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David Borwein, who passed away on September 3, 2021 at the age of 97 was my PhD supervisor at Western University from 1970 to 1974. I have been asked to make a few remarks about my experience as his student. But there was rather more to our involvement than that. I was pleased to have David as an undergraduate (as well as graduate) lecturer, research collaborator and friend in addition to doctoral supervisor.

In 1968 I sat his fourth-year undergraduate course in Lebesgue Measure and Integration in  $\mathbb{R}^n$ . His lecture notes, which he seemed to reproduce on the blackboard verbatim, were complete and appeared hand-written. I thought at the time that he must be planning to write a textbook. But, as far as I know, he never followed up. Years later, when I taught the same subject matter, I realized that he had made clever use of ordinate sets rather than the more common simple function approach to measurability. This saved a lot of generalization when moving from  $\mathbb{R}^1$  to  $\mathbb{R}^n$ .

The following year I registered for David's graduate course called "Introduction to Summability Theory." At that time, summability theory was David's main research area. And he used classical, as opposed to functional analytic, techniques. This course was a scary experience for the four graduate students registered. For starters, the textbook was G. H. Hardy's classic book "Divergent Series," which is not the easiest read. Then, the students did all the lecturing and the audience consisted not only of David, but of all faculty, visiting faculty and post-docs in the summability research group at Western. Each student felt considerable angst before and during his or her turn at the blackboard.

Following that year I asked, with some trepidation, if David would take me on as a doctoral student. He agreed, suggesting I apply to various sources for funding. At our initial meeting he mentioned that there were some tauberian questions involving the  $A_\lambda$  scale of summability methods that I could look at if I didn't have a problem picked out. I didn't.  $A_\lambda$  methods are examples of what are called power series methods. Without going into the definitions,  $A_0$  is the ordinary Abel method discussed in some undergraduate analysis books. The Logarithmic method,  $L$ , is another power series method and it also played a role in my thesis. I have always admired David's 1957 paper "On methods of summability based on power series" *Proc. Royal Soc. Edinburgh*, **64**. David had been studying power series methods since the mid-1950s, developing their abelian properties. It was known, for example, for  $\lambda > -1$  and  $\epsilon > 0$  that  $A_{\lambda+\epsilon} \subseteq A_\lambda \subseteq L$  and that the inclusions are strict. That is, any series summable to  $s$  by one method in the list is summable to  $s$  by any method to the right. A tauberian condition is an extra requirement which forces inclusion in the reverse direction.

To get me started, he gave me a few papers to read and suggested I look at some other results in the literature. David, in my experience, was a laissez-faire type of supervisor. Rather than having regular meetings, he expected me to make an appointment (he was department head at this time) whenever I had something to discuss.

During my second year into the program he gave me my introduction to undergraduate teaching from the instructor's point of view. He had a meeting conflict during his calculus lab and asked me to take it. I asked what I would have to do. "Nothing" he said. "Just answer a few questions. You might want to review what they are working on now." In the first minute or so I realized that Peter Borwein was registered for the class. "Don't screw up" I thought. "Otherwise, this is getting back to dad!"

I did get some early results for the thesis. But things dragged in the middle. The proof technique I had used for the  $A_\lambda$  scale did not work between  $A_\lambda$  and  $L$ . Plus, David knew I was addicted to playing bridge. Rather than sit me down for a stern talking-to, he invited me over to his house for a rubber bridge session. He and his other son, Jon, were both good players. Anyway, I eventually found another approach to the crucial step and finished the degree.

David and I collaborated on some papers in the days before LATEX. Before anything went to the typist (IBM Selectric) he insisted on hand-written pages which he would go through line-by-line with me. This was, I suspected, because of his low regard for my writing skills. But I think I did learn to be precise and, when possible, to be concise with written mathematics. Subsequently, I tried to remember his teachings with my own writing. I remember my time as his student as a turning point in my career. He gave me many life lessons, not all mathematical.