Why Everyone Loves History of Mathematics . . . But Philosophy of Mathematics is an Acquired Taste

Notes de la SCHPM

Novembre 2022 (tome 54, no. 5)

Thomas Drucker (University of Wisconsin-Whitewater)

CSHPM Notes bring scholarly work on the history and philosophy of mathematics to the broader mathematics community. Authors are members of the Canadian Society for History and Philosophy of Mathematics (CSHPM). Comments and suggestions are welcome; they may be directed to either of the column's co-editors:

Amy Ackerberg-Hastings, *Independent Scholar (aackerbe@verizon.net)*
Hardy Grant, *York University [retired] (hardygrant@yahoo.com)*

Statistics suggest that history of mathematics is perhaps three times more popular among mathematicians than philosophy of mathematics (based on membership in various groups). Those who wear both hats are likely to find larger audiences at their historical talks than their philosophical ones. Unless one is inclined to argue that historians are more genial by nature than philosophers, there is likely to be something about the disciplines and how they are practiced that leads to the disparity. Recent work in both fields suggests that the gap is not likely to be narrowed, although some attempts are being made.

History (as it is taught) is often a matter of stories about what happened, the kind of things that the late Ivor Grattan-Guinness described as 'heritage' [5]. Such stories have an appeal that goes beyond the question of whether they are, in fact, history. Luring students into an area can start with a judicious choice of story, even if such tales have to be told with mental reservations.

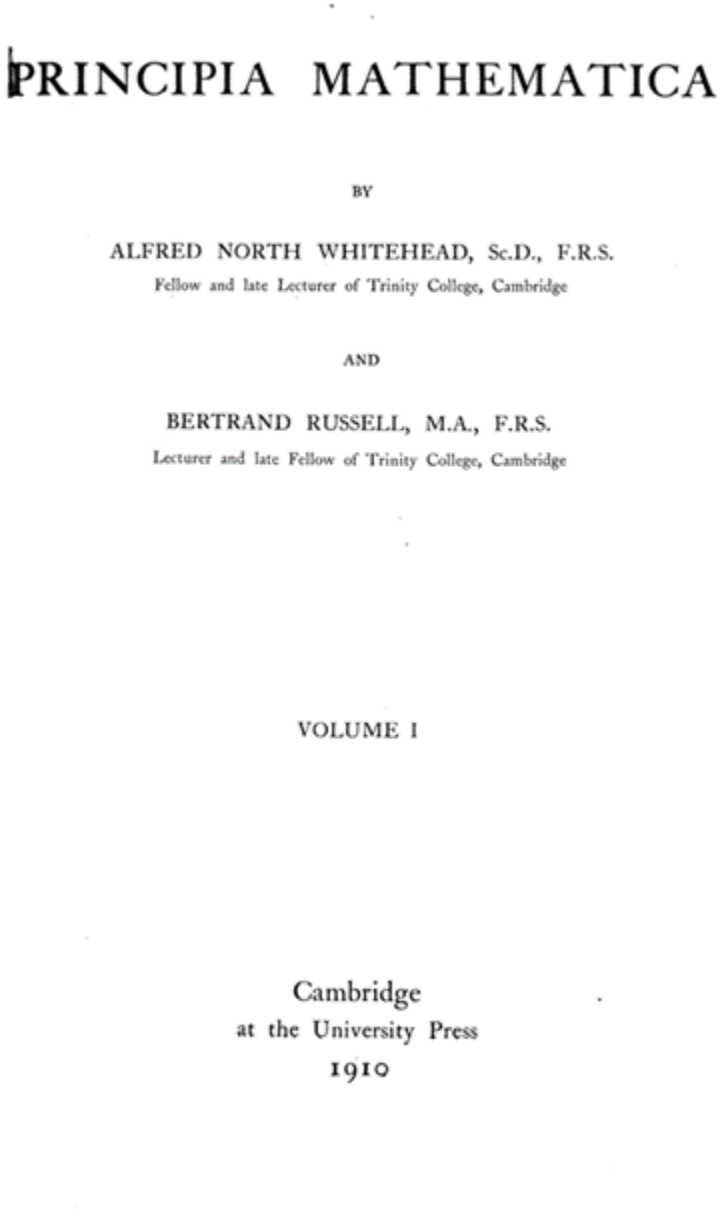
Once the audience is there, one can proceed to disabuse them of some of those stories. Getting to 'what really happened' is harder work than passing along stories. A good example is Tony Rothman's essay 'Genius and Biographers' [9]. He takes down the image of Galois as built up by Eric Temple Bell [1] and Leopold Infeld [6] and tries to leave the historical Galois in its place. I have yet to encounter a student who preferred Rothman's account to Bell's, but historians admit that *ben trovato* stories might be just the lure to get students interested enough to work harder.

