Communication Score. Some students felt like the essays, written tasks, and presentation were futile, because they did not directly relate to content that would help them do well on the final exam. However, I found that these comments began to change as I put more effort into motivating why I was asking students to complete these tasks. For example, on the course's Communication Score Description document, I included this disclaimer and discussed these points in class:

Motivation: You may be asking: "Why am I being asked to write an essay in a math class?!" or "This is stupid... how will this help me on the final exam?" It's true that these Essay/Bonus Reflections will most likely not directly help in preparing for your final exam. But this doesn't mean that they don't have value. The Essay/Bonus Reflections will give you the opportunity to take a step back and reflect on the skills you are developing in this course. You are all very busy, and it can be difficult to take the time to think critically about "Why on earth is this content useful?" or "What skills might I be developing?" Moreover, your future self will thank you for becoming a better writer, since in almost any job you will be required to write formal documents. If you are a science major, then you may not have had many opportunities to write formally, and so my hope is that the Essay/Bonus Reflections will allow you to practice.

After doing this, I found that almost all students who disliked the Communication Score added the qualifier, "...but I understand why it is necessary." Learning is like exercise; it is uncomfortable and we may not enjoy it, but most of us are thankful that we did it.

Will my students remember how to orthogonally diagonalize a matrix ten years from now? Probably not. But I hope the communication components of my course have instilled in some students the feeling that mathematics has deep connections to many disciplines, the sense that writing and talking through a problem with others can accelerate learning, the care one must take when explaining complex topics to non-experts, and other intangible skills that help to unlock long-term success.

Lauren DeDieu is an instructor in the Department of Mathematics and Statistics at the University of Calgary. She is originally from Cape Breton, N.S., and completed her Ph.D. in mathematics at McMaster University. Lauren has a strong interest in tertiary math education and is heavily involved in K-12 mathematics outreach initiatives.

References

- Herreid, C. F. (1998). Why isn't cooperative learning used to teach science? *BioScience*, 48, 553-559.
- Springer, L., Stanne, M., & Donovan, S. (1999). Effects of small-group learning on undergraduates in science, mathematics, engineering, and technology: A meta-analysis. *Review of Educational Research*, 69(1), 21-51.
- Treisman, U. (1992). Studying students studying calculus: A look at the lives of minority mathematics students in college. *The College Mathematics Journal*, 23(5), 362-372.

William Hackborn (University of Alberta)

CSHPM Notes bring scholarly work on the history and philosophy of mathematics to the broader mathematics community. Authors are members of the Canadian Society for History and Philosophy of Mathematics (CSHPM). Comments and suggestions are welcome; they may be directed to either of the column's co-editors:

Amy Ackerberg-Hastings, Independent Scholar (aackerbe@verizon.net)

Hardy Grant, York University [retired] (hardygrant@yahoo.com)

Like the two cultures of the sciences and the humanities, as lamented by C. P. Snow in his influential and controversial Rede Lecture [4], there are two cultures within the mathematics community itself. Snow used specific examples of cultural clashes to illuminate his argument, and similarly here I will attempt to shed some light on the different cultures of pure and applied mathematics by recounting a 1950–51 conflict involving Harvard mathematician Garrett Birkhoff and J. J. Stoker, one of the founders of the Courant Institute of Mathematical Sciences.

Garrett Birkhoff, as most readers will know, co-authored (with Saunders Mac Lane) the classic text, *A Survey of Modern Algebra*. However, during and after the Second World War, he began working in more applied areas. I first encountered his monograph on fluid dynamics [1] when I was exploring Euler's role in the history of d'Alembert's paradox [2]. As an applied mathematician who had done research in fluid flow (and later developed an interest in the history of mechanics), I was struck by the fresh approach and clarity of Birkhoff's writing on the subject. The entire first chapter [1, pp. 3–39] of his monograph is devoted to paradoxes of fluid flow; d'Alembert's paradox is the first discussed [1, pp. 10–13].

As Birkhoff explained, this paradox involves the steady, uniform flow of a non-viscous, incompressible fluid (often called an ideal fluid) past a smooth, finite body (such as a sphere). Flows of this kind can be described by the gradient of a potential function, which implies the flow exerts no drag on the body, a result contradicted by the physical fact of substantial drag exerted by actual flows. For Birkhoff, d'Alembert's paradox and others are "in part at least, paradoxes of topological oversimplification and symmetry paradoxes" [1, p. 22]. Euler's resolution of d'Alembert's paradox challenged the assumption of an incompressible fluid (especially for a ball shot through the air) [2]; two other plausible resolutions are considered below.

