Prove It!
Emphasizing Mathematical Communication in a Linear Algebra Class

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Education Notes bring mathematical and educational ideas forth to the CMS readership in a manner that promotes discussion of relevant topics including research, activities, issues, and noteworthy news items. Comments, suggestions, and submissions are welcome.

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"Writing in a math class?!" Many of our students tend to view communication and mathematics as two distinct entities. They enter our classroom with a preconceived notion of what a math class is and how they will be assessed. However, many of us decide to push back against this norm in our classrooms. Three factors which have motivated me to emphasize mathematical communication in my classes are: authenticity, student learning, and long-term student success.

A crucial part of a mathematician's job is to find the best ways to communicate their results both verbally and in writing. Incorporating mathematical communication into our assessments can help make our classrooms more authentic. Communication-based assessments can also accelerate student learning in a variety of ways. Writing provides a natural venue for enabling students to combine and unify course concepts. Reflective writing assignments can help boost student motivation and refine metacognitive skills. Moreover, research suggests that oral communication activities in a group setting boost achievement, persistence, and attitudes among undergraduate math students (Springer, Stanne, & Donovan, 1999). These benefits have been shown to be especially effective for female students (Herreid, 1998) and groups underrepresented in STEM (Treisman, 1992). Finally, some of us may be compelled to help students cultivate skills they will need in the workplace, which can lead to long-term student success. Mathematical communication exercises can help train our students to become logical thinkers, construct rigorous arguments, and effectively communicate complex ideas to non-experts.

In 2018/2019, I taught a second-year linear algebra course three semesters in a row which introduced the theory of abstract vector spaces. I took this opportunity to develop communication-based course components. I packaged these components into a Communication Score worth 25% of the course grade. For context, the other course components were: online Lyryx assignments (10%), tests (30%), and a final exam (35%). Each component of the Communication Score is outlined below.

Essay/Written Tasks (5%). Halfway through the course, students submitted a 500-word "Applications of Linear Algebra" essay. Students were encouraged to choose an application applicable to their major. They were instructed to avoid field-specific jargon; their audience was any person who had taken this course. This essay gave students the opportunity to practice communicating complex idea to non-experts. It was also meant to help prepare students for their upcoming poster presentation.

Essays were graded according to the following rubric:

	Essay Rubric	Possible Points
A	Appropriate Mathematical Depth/ Contains an Explicit Example	40
B	Clarity	40
C	Narrative/Organization	10
D	Grammar/Spelling/Punctuation/Legibility	10

Note: Your B score cannot exceed your A score. Moreover, your scores in categories C/D cannot exceed your A score divided by 2. (e.g. If you receive 10/40 in category A, then you can receive at most 10/40 in B, 5/10 in C, and 5/10 in D.)

I designed this rubric in response to the common mistakes I saw the first time around. The weakest essays were superficial in nature and said something along the lines of, "Linear algebra is awesome. It's used in quantum mechanics, robotics, and signal processing. The end." To help prevent essays like this, clarify expectations, and streamline grading, I anchored the rubric so that one needs appropriate mathematical depth to do well.

Another common critique was that some students were too ambitious, which made their essay difficult to understand; explaining a concept in complete generality in only 500 words so that an average classmate can understand can be extremely difficult! I found that essays which led with an example were a

lot clearer and helped to demonstrate that students understood their topic. As such, I included "Contains an Explicit Example" in the description of the anchoring category "A" of the rubric.

Presentation (5%). The Presentation component was a group poster presentation based on the "Applications of Linear Algebra" essay. Students worked in groups of four. Most groups chose to report on one or two of their group members' essay topics. The class had two 50-minute tutorial sections consisting of ~60 students. The last two tutorials were dedicated to poster presentations.

Eight groups presented at each tutorial. They were instructed to prepare a three-minute presentation and to allow two minutes for questions. Posters were set up around the perimeter of the classroom. The eight non-presenting groups were assigned a starting position. They spent five minutes with each poster, then moved clockwise (so each group gave their presentation eight times). Non-presenting groups kept track of the questions they asked and handed their questions in at the end, as part of their Tutorial Quiz score. This format was meant to mirror a conference poster session and gave me the opportunity to view each presentation. Posters were graded based on content, organization, visual design, and oral presentation.

Tutorial Tasks (10%). Tutorials were designed to cultivate students' ability to communicate mathematics orally and to gain practice in proof writing. The first five minutes were dedicated to a short quiz concerning the previous week's material. Students brought their own paper to complete the quiz. The quiz was graded out of one point; students received 0.25 points for completion and 0.75 points for correctness. These quizzes were included to encourage students to review their notes before coming to tutorial so that they could productively engage in group work activities.

