## Bienvenue au numéro de décembre des Notes de la SMC

Table des matières Décembre 2022 : tome 54, no. 6

Article de couverture

2022 : La fin d'une époque — Termeh Kousha

Éditorial

Des recherches non recherchées — Robert Dawson

Notes pédagogiques

Recurring decimals, proof, and ice floes - Brian McMaster

Notes de la SCHPM

Quaternions at Twilight: Remembering Mary Somerville 150 years after her death — Brigitte Stenhouse

Comptes rendus

The Best Writing on Mathematics, 2021 — Karl Dilcher

MOSAIC

Appel à soumissions : MOSAIC (Mathématiques ouvertes sur la société, accessibles et inclusives en chroniques) des Notes de la SMC

Réunions de la SMC

Réunion d'été 2023 de la SMC

Appel de Sessions scientifiques : Réunion d'été de la SMC 2023

Appel à propositions de Sessions en matière d'éducation : Réunion d'été 2023

Annonces

Rejoignez votre propre société!

Appel de candidatures

Appel de candidatures : Prix de doctorat SMC Blair Spearman 2023

## 2022 : La fin d'une époque



Article de couverture Décembre 2022 (tome 54, no

**Termeh Kousha** (Canadian Mathematical Society) *Executive Director* 

Alors que je suis assise ici et que je réfléchis à l'année écoulée, je suis reconnaissante pour mon bureau à la Maison canadienne des mathématiques d'où j'écris cet article. Malgré les nombreux hauts et bas auxquels la SMC a été confrontée au cours de cette année (ou de plusieurs années), je suis heureuse des progrès que nous avons réalisés et que nous continuons à réaliser dans le cadre de nos activités. Cette année a été passionnante, avec la reprise de nombreuses activités en personne, qui ont permis à de jeunes mathématiciens de vivre l'atmosphère émouvante des Olympiades mathématiques et de participer à divers camps de mathématiques dans tout le Canada. De plus, les mathématiciens professionnels de tous les domaines ont pu se réunir et collaborer en face à face (hors ligne).

C'est un tel plaisir de faire partie de la Société mathématique du Canada, un groupe de mathématiciens diversifiés qui est toujours prêt à se tenir aux côtés de ses collègues face à l'injustice. Le Conseil d'administration a travaillé fort pour créer de nouveaux comités et sous-comités afin de répondre aux besoins de nos membres et des mathématiciens. Cette année, deux nouveaux comités ont été créés : le comité des droits de l'homme en mathématiques et le comité des prix internationaux.

C'était la première année depuis 2019 que nous avons pu nous réunir sur site avec de vieux amis et collègues pour discuter de questions importantes concernant le domaine et ceux qui le composent. Lors de la Réunion d'été, nous avons ressenti les effets résiduels de la pandémie, notre envoi de fournitures de réunion étant resté bloqué dans un entrepôt au Nouveau-Brunswick et étant arrivé alors que la réunion était déjà terminée. Pas de badges ? Pas de problème! Malgré ce petit contretemps, tout le monde était heureux de pouvoir enfin assister à une conférence de mathématiques en personne. Je ne saurais dire si le personnel du bureau exécutif a tiré des leçons de cette expérience ou s'il est devenu paranoīaque à cause de cette expérience; nous avons chargé notre voiture et sommes partis en direction de Toronto pour la Réunion d'hiver. Badges ou pas, les mathématiciens savent comment profiter d'une conférence! C'était formidable de revoir tout le monde après un semestre réussi et de féliciter nos lauréats et notre nouvelle classe de Fellows de la SMC.

Parmi ceux qui ont profité de l'enthousiasme de participer à un événement mathématique, il y a les élèves de l'Équipe mathématique du Canada qui se sont rendus à l'Olympiade mathématique internationale et à l'Olympiade mathématique des filles européennes. J'aimerais profiter de l'occasion pour vous vanter les merveilleux résultats de notre équipe et vous encourager à consulter les résultats officiels des concours[1]. Bien sûr, rien de tout cela ne serait possible sans Dorette Pronk, Robert Woodrow et le reste de l'équipe qui ont passé des heures interminables en coulisses pour former les Olympiens, coordonner leur voyage et leur offrir une expérience positive.

Cette année a été une année chargée pour les concours. Le Bureau exécutif a mené une campagne pour augmenter la participation des écoles au DOCM et au CMMC. 14792 invitations ont été envoyées aux écoles canadiennes pour les inviter à participer à nos concours cet automne. Notre réseau de concours s'est étendu cette année grâce à des partenariats nouveaux et existants, tant au Canada qu'à l'étranger[2]. À l'aide de ces efforts, la participation au CMMC a augmenté de près de 50 %. Nous terminons une saison de concours très chargée et nous avons hâte d'analyser les résultats de cette campagne de marketing direct au cours de la nouvelle année.

L'été 2022 a apporté la reprise de nombreux programmes de camps de mathématiques à travers le pays et la création de plusieurs nouveaux programmes. Nous avons reçu d'excellents commentaires des étudiants et des organisateurs concernant les camps et nous sommes certains que 2023 apportera d'autres opportunités passionnantes pour les jeunes dans le domaine des mathématiques.

Ici, au bureau exécutif de la SMC, nous ne sommes pas étrangers aux joyeuses retrouvailles. Après plus de deux ans de travail à distance, nous sommes de retour au bureau. Il convient de noter que ce bureau n'est pas n'importe quel bureau, mais la nouvellement baptisée Maison canadienne des mathématiques: un foyer permanent pour la SMC et les mathématiques au Canada.

Ces retrouvailles ont été douces-amères, car l'un des membres de notre équipe n'est plus parmi nous. Le 30 septembre, l'expert résident de tout ce qui concerne la SMC, notre ami et mentor, Alan Kelm, est décédé. Il a été un pilier de la Société mathématique du Canada pendant près de 30 ans, et il manque cruellement à toute notre équipe.