One topological oversimplification associated with "the hypothesis of an 'ideal fluid'" is "that a locally single-valued velocity potential U is single-valued in the large" for two-dimensional flow past a body. To avoid symmetry paradoxes, Birkhoff advised the reader to "admit the possibility that a *symmetrically stated problem may not have any stable symmetric solution*"; in the case of the uniform flow of an ideal fluid past a sphere, for example, a steady, axially-symmetric solution exists mathematically, but "there is no reason to suppose that any steady flow is *stable*." An instability makes a steady, mathematical flow physically unrealizable, and "irregular, turbulent 'eddies' ... in the 'wake' of an obstacle" might therefore occur in actual flows [1, pp. 20–21; italics are Birkhoff's]. An instability of this kind might thus resolve d'Alembert's paradox.



Garrett Birkhoff, MacTutor.

Birkhoff opined that theories of fluid dynamics can be learned "more effectively ... by studying the paradoxes" he described. He criticized textbooks that attributed the gap between theory and experiment to the difference between real fluids with "small but finite viscosity" and ideal fluids of "zero viscosity," and he thought "that to attribute them all [the paradoxes he describes] to the neglect of viscosity is an unwarranted oversimplification"—the "root lies deeper, in lack of precisely that deductive rigor whose importance is so commonly minimized by physicists and engineers" [1, pp. 3–4]. The paradoxes warn against "the impression ... that mathematical deduction should be supplanted by 'physical' reasoning," which can lead to flawed approximations and oversimplifications, though Birkhoff admitted the usefulness of "oversimplifications based on the 'right' approximations." He continued, "mathematicians can perform a useful service if they will analyze critically these oversimplifications, by the deductive method, and so establish their limitations more clearly" [1, p. 37]. Birkhoff offered his paradoxes to a subject that is primarily the domain of engineers and applied mathematicians (like me, in my past life as a fluid dynamics specialist). His criticisms were severe and, in fact, he named J. J. Stoker as one who "effectively exploited" an "analogy" between two kinds of waves, even though another paradox (not d'Alembert's) made one kind "*mathematically impossible*" [1, pp. 22–24; italics are Birkhoff's].

As life and luck would have it, J. J. Stoker wrote a review [5] of Birkhoff's 1950 monograph. Stoker's assessment of Birkhoff's Chapters 2 through 5 was balanced, even complimentary in the cases of Chapter 2 (on problems with free boundaries) and Chapter 3 (on modelling and dimensional analysis). Stoker's review of Chapter 1, however, was withering. He found "it difficult to understand for what class of readers the first chapter was written"; indicated



J. J. Stoker, ca 1960, Courant Institute

that “the majority of cases cited as paradoxes” were either “mistakes long since rectified” or “discrepancies between theory and experiment the reasons for which are also well understood”; and worried that “the uninitiated would be very likely to get wrong ideas about some of the important and useful achievements in hydrodynamics from reading this chapter.” Referring to “some general observations regarding the philosophy and correct attitude toward applied mathematics” made by Birkhoff, Stoker allowed that most “workers in the field would agree quite well with the author’s observations,” but he thought that “they are perhaps better informed in some cases than the author would seem to imply” [5, pp. 497–498].

To illustrate this last point, Stoker offered salient mathematical reasoning underlying the generally accepted resolution of d’Alembert’s paradox: “the small coefficients involving viscosity occur in terms containing derivatives of the highest order in the system of differential equations, and thus developments in the neighborhood of zero viscosity involve boundary layer effects because of the loss of order of the differential equations in the limit” Stoker was referring to the fact that the Navier-Stokes equations, which describe viscous, incompressible flow past a body, are second-order partial differential equations that permit the so-called no-slip condition (that the fluid’s velocity vanish on the body’s boundary) to be satisfied; in the case of zero viscosity, the Navier-Stokes equations become Euler’s equations, which are first-order and allow the flow to be described by a potential function but permit the vanishing of only the flow velocity normal to the body’s boundary; the transition from

viscous flow (with the no-slip condition on the boundary) to non-viscous flow (with the corresponding loss of second-order terms) farther from the boundary occurs in what is known as a boundary layer, in which the fluid’s velocity is approximated mathematically using matched asymptotic expansions (or computationally using a very fine mesh).