Figure 1. Evariste Galois (1811–1832). Wikimedia Commons.

Philosophy, on the other hand, addresses certain kinds of questions, and it is simply more difficult to get students to acknowledge an interest in some of those areas. One can trot out terms like 'metaphysics' or 'epistemology' without seeing light in the eyes of listeners. When students sign up for philosophy courses, they are often looking forward to the kind of pop philosophy that

populates so many discussions of medical ethics. Even writers as felicitous as Bertrand Russell can be a hard slog compared to what is expected.

Further, convincing arguments in philosophy can be complicated. Gödel's case for the incompleteness of *Principia Mathematica* and 'related systems' has been rephrased many times, but it is not bedtime reading. Then again neither are many mathematical arguments, and one might think that mathematicians, in particular, would find the philosophical use of detailed arguments quite familiar.



PRINCIPIA MATHEMATICA

BY

ALFRED NORTH WHITEHEAD, Sc.D., F.R.S.
Fellow and late Lecturer of Trinity College, Cambridge

AND

BERTRAND RUSSELL, M.A., F.R.S.
Lecturer and late Fellow of Trinity College, Cambridge

VOLUME I

Cambridge
at the University Press
1910

Figure 2. First edition of Whitehead and Russell's *Principia Mathematica*.
Bertrand Russell Archives.

What explains the lack of appeal to many mathematicians of the kind of argument that shows up in the setting of philosophy of mathematics isn't only the reader's looking for something lighter. It can be argued that the reason for wading through long arguments such as the classification of finite simple groups is that one ends with a conclusion that is indisputable. The same could apply to, say, Andrew Wiles's proof of Fermat's Last Theorem. In mathematical reasoning, one reaches the top of a mountain.

By contrast, after a heavy dose of closely reasoned philosophy, the conclusion at which one arrives is likely to be the object of a symposium in which various reasons for doubting it are advanced. After the letter Russell wrote to Frege in 1902 pointing out the paradox named for him, one might have thought (with Frege) that his system was in ruins. The prevalence of neo-Fregean attempts to resuscitate the Fregean system (see, for example, John Burgess's *Fixing Frege* [2], as well as many papers by Crispin Wright and Bob Hale) shows that Frege may have been buried prematurely. In view of the shortage of knock-down arguments in philosophy, the value of investing many hours of effort in any alternative system can be questioned. Those who spent their time working through *Principia Mathematica* may have decided against similar investments in other systems, e.g., W.V.O. Quine's New Foundations.

Another consideration that may go some distance to explain the relative popularity of history over philosophy within mathematics is the areas of mathematics on which those two disciplines are characteristically brought to bear. There are histories of mathematics at large. There are histories of specialties within mathematics (like algebra or analysis). There are histories of sub-specialties

within the specialties (like combinatorial group theory or Fourier analysis). Mathematicians working in any area can look back at the history of that area to see what has led to the current state of knowledge.

Philosophy of mathematics does not cast its net so widely. For centuries, arithmetic and geometry may have been the sole subjects of philosophical interest. With the rise of calculus, the issue of how to provide a suitable foundation became a matter of philosophical interest, and generally questions about foundations became central to investigations into philosophy. More recently, there has been a parting of the ways among philosophers between further work in foundations and more general philosophical interest in mathematics. Those working in philosophy departments are ready to turn back to arithmetic in a search for answers to questions about metaphysics and epistemology. Those working in mathematics departments are more likely to be pursuing issues such as whether set theory is a satisfactory foundation for mathematics at large.

Tastes in mathematics do make a difference in the kind of question at the center of such foundational discussions. The ascendancy of category theory, including its increasing presence in undergraduate and graduate curricula, certainly contributed to the appeal of its use as a foundation for mathematics, in competition with the traditional role of set theory. Readers of that kind of discussion should have picked up some of the language and techniques of both areas in order to make a comparison. However, if the question is which approach makes for the more satisfactory foundation, it is not clear that an answer even exists. In the meantime, perhaps there is some chance of making category theory more appealing to nonphilosophers, as in the recent work of Eugenia Cheng [3], but it is too early to predict the extent to which her readership will increase the audience for philosophy talks. Similarly, as long as the philosophical view known as 'structuralism' remains of interest, category theory will continue to be relevant. Leo Corry's *Modern Algebra and the Rise of Mathematical Structures* [4] gives a historical background for the philosophical arguments.