The next 10 minutes of tutorial was dedicated to proof writing. Some weeks students were given several possibly faulty proofs to evaluate according to a rubric; in tutorial, students would discuss the score they gave each proof and the TA would reveal their score. Other weeks, students were given a prompt ahead of time and asked to write a proof; in tutorial, the TA would go through the proof at the board.

The remaining 35 minutes was dedicated to group work activities. Students moved their desks into groups of four and worked on worksheets. The TA and I floated around to answer questions. Students received an Oral Communication Score based on their participation. To provide incentive to complete the worksheets, a quiz was given in class at the end of the week worth bonus points; students received one bonus point towards their test score if they got a 5/5 on this quiz.

One interesting note about the worksheets is that I found that the way I packaged these materials mattered. When I called these exercises "Worksheets", many students complained that the exercises were too long and that they did not have enough time to complete them in tutorial – even though I had emphasized in class that they were not expected to finish the worksheets in tutorial. The second semester, I renamed the worksheets "Homework" and I did not receive a single criticism about the exercises being too long.

The Tutorial Quizzes and Oral Communication Score were worth 8% and 2%, respectively. Although, since students receive 25% on each quiz for compelition, I really considered them each to be worth 6% and 4%, respectively. The quizzes provided a means to take attendance. Students received full points on their Oral Communication Score if they actively worked in tutorial, which meant I only needed to make note of students who were not working (which almost never happened). Two tutorial absences were excused.

Final Proof (5%). This course served as an introduction to proofs for many students. Proof writing was emphasized throughout the semester and was evaluated on four term tests (best 3 of 4 were counted). The Final Proof appeared on the last page of the final exam. The marking scheme and difficulty mirrored the term-test proofs.

I chose to package this final proof as part of the Communication Score (instead of making the final exam worth 40%) in order to underline the critical role proof writing would play in the course. Putting the Final Proof on the Course Outline helped to give the course direction and gave students a goal to work towards.

Written Tasks (bonus). There were two optional bonus reflections which could each bump students' 25-point Communication Score by 0.25 points, so long as they appeared to be genuine and well thought out. These reflections were originally mandatory, but I decided to make them optional based on student feedback that the course had too many due dates.

The first reflection was designed to help students find value in proof writing:

> *In this course, we have been gaining experience in formal proof-writing. With your future career goals in mind, reflect upon if/how engaging in this formal proof-writing may help or influence you in your post-university life/career.*

The second reflection was designed to push students to reflect on their learning. It had the added benefit of providing insights to me about what students felt like they were taking from the course:

> *What were 3 of the most important discoveries or realizations you made in this class? In other words, what are you taking away from this class that you think might stick with you and/or influence you in the future? These can include things you had not realized about*

> *mathematics, specific homework problems or theorems, discussions with other students or the instructor, connections to other courses, etc. Explain why these three discoveries or realizations are important to you.*

What did students think about the Communication Score? The most memorable component appeared to be the poster presentation; the first time I read students' "most important discoveries or realizations" in the reflective prompt above, I was surprised to see that the majority of students commented on the widespread applications of linear algebra. We did not do any applications in class, so this takeaway is most likely due to the poster presentations they viewed – or possibility the research they did for their essay. I was also delighted to hear students report in this reflection and in unsolicited feedback – sometimes months later – their appreciation for the emphasis this course placed on group work and writing, because it led to deeper understanding of course material.

As can be expected, there were many students who were not fans of the Communication Score. Some students felt like the essays, written tasks, and presentation were futile, because they did not directly relate to content that would help them do well on the final exam. However, I found that these comments began to change as I put more effort into motivating why I was asking students to complete these tasks. For example, on the course's Communication Score Description document, I included this disclaimer and discussed these points in class:

> *Motivation: You may be asking: "Why am I being asked to write an essay in a math class?!" or "This is stupid… how will this help me on the final exam?" It's true that these Essay/Bonus Reflections will most likely not directly help in preparing for your final exam. But this doesn't mean that they don't have value. The Essay/Bonus Reflections will give you the opportunity to take a step back and reflect on the skills you are developing in this course. You are all very busy, and it can be difficult to take the time to think critically about "Why on earth is this content useful?" or "What skills might I be developing?" Moreover, your future self will thank you for becoming a better writer, since in almost any job you will be required to write formal documents. If you are a science major, then you may not have had many opportunities to write formally, and so my hope is that the Essay/Bonus Reflections will allow you to practice.*

After doing this, I found that almost all students who disliked the Communication Score added the qualifier, "…but I understand why it is necessary." Learning is like exercise; it is uncomfortable and we may not enjoy it, but most of us are thankful that we did it.

Will my students remember how to orthogonally diagonalize a matrix ten years from now? Probably not. But I hope the communication components of my course have instilled in some students the feeling that mathematics has deep connections to many disciplines, the sense that writing and talking through a problem with others can accelerate learning, the care one must take when explaining complex topics to non-experts, and other intangible skills that help to unlock long-term success.

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