En parlant de notre équipe, je ne saurais trop insister sur la résilience du personnel du bureau exécutif de la SMC au cours de la dernière année. Malgré le manque de personnel, les problèmes technologiques, la maladie et tout ce que la vie a pu leur réserver, tous les membres du bureau exécutif sont restés dévoués à la Société et au rôle qu'ils y jouent. Le travail acharné de Kaileigh, Jessica, Sarah, Steve et Xinxin n'est pas passé inaperçu. J'ai beaucoup de chance de les avoir comme collègues et je souhaite leur exprimer mes remerciements les plus sincères pour tous leurs efforts cette année. Enfin, et surtout, je voudrais remercier Yvette pour tout ce qu'elle a fait pour moi et pour la SMC cette année. Son travail acharné et son dévouement ont assuré la survie de la SMC pendant des périodes difficiles. Je n'aurais pas pu demander une meilleure équipe.

Enfin, j'aimerais remercier David Pike, Javad Mashreghi, Monica Nevins, le Conseil d'administration et les membres de cette merveilleuse Société pour leur soutien et leurs conseils cette année. J'apprécie grandement leur mentorat et leur amitié alors que j'apprends et grandis dans mon rôle à la SMC.

De ma position privilégiée, je dois reconnaître la tristesse et la colère que j'éprouve en tant que Canadienne iranienne lorsque je pense aux injustices que mes compatriotes iraniennes subissent (et ont subies) sous le régime actuel. La bravoure de ceux qui protestent pacifiquement pour leurs libertés me rend fière d'être une femme iranienne. [3]

En même temps, la Société mathématique du Canada et le dévouement de ses merveilleux membres envers la communauté mathématique me rendent fière d'être une mathématicienne canadienne.

Je vous souhaite à tous une merveilleuse période de repos, et une très heureuse année 2023 !

Pour les femmes, la vie, la liberté

برای زن، زندگی، آزادی

[1] OMC/OJMC : https://smc.math.ca/news-item/2022-resultats-omc-omjc/

OEMF: https://smc.math.ca/news-item/resultats-oemf-2022/

OMI: https://smc.math.ca/news-item/equipe-canada-oim-2022/

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 $[3] \ Vous \ trouverez \ de \ plus \ amples \ informations \ sur \ les \ \'ev\'enements \ actuels \ en \ Iran \ \`a \ l'adresse suivante \ (en \ anglais): \ https://www.cbc.ca/news/topic/Tag/Iran. \ les \ respectives \ respecti$ 

## Des recherches non recherchées



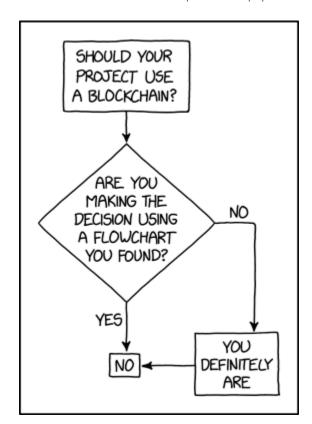
Éditorial Décembre 2022 (tome 54, no. 6)

**Robert Dawson** (Saint Mary's University) *Editor-in-Chief, CMS Notes* 

Je ne suis sans doute pas le seul membre de la SMC à recevoir un flux constant de courriels du même genre que celui-ci :

Cher professeur: Je suis sur le point d'obtenir une maîtrise en informatique de l'Université nationale d'Utopie, où j'ai également obtenu mon B.Sc. J'ai obtenu une moyenne de 3,8 dans mes cours de premier cycle. J'ai bon espoir de travailler avec vous sur un doctorat dans votre université. ... Mon mémoire de maîtrise portait sur « les paradigmes de l'informatique en nuage pour la technologie chaîne de blocs dans la conception de jouets intelligents pour chats. » J'ai eu des emplois d'été avec une variété d'entreprises. Je pense que mes compétences logicielles seront un excellent ajout à votre équipe de recherche. Je serais très reconnaissant de l'occasion de travailler sous votre supervision pour obtenir mon doctorat. Je joins un curriculum vitae à ce courriel, résumant mes réalisations scolaires et mon expérience d'enseignement. N'hésitez pas à me contacter si vous avez des questions ou si vous souhaitez lire mon mémoire. Je suis disponible à tout moment pour un entretien. Je vous remercie par avance, (etc.)

Avant que vous ne pensiez avoir trouvé le cadeau de Noël idéal pour Minou, je dois avouer que les jouets intelligents pour chats sont aussi éloignés de mon domaine d'expertise que l'infonuagique. Quant à la chaîne de blocs, je crois que la bande dessinée xkcd de Randall Munroe serait précise dans la plupart des cas :



https://imgs.xkcd.com/comics/blockchain.png

- surtout le texte au passage de la souris. En bref, je suis à peu près aussi éloigné d'un superviseur potentiel pour cette jeune personne que vous pouvez l'être sans aller au département des lettres classiques (par exemple). (Mon manque de financement de la recherche serait la cerise sur la coupe glacée, s'il y en avait.)

Mais supposons que mon correspondant ait écrit un mémoire de maîtrise qui ne soit pas très éloigné de l'un des sujets obscurs de géométrie, de théorie des nombres ou de théorie des catégories qui m'occupent entre les cours ; et supposons que j'aie suffisamment de fonds pour soutenir un étudiant (ce qui est rarement le cas, même lorsque j'ai une bourse de recherche). Cette demande hors norme me convaincrait-elle?

Probablement pas: même avec une chance de succès, elle porte encore les marques d'une demande à la sauvette. Il est difficile de faire valoir de manière convaincante que votre travail est adapté au projet de quelqu'un d'autre sans mentionner ce projet! Je conseille aux étudiants potentiels à la recherche d'un superviseur de rechercher soigneusement les personnes qui font du bon

travail dans leur domaine d'intérêt, et d'envoyer des courriels personnalisés présentant en détail leurs arguments pour travailler avec ce superviseur particulier. Personne n'aime les spammeurs	S.
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## Recurring decimals, proof, and ice floes



Notes pédagogiques Décembre 2022 (tome 54, no. 6)

Brian McMaster (Queen's University, Belfast)

Aisling McCluskey (University of Galway)

Education Notes bring mathematical and educational ideas forth to the CMS readership in a manner that promotes discussion of relevant topics including research, activities, issues, and noteworthy news items. Comments, suggestions, and submissions are welcome.

**John Grant McLoughlin**, University of New Brunswick (johngm@unb.ca)

Kseniya Garaschuk, University of the Fraser Valley (kseniya.garaschuk@ufv.ca)

## Or: Why do we teach students how to prove things we all know already, such as 0.9999... = 1?

Partly, of course, so that they develop thinking skills to use on questions whose truth-status they won't know in advance. Another part, however, concerns the dialogue nature of proof: a proof must be not only correct, but also persuasive: and persuasiveness is not objective and absolute, it's a two-body problem. Not only to tango does one need two.



