NOTES de la SMC

Taking a Cue from Cauchy

Maritza M. Branker (Niagara University)

Les articles de la SCHPM présentent des travaux de recherche en histoire et en philosophie des mathématiques à la communauté mathématique élargie. Les auteurs sont membres de la Société canadienne d'histoire et de philosophie des mathématiques (SCHPM). Vos commentaires et suggestions sont le bienvenue; ils peuvent être adressées à l'une des co-rédacteurs:

Amy Ackerberg-Hastings, *chercheuse indépendante (aackerbe@verizon.net)*
Hardy Grant, *York University [retraité] (hardygrant@yahoo.com)*

Figure 1. Augustin-Louis Cauchy around 1840.

Most of us think of complex analysis when we hear the name of Augustin-Louis Cauchy (1789–1857)—rightfully so, since he was the founder of complex function theory of one variable. Cauchy has been described as "the greatest French mathematician of his time, the heir of Euler, Lagrange, Laplace and other great mathematicians in the eighteenth century. He led what has been described as the first revolution of rigour in mathematics" [4, p. 80].

What tends to be overlooked is his appealing approach to *real* analysis, as presented in an 1821 text based on his lectures at the École Polytechnique in Paris. The *Analyse algébrique* is the first part of a projected *Cours d'analyse* which Cauchy never completed. Curriculum changes made in 1822 rendered his planned second volume obsolete before it was written, since it was not aligned with the reduction of the foundational emphasis. Thus we follow the common parlance and refer to *Analyse algébrique* as Cauchy's *Cours d'analyse*. It is also worth mentioning that we rely on the excellent English translation by Bradley and Sandifer [1] for this discussion, rather than quoting the original French text, which is available in digitized form at [2].

In 1821 the École Polytechnique maintained the position it had held since 1794, as the first and most prestigious institution in the world for training engineers. Its curriculum in physics and mathematics was renowned, and students competed vigorously for the 200 seats available in each year's class [3, p. 5]. Cauchy had been appointed to the faculty in 1815 as a substitute for Louis Poinsot; in 1816 he was elevated to full professor as a consequence of political alignment. Specifically, royalists were removing the influence of the Napoleonic regime and, given the existing Bonapartist sympathies of the school, "seized on a pretext in April 1815 to send the students home and reform the school" [3, p. 35]. Since Cauchy was a staunch royalist, he was an obvious candidate to be promoted after the purge of liberal professors.

Cauchy's task was to teach calculus to future engineers, which included an expectation to prepare textbooks [3, p. 35]. However, Ioan James and other scholars have argued that his true purpose was more theoretical, with the result that the book exerted a profound impact on the discipline of mathematics: "Despite its name it was never intended as a textbook, and never used as such; however Cauchy's work is the source from which the classic *Cours d'analyse* of Jordan and others are ultimately derived" [4, p. 84].

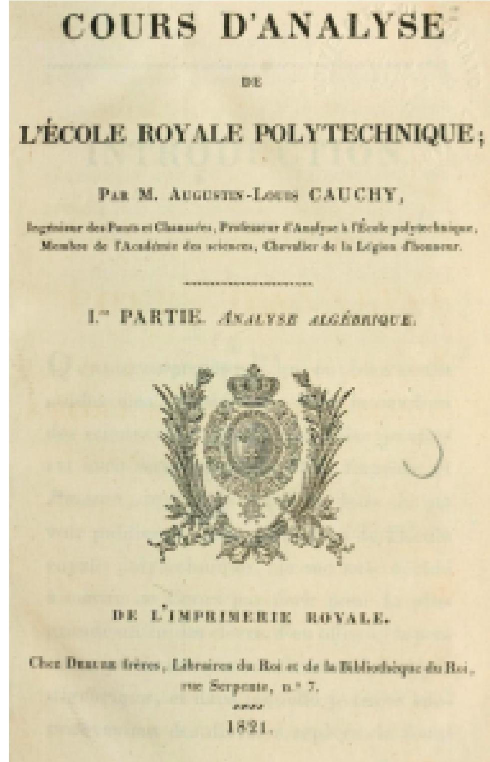
Keeping the environment at École Polytechnique and these discrepancies in audience in mind, here is Cauchy's chosen approach for deriving the addition formula for cos(x):

> *Problem: To determine the function* $\varphi(x)$ *in such a manner that it remains continuous between any two real limits of the variable* x *and so that for all real values of the variables* x *and* y *we have* $\varphi(y+x)+\varphi(y-x)=2\,\varphi(x)\,\varphi(y)$ [1, p. 77].

