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CMS NOTES de la SMC

IN THIS ISSUE

Reconciliation in Mathematics

Cover Article

Keith F. Taylor (Dalhousie University)

On September 19, 2007, the *Indian Residential Schools Settlement Agreement* (IRSSA) came into effect. It arose from a lengthy process involving class action lawsuits in several jurisdictions across Canada. The IRSSA was signed by the National Consortium (formed from twenty law firms representing former residential school students), several Canadian church entities, and the Government of Canada. The provincial and territorial courts approved the IRSSA, which was, at that time, the largest out of court settlement in Canadian history. See [1] for a detailed account of the background and consequences to date of the IRSSA.

In addition to providing a process of financial compensation to individuals who had attended residential schools, the IRSSA laid out a number of other actions intended to redress damages to the Indigenous nations of Canada resulting from the residential schools program. One significant action was the establishment of the *Truth and Reconciliation Commission* (TRC) with a budget of $60 million. After five years of information gathering and consultations, the TRC submitted its final reports in 2015; they can be found at the website for the *National Centre for Truth and Reconciliation* [2]. Among these reports is the critical *Calls to Action*. The Calls to Action is a list of 94 specific "calls". Number 7 on the list is:

> *We call upon the federal government to develop with Aboriginal groups a joint strategy to eliminate educational and employment gaps between Aboriginal and non-Aboriginal Canadians.*

It is clear that there are many aspects to these "educational and employment gaps" and improvements in multiple areas (for example, per-student funding in K-12 education) that don't directly involve mathematics are needed. However, it is also clear, to us at least, that mathematics education levels are a significant component in education levels, and closing any overall gap will require meaningful improvement in mathematics education in many ways. Although call number 7 is directed at the federal government, the intellectual resources necessary to respond to the call are mostly in the Canadian mathematics and mathematics education communities, as represented by the CMS/SMC, CAIMS/SCMAI [3], and CMESG/GCEDM [4]. There are important ways in which our community is responding. For example, special sessions at CMS meetings with a theme related to Indigenous education attract speakers presenting a variety of initiatives aimed at enhancing mathematics interest and learning in various regions of the country, as well as enthusiastic and engaged attendees.

The CMS Reconciliation in Mathematics Committee was established to ensure such communication and sharing of ideas remains effective and to generate national-scale initiatives for improving the opportunities for Indigenous students and scholars to access the power of mathematics. Some examples of how opportunities might be improved are: Developing systematic support for preparation, and in-service training, of elementary school teachers in their command of mathematics; especially for those planning to teach in First Nations schools; building pedagogy, based in appropriate Indigenous cultural examples, that develops mathematical thinking; enhancing the widespread availability of enrichment opportunities – every school should have a math circles program; and building a network of Indigenous Mathematics Knowledge Keepers, be they ecologists, land surveyors, engineers, or doctors, to serve as role models for living a life enhanced by mathematical tools. This was just a sampling of ideas. Moving forward with any of these is challenging.

A core principle of reconciliation is summed up in the phrase "nothing about us without us". This means building trusting relationships and responding to the guidance provided by the leadership of Indigenous communities. One challenge is that there are more than 50 First Nations in Canada spread over 650+ communities as well as significant off-reserve populations. There are many different "Indigenous cultures" and languages. That being said, the Canadian mathematics community, broadly defined, should strive to have a relationship with every community as well as the national Indigenous leadership. Building relationships requires money. We must develop partnerships with corporations that have natural interests in northern Canada where the communities are particularly expensive to visit. We must build trust with the federal government – the target of Call 7 – to obtain funding on the scale necessary for sustained activities.

Returning to building trust with Indigenous communities, the Reconciliation in Mathematics Committee could arrange for an activity, coordinated with Elders from the Nation on whose traditional lands a summer or winter meeting is held, that will raise awareness within the mathematics community of the meaning of reconciliation or, more generally, the impact of colonization on the First Nations. A Blanket Exercise [5], if available, at an annual general meeting would also be helpful.

Finally, on the scale of each of us, if you are doing something, or thinking of doing something, that might be related to the mandate of the Reconciliation in Mathematics Committee, let them know so your good ideas can spread.

