Book Reviews bring interesting mathematical sciences and education publications drawn from across the entire spectrum of mathematics to the attention of the CMS readership. Comments, suggestions, and submissions are welcome.

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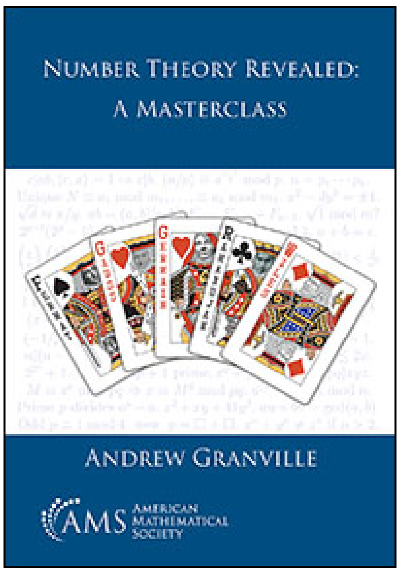

Number Theory Revealed: A Masterclass

by Andrew Granville
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Reviewed by Patrick Ingram (York University)

If I'm completely honest, my initial reaction upon receiving this book in the mail (while blissfully quarantined on a small island in Northern Ontario) was "But why do we need yet another introductory number theory textbook?" Research monographs are easy to justify, as they are usually the first cohesive thing written on a new body of literature, and graduate textbooks often fill a conspicuous gap. Authors of undergraduate textbooks, though, have a high bar to clamber over, since there is always a shelfful of books already at hand. I thought back to a colleague at a previous institution who was writing an introductory linear algebra text; he assured me he had a new way of presenting the subject, but I couldn't help wondering if it was a new new way, or one of the old new ways. As I thought back to teaching various first courses in number theory, though, I remembered consistently having trouble finding a suitable book, and so I decided to give the author the benefit of the doubt, sit on the dock with a beer in hand, and flip through *Number Theory Revealed: A Masterclass*, by Andrew Granville.

This book is 587 pages long, with 17 chapters, and several appendices attached to each chapter. Although not as voluminous as Rosen's 752-page *Elementary Number Theory*, an obvious comparator, there is no shortage of material in this book, and the author makes no pretense of it being a packaged 12-week undergraduate semester (although a few outlines of such are suggested as subsets of the book). Another book I have used in courses like this is Silverman's *A Friendly Introduction to Number Theory*, which is shorter, and broken up into natural one-class chunks. Silverman's book is really aimed at a liberal arts audience, though, and Granville's would be more appropriate for promising math students.

Number Theory Revealed is written in a conversational tone, and the author takes an intentional approach of motivating theorems and proofs in advance, with examples and computations, which is how we all learn (but not necessarily how we all teach). Not just a list of results and definitions that were hopefully explained in class, this a book for sitting down and engaging with, pen in hand, ready for the exercises interlaced with the exposition. Students learn not just by seeing examples and special cases worked out in advance of a general statement, but by working things out for themselves, and as much as I'm intuitively drawn to books that separate the exercises out into a nice hermetically sealed section at the end of a chapter (just as I'm intuitively drawn to the tidiness of "Definition, Theorem, Proof"), this intuition is wrong. Granville's copious exercises (with hints at the back, to give students a nudge in the right direction) are right where they should be to help the students get a feel for the material.

This book is well-organized and the chapters progress coherently. The range of topics in the main text is fairly standard, with perhaps a very slight slant towards the analytic, but the trend of the book is to go at least a bit deeper on everything. The book begins with a review of induction, covers the Euclidean algorithm and congruences, multiplicative functions, and then introduces the prime number theorem, perhaps a bit earlier than some other books. Next the author introduces Diophantine problems (starting, as one should, with Pythagorean triples) and the multiplicative structure of the integers modulo m, followed by quadratic reciprocity and consequences for quadratic equations. Beyond this, the book starts to enter "additional topics" territory, with chapters on factoring and cryptography, rational approximation and transcendence, binary quadratic forms, combinatorial number theory, p-adic numbers, and rational points on elliptic curves. In addition to having more topics, each of the topics is itself more fleshed out with logical consequences, such as continued fractions for the Euclidean algorithm, card-shuffling for orders of elements modulo m, and running times of algorithms as a key feature in cryptography. The 17 chapters either form a single introductory course, with plenty of choices for the last few topics, or a two-term course with material to spare.

The chapters represent less than half the book, though, and each is supplemented by a number of appendices, which more loosely meander through various offshoots of the main text. In some cases this means going deeper into a topic, or providing applications, and in others it indicates how some of the ideas are generalized in abstract algebra, or link up with questions in analysis or combinatorics. This style does lead to some repetition, which is perhaps unavoidable, but may frustrate the reader who plans to sit down and read everything in order; this aspect of the book is clearly for the reader who will jump around a bit, and choose what to explore. If this seems to be a little more than you need for your course, a pared-down version of the book *Number Theory Revealed: An Introduction* is more focussed, leaving out several chapters and most of the appendices. Readers who want to go even deeper, on the other hand, can look forward to *The Distribution Of Primes: Analytic Number Theory Revealed*, and *Rational Points On Curves: Arithmetic Geometry Revealed*, forthcoming books which I assume will straddle the advanced undergraduate and graduate curricula. Although the book under review clearly shows the influence of Gauss, those who want to see number theory in a more classical light can look forward to *Gauss's Disquisitiones Arithmeticae Revealed*, the fifth book in another increasingly inaccurately named trilogy.

One striking feature of this book (and perhaps one argument for a stream of new books on old subjects) is its inclusion of current research in its additional topics. Undergraduate textbooks in pure mathematics don't often cover modern research, and while number theory is billed as one of the few subjects wherein open problems are readily explicable to students seeing the field for the first time, in practice the examples given are often things like Goldbach's Conjecture (which, at over 275 years old, is perhaps no longer the cutting edge). Granville includes a plethora of more recent speculation, such as Zaremba's Conjecture, the Erdős-Strauss Conjecture on Egyptian fractions, and many more, as well as some more classical non-elementary conjectures (like the Riemann Hypothesis). On top of this, Granville offers the reader innumerable windows into modern number theory, including breakthroughs like Green and Tao's 2004 result on arithmetic progressions of primes, Zhang and Maynard and Tao's progress on small gaps between primes, and Helfgott's 2013 work on (yes) the Golbach Conjecture. An impressive feat for a book published in 2019, the text even mentions progress on Waring's problem from September of the same year. This book will surely address the precocious student's desire to know what number theory is about now.

So, will I assign this book next time I teach an introductory number theory course? Not exactly. Despite how much I like the book, I will probably choose the abridged version, which offers ample material for our one-semester introduction, and is better aimed at the typical student, who might not be inclined to go so far beyond the syllabus. I will, though, advise students who are really interested in going deeper that they should buy the full version instead. Students working on independent study, or undergraduate theses, will benefit as well from the longer form, with more rabbit holes to get happily lost in. All in all, this is a wonderful book for engaging enthusiastic entrants into the field of number theory, and I look forward to further instalments in the series.