The statements —

- (1) ice floats on water,
- (2) ice is less dense than water

— are widely acknowledged as facts and, usually, as interchangeable facts. But although rooted in everyday experience, they are not that experience. We have firstly represented stuffs of experience by sounds English speakers use to stand for them, then represented these sounds by word-processor symbols that, by common agreement, stand for them. Two steps away from reality already! This is what humans do: we invent symbols for perceived realities and, eventually, evolve procedures for manipulating them in ways that mirror how their real-world origins behave. Virtually no communication between two persons, and possibly not much internal dialogue within one mind, can proceed without this. Man is a symbol-using animal.

Statement (1) counts as fact because folk living in cooler climates have directly observed it throughout history (and because conflicting evidence is lacking). Statement (2) is factual in a significantly different sense, arising by

further abstraction from (1) and from a million similar experiential observations: partly to explain (1) and its many cousins, we have conceived ideas like mass, volume, ratio of mass to volume, and explored for generations towards the conclusion that mass-to-volume works out the same for similar materials under similar conditions, and that the comparison of mass-to-volume ratios predicts which materials will float upon others.

Statement (3): 19 is a prime number. In what sense is this a fact? Its roots are deep in direct experience: the hunter-gatherer wishing to share nineteen apples equally with his two brothers or his three sons or his five children must have discovered that he couldn't, without extending his circle of acquaintance so far that each got only one, long before he had a name for what we call 'nineteen'. But (3) is many steps away from the experience where it is grounded. It involves conceptualisation of numerical measurements of sets one encounters, and millennia of thought to acquire symbols for these and codify procedures for manipulating them in ways that mirror how reality functions. We've done this so successfully that it's easy to forget how far from the tangibles of experience they stand.

Statement (4):  $\sqrt{2}$  is not exactly the ratio of two whole numbers. Most first-year mathematics students know this. But by this stage of abstraction, separating its fact-ness from its demonstration is impossible: the property of being exactly a fraction is not detectable by physical experience. It is a property of how we abstracted and systematised the numbers that proved useful in modelling

reality, not of our hands-on experience of reality. The reason we regard  $\sqrt{2}$ 's irrationality as factual is precisely because we can give a demonstration within an accepted logical framework.

What then about recurring decimals? For persuasive argument, we must first ascertain the distance from reality at which the question arises: not, in this case, the rarified atmosphere of undergraduate mathematics but the primary school classroom. Once a child has learned rituals for dividing whole numbers and the convenience of decimal notation, she will try to divide, say, 2 by 3 and will hit a problem: the decimal representation of the answer does not cease to spew out digits of lesser and lesser significance no matter how long she keeps turning the handle.

What should we reply when she asks whether zero point infinitely many 6's is or is not two thirds, or even — as a thoughtful child should — whether zero point infinitely many 6's is a legitimate symbol at all?

The answer must be tailored to the questioner's needs, but the natural way forward — though it took us centuries to make it logically watertight — is the nineteenth-century definition of sum of an infinite series. For the primary school kid it may suffice to say that, by writing down enough 6's, we'd get as close to 2/3 as we'd need for any practical purpose. For differential calculus we'd need something better, and for model-theoretic discourse involving infinitesimals something better again. Yet the underpinning mathematics for equalities like  $0.6666\cdots = 2/3$  where the question arises is the nineteenth-century one. Its fact-ness therefore resembles that of ice being less dense than water, of 19 being prime or of  $\sqrt{2}$  being irrational: it can be demonstrated within a logical framework that systematises our observations of real-world experiences. So it is a fact not about reality but about the models we build to explain reality. Demonstration is the only tool available for establishing its truth.

Mathematics without proof is not like an omelette without salt and pepper; it is like an omelette without egg.

#### About the Authors



Brian McMaster has served his alma mater, Queen's University Belfast (QUB), in multiple capacities including Adviser of Studies, Head of Research and Associate Director of Education. His publication profile covers over sixty refereed journal articles and his teaching interests focus around analysis and analytic topology. Formally retired since 2011, he continues to deliver ad hoc undergraduate teaching on an honorary basis. In 2018 he received the accolade for Special Recognition for Inspirational Teaching awarded by QUB Students' Union.



Aisling McCluskey is a Personal Professor in Mathematics at the University of Galway, Ireland. She maintains an active research profile whilst holding the teaching and learning of mathematics central to her academic endeavour. She has received several institutional and national awards for excellence in teaching, most recently in 2020 and 2021. She is currently a Governor at the University of Galway and Head of the School of Mathematical and Statistical Sciences there.

Aisling McCluskey and Brian McMaster have co-authored several textbooks in the areas of topology, analysis and complex numbers.

## Quaternions at Twilight: Remembering Mary Somerville 150 years after her death



Notes de la SCHPM Décembre 2022 (tome 54, no. 6)

**Brigitte Stenhouse** (University of Toronto)

CSHPM Notes brings scholarly work on the history and philosophy of mathematics to the broader mathematics community. Authors are members of the Canadian Society for History and Philosophy of Mathematics (CSHPM). Comments and suggestions are welcome; they may be directed to either of the column's co-editors:

**Amy Ackerberg-Hastings**, independent scholar (aackerbe@verizon.net)

Hardy Grant, York University [retired] (hardygrant@yahoo.com)

November 29, 2022, marks 150 years since the death of Mary Somerville (1780–1872), one of the most iconic mathematicians of 19th-century Britain. Somerville built her reputation in the drawing rooms of Edinburgh, London, and Paris, in which she demonstrated her expert knowledge of the analytical mathematics recently developed in France. This mathematics was widely believed to be the answer to a long-running decline in British science, owing to the fruitful applications of methods from the differential calculus and the calculus of variations. Somerville's earliest publications were solutions to puzzles printed in *The New Series of the Mathematical Repository*, through which she was one of the earliest adopters of the differential notation in Britain. The work for which she is most famous is undoubtedly her 1831 *Mechanism of the Heavens*, a translation of Laplace's formative *Traité de Mécanique Céleste*, in which he gave an analytical, algebraic treatment of physical astronomy.