That's an interesting question; given the fact that students consistently forget the trigonometric identities, a solution would not be immediately obvious even if we posed this problem to students of real analysis in the 21st century. Indeed, it takes Cauchy many pages of methodical calculations to be able to say the two functions cos(ax) and ½ ($A^x + A^{-x}$) have the common property of satisfying the given equation, so perhaps one would not want to actually grade the results of assigning this as an actual problem, either now or in the past.

The framing of Cauchy's conclusion is also noteworthy. He begins his solution to the problem by taking $x = 0$, so he closes with

> *Both of these functions still reduce to one for x = 0. But one essential difference between the first and second is that the numerical value of the first is constantly less than the limit 1, whenever it does reach this limit, while under the same hypothesis, the numerical value of the second is constantly above the limit 1* [1, p. 83].



COURS D'ANALYSE

DE

L'ÉCOLE ROYALE POLYTECHNIQUE;

Par M. Augustin-Louis CAUCHY,

Ingénieur des Ponts et Chaussées, Professeur d'Analyse à l'École polytechnique, Membre de l'Académie des sciences, Chevalier de la Légion d'honneur.

I.re PARTIE. Analyse algébrique.

DE L'IMPRIMERIE ROYALE.

Chez Debure frères, Libraires du Roi et de la Bibliothèque du Roi, rue Serpente, n.° 7.

1821.

Figure 2. Title page of *Cours d'analyse*.

For the record, Cauchy was not the first to note the parallels between the trigonometric and hyperbolic functions—Lambert had observed them in 1768. But this parallel is not mentioned in any methodical way in the current mathematics curriculum; in fact, it's possible not to encounter hyperbolic functions until graduate school. Specifically, this connection is not highlighted in modern introductory texts, where hyperbolic functions are rarely mentioned and the parallel between the trigonometric and hyperbolic functions is glossed over.

Jeremy Gray has described Cauchy's teaching in 1821 as "defin[ing] and discuss[ing] a number of concepts with a remarkable, novel, degree of precision" [3, p. 35]. That's a laudable goal for any mathematical lecture, although it also carries the implicit assumption that the lecture is appropriate for the target audience. Cauchy sometimes failed in this respect: James noted that "there were objections that his courses were over-ambitious, and that he gave too much time to pure rather than applied mathematics" [4, p. 86].

This episode may therefore be connected with the following personal observation from my 20 years of teaching undergraduates: *Students benefit from more diverse methods in presenting material*; a holistic approach may succeed in providing a gateway to traditionally underrepresented groups into mathematics. Thus, it would be worthwhile to consider taking a cue from Cauchy by choosing to show the hidden connections early on, rather than waiting until students progress further in their mathematical journeys.

Maritza M. Branker is an associate professor at Niagara University, New York and has been a member of the CSHPM since 2016. Her research interests currently include pluripotential theory, history of mathematics, mathematics education, and higher education and leadership.

References

[1] Bradley, R. E., and C. E. Sandifer. (2009) *Cauchy's Cours d'analyse: An Annotated Translation. Sources and Studies in the History of Mathematics and Physical Sciences*. Springer.

[2] Cauchy, A.-L. (1821) *Cours d'analyse de l'École Royale Polytechnique . . . 1re Partie. Analyse algébrique*. Debure frères, Libraires du Roi et de la Bibliothèque du Roi. https://archive.org/details/coursdanalysedel00cauc.

[3] Gray, J. (2015) *The Real and the Complex: A History of Analysis in the 19th Century. Undergraduate Mathematics Series*. Springer.

[4] James, I. (2002) *Remarkable Mathematicians From Euler to von Neumann. MAA Spectrum Series*. Cambridge University Press.