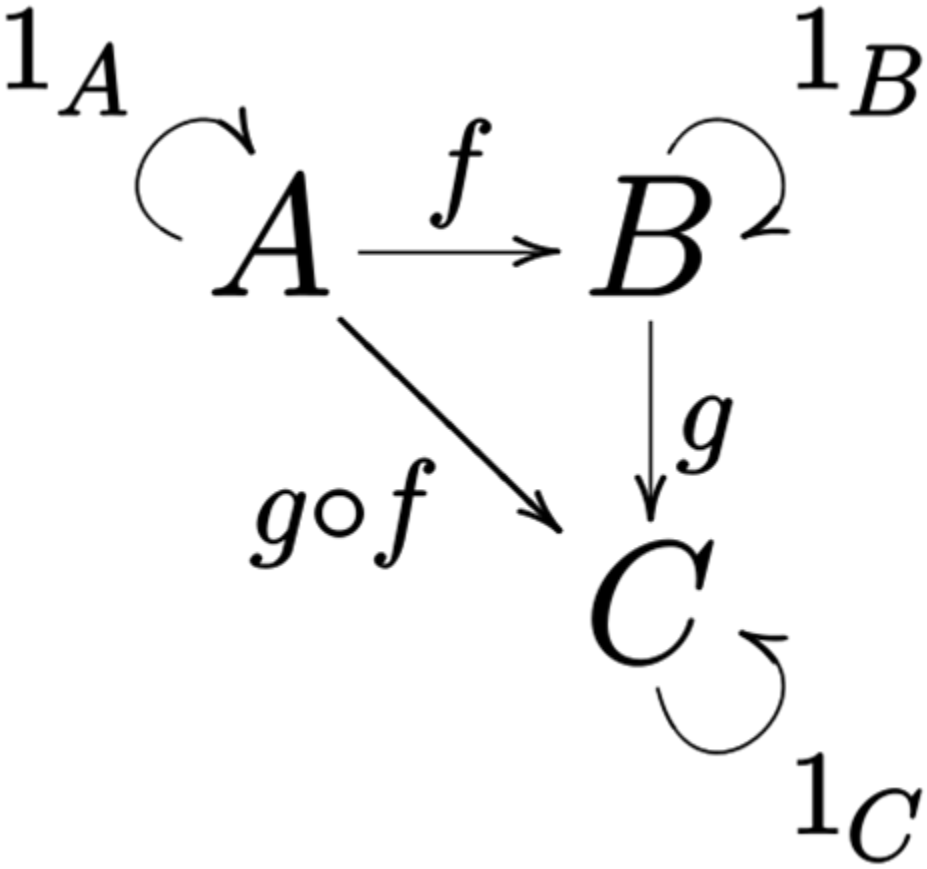


Figure 3. Diagram of a category by IkamusumeFan, CC BY-SA 4.0, Wikimedia Commons.

After all, there have been similar situations where there is not an obvious way to make a choice between two different approaches to an area. In the nineteenth century, the prolonged discussion of the merits of non-Euclidean geometry tended to involve the argument that Euclidean geometry was better for the mind. In the twentieth century, the conflict between constructability and Cohen's indiscernibles with respect to resolving the Continuum Hypothesis has not had a resolution. This outcome seems more typical of the resolution (or absence thereof) of philosophical arguments than of mathematical ones.

If we turn to examining biography, history again comes off as more appealing. It is possible to write dully about the life of a mathematician, but there are plenty of examples of the contrary, e.g., Siobhan Roberts's *Genius at Play* [8], which does not attempt to do justice to all the aspects of J.H. Conway's mathematics but which successfully lures readers into Conway's mind with examples of his approach to a variety of topics. The lives of mathematicians are parts of their subject's history.

In contrast, the lives of philosophers of mathematics are not themselves monuments of philosophy. Ever since Diogenes Laertius's *Lives and Opinions of Eminent Philosophers* was criticized as not even good history, let alone sound philosophy, there has been reluctance to turn to the life of a philosopher for revelation of the nature and value of his or her thought. A look at philosophical autobiography (e.g., that of W.V.O. Quine [7]) suggests that even a philosopher may not be able to tell a life story with philosophical content.