Mary Somerville's mathematical works were the focus of my doctoral thesis, which sought to understand what it meant for Somerville, as a woman in 19th-century Europe, to be a mathematician [8]. In early 2022, I was delighted to again have access to the Somerville papers at the Bodleian Library, which I had been unable to consult for the final 18 months of my PhD owing to the Covid-19 pandemic. I especially wanted to resolve the footnotes in my thesis that said, "I know relevant material exists, I just haven't been able to look at it yet!" According to the online catalogue, one folder held a "notebook containing notes and comments in Mary's hand on The Mechanism of the Heavens, n.d. (c.1831)". Much to my surprise, this notebook was accompanied by 70 loose sheets in the shaky handwriting typical of Somerville much later in her life and full of algebraic calculations. One page in particular took my breath away: a draft page from her autobiography, again featuring algebraic formulas down the side, in which she acknowledged that the end of her life was approaching but noted that she was perfectly content in the care and company of her beloved daughters [4]. I was immediately hooked, and could clearly picture Somerville at her writing table, expressing her gratitude that her "intellect [was] still unimpaired", before turning to mathematics, a subject that she had enjoyed for nearly eighty years. I needed to know more about how and why Somerville returned to serious mathematical study when she was nearly ninety.



**Figure 1.** Self-Portrait of Mary Somerville, undated. Reproduced with the kind permission of the Principal and Fellows of Somerville College, Oxford.

By 1870, when this tale begins, Somerville had been living a peripatetic life on the Italian peninsula for around thirty years. Moving to Italy had been decided upon for health reasons, to seek a better climate than the UK offered, and also in hope of living at a lower cost than in London. During this lengthy sojourn, Somerville published multiple highly successful books—giving surveys of recent scientific developments in the physical sciences and physical geography—yet she still felt on the outside of the scientific community. Individuals such as Giovanni Plana, Professor of Astronomy at the University in Turin, offered her access to their personal libraries, but this was nonetheless a sharp contrast to the lively, sociable scientific community she had been a part of in London. Somerville described how she felt at "a great disadvantage being so entirely deprived of scientific society and of the means of hearing of recent discoveries and new publications" [3]. Both the Royal Institution and the Royal Astronomical Society decreed that Somerville was to be sent copies of their *Proceedings* and the *Greenwich Observations*, respectively, but these did not always materialize and instead she often relied on the goodwill of contemporaries—for example, mathematician Augustus De Morgan—to send her books and papers from Britain.

Beyond communication with her scientific acquaintances via letter, Somerville often hosted visitors in whichever city she was then living. One such visitor was Benjamin Peirce, Professor of Mathematics and Astronomy at Harvard University, USA, who came to Europe in 1870 to view an eclipse. Whilst in Naples he paid Somerville a visit and was so enamoured with his host that, on his return home, he sent her a privately-printed copy of his *Linear Associative Algebra*. In testament to her ongoing reputation, Peirce inscribed the work with: "To the brightest glory of her sex, Mrs Mary Somerville, with the sincere admiration and the profound respect of the Author" [2].

The mathematics in Peirce's *Linear Associative Algebra* would have been entirely new to Somerville. Building on the work of William Rowan Hamilton on quaternions (to which we return later), Peirce considered hyper-complex numbers and presented 162 different algebras in this volume [1, p. 127]. Hamilton's quaternions were notable in not satisfying commutativity of multiplication, and Peirce went further in allowing systems in which associativity did not hold, and even those in which division was not well defined.

Apparently struggling to understand these new ideas, which marked a distinctive conceptual shift towards perceiving algebra as the study of structures, Somerville soon began soliciting books to aid her reading of Peirce. In April 1871 she wrote to her publisher, John Murray, telling him of Peirce's book and asking him to send her a copy of Hamilton's work on quaternions. A few months later Hamilton's 1853 *Lectures on Quaternions* were sent to her by the Reverend Whitwell Elwin, a close acquaintance of Murray who was not previously known to Somerville.

It is somewhat unfortunate that Somerville came to the study of quaternions only in 1870, five years after Hamilton's death. The two had met in Cambridge 38 years earlier when Somerville was received at Trinity College, and Hamilton subsequently oversaw her election in 1834 as an honorary member of the Royal Irish Academy. At this time Hamilton was still at the very beginning of his work on quaternions. He was interested in studying complex numbers as 'algebraic couples', or ordered pairs of real numbers on which he defined operations of multiplication and addition. He then began searching for an analogous system of triplets of real numbers, but he struggled to define such a system in which the properties of commutativity, associativity, and distributivity held, and where division (the inverse of multiplication) was well-defined. Eventually, in 1843, Hamilton developed his system of quaternions, namely numbers of the form a + bi + cj + dk, where i, j, k are unit vectors such that  $i^2 = j^2 = k^2 = -1$ . These hypercomplex numbers satisfied all of the properties Hamilton desired, except for commutativity of multiplication; the abandonment of commutativity was revolutionary at the time. In 1848 Hamilton gave a series of four lectures on quaternions at Trinity College Dublin, and these were subsequently expanded into his 1853 book that was sent to Somerville [1, pp. 28–35].

Hamilton's *Lectures* were apparently not sufficient, as Somerville soon reached out to William Spottiswoode, then President of the London Mathematical Society, for further assistance. Like Peirce, Spottiswoode had made Somerville's acquaintance while visiting Naples a few years earlier. In August 1871 Spottiswoode wrote to Somerville:

I was glad to hear of you again, & especially so as you are still pursuing your studies. As you do not mention the exact subject of the American book, I am not quite sure as to the work best leading to it. But I send you three which I think must cover the ground, & with which I feel sure that you will in any case be interested. . . . The third is [Peter Guthrie] Tait's work on Quaternions. I have sent this rather than Sir William Hamilton's works as the latter are intolerably diffuse, & Tait has carried out the applications of the subject much further than anyone else [7].

During the winter of 1858–59 Tait and Hamilton had shared an intense exchange of letters discussing ideas around quaternions. After Hamilton's death, Tait became one of the leading advocates of quaternions, developing them into a tool with applications in the physical sciences.