Birkhoff had anticipated this persuasive argument from applied mathematics “in support of the view that the paradoxes of fluid mechanics are due to an unjustified neglect of viscosity.” He conceded that the argument had “some merit” but thought that it was “inconclusive.” For him, “the real question is, why does separation of the boundary layer occur?” This question alluded to observations that the boundary layer adjacent to a body immersed in a flow often separates from that body (downstream from where it begins) to become the border of a turbulent wake behind the body. Birkhoff believed this question “concerns the *stability* of nearly non-viscous flows” [1, p. 27; italics are Birkhoff’s]. It seems that Stoker’s argument was inconclusive for Birkhoff because it failed to rule out, deductively, the stability question raised by d’Alembert’s paradox and others.

Birkhoff, as one steeped in the culture of pure mathematics, saw his paradoxes as guidelines to sharpen the deductive skills of fluid dynamics researchers. Stoker rejected the applicability of those guidelines. In the second (1960) edition of Birkhoff’s monograph, the first chapter grew to two chapters; these chapters doubled down on paradoxes (one covered those of non-viscous flow, the other viscous flow), but Birkhoff’s earlier criticisms of physicists, engineers, their lack of deductive rigor, and J. J. Stoker were removed. Further, Birkhoff made no suggestion that an instability in the flow of an ideal fluid might resolve d’Alembert’s paradox. I prefer to believe that these omissions are evidence of Birkhoff’s attempt to reconcile with those working within the culture of applied mathematics. Ironically, however, recent (not yet mainstream) research [3] indicates that Birkhoff’s suggested resolution to d’Alembert’s paradox might be the best one.

William Hackborn (who prefers to be addressed simply as “Bill”) is Professor of Mathematics and Computing Science at the Augustana (Camrose) Campus of the University of Alberta. He has dabbled in various mathematical areas over the years, including fluid dynamics, dynamical systems, mathematical biology, and most recently the history of mathematics and physics. Bill anticipates, with mixed emotions, his upcoming retirement on 30 June 2021.

References

- [1] Birkhoff, G. (1950) *Hydrodynamics: A Study in Logic, Fact, and Similitude*. Princeton University Press.
- [2] Hackborn, W. W. (2018) Euler’s Discovery and Resolution of D’Alembert’s Paradox. In M. Zack and D. Schlimm (eds), *Research in History and Philosophy of Mathematics*, 43–57. Proceedings of the Canadian Society for History and Philosophy of Mathematics. Birkhäuser. doi.org/10.1007/978-3-319-90983-7_3.
- [3] Hoffman, J. and C. Johnson. (2010) Resolution of d’Alembert’s Paradox. *J. Math. Fluid Mech.* 12, 321–334. doi.org/10.1007/s00021-008-0290-1.
- [4] Snow, C. P. and S. Collini. (1993) The Rede Lecture (1959). In *The Two Cultures*, 1–52. Cambridge University Press. doi.org/10.1017/CBO9780511819940.002.
- [5] Stoker, J. J. (1951) Review of *Hydrodynamics: A Study in Logic, Fact, and Similitude*, by Garrett Birkhoff. *Bull. Amer. Math. Soc.* 57, 497–499. doi.org/10.1090/S0002-9904-1951-09552-X.

2020 CMS Fellows

Calls for Nominations

March 2020 (Vol. 52, No. 2)

The CMS Fellows program recognises CMS members who have made excellent contributions to mathematical research, teaching, or exposition; as well as having distinguished themselves in service to Canada's mathematical community. In exceptional cases, outstanding contributions to one of the below areas may be recognised by fellowship.

- Making significant contributions to the profession and to the Canadian mathematical community.
- Increasing the relevance and visibility of the CMS.

For a nomination to be complete, all [nomination requirements and eligibility](#) should be included. A CMS member may nominate a maximum of two Fellows in a calendar year. Any person who is nominated and is not selected a Fellow will remain an active nominee for a further two years.

The CMS aims to promote and celebrate diversity in the broadest sense. Nominations for outstanding colleagues are encouraged regardless of race, gender, ethnicity or sexual orientation.

All documentation, including letters of support, should be submitted electronically, preferably in PDF format, to fellows@cms.math.ca **no later than March 31, 2020**.

For the full program description, please visit [here](#).



