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Euclid and the Eclipse

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On the morning of November 8th a total eclipse of the moon was visible across Canada. In Nova Scotia it happened a little before sunrise, a convenient time for viewing. So I woke up early and looked out the window: the moon's disc was already half darkened, with a faint coppery glow just visible on the eclipsed half. I woke my wife, and we watched though binoculars as the rest of the moon slid into shadow. While the half-disc that we saw at first did not look very different from an ordinary waxing half-moon (except that it was a week late), as the eclipse progressed the shape was not the familiar crescent but almost a perfect segment, one edge straight and one curved.

At totality, the moon looked startlingly three-dimensional – the soft sunset light that still lit it showed its shape in a way that stark sunlight never does. And then the sun rose, the sky lightened, and the ochre ball faded into the dawn sky like a Cheshire cat.

A beautiful sight: and it prompted me to think about the geometry involved. It's clear that a full moon is a necessary condition for a lunar eclipe (and a new moon for a solar eclipse): but why isn't it sufficient? The answer, of course, is that the moon's orbit around the earth is not in the same plane as the earth's orbit around the sun. The three bodies can line up only when the moon crosses the ecliptic plane (that's why it's called that!) So eclipses ought to happen twice a year.

And, roughly, that's what happens. But the plane of the moon's orbit precesses over about 18.6 years, so the interval between eclipses is just a bit less than six months. And sometimes the moon isn't full quite as it crosses the ecliptic: a near miss gives a partial eclipse, a bigger miss an almost-undetectable "penumbral eclipse." Solar eclipses, which require the three bodies to line up to a much higher precision, follow similar cycles but with fewer "hits." It gets complicated!

These interacting cycles rapidly lead us into number theory. And they've had that effect on people for a long time: witness the vocabulary associated with eclipse chronology. "Saros cycle," "exeligmos," "draconic month": these are names, if not from Harry Potter, certainly from antiquity. After our ancestors had got used to predicting the return of the various seasons, predicting eclipses was the obvious next project.

And I presume that that's why Euclid's _Elements_, though primarily a work on geometry, detours for four of its thirteen books into number theory – rather than, say, calculus or the theory of quadratic equations. Number theory answered an important question–"when will the next eclipse be?"–of interest to all. So it's possible that eclipses go some way to explaining why one of the best-known results in the _Elements_ — the only one to which Euclid's name is widely attached — is an algorithm in number theory.