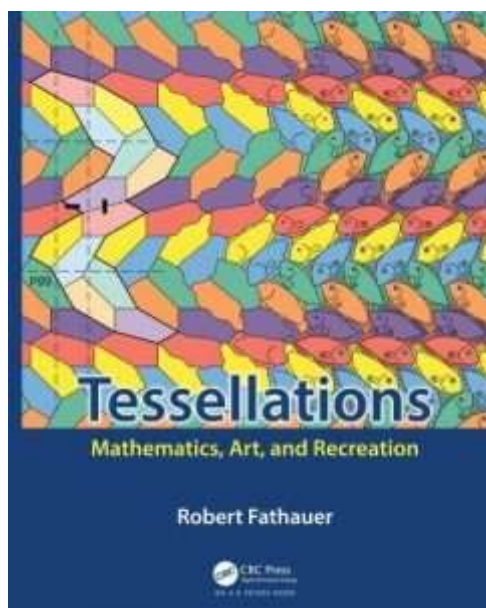


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Book Reviews bring interesting mathematical sciences and education publications drawn from across the entire spectrum of mathematics to the attention of the CMS readership. Comments, suggestions, and submissions are welcome.

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Tessellations: Mathematics, Art, and Recreation

by Robert Fathauer

A K Peters/CRC Press, 2020

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Robert Fathauer's *Tessellations: Mathematics, Art, and Recreation* is a gorgeous book. It's lavishly illustrated with photographs of tessellations and related patterns from nature and architecture; with reproductions of artwork by M. C. Escher and other artists who have found inspiration in tessellations; with the tilings of geometers such as Sir Roger Penrose, Robert Ammann, and Casey Mann; and, most of all, with the author's own creations.

The reader will need an interest in mathematics, but no great background; the level of rigor is only a little higher than what aficionados of the late Martin Gardner will remember. To take advantage of this, the book is liberally sprinkled with activities aimed at the K-12 classroom, including handouts, lists of vocabulary words, and (where relevant) CCSSM standards. These exercises are interesting and have solid mathematical content, but their level of difficulty varies widely. An appendix listing the activities by approximate grade level (or cross-tabulated with topic) might make it easier for a teacher dipping into the book to find an activity. In some cases (for instance, worksheet 8.1 on page 148) opportunities are missed for older students to use algebra rather than the too-ubiquitous hand calculator. The range of topics is wide, and each one is explored fairly deeply, with its relevant history. The second chapter, for instance, begins with regular

tessellations of the plane. It goes on to the semiregular tessellations, pausing to consider the possible ways in which regular polygons can be arranged around a point. After that, it considers tilings with a single shape of polygonal tile, including the Laves tessellations, dual to the regular and semiregular tessellations. The fifteen classes of tilings with convex pentagons are shown (without proof, but with information about the discoverers.) We then consider the tessellations involving star polygons (the reader must be careful here: the definition on page 31 is more restrictive than what is implied by the tilings on pages 33-35.) Then there's a short look at non-edge-to-edge tessellations with squares and triangles, including "squared squares." Four pages on creating new tessellations from old, and a look at Apollonian packings, bring us to the end of the chapter. You will have to read the other twenty-four chapters yourself!

Chapter 8, on rosettes and spirals has a good introduction to Fibonacci spirals and phyllotaxis. I would argue with the author's decision (p. 122) to use "golden number" rather than "golden ratio" on the grounds that

"The term 'golden ratio' implies a ratio of two objects, which is not how it will generally be used in this book."

While I agree with him that

"[t]he number has a long history of being applied to objects that it doesn't really fit very well, such as the nautilus shell and the Acropolis,"

the dimensionless number ϕ in a tiling is indeed a ratio of lengths (or of tile populations in the Penrose tiling) and this is not changed by other writers' over-eagerness to spot it in architecture and nature.

Chapter 9, on fractal tiles and fractal tilings, contains material of considerable beauty that will be new to most mathematicians. Many of these tilings have a deep connection to the hyperbolic plane that clearly cannot be explored at the level of this book! The author attempts on page 152 to draw a distinction between fractal and hyperbolic tilings, with Escher's "Square Limit" woodcut on one side of the line and his "Circle Limit" works on the other. I confess that I do not see the basis for this distinction. But this is a minor detail, that in no way detracts from the overall presentation.

Finally, Fathauer is not just a mathematician but also an artist. He shares his artistic tips freely here, including, in Chapter 13-19, some really good instructions on how to create an Escher-style tiling based on various symmetries. If you are a high school math teacher and you bring a copy of this book in to work, you may need to hide it from the art teacher!