Second Inaugural Class of Fellows

2019 Winter Meeting Banquet, Toronto, ON



First Inaugural Class of Fellows

2018 Winter Meeting Banquet, Vancouver, BC

Copyright 2020 © Canadian Mathematical Society. All rights reserved.

2020 Adrien Pouliot Award

Calls for Nominations

March 2020 (Vol. 52, No. 2)

Nominations of individuals or teams of individuals who have made significant and sustained contributions to mathematics education in Canada are solicited. Such contributions are to be interpreted in the broadest possible sense and might include: community outreach programs, the development of a new program in either an academic or industrial setting, publicizing mathematics so as to make mathematics accessible to the general public, developing mathematics displays, establishing and supporting mathematics conferences and competitions for students, etc.

CMS aims to promote and celebrate diversity in the broadest sense. We strongly encourage department chairs and nominating committees to put forward nominations for outstanding colleagues regardless of race, gender, ethnicity or sexual orientation.

Nominations must be received by the CMS Office **no later than April 30, 2020**.

Please submit your nomination electronically, preferably in PDF format, to apaward@cms.math.ca.

Nomination requirements

- Include contact information for both nominee and nominator.
- Describe the nominated individual's or team's sustained contributions to mathematics education. This description should provide some indication of the time period over which these activities have been undertaken and some evidence of the success of these contributions. This information must not exceed four pages.
- Two letters of support from individuals other than the nominator should be included with the nomination.
- Curricula vitae should not be submitted since the information from them relevant to contributions to mathematics education should be included in the nomination form and the other documents mentioned above.
- If nomination was made in the previous year, please indicate this.
- Members of the CMS Education Committee will not be considered for the award during their tenure on the committee.

Renewals

Individuals who made a nomination last year can renew this nomination by simply indicating their wish to do so by the deadline date. In this case, only updating materials need be provided as the original has been retained.



2019 Adrien Pouliot Award Recipient



Tiina Hohn
MacEwan University

Prof. Hohn is the most recent recipient of the award. Please read the [Media Release](#) or her [citation](#). For a list of past recipients and to read their citations, please visit the official [Adrien Pouliot Award](#) page.

2020 Graham Wright Award for Distinguished Service

Calls for Nominations

March 2020 (Vol. 52, No. 2)

In 1995, the Society established this award to recognize individuals who have made sustained and significant contributions to the Canadian mathematical community and, in particular, to the Canadian Mathematical Society. The award was renamed in 2008, in recognition of Graham Wright's 30 years of service to the Society as the Executive Director and Secretary.

CMS aims to promote and celebrate diversity in the broadest sense. We strongly encourage department chairs and nominating committees to put forward nominations for outstanding colleagues regardless of race, gender, ethnicity or sexual orientation.

Nominations should include a reasonably detailed rationale including three support letters and be submitted **by March 31, 2020**.

All documentation should be submitted electronically, preferably in PDF format, by the appropriate deadline, to gwaward@cms.math.ca.

Renewals

Individuals who made a nomination last year can renew this nomination by simply indicating their wish to do so by the deadline date. In this case, only updating materials need be provided as the original has been retained.

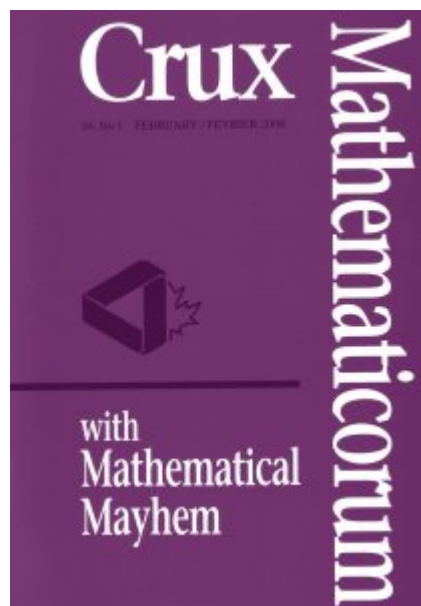


2019 Graham Wright Award for Distinguished Service Recipient



Karl Dilcher
Dalhousie University

Prof. Dilcher is the most recent recipient of the award. Please read the [Media Release](#). For a list of past recipients and to read their citations, please visit the official [Graham Wright Award](#) page.



The CMS invites expressions of interest to fill Associate Editor positions for *Crux Mathematicorum* (CRUX), the CMS international problem solving journal. CRUX is in the process of expanding the current compliment of editors on its [editorial board](#) to help with the growing number of submissions.

