Weaving Together Research and Teaching

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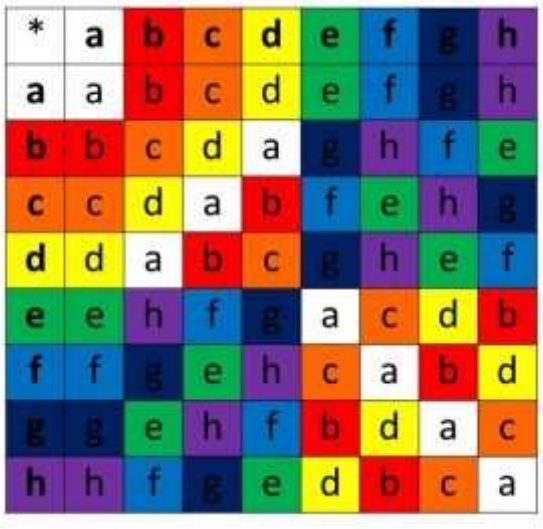
I believe that research and teaching are interconnected. I've been fortunate to be part of a research collaboration that has greatly enriched my teaching in diverse and surprising ways. I wanted to share this to encourage others to be open to collaborations outside of their main research interests.

In 2012, I was asked to join the research group *MathWeave*, by Dr. Eva Knoll (formerly of MSVU, now at UQAM). Eva's work ranges over many areas including math education and connections between math and art. She was working with a master weaver, Wendy Landry. They asked me to join in the role of "mathematician". I initially thought that I would be able to provide a mathematical lens to analyze their work. I had no idea how much I would learn from our research collaborations. Our group grew to include other members (an elementary school math and art teacher and a librarian with a math background). According to the group website (http://mathweave.teknollogy.com/), our work is focused on the question "Is there a 'Making Way' to show or learn mathematical concepts, skills and procedures by engaging with the struggle that is making art?" This includes the creation of a range of artefacts that involve mathematical thinking, research papers discussing the math and art connections we work with, development of curriculum materials for many different levels, workshops and presentations for different audiences (math and/or art conferences, math and/or art teachers), and outreach activities to various community groups like Girl Guides.

For example, consider the series of artefacts in Figure 1. Eva created these artefacts inspired by plaited mats produced in Southeast Asia. Mats can take advantage of two kinds of decorative effects. The first one involves patterns of in-woven holes in the surface. The second one involves patterns incorporating colours. Eva showed me this series and I was intrigued by the mathematics underlying the colour choices. There are 14 distinct strips in each artefact. Colour is one way to convey equivalence of strips, and we can use different definitions of equivalence. At one extreme, the strips all have the same colour. At the other extreme, the strips are all distinct. Figure 1(b) displays an artefact where the equivalence is based on which region of the artefact the strip travels. In Figure 1(c), the equivalence is based on which paths are isometric. I think the artefacts are beautiful by themselves but even more so as a series. Different equivalences can help to see different mathematical aspects.

Figure 1: Series of Open Squares

The use of colour is an important consideration for the creation of art. I have found it helpful to incorporate colour in different ways to help students understand mathematical concepts. We want our students to see relationships and patterns. I often teach abstract algebra, and the symmetries of the square is one of the main examples that we see throughout the course. There are 8 symmetries of the square (the identity, 3 rotations, 4 reflections). The corresponding binary table is in Figure 2. The colour helps the students make observations about the structure of the group.



*	a	b	c	d	e	f	g	h
a	a	b	c	d	e	f	g	h
b	b	c	d	a	g	h	f	e
c	c	d	a	b	f	e	h	g
d	d	a	b	c	g	h	e	f
e	e	h	f	g	a	c	d	b
f	f	g	e	h	c	a	b	d
g	g	e	h	f	b	d	a	c
h	h	f	g	e	d	b	c	a

Figure 2: Binary table for the Symmetries of the Square

The symmetries of the square is a subgroup of the group of permutations on 4 elements. We presented a workshop on another subgroup of the permutations on 4 elements at a math and art conference. This workshop included different ways to experience the subgroup- through music, poetry, visual art and culinary art. The corresponding paper is available at https://archive.bridgesmathart.org/2018/bridges2018-659.html.

The actual act of making art has deepened my own mathematical understanding of different concepts, so I try to bring more hands-on activities into my classes. I introduce topology in a first year math concepts class. In the first few years I taught the course, I would tell the students the joke about how a topologist can't tell the difference between a donut and a coffee cup. Now I have an assignment that has a component that requires the students to make a donut out of play-doh and transform it to the coffee cup. The students take photos of the transformation and include the photos in their assignment. They generally love this assignment. I have heard students say that they don't usually get to do anything like that or that they haven't played with play-doh since they were kids. The photos are amusing too, which helps make the marking more fun.

I have learned a great deal about pedagogy from being in the group. One thing that was new to me is the idea of embodied cognition (using the whole body in the learning process). One example I have used is in the proof that the exterior angles of a convex polygon always add to 360. I expect the students to be able to prove it algebraically, but to also feel what it really means. They can use painters' tape to make big polygons on the floor and then walk around them. No matter how many sides there are, their body will do one complete rotation as they move around.

There are other ways that my teaching has benefitted from my involvement with *MathWeave*. I am a complete novice when it comes to weaving. I can get frustrated by how slow I am and how much of a struggle it can be. I had an epiphany about this though- it helps me to imagine what many of my students are going through when they are struggling with new concepts. In my first year math concepts class, they have a final project where they have to research a way to connect math to something they are interested in. Originally it had to be a paper. Then it could be a paper or poster. Over time, I've gotten more flexible in how the students present their projects (and the pandemic also forced me to be more creative). Some will create music or visual art. I've also noticed that the more artistic activities often show different strengths of students compared to what we typically assess. This helps the students see themselves and each other differently. It has also helped me to see many of them in a new light. Every year I am blown away by what they come up with. The *MathWeave* group continues to be a respectful community of learners, and that is one my goals when I am teaching- that the classroom is a community of learners where we can all contribute and we can all learn.