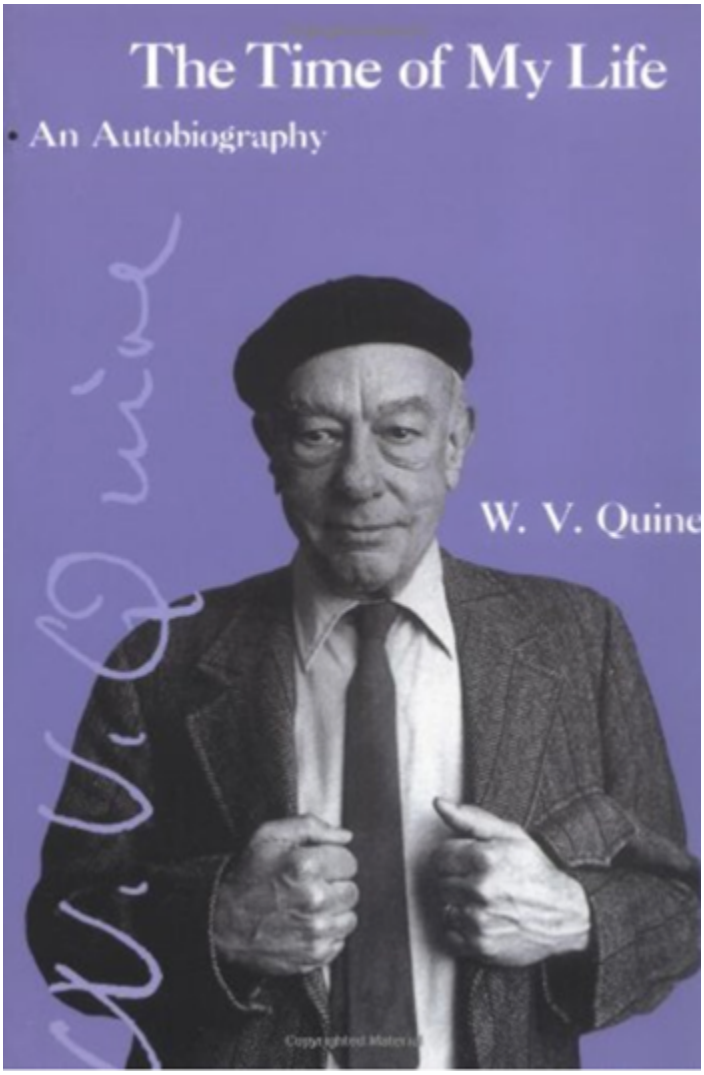


Figure 4. Quine's autobiography. MIT Press.

The deck seems stacked against the philosophers. Historians can tell good stories to interest audiences in what actually happened. Those historians can point to monuments of enduring value in the mathematics of the past. Historians can tell the lives of mathematicians like Gödel who displayed oddities in many situations. Philosophers do not have anything similar to offer.

Yet, for those attracted to the inherently challenging nature of the subject, philosophy offers intellectual rewards and professional opportunities. Particularly good news is that philosophers of mathematics in philosophy departments outnumber historians of mathematics in history departments. As a result, mathematicians with philosophical interests can usually readily secure places on the programs of philosophy meetings, even if the audiences turning out for such sessions during mathematics conferences remain relatively small in comparison to those in the rooms offering history presentations.

Thomas Drucker studied history of mathematics at Princeton under Michael S. Mahoney and at Toronto under Kenneth O. May. He has been Chair of the Philosophy of Mathematics Special Interest Group of the Mathematical Association of America. He retired from teaching at the University of Wisconsin–Whitewater in 2021.

References

[1] Bell, Eric Temple. (1937) *Men of Mathematics*. New York: Simon & Schuster.

[2] Burgess, John. (2005) *Fixing Frege*. Princeton: Princeton University Press.

[3] Cheng, Eugenia. (2022) *The Joy of Abstraction: An Exploration of Math, Category Theory, and Life*. Cambridge: Cambridge University Press.

[4] Corry, Leo. (2004) *Modern Algebra and the Rise of Mathematical Structures*. 2nd ed. Basel: Birkhäuser.

[5] Grattan-Guinness, Ivor. (2004) History or Heritage? An Important Distinction in Mathematics and for Mathematics Education. *The American Mathematical Monthly* 111(1), 1–12.

[6] Infeld, Leopold. (1948) *Whom the Gods Love: The Story of Evariste Galois*. New York and London: Whittlesey House.

[7] Quine, Willard Van Orman. (1985) *The Time of My Life: An Autobiography*. Cambridge: MIT Press.

[8] Roberts, Siobhan. (2015) *Genius at Play: The Curious Mind of John Conway*. New York: Bloomsbury.

[9] Rothman, Tony. (1989) Genius and Biographers: The Fictionalization of Evariste Galois. In *Science à la Mode: Physical Fashions and Fictions*, 148–194. Princeton: Princeton University Press.

Copyright 2020 © Société mathématique du Canada.