Anyone with an interest in problem solving is invited to forward an expression of interest, including a covering letter with, curriculum vitae, and an expression of views regarding the publication. The appointment will begin on May 1, 2020 until December 31, 2024.

Please submit your expression of interest to the Editor-in-Chief at crux.eic@gmail.com **no later than March 31, 2020.**

The CMS Research Committee is inviting nominations for three prize lectureships. These prize lectureships are intended to recognize members of the Canadian mathematical community.

Coxeter-James Prize

The **Coxeter-James Prize** Lectureship recognizes young mathematicians who have made outstanding contributions to mathematical research. The recipient shall be a member of the Canadian mathematical community. Nominations may be made up to ten years from the candidate's Ph.D. A nomination can be updated and will remain active for a second year unless the original nomination is made in the tenth year from the candidate's Ph.D. The selected candidate will deliver the prize lecture at the 2021 Winter Meeting.

Jeffery Williams Prize

The **Jeffery-Williams Prize** Lectureship recognizes mathematicians who have made outstanding contributions to mathematical research. The recipient shall be a member of the Canadian mathematical community. A nomination can be updated and will remain active for three years. The prize lecture will be delivered at the 2021 Summer Meeting.

Krieger-Nelson Prize

The **Krieger-Nelson Prize** Lectureship recognizes outstanding research by a female mathematician. The recipient shall be a member of the Canadian mathematical community. A nomination can be updated and will remain active for two years. The selected candidate will deliver the prize lecture at the 2021 Summer Meeting.

CMS aims to promote and celebrate diversity in the broadest sense. We strongly encourage department chairs and nominating committees to put forward nominations for outstanding colleagues for research in the mathematical sciences regardless of race, gender, ethnicity or sexual orientation. A candidate can be nominated for more than one research prize in the applicable categories; several candidates from the same institution can be nominated for the same research prize.

CMS research prizes are gender-neutral, except for the Krieger-Nelson prize, which is awarded to women only. Nominations of eligible women for the general research prizes in addition to the Krieger-Nelson Prize are strongly encouraged.

Nominations Requirements

The deadline for nominations, including at least three letters of reference, is **September 30, 2020**. Nomination letters should list the chosen referees and include a recent curriculum vitae for the nominee. Some arms-length referees are strongly encouraged. Nominations and the reference letters from the chosen referees should be submitted electronically, preferably in PDF format, to the corresponding email address and **no later than September 30, 2020**:

Coxeter-James: cjprize@cms.math.ca

Jeffery-Williams: jwprize@cms.math.ca

Krieger-Nelson: knprize@cms.math.ca



The Publications Committee of the CMS solicits nominations for Associate Editors for the *Canadian Journal of Mathematics* (CJM) and the *Canadian Mathematical Bulletin* (CMB). The appointment will be for five years beginning January 1, 2021. There are eight associate editors on the *CJM/CMB Editorial Board* whose mandates are ending at the end of December.

For over fifty years, the *Canadian Journal of Mathematics* (CJM) and the *Canadian Mathematical Bulletin* (CMB) have been the flagship research journals of the Society, devoted to publishing original research works of high standard. The CJM publishes longer papers with six issues per year and the CMB publishes shorter papers with four issues per year. CJM and CMB are supported by respective Editors-in-Chief and share a common Editorial Board.

Expressions of interest should include your curriculum vitae and your cover letter and sent electronically to: cjmcmb-ednom-2020@cms.math.ca **before September 15, 2020.**



CMS *75th Anniversary* SUMMER MEETING RÉUNION D'ÉTÉ du *75^e anniversaire* de la SMC

JUNE 5-8 JUIN, 2020 | OTTAWA, ON

Plenary Speakers | Conférences plénières

Henri Darmon (McGill)
Moon Duchin (Tufts)
Matilde Marcolli (Toronto)
Aaron Naber (Northwestern)
Ian Putnam (Victoria)

Public Lecture | Conférence publique

Anne Broadbent (Ottawa)

Prizes | Prix

Excellence in Teaching Award | Prix d'excellence en enseignement
Joseph Khoury (Ottawa)

Jeffery-Williams Prize | Prix Jeffery-Williams
Juncheng Wei (UBC)

Krieger-Nelson Prize | Prix Krieger-Nelson
Sujatha Ramdorai (UBC)

