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Book *Reviews bring interesting mathematical sciences and education publications drawn from across the entire spectrum of mathematics to the attention of the CMS readership. Comments, suggestions, and submissions are welcome.*

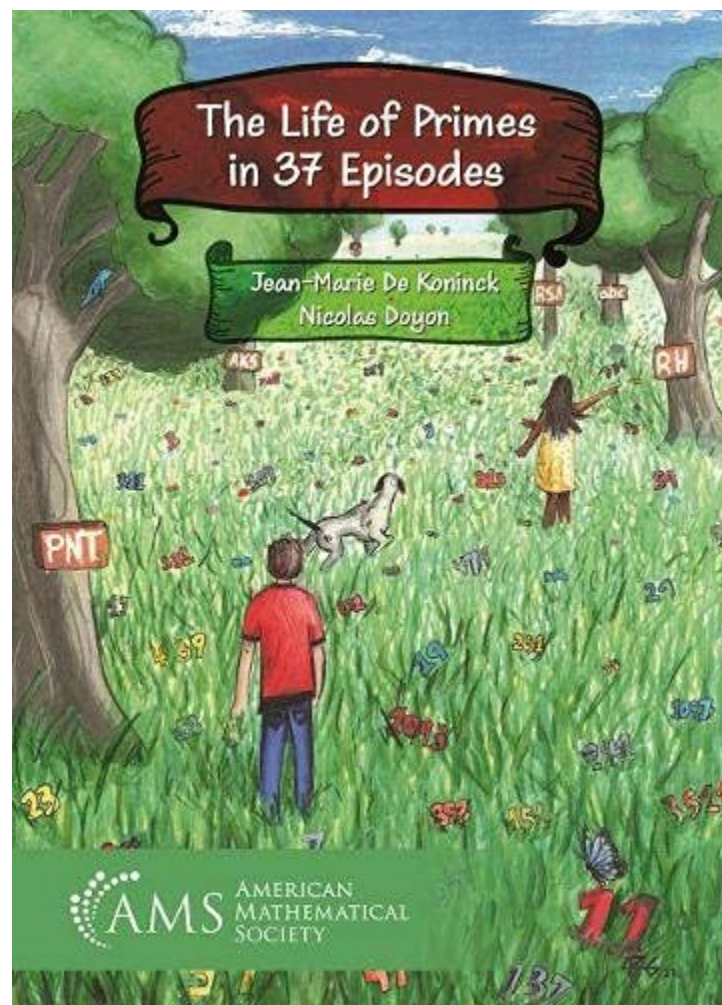**Karl Dilcher,** *Dalhousie University* (*notes-reviews@cms.math.ca*)

The Life of Primes in 37 Episodes
by Jean-Marie De Koninck and Nicolas Doyon
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Reviewed by Karl Dilcher

Numerous books on prime numbers have been published over the years, as a quick search of MathSciNet will show. These books range from expositions for the general reader (for instance, *The Music of the Primes* by Marcus du Sautoy) through higher-level expositions (e.g., Paulo Ribenboim's *The Little Book of Bigger Primes*) to more specialized graduate-level textbooks (e.g., *The Distribution of Prime Numbers* by Dimitris Koukoulopoulos) and numerous research monographs on special topics related to primes. There is even a graphic detective novel, *Prime Suspects*, by

Andrew and Jennifer Granville, published in 2019.The reader will have noticed that three of the names mentioned above are prominent Canadian number theorists. The book under review, *The Life of Primes in 37 Episodes*, was also written by well-known Canadian number theorists, Jean-Marie De Koninck and Nicolas Doyon of Laval University.

In their Preface, the authors recall a key idea from the movie *Contact* (based on Carl Sagan's novel), namely the universality of prime numbers and mathematics as a universal language. To explain the book's title, the authors then state the thought that once primes are defined, they begin a life of their own: "The mysteries surrounding primes begin multiplying just like living cells reproduce themselves, and there seems to be no end to it." This will explain the title of the book, and it may or may not be a coincidence that 37 is a prime.

The purpose of this book is best explained by further quoting from the Preface: "Many number theory books include the study of prime numbers. Most of them were written for teaching purposes, and others for the pleasure of the general public. [...] So why another book on primes? Our monograph offers a somewhat different perspective. Besides covering some of the most important results regarding prime numbers, we present a range of problems number theorists are currently working on and the references that will allow the curious reader to further investigate some of these problems. Moreover, we selected topics related to primes that will appeal to those mathematicians who wish to enrich their general mathematical culture. We have also chosen to present the topics in a chronological order, as they have emerged throughout history.

"Although we do not claim to provide a thorough history of number theory, we do shed light on the humans who contributed to the life of primes. Indeed, history does help understand how mathematical results evolved over time. [...] Indeed, theorems do not pop up suddenly. They are for the most part the final outcome of many attempts by various mathematicians. This is why in this book we also write about the people behind the results, mentioning their successes and sometimes their failures."

The character and contents of this book are also well described by the text on the back cover, which partly reads, "This monograph takes the reader on a journey through time, providing an accessible overview of the numerous prime number theory problems that mathematicians have been working on since Euclid. Topics are presented in chronological order as episodes. These include results on the distribution of primes, from the most elementary to the proof of the prime number theorem. The book also covers various primality tests and factorisation algorithms."

The book contains a brief (6-page) timeline, covering advances in the theory of prime numbers from the Sieve of Eratosthenes of around 300 BCE to results from 2018. Throughout the book there are also short biographies of the key players in the history of number theory, often accompanied by interesting anecdotes.

The book is organized along five general themes, which are further divided into a total of 37 "episodes". These themes are as follows:

1. Counting primes, the road to the prime number theorem.
2. Counting primes, beyond the prime number theorem.
3. Is it a prime?
4. Finding the prime factors of a given integer.
5. Making good use of the primes and moving forward.

I will just mention a few of the episodes as examples. Theme 1 begins with Euclid's proof of the infinitude of primes, as one would expect (*An Infinite Family*, Episode 1), and ends with an outline of an elementary proof of the prime number theorem (Episode 13). Theme 2 begins with *Sieve Methods* (Episode 14), and also contains *Small and Large Gaps Between Consecutive Primes* (Episode 17), which has gained particular prominence in recent years. As their titles indicate, Themes 3 and 4 contain the most important primality tests and factorisation methods in six, resp. seven episodes. The two episodes comprising Theme 5 are *Cryptography, From Julius Caesar to the RSA Cryptosystem* (Episode 36), and *The Present and Future Life of Primes* (Episode 37).

Each episode ends with a number of interesting problems, some of which are quite challenging. An appendix of over 50 pages contains hints, sketches, and solutions to selected problems. A final appendix lists some basic results from number theory, algebra and analysis; they may be helpful to the reader as quick reference.

This is a beautiful book, very well written and edited. It should appeal to number theorists as well as interested mathematicians in other fields. It is also a rich source of supplementary readings for any undergraduate or graduate course in number theory.