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Before Forster's Education Act of 1870 mandated compulsory education, primary schooling in Great Britain was irregular, with many children receiving little or no systematic instruction [6, p. 184]. When education became required, the curriculum was typically centered on the Three R's: reading, writing, and arithmetic. Teaching had undergone professionalization with the opening of James Phillips Kay-Shuttleworth and Edward Carleton Tufnell's Battersea College teaching institute in the 1830s, but Robert Lowe, Vice President of the Committee of the Council on Education from 1859 to 1867, believed that greater emphasis should be placed on exam results. This undermined the implementation of active-learning pedagogies such as Pestalozzian approaches. Student performance on the General Inspector's exams determined the amount of grant money awarded to a school and consequently impacted a teacher's salary. The vast majority of teachers thus saw student memorization without understanding as the best means to securing the grant money [6, p. 188]. While their inference impacted all subjects, this development was especially problematic for mathematics because students had to learn mathematical facts in a sterile, repetitive regime, which unsurprisingly was difficult and unenjoyable [4, p. 76].

Victorian education, and its rote learning, came to be criticized by mathematical pedagogues. Specifically, they observed that anxieties about learning mathematics were prevalent, and they conceived that the dominant pedagogical approach failed to address this concern. For instance, Augustus De Morgan believed that presenting students with too much new material at once would overwhelm and "embarrass" them. In turn, he feared their embarrassment would deter them from further mathematical study [3, p. 5]. Decades later, Bertrand Russell complained: "Even the most intelligent child finds, as a rule, great difficulty" in learning algebra; for such a child it was "almost impossible, at first, not to think that every letter stands for some particular number" and become frustrated [8, p. 63].

One of the first Victorian mathematics educators to attempt to counter the damage inflicted by rote learning by implementing anti-anxiety techniques in her pedagogy was Mary Everest Boole (1832–1916). As a child, Boole was educated at home and took arithmetic lessons from a Monsieur De'place. She later deemed Monsieur De'place her 'hero' because, instead of forcing her into rote memorization of arithmetical principles, he "asked [her] a succession of questions and made [her] write down each answer as [she gave it]" [quoted in 5, p. 36]. He engaged in an effective mathematical conversation with her and guided her through the process of mathematical discovery to develop her intuition. But her lessons with respect to approaches to learning mathematics did not end there. At the age of sixteen, while she was learning differential calculus, she realized how non-intuitive theoretical textbooks were and instead taught herself the subject from an older book on fluxions [5, p. 36], which placed a greater emphasis on the discovery process in a more natural setting. When she began to teach in 1864, Boole adapted her own favorable learning experiences in her pedagogy. She "discourage[d] all formulae" until the students had constructed them for themselves. Only then were they allowed to write in their form[ula] books, from which they could build their mathematical knowledge [1, p. 807].



Boole required this process because she noticed “nerve storms” [2, p. 910] that happened in a child’s brain if it were overloaded with too much new material at once. She also recognized a generally widespread malaise with arithmetic, which “seems to some people dry and un-beautiful, but that is because they have not soaked it in the solvent which is called sympathy” [1, p. 815]. Sympathy, for Boole, was achieved by recreating the mathematical discovery process through allowing students to experiment, record their experiments, and gain an intuitive understanding of the concept in question. Showing very young students the aesthetically pleasing discovery process of mathematics was critical to avoiding the common mental blocks—associated with “embarrassment,” among other feelings of frustration—to their mathematical progress.

To prevent these ill feelings toward mathematics, Boole began a child’s mathematical education by stimulating their discovery processes from infancy. Instead of teaching babies to say “one, two, three like a parrot,” she taught them to count objects such as bricks, pebbles, or buttons, and expanded that materiality to higher numbers such as eleven and twelve by breaking them down into “ten-one, ten-two, etc.” [1, p. 823]. The underlying concept for this approach, which involves narration of each step, is simple. A bijective assignment of a number term such as “one, two, . . . ten” narrated the cardinal sequence of the numbers, and breaking higher numbers into their constituent parts helped children discover the underpinnings of the base-ten number system. Boole acknowledged that number words should reveal these underlying concepts. Although the learner was allowed to memorize the numbers one through nine, rote memorization was not the most critical pedagogical tool involved in the development of the child’s intuition of the concept of numbers. Rather, the act of picking up a brick, pebble, or button and creating an aggregated pile physically represented the counting process. It surpassed rote memorization because it emphasized the material quantity behind the abstracted counting process. This process allowed the learner to create a representation of a number and gain intuition of it, which helped prevent ill feelings toward the subject later because pupils better understood the concept of a number by discovering it for themselves.

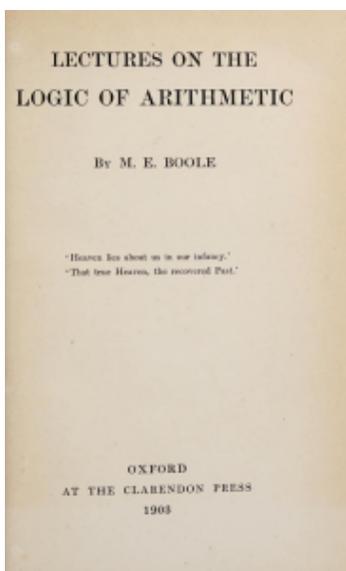


Figure 2. Title page of Boole’s *Lectures on the Logic of Arithmetic* (1903). Internet Archive.

When children were old enough to participate in monetary transactions, Mary Boole emphasized the importance of learning sums with money naturally. She acknowledged that it was possible to go out shopping, exchange currency, and then be given change “in a muddled order” [1, p. 826]. She believed that students needed to be prepared for this messy situation instead of experiencing only the orderly situations that were presented in the vast majority of their textbooks. A typical sum from her 1903 *Lectures in the Logic of Arithmetic* is as follows: If you have nine pence in your purse and spend three pence on flower-roots, what do you have left? On the surface, there seems to be a simple answer of sixpence. However, Boole extended the narrative to include the value and cost beyond the transaction. The true cost and value, she argued, depended on the lives of the flowers. If the flowers lived, sixpence and flowers are left. However, if they died, all that was left was the sixpence, and money was wasted [1, p. 828]. Thinking about all these potential outcomes, she contended, helped students not only to gain a stronger understanding of arithmetic but also to appreciate the aesthetics of arithmetic as they anticipated and narrated multiple possible outcomes of the transaction. In this case, the student was the determining agent in which way the sum ended. The student narrated the outcome of the flowers, calculated the amount of change they got back, and by caring or not caring for the flowers, they participated in discovering the true arithmetical results from their transaction.

Boole believed that understanding arithmetic in this way allowed a more natural progression into algebra, geometry, and the mathematical discovery process in general. Her ideas have continued to resonate with education theorists. The American anthropologist, Leslie White, posited in 1947 that “mathematical truths exist in the cultural tradition into which the individual is born, and so enter his mind from the outside” [9, p. 2350]. White’s views of mathematics as a cultural tradition coincide with Boole’s discovery process that relied on the child’s access to material objects such as the pebble, brick, button, coin, and the narratives they created with them in natural language. Taking that language and putting it into mathematical terms fit for their form book directly engaged them with this deeply culturally entrenched discovery process, which still has important implications in the philosophy of mathematics today. For example, Imre Lakatos’ *Proofs and Refutations* [7] also focused on utilizing the narrativized dialogue in the classroom discovery process, which emphasizes the role of the person and their background in shaping mathematics.

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