Scientific Directors | Directeurs scientifiques

Ailana Fraser (UBC)
Monica Nevins (Ottawa)
Mateja Šajna (Ottawa)

Scientific Organizing Committee | Comité scientifique

Benoit Collins (Kyoto)
Anita Layton (Waterloo)
Kathryn Mann (Brown)
Robert McCann (Toronto)
Ram Murty (Queen's)

Sponsors | Commandites



FIELDS



PIMS



AARMS



CRM



uOttawa

WWW.SUMMER20.CMS.MATH.CA
WWW.HIVER20.SMC.MATH.CA



Call for Pictures and Anecdotes

CMS Meetings

March 2020 (Vol. 52, No. 2)

The Canadian Mathematical Society (CMS) invites members to share pictures and anecdotes relevant to the Society and its activities for the 2020 CMS 75th Anniversary Summer Meeting that will take place in Ottawa from **June 4-8**.

Call for Pictures and Anecdotes

To mark the 75th anniversary of the CMS, the CMS Meetings team is collecting pictures and memories from members, old and new, to display and share with other members. If you have pictures of CMS events or anecdotes and memories that you would like to share, we encourage you to send them to finances@cms.math.ca **no later than May 1st, 2020**

If you or someone you know could identify the mathematicians on the original image of 1945 Canadian Mathematical Congress below, please let us know so we can complete our archives. Visit our [website](#) for a list of already identified participants.



The Canadian Mathematical Society (CMS) welcomes and invites session proposals and mini course proposals for the 2020 CMS Winter Meeting in Montreal from **December 4-7**.

Call for Sessions

Proposals should include (1) names, affiliations and contact information for all session co-organizers, (2) title and brief description of the focus and purpose of session, (3) a preliminary list of potential speakers, with their affiliation and if they have agreed to participate, along with a total number of expected speakers.

Session will take place December 5, 6, and 7. The meeting schedule will accommodate 12 speakers per full day, and 5 or 7 per half day. Sessions will be advertised in the *CMS Notes*, on the web site and in the *AMS Notices*. Speakers will be requested to submit abstracts which will be published on the web site and in the meeting program. Those wishing to organise a session should send a proposal to the Scientific Directors and copy the CMS office. Those submitting proposals are encouraged to pay attention to the diversity of both the session invitees and the proposed session organisers.

Proposals should be submitted by **March 30, 2020**.

Call for Mini-Courses

The CMS is organising three-hour mini-courses to add more value to meetings and make them attractive for students and teachers to attend.

The mini-courses will be held on Friday afternoon, December 4th, before the public lecture, and include topics suitable for graduate students, postdocs and other interested parties.

Proposals should include names, affiliations, and contact information for all the mini course co-organizers and title and brief description of the focus of the mini course.

Scientific Directors

Michael Lipnowski (McGill University)
michael.lipnowski@mcgill.ca

Brent Pym (McGill University)
brent.pym@mcgill.ca

CMS Office

meetings@cms.math.ca

MCA 2021

Mathematical Congress of the Americas

19–24 July | Buenos Aires Argentina | mca2021.org

The goal of the Mathematical Congress of the Americas (MCA) is to internationally highlight the excellence of mathematical achievements in the Americas and foster collaborations among researchers, students, institutions and mathematical societies in the Americas.

Plenary speakers



Ian Agol, University of California, Berkeley

Julia Chuzhoy, Toyota Technological Institute at Chicago

Carlos Kenig, University of Chicago

Allan Sly, Princeton University

Claire Voisin, Collège de France

Miguel Walsh, Universidad de Buenos Aires

CALL FOR SPECIAL SESSIONS

Proposals of special sessions at MCA 2021 are welcomed by the Special Sessions Committee. Early submission of proposals is encouraged: good proposals will be approved on a regular basis before the deadline of **July 31, 2020**.