A copy of Tait's 1867 An Elementary Treatise on Quaternions was part of Somerville's personal scientific library at the time of her death [9]. These books were donated by her daughters as a single collection to Girton College, Cambridge, and the copy of Tait still contains a sheet of handwritten notes by Somerville placed between pages 294 and 295 [10]. This page of formulas hints at the volume of time Somerville spent studying Tait, her last mathematical project.

In the final three years of her life, 1870–72, Somerville was working on numerous publication projects. She revised two mathematical manuscripts that she had first written in the 1830s, prepared new editions of her scientific survey books, and authored her autobiographical *Personal Recollections*. All of these works were left to be published after Somerville's death, for then her government pension would cease, and she intended the royalties from book sales to provide a vital income for her two unmarried daughters.

That Somerville was studying quaternions alongside these authorial projects is clear from manuscript drafts of her *Personal Recollections*. Interspersed with a draft from circa 1872 are sheets on which Somerville has made notes on Peirce's *Linear Associative Algebra*, and some of the draft sheets themselves feature brief jottings of calculations and diagrams; see, for example, Figure 2 [5]. Moreover, in a draft of a letter to Murray from November 1872 Somerville described her morning routine which involved "solv[ing] problems by the higher algebra or add[ing] to the narrative of [her] life" [4].

As can be seen in Figure 3, the draft of this letter was used by Somerville for jotting down calculations and ideas about quaternions. This folio is held with the aforementioned 70 loose sheets, which are a mixture of scrap paper used for rough workings-out and neat pages of notes. Tait provided exercises with no solutions at the end of each chapter of his *Quaternions*, and we can here witness Somerville preparing her own solutions to these problems. Other pages contain summaries of key ideas, and cross-references to results contained in works by Hamilton. One particularly interesting sheet contains an attempt at a proof of a result mentioned but not demonstrated by Tait.

It is highly likely that Somerville was producing these notes for an imagined reader other than herself. As can be seen in the bottom right-hand corner of the sheet in Figure 2, she felt that the quaternion system heralded a new, more powerful age in the mathematical study of the physical sciences. Thus it was a natural continuation of her previous mathematical work advocating for the adoption of analytical methods by those studying the natural world, for instance in her translation of Laplace. In addition, she may have felt that with the recent death of Hamilton, and Tait's declaration that he had moved on from his studies of quaternions, there was a lucrative gap in the book market that she could profit from and aid her daughters. The archival materials provide

further textual evidence that Somerville was in the early stages of preparing a book, possibly a companion to Tait's own. She wrote up multiple copies of the same sheets, returned to others to make edits, and at the beginning of her notes on Chapter 3 she declared that "the whole of this chapter is difficult and requires explanation which I have attempted" [4]. This justification for the material that she has produced and the neatly copied-up pages of notes would be unnecessary, were Somerville writing only as part of her own mathematical learning practice.

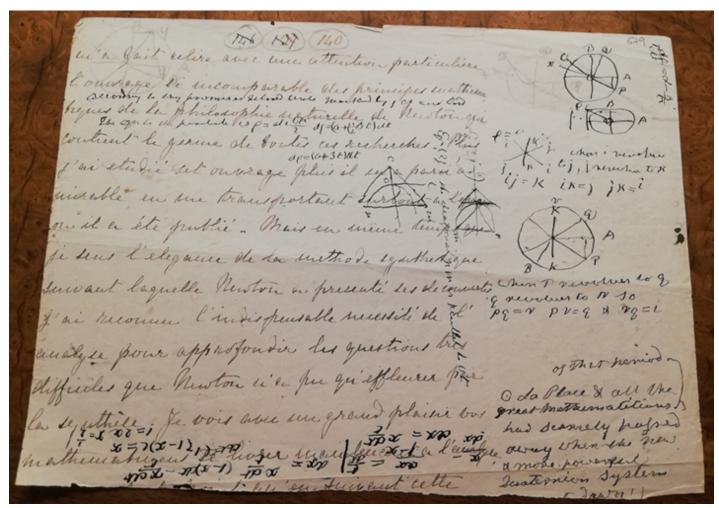
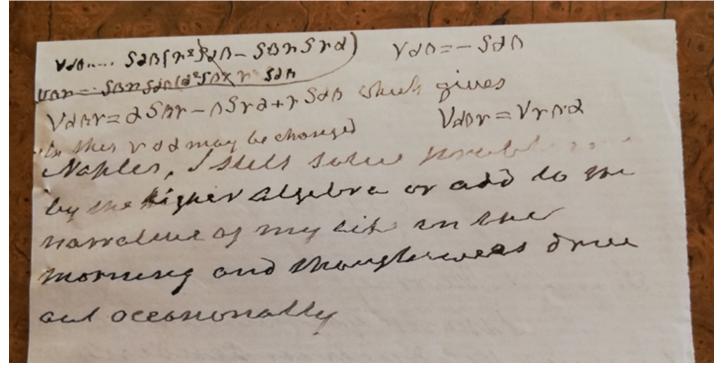


Figure 2. Image of a sheet held with a draft of Somerville's Personal Recollections. This page features a letter from Laplace copied up in Somerville's hand, which was included in the final publication [6, p.181]. Reproduced with the kind permission of the Principal and Fellows of Somerville College, Oxford.



 $\textbf{Figure 3.} \ Draft of letter written to publisher \c) on Murray, dated November 1872. \ Reproduced with the kind permission of the Principal and Fellows of Somerville College, Oxford.$ 

Unfortunately, Somerville's work on the quaternions never came to fruition. She continued studying Tait's book until the day she died, November 29, 1872, at which point she had only written notes up to Chapter 3 [6, p. 376]. Her two other mathematical manuscripts were also left unpublished, but her autobiography was well received and two of her scientific survey books were reissued after her death, providing some income for her daughters.

These loose sheets full of letter drafts, notes, and calculations raise questions about opportunities for publishing mathematical books in nineteenth-century Britain. Throughout her life Somerville was described as a mathematician, and she described mathematics as the subject she found most congenial. Yet her career as an author was focused on books which detailed results in the physical sciences, without delving into the mathematics used to reach them. It is impossible to say for certain that Somerville was preparing a work on quaternions for publication, but it is clear that at the end of her life she again hoped to reconcile her need for financial stability and her own intellectual gratification through mathematical writing.

