Book Reviews bring interesting mathematical sciences and education publications drawn from across the entire spectrum of mathematics to the attention of the CMS readership. Comments, suggestions, and submissions are welcome.

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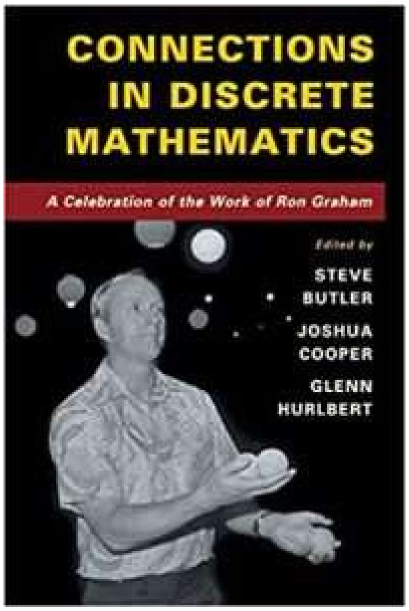


Connections in Discrete Mathematics

A Celebration of the Work of Ron Graham

Edited by S. Butler, J. Cooper, and G. Hurlbert
Cambridge University Press, 2018.
ISBN: 978-1316607886

Reviewed by Karl Dilcher

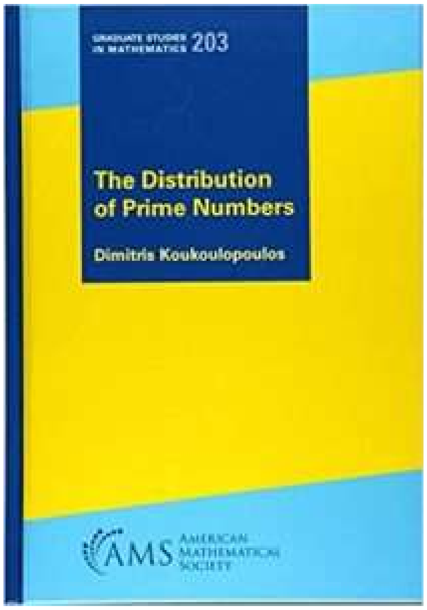
In 2020 the mathematical community lost three of the most influential and best-known researchers in Discrete Mathematics, namely Richard Guy in March, John Conway in April, and Ron Graham in July of that year. All three were well-known beyond the confines of their research areas, and even beyond mathematics.

The volume under review came out of a conference, called *Connections in Discrete Mathematics*, which was held in 2015 at Simon Fraser University, in honour of Ron Graham and at the occasion of his 80th birthday. To quote from the Preface: "This was a chance to bring together many of his friends and colleagues, the best and brightest in discrete mathematics, to celebrate Ron, and also to celebrate discrete mathematics. A major theme of the conference was connections, both the personal connections (as Ron had with so many speakers and participants) was well as the connections between mathematical topics. Both types of connections are what lead to advances in mathematics and open up new ideas for exploration.

"This book came out of the conference, with many of the authors having been featured speakers. The chapters here are across the spectrum of discrete mathematics, with topics in number theory, probability, graph theory, Ramsey theory, discrete geometry, algebraic combinatorics, and, of course, juggling. A beautiful mix of topics and also of writing styles, this book has something for everyone."

I fully agree. The book is also handsomely produced, which is important – at least for me. I already tagged a few of the articles for reading in detail.

The 20 individual articles and their authors are as follows: *Probabilizing Fibonacci numbers*, by Persi Diaconis; *On the number of ON cells in cellular automata*, by N. J. A. Sloane; *Search for ultraflat polynomials with plus and minus one coefficients*, by Andrew Odlyzko; *Generalized Goncharov polynomials*, by Rudolph Lorenz, Salvatore Tringall and Catherine H. Yan; *The digraph drop polynomial*, by Fan Chung and Ron Graham; *Unramified graph covers of finite degree*, by Hau-Wen Huang and Wen-Ching Winnie Li; *The first function and its iterates*, by Carl Pomerance; *Erdős, Klarner, and the 3x+1 problem*, by Jeffrey C. Lagarias; *A short proof for an extension of the Erdős–Ko–Rado theorem*, by Peter Frankl and Andrey Kupavskii; *The Haight–Ruzsa method for sets with more differences than multiple sums*, by Melvyn B. Nathanson; *Dimension and cut vertices: An Application of Ramsey Theory*, by William T. Trotter, Bartosz Walczak and Ruidong Wang; *Recent results on partition regularity of infinite matrices*, by Neil Hindman; *Some remarks on pi*, by Christian Reiher, Vojtěch Rödl and Mathias Schacht; *Ramsey classes with closure operations*, by Jan Hubička and Jaroslav Nešetřil; *Borsuk and Ramsey type questions in Euclidean space*, by Peter Frankl, János Pach, Christian Reiher and Vojtěch Rödl; *Pick's theorem and sums of lattice points*, by Karl Levy and Melvyn B. Nathanson; *Apollonian ring packings*, by Adrian Bolt, Steve Butler and Espen Hovland; *Juggling and card shuffling meet mathematical fonts*, by Erik D. Demaine and Martin L. Demaine; *Randomly juggling backwards*, by Allen Knutson; *Explicit error bounds for lattice Edgeworth expansions*, by J. P. Buhler, A. C. Gamst, Ron Graham and Alfred W. Hales.