Learn more at mca2021.org/news/item/16-mca-2021-call-for-special-sessions



The goal of the Mathematical Congress of the Americas (MCA) is to internationally highlight the excellence of mathematical achievements in the Americas and foster collaborations among researchers, students, institutions and mathematical societies in the Americas. We are very happy to bring to your attention the next MCA which will be held in 2021 in Buenos Aires and urge you to look at the web site www.mca2021.org

The decision to launch these quadrennial Congresses was taken in New Orleans in January, 2011 at a meeting of the founding mathematical societies, namely the AMS (US), CMS (Canada), SBM (Brazil), SIAM (US), SMM (Mexico) and UMALCA (Latin America Union). The inaugural MCA was held in the summer of 2013 in Guanajuato, Mexico. It was an important mathematical event that greatly increased communication and cooperation among mathematicians throughout the Americas. The setting of the lovely town of Guanajuato and the gracious hospitality of the Mexican hosts ensured a delightful background for excellent mathematics. The Canadian mathematical community played host to the 2017 Mathematical Congress of the Americas in Montreal, from July 24 to the 28th. By all accounts, the event was a great success with close to 1100 participants.

The third MCA 2021 will take place in the historic capital city of Argentina and will be hosted by the Departamento de Matemática, Facultad de Ciencias Exactas y Naturales of Universidad de Buenos Aires. We are pleased to report that a large new building on the campus has recently been completed and will provide a spacious and convenient setting for all the activities of the MCA2021. The scientific program of the Congress is well underway with the selection of the plenary and invited speakers. Their selection by the international scientific program committee is based on excellence in research and very good expository skills. These outstanding mathematicians include Ian Agol, Julia Chuzhoy, Carlos Kenig, Alan Sly, Claire Voisin and Miguel Walsh. We strongly encourage mathematicians from all over the Americas to submit proposals for special sessions following the information found at www.mca2021.org. We seek to host many special sessions on a very broad selection of topics and with diverse participation.

Argentina has a very strong tradition of mathematical research. Pioneering work in the early 1900's set the foundations for the development of a wide variety of areas where Argentinian mathematicians have a strong presence. Some Argentinians established major centers in mathematical institutes in Buenos Aires, Cordoba, Santa Fe, Bahia Blanca and La Plata. Many Argentinian mathematicians are well-known for their results, with A. Calderon, L. Caffarelli and C. Kenig being prominent examples in the field of analysis. The country of MCA2021 is an excellent site to host a major international mathematical event, both because of its strong culture of mathematics and its very attractive location. We urge our colleagues in all parts of the Americas to place Buenos Aires in July 2021 on their calendars for a very special Congress.

Award type: Scholarships

Affiliation: Indigenous

Value: \$15,000



Award description: up to five scholarships, each valued at a maximum of \$15,000, will be awarded annually to students entering into the Master of Mathematics for Teachers (MMT) program at the University of Waterloo. In order to be considered for the scholarship, candidates must be active mathematics teachers, demonstrate a strong connection to an Indigenous community, and show an ability to impact Indigenous students. Preference will be given to Indigenous candidates. Each scholarship is designed to cover the full cost of tuition while the student is registered in the MMT program. Candidates interested in being considered for the scholarship must apply for both admission to the program and for the scholarship by May 1. Scholarship application forms are available on the [MMT website](#).

Value description: Each scholarship will have a value of the total cost of tuition and incidental fees up to a maximum value of \$15,000.

Eligibility & selection criteria:

Applicants must:

- Be a Canadian citizen or permanent resident.
- Be accepted into the Master of Mathematics for Teachers in the Centre for Education in Mathematics and Computing (CEMC) at the University of Waterloo.
- Teach mathematics at a school in Canada.
- Demonstrate a strong connection to an Indigenous community in Canada.
- Show an ability to impact Indigenous students in Canada.
- Apply for admission to the MMT and for the scholarship by May 1.

Preference will be given to Indigenous candidates.

Level: Masters

Program: Mathematics – Mathematics for Teachers

Citizenship: Canadian/Permanent resident

Selection process: application required

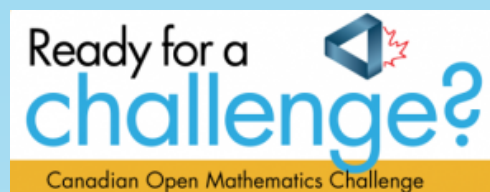
Term: Spring

Application deadline: May 1

Additional instructions: Candidates interested in being considered for the scholarship must apply for both admission to the program and for the scholarship by May 1. Scholarship application forms are available on the [MMT website](#).

Contact person: for further information about this award please contact the [MMT](#).

The CMS is looking for assistance from the Canadian mathematical community in marking the Canadian Open Mathematical Challenge (COMC). The number of students writing the COMC has been growing and we are currently looking for additional marking partners to help with the influx of exams. The Canadian Open Mathematics Challenge (COMC) is Canada's premier national mathematics competition open to any student with an interest in, and grasp of high school math.