Somerville's exposure to emerging ideas in algebra was contingent on the respect and recognition she commanded in scientific circles, which led men such as Peirce and Spottiswoode not only to visit her, but to provide her with recent mathematical works. When she faced difficulties, she did what she had done for over sixty years and turned to her acquaintances to ask for help. It is a testament to her tenacity that rather than being dismayed and discouraged by her difficulties in understanding the works of Peirce, Tait, and Hamilton, Somerville instead enjoyed the opportunity to study an entirely new branch of mathematics. When writing about solving the exercises in Tait, she admitted:

Sometimes I find them difficult, but my old obstinacy remains, for if I do not succeed to-day, I attack them again on the morrow [6, p. 364].

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Dr Brigitte Stenhouse is currently a postdoctoral researcher at the IHPST at the University of Toronto, and in January 2023 will take up the post of Lecturer in the History of Mathematics at The Open University, UK. Her research looks at the role of marriage and domesticity in mathematics, and 19th-century mathematics in Britain more generally.

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## The Best Writing on Mathematics, 2021

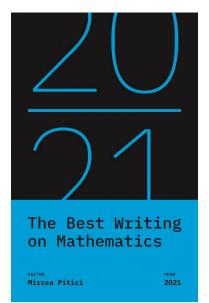


Comptes rendus Décembre 2022 (tome 54, no. 6)

Karl Dilcher (Dalhousie University)

Book Reviews bring interesting mathematical sciences and education publications drawn from across the entire spectrum of mathematics to the attention of the CMS readership. Comments, suggestions, and submissions are welcome.

Karl Dilcher, Dalhousie University (notes-reviews@cms.math.ca)



### The Best Writing on Mathematics, 2021

Edited by Mircea Pitici Princeton University Press, 2022 ISBN: 978-0-691-22570-8 Paperback: US \$25, CAN \$32

Reviewed by Karl Dilcher

Reviews of past issues of The Best Writing on Mathematics can be found in the 2019, 2020, and 2022 volumes of CMS Notes.

"So much to read, so little time!" is a common complaint that is known in several variations. Different solutions have been suggested, including speed reading. I tried learning a technique once when I was younger; but in the end, I had to agree with Woody Allen who reportedly said in one of his early comedy routines, "I read War and Peace in 20 minutes—It's about Russia."

A much better solution is to get advice from a trusted friend or colleague on what to read, and then read it slowly and in depth. When it comes to the large number of shorter pieces of writing in any given field, including mathematics, such a friend or colleague can also be a trusted and knowledgeable editor and/or anthologist.

This is the solution I have now happily relied on for several years, with the help of the series *The Best Writing on Mathematics*, edited by Mircea Pitici and published by Princeton University Press since 2010. The book under review is the twelfth volume in this remarkable series of annual anthologies. In the brief review of *The Best Writing on Mathematics*, 2018, I addressed some general features shared by all volumes. I will not repeat these remarks here; the interested reader will find them in the September 2019, issue. Instead, I quote from the Introduction to the current volume, where the Editor recalls that the series brings together "diverse perspectives on mathematics, its application, and their interpretation—as well as on their social, historical, philosophical, educational, and interdisciplinary contexts. The volume should be seen as a continuation of the previous volumes."

A bit later in the Introduction, Pitici writes, "The pieces offered this time originally appeared during 2020 in professional publications and/or in online sources. The content of the volume is the result of a subjective selection process that started with many more candidate articles. [...] Once again, this anthology contains an eclectic mix of writings on mathematics, with a few even alluding to the events that just changed our lives in major ways."

I will now quote from the overview of this current volume and add the titles of the 26 individual pieces of writing, as I did in the two previous years. The average length of the pieces is almost exactly 10 pages.

"To start, Viktor Blåsjö takes a cue from our present circumstances and reviews historical episodes of remarkable mathematical work done in confinement, mostly during wars and imprisonment. [Lockdown Mathematics: A Historical Perspective].

"Andrew Lewis-Pye explains the basic algorithmic rules and computational procedures underlying cryptocurrencies and other blockchain applications, then discusses possible future developments that can make these instruments widely accepted. [Cryptocurrencies: Protocols for Consensus].

"Michael Duddy points out that the ascendancy of computational design in architecture leads to an inevitable clash between logic, intellect, and truth on one side—and intuition, feeling, and beauty on the other side. He explains that this trend pushes the decisions traditionally made by the human architect out of the resolutions demanded by the inherent geometry of architecture. [Logical Accidents and the Problem of the Inside Corner].

"Steve Pomerantz combines elements of basic complex function mapping to reproduce marble mosaic patterns built during the Roman Renaissance of the twelfth and thirteenth centuries. [Cosmatesque Design and Complex Analysis].

"Ben Logsdon, Anya Michaelsen, and Ralph Morrison construct equations in two variables that represent, in algebraic form, geometric renderings of alphabet letters—thus making it possible to generate word-like figures, successions of words, and even full sentences through algebraic equations. [Nullstellenfont].

"Maria Trnkova elaborates on crocheting as a medium for building models in hyperbolic geometry and uses it to find results of mathematical interest. [Hyperbolic Flowers].

"Yelda Nasifoglu decodes the political substrates of an anonymous seventeenth century play allegorically performed by geometric shapes. [Embodied Geometry in Early Modern Theatre].

"In the next piece, Stephen K. Lucas, Evelyn Sander, and Laura Taalman present two methods for generating three-dimensional objects, show how these methods can be used to print models useful in teaching multivariable calculus, and sketch new directions pointing toward applications to dynamical systems. [Modeling Dynamical Systems for 3D Printing].

"Joshua Sokol tells the story of a quest to classify geological shapes mathematically—and how the long-lasting collaboration of a mathematician with a geologist led to the persuasive argument that, statistically, the most common shape encountered in the structure of the (under) ground is cube-like. [Scientists Uncover the Universal Geometry of Geology].

"Don Monroe describes the perfect similarity between foundational algorithms in quantum computing and an experimental method for approximating the constant p, then asks whether it is indicative of a deeper connection between phenomena in physics and mathematics or it is a mere (yet striking) coincidence. [Bouncing Balls and Quantum Computing].

"Kevin Hartnett relates recent developments in computer science and their unforeseen consequences for physics and mathematics. He explains that the equivalence of two classes of problems that arise in computation, recently proved, answers in the negative two long-standing conjectures: one in physics, on the causality of distant-particle entanglement, the other in mathematics, on the limit approximation of matrices of infinite dimension with finite-dimension matrices. [Landmark Computer Science Proof Cascades through Physics and Math].