The Distribution of Prime Numbers
by Dimitris Koukoulopoulos
Graduate Studies in Mathematics 203, AMS, 2019
ISBN 978-1-4704-6285-7

Reviewed by Karl Dilcher

There is no shortage of books on analytic number theory, from advanced undergraduate textbooks to research monographs, and many of these books are excellent. At least one of them has the same title as the one under review, namely Ingham's classical treatise of 1932, reprinted in 1990.

Of course, there have been substantial advances since Ingham's book and most of the others were published, and it is one of the great strengths of Dimitris Koukoulopoulos's excellent book to make some of the recent spectacular results accessible in textbook form. The author is Professor of Mathematics at the Université de Montréal, and is therefore a member of the extremely strong and active Number Theory Group in Montréal.

Since this is a brief review, I will borrow freely from front and back matter of this book, beginning with the publisher's description: "Prime numbers have fascinated mathematicians since the time of Euclid. This book presents some of our best tools to capture the properties of these fundamental objects, beginning with the most basic notions of asymptotic estimates and arriving at the forefront of mathematical research. Detailed proofs of the recent spectacular advances on small and large gaps between primes are made accessible for the first time in textbook form. Some other highlights include an introduction to probabilistic methods, a detailed study of sieves, and elements of the theory of pretentious multiplicative functions leading to a proof of Linnik's theorem.

> *Throughout, the emphasis has been placed on explaining the main ideas rather than the most general results available. As a result, several methods are presented in terms of concrete examples that simplify technical details, and theorems are stated in a form that facilitates the understanding of their proof at the cost of sacrificing some generality. Each chapter concludes with numerous exercises of various levels of difficulty aimed to exemplify the material, as well as to expose the readers to more advanced topics and point them to further reading sources.*

As far as the goal and possible uses of this book are concerned, the Preface states,

> *The main goal of this book is to introduce beginning graduate students to analytic number theory. In addition, large parts of it are suitable for advanced undergraduate students with a good grasp of analytic techniques.*
>
> *The book borrows the structure of Davenport's masterpiece Multiplicative Number Theory with several short- to medium-length chapters. Each chapter is accompanied by various exercises. Some of them aim to exemplify the concepts discussed, while others are used to guide the students to self-discover certain more advanced topics.*
>
> *The contents of the book are naturally divided into six parts […]. The first two parts study elementary and classical complex-analytic methods. They could thus serve as the manual for an introductory graduate course to analytic number theory. The last three parts of the book are devoted to the theory of sieves: Part 4 presents the basic elements of the theory of the small sieve, whereas Part 5 explores the method of bilinear sums and develops the large sieve. These techniques are then combined in Part 6 to study the spacing distribution of prime numbers and prove some of the recent spectacular results about small and large gaps between primes. Finally, Part 3 studies multiplicative functions and the anatomy of integers, and serves as a bridge between the complex-analytic and the more elementary theory of sieves. Topics from it could be presented either in the end of an introductory course to analytic number theory, or in the beginning of a more advanced course on sieves.*

The author then mentions that certain portions of the book can be used as a reference for an undergraduate course. The Preface also contains a list of the main results proven in the book, along with their prerequisites, from Chebyshev's and Mertens' estimates in the first few chapters, through the Prime Number Theorem, also quite early on, to the breakthrough of Zhang-Maynard-Tao about the existence of infinitely many bounded gaps between primes. The book then ends with recent work by Ford-Green-Konyagin-Tao and Maynard about large gaps between primes, and work by Maier from the 1980s about irregularities in the distribution of primes.

Let me finish by quoting from the end of an MAA review by Michael Berg (Loyola Marymount University):

> *It's clear that Koukoulopoulos had a marvelous time putting together this beautiful material, and producing a very readable and pedagogically sound text (replete with good exercises). The book is well-paced and reads very well. The careful reader, with pencil and paper in hand, keen to do exercises galore and have fun doing so, will learn a lot of beautiful number theory and find out marvels about the secret life of the set of primes: they are elusive but not unyielding.*