Current marking partners are:

- BC – University of British Columbia
- Alberta – University of Calgary
- Saskatchewan – University of Saskatchewan
- Manitoba – University of Manitoba
- Ontario – University of Toronto and York University (York marks our international papers)
- Newfoundland – Memorial University
- New Brunswick – University of New Brunswick
- Nova Scotia – Dalhousie University
- PEI – University of Prince Edward Island

We would like to involve more universities in Canada to become official marking sites in order to accommodate the growing number of students. The CMS can tailor the number of exams an institution may want to mark.

What does being a partner entail?

The exam is typically written on the first Thursday in November and the CMS ships out all COMC materials to the writing centres. The schools are provided with a return postage envelope which would go to the Marking Lead designated by the university. Marking Leads recruit volunteers such as faculty members, PDF's, grad students and strong undergraduates to help with the marking. Once the exams have been received (roughly November 10-15) they need to be scanned into Crowdmark which is our collaborative online grading and analytics platform. Markers can be assigned to grade an exam, with Part A and Part B questions taking on average less time because full marks are assigned when the correct answer appears. The data on the front page (student name, email, etc) needs to be entered into a database. This can be done by anyone. The Marking Lead provides access to the tasks by entering the name and email address of the individual on the Crowdmark page. On average, it takes 6.96 minutes (0.116 hours) to grade an exam. Once the exams are received, your team has roughly 15 days to mark the exams, and enter the metadata information. We like to have the preliminary marks back to teachers before the Christmas break. Since the marking platform is online, your volunteers don't have to be in one place to mark the exams. Once the exams are marked and the grades are entered, CMS will select the top 200+ exams from across the country and have them vetted to see who will go on to write the Canadian Mathematical Olympiad (CMO) and to receive public recognition for their efforts.

By being a marking partner, your university will receive a list of all the top students in Canada to use to offer scholarships or recruitment. We will feature your logo on our COMC website and include your logo on every exam that goes to your province.

CMS would like to accommodate the increase in the number of Students writing the COMC competition and give them the chance to represent Canada at the International Mathematical Olympiad. If you are interested in this opportunity or would like to discuss this further, please contact the COMC committee chair or [the CMS Competition team](#).

Dr. Robert Woodrow
Chair of COMC Committee
Canadian Mathematical Society

Dr. Termeh Kousha
Executive Director
Canadian Mathematical Society

CMS Notes

Editors-in-Chief

Robert Dawson and Srinivasa Swaminathan

notes-editors@cms.math.ca

Editor

Zishad Lak

zlak@cms.math.ca

Contributing Editors:

Calendar and Member Relations:

Denise Charron

mpagent@cms.math.ca

CSHPM:

Amy Ackenberg-Hastings and Hardy Grant

aackerbe@verizon.net and hardygrant@yahoo.com

Book Reviews:

Karl Dilcher

notes-reviews@cms.math.ca

Education:

John McLoughlin and Kseniya Garaschuk

johngm@unb.ca and kseniya.garaschuk@ufv.ca

Meetings:

Sarah Watson

notes-meetings@cms.math.ca

Research:

Patrick Ingram

notes-research@cms.math.ca

Executive Committee

President:

Mark Lewis (Alberta)

president@cms.math.ca

President-Elect:

Javad Mashreghi (Laval)

pres-elect@cms.math.ca

Vice-President – Atlantic:

Sara Faridi (Dalhousie)

vp-atl@cms.math.ca

Vice-President – Quebec:

Matilde Lalin (Montréal)

vp-que@cms.math.ca

Vice-President – Ontario:

Monica Nevins (Ottawa)

vp-ont@cms.math.ca

Vice-President – West:

Gerda de Vries (Alberta)

vp-west@cms.math.ca

Vice-President – Pacific:

Malabika Pramanik (UBC Vancouver)

vp-pac@cms.math.ca

Treasurer: David Oakden

treasurer@cms.math.ca

Corporate Secretary:

Termeh Kousha

corpsec@cms.math.ca

The editors welcome articles, letters and announcements. Indicate the section chosen for your article, and send it to *CMS Notes* at the appropriate email address indicated above.

No responsibility for the views expressed by authors is assumed by the *CMS Notes*, the editors or the [CMS](#).

Copyright 2020 © Canadian Mathematical Society. All rights reserved.