"David Hand reviews the risks, distortions, and misinterpretations caused by missing data, by ignoring existing accurate information, or by falling for deliberately altered information and/or data. [Dark Data].

"In the same vein, Michael Wallace discusses the insidious perils introduced in experimental and statistical analyses by measurement errors and argues that the assumption of accuracy of data collected from observations must be recognized and questioned. [Analysis in an Imperfect World].

"In the midst of our book—like a big jolt on a slightly bumpy road—John Conway, Mike Paterson, and their fictive co-author Moscow, bring inimitable playfulness, multiple puns, and nonexistent self-references to bear on an easy game of numbers that (dis) proves to be trickier than it seems! [A Headache-Causing Problem].

"Next, Sanjoy Mahajan explains (and illustrates with examples) why some mathematical formulas and some physical phenomena change expression at certain singular points. [A Zeroth Power is Often a Logarithm Yearning to Be Free].

"Stan Wagon describes the counterintuitive movement of a bicycle pedal relatively to the ground, also known as the "bicycle paradox", and uses basic trigonometry to elucidate the mathematics underlying the puzzle. [The Bicycle Paradox].

"Jacob Siehler combines modular arithmetic and the theory of linear systems to solve a pyramid-coloring challenge. [Tricolor Pyramids].

"Natalie Wolchover untangles threads that connect foundational aspects of numbers with logic, information, and physical laws. [Does Time Really Flow? New Clues Come from a Century-Old Approach to Math].

"The late Harold Edwards pleads for a reading of the classics of mathematics on their own terms, not in the altered "Whig" interpretation given to them by the historians of mathematics. [The Role of History in the Study of Mathematics].

"Michael Barany uncovers archival materials surrounding the birth circumstances, the growing pains, and the political dilemmas of the Notices of the American Mathematical Society—a publication initially meant to facilitate internal communication among the members of the world's foremost mathematical society. ["All of These Political Questions": Anticommunism, Racism, and the Origin of the Notices of the American Mathematical Society].

"Mike Askew pleads for raising reasoning in mathematics education at least to the same importance give to procedural competence—and describes the various kinds of reasoning involve in the teaching and learning of mathematics. [Reasoning as a Mathematical Habit of Mind].

"Roger Howe compares the professional opportunities for improvement and the career structure of mathematics teachers in China and in the United States—and finds that in many respects the Chinese ways are superior to the American practices. [Knowing and Teaching Elementary Mathematics—How are We Doing?].

"Stephen Ramon Carcia draws on his work experience with senior undergraduate students engaged in year-end projects to distill two dozen points of advice for instructors who supervise mathematics research done by undergraduates. [Tips for Undergraduate Research Supervisors].

"Adam Glesser, Bogdan Suceavă, and Mihaela B. Vâjiac read (and copiously quote) Sophie Germain's French Essays (not yet translated into English) to unveil a mind not only brilliant in original mathematical contributions that stand through time, but also insightful in humanistic vision. ["The Infinite Is the Chasm in Which Our Thoughts Are Lost": Reflections on Sophie Germain's Essays].

"Melvyn Nathanson raises the puzzling issues of authorship, copyright, and secrecy in mathematics research, together with many related ethical and practical questions; he comes down uncompromisingly on the side of maximum openness in sharing ideas. [Who Owns the Theorem?]

"In the end piece of the volume, Terence Tao candidly recalls selected adventures and misadventures of growing into one of the world's foremost mathematicians." [A Close Call: How a Near Failure Propelled Me to Succeed].

Returning to the Introduction, the Editor mentions the difficulties of compiling this volume, the main work of which was done during the height of the COVID crisis. This meant that many print-only resources were not available to him. Still, in addition to the fascinating 26 pieces of writing published here, the volume ends with a chapter on *Notable Writings*, containing a sizeable list of *Notable Journal Articles* and a list of close to 30 *Notable Journal Issues* which are "fully or partly dedicated to the specified topics—or contain symposia on the respective theme".

The tone of the Introduction to this volume is more subdued than that of earlier volumes, and the Editor also mentions that it is shorter than in the past, and that, due to the COVID crisis, some additional material (e.g., a book list) is lacking. More concerning, the Editor indicates that the series faces an uncertain future. I sincerely hope that any difficulties facing further publication have been, or will be, overcome, and that Princeton University Press will make it possible for Mircea Pitici to continue providing this wonderful service to the community.

# Appel à soumissions : MOSAIC (Mathématiques ouvertes sur la société, accessibles et inclusives en chroniques) des Notes de la SMC



MOSAIC Décembre 2022 (tome 54, no. 6)

La Société mathématique du Canada vous invite à soumettre des articles pour figurer dans la chronique MOSAIC (Mathématiques ouvertes sur la société, accessibles et inclusives en chroniques) des Notes de la SMC.

MOSAIC est dirigée par le Comité d'équité, de diversité et d'inclusivité (EDI) de la SMC.

Cette chronique vous offre un endroit où vous pouvez poser des questions, écouter, apprendre, partager les expériences, et proposer les solutions pour créer une communauté mathématique qui est plus diversifiée, juste, et forte. Par exemple, vous êtes invités à soumettre un article qui décrit les défis et les succès dans la mise en place des initiatives liées aux concours, la diffusion des services, ou d'autres événements à votre université.

Votre soumission par courriel devra comprendre votre article en fichiers Word et PDF. Veuillez soumettre votre article au Comité EDI à edic@cms.math.ca.

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Décembre 2022 (tome 54, no. 6)



## Appel de Sessions scientifiques : Réunion d'été de la SMC 2023



Décembre 2022 (tome 54, no. 6)

Réunions de la SMC

Appel de Sessions scientifiques

La Société mathématique du Canada (SMC) lance un appel de propositions de sessions pour la Réunion d'été 2023 de la SMC qui se tiendra à Ottawa du 2 au 5 juin 2023.

- Les sessions sont programmées en blocs de 2,5 heures, et auront lieu du 3 au 5 juin 2023. Les sessions scientifiques comprennent généralement entre 10 et 20 exposés de 30 minutes chacun, mais des exposés d'une heure sont possibles.
- Conformément au mandat de la SMC de proposer des conférences accessibles et accueillantes pour tous les groupes, la diversité parmi les organisateurs et les conférenciers est fortement encouragée. Afin de soutenir les organisateurs dans leur important travail et dans leurs efforts en faveur de l'inclusion et de la diversité, la SMC organisera un appel à résumés ouvert pour toutes les sessions, et demande aux organisateurs de prendre en compte toutes les soumissions de résumés éligibles pour leur session.
- La diversité comprend les sujets d'intérêt, les étapes de la carrière, l'emplacement géographique et les données démographiques; les groupes sous-représentés désignés comprennent, sans s'y limiter, les femmes, les peuples autochtones, les personnes handicapées, les membres des minorités visibles/groupes racialisés et les membres des communautés LGBTQ2+.
- Il y aura un appel à propositions distinct pour les sessions d'éducation.

## Les propositions doivent comprendre :

- 1. Les noms, affiliations et coordonnées des co-organisateurs de la session. Les chercheurs en début de carrière sont encouragés à proposer des sessions.
- 2. Un titre et une brève description du sujet et de l'objectif de la session. Cela peut inclure un bref aperçu du sujet. Incluez un résumé de deux à trois phrases qui sera affiché sur le site Web à l'intention des conférenciers potentiels.
- 3. Le nombre de blocs de 2,5 heures prévus, avec une liste de conférenciers possibles.

Les propositions seront sélectionnées par le comité d'organisation scientifique, dans la limite de l'espace disponible en salle de classe, avec une priorité pour les sessions qui montrent l'intention d'inclure un mélange de chercheurs seniors et juniors, de rendre certaines parties de leur session accessibles aux étudiants diplômés, et d'inclure des orateurs de groupes sous-représentés désignés.

## Dates limites:

Il y aura deux tours de soumissions. Les propositions soumises avant le mercredi 14 décembre 2022 aux directeurs scientifiques avec le bureau de la SMC en copie conforme seront considérées dans le premier tour, avec des réponses début janvier. La date limite pour le deuxième tour sera le 28 février 2023.

Monica Nevins mnevins@uottawa.ca

Aaron Tikuisis Aaron. Tikuisis @uottawa.ca

Bureau de la SMC: meetings@cms.math.ca





Réunions de la SMC

Décembre 2022 (tome 54, no. 6)

La Société mathématique du Canada (SMC) sollicite des propositions de sessions en matière d'éducation pour la Réunion d'été 2023 de la SMC qui aura lieu à Ottawa du 2 au 5 juin 2023.

Cette année, les propositions de sessions en matière d'éducation seront sélectionnées par le Comité des sessions en matière d'éducation de la réunion de la SMC, qui établira également le calendrier des sessions acceptées, en communication avec leurs coorganisateurs.

Chaque proposition doit suivre les directives indiquées dans l'appel à sessions scientifiques. En outre, les organisateurs sont priés de préciser la structure de leur session (par exemple, un exposé de 20 minutes suivi de 5 minutes de questions-réponses et de 5 minutes de transition, ou un panel, ou une session/un atelier interactif, etc.)

Conformément aux propositions de sessions scientifiques, il y aura deux tours de soumission. Les propositions soumises avant le mercredi 14 décembre 2022 seront prises en compte lors du premier tour, avec des réponses début janvier. La date limite pour le deuxième tour sera le 28 février 2023.

Envoyez vos propositions de sessions en matière d'éducation (et vos questions) à :

Andie Burazin a.burazin@utoronto.ca

Avec Monica Nevins mnevins@uottawa.ca , Aaron Tikuisis Aaron.Tikuisis@uottawa.ca , et le bureau de la SMC: meetings@cms.math.ca en copie conforme.

de la SMC

Annonces Décembre 2022 (tome 54, no. 6)





Société mathématique du Canada

Numéro d'organisme de bienfaisance ARC: 118833979RR0001

## Appel de candidatures: Prix de doctorat SMC Blair Spearman 2023



Appel de candidatures Décembre 2022 (tome 54, no. é

Le Prix de doctorat Blair-Spearman de la SMC récompense le travail exceptionnel d'un étudiant au doctorat.



Nous acceptons actuellement les mises en candidature pour le prix 2023. Date limite : 31 janvier

Le prix sera décerné à une personne qui aura reçu son diplôme de troisième cycle d'une université canadienne l'année précédant sa mise en candidature (entre le 1er janvier et le 31 décembre) et dont les résultats pour l'ensemble des études supérieures seront jugés les meilleurs. La dissertation constituera le principal critère de sélection (impact des résultats, créativité, qualité de l'exposition, etc.), mais ne sera pas le seul aspect évalué. On tiendra également compte des publications de l'étudiante, de son engagement dans la vie étudiante et de ses autres réalisations.

Les mises en candidature qui ne seront pas choisies dans leur première compétition seront considérées pour une deuxième année (sans possibilité de mise à jour du dossier), et seront révisées par le Comité de sélection du prix de doctorat l'an prochain.

#### **Candidatures**

La SMC a pour but de promouvoir et de célébrer la diversité au sens le plus large. Nous encourageons fortement les directrices et les directeurs de départements et les comités de mise en candidature à proposer des candidate.s exceptionnel.le.s sans distinction de race, de genre, d'appartenance ethnique ou d'orientation sexuelle.

Les candidats doivent être nommés par leur université; la personne qui propose un e candidate doit se charger de regrouper les documents décrits dans les paragraphes suivants et de faire parvenir la candidature à l'adresse ci-dessous. La date limite pour recevoir la candidature est indiqué ci-dessus.

Le dossier sera constitué des documents suivants :

- Un curriculum vitae rédigé par l'étudiant
- Un résumé du travail du candidat d'au plus dix pages, rédigé par l'étudiant, où celui-ci décrira brièvement sa thèse et en expliquera l'importance, et énumérera toutes ses autres réalisations pendant ses études de doctorat.
- Trois lettres de recommandation, dont une du directeur de thèse et une d'un examinateur de l'extérieur (une copie de son rapport serait aussi acceptable). Le comité n'acceptera pas plus de trois lettres de recommandation.

Veuillez faire parvenir tous les documents par voie électronique, de préférence en format PDF, avant la date limite à prixdoc@smc.math.ca.