[1] https://www.residentialschoolsettlement.ca/IAP_Final_Report_English.pdf

[2] https://nctr.ca/records/reports/

[5] https://www.kairosblanketexercise.org/

[3] https://caims.ca/

[4] https://www.cmesg.org/

A Little Man Who Wasn't There

Editorial

Robert Dawson (St. Mary's University) *Editor, Notes*

Last June I wrote about major wrong turns in various sciences: things like angle trisection, alchemy, and perpetual motion. Recently I found myself thinking about the role, especially in mathematics, of minor wrong turns: the things researchers try to do, and can't, but end up explaining why it can't be done.

As with so much of mathematics, we can trace this back at least to that apocryphal boat cruise by the Pythagorean Brotherhood, during which somebody possibly called Hippasus was (or maybe wasn't) sent to do geometry with the rays and angler fish for proving (or maybe revealing) that the diagonal of a square wasn't commeasurable with its side. If this simple little result was such a surprise to the Tenure and Survival Committee, it must have been very nearly the first of its kind.

This leads us to suspect that Hippasus (or whoever it was) started out trying to prove the opposite–to find the (rational, because those were the only numbers they had back then) ratio between side and diagonal. When this attempt failed, he (or maybe she: it's said that the Pythagoreans were coeducational) had a lash of inspiration and did what uncounted thousands of us have done since: changed sides and declared–nay, proved–that the grapes were sour. The rest is history, and we are the richer for it.

It's hard to tell what early mathematicians beat their heads against until they realized their error, because it wasn't the fashion until recently to discuss lacunae. There is no note in Euclid asking whether some clever soul might prove the parallel postulate! Nonetheless, when we look at the structure of the first book, with the use of that postulate being delayed for very nearly as long as possible, we feel that Euclid must have thought about the matter, even if he did not make the breakthrough and invent non-Euclidean geometry.

Wanzel on constructibility, Galois on quintics, Gödel on decidability: the history of mathematics is full of results like these. Some may say "if you can't beat 'em, join 'em." We say, "if you can't prove it, prove you can't prove it."

*This familiar phrase comes from an 1899 poem by William Hughes Mearns entitled (really!) "Antigonish." X-persons may make of this what they will.

The Birth of Modern Cosmology

CSHPM Notes

Craig Fraser (Institute for the History and Philosophy of Science and Technology, University of Toronto)

CSHPM Notes bring scholarly work on the history and philosophy of mathematics to the broader mathematics community. The authors are members of the Canadian Society for History and Philosophy of Mathematics (CSHPM). Comments and suggestions are welcome; they may be directed to either of the column's co-editors:

Amy Ackerberg-Hastings, *Independent Scholar (aackerbe@verizon.net)*
Hardy Grant, *York University [retired] (hardygrant@yahoo.com)*

In the late 1990s I began to teach courses in the history of astronomy and cosmology as a complement to my existing courses in the history of mathematics. I was very impressed by John North's *Measure of the Universe* [14], a history of cosmology from the end of the 19th century to the 1960s. The book explores its subject from historical, scientific and philosophical viewpoints. In terms of historical sophistication and seriousness, it vastly betters popular works on astronomy that include historical surveys. It is one of the finest books of professional history of science over the past sixty years.

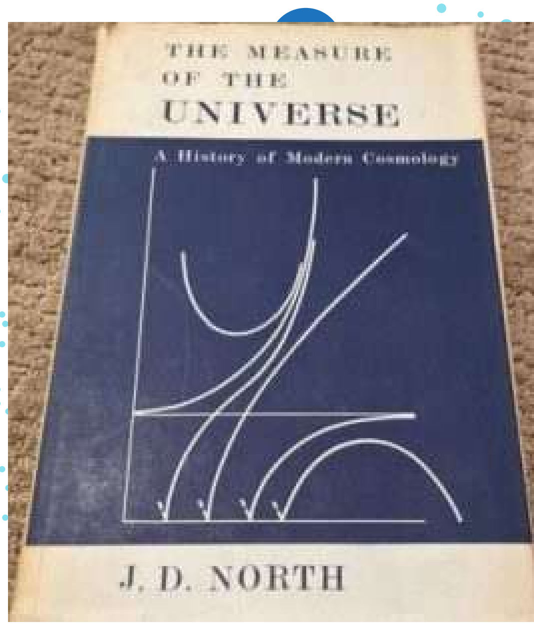

Figure 1. John North (1934–2008) and *The Measure of the Universe* (1965). *The Independent*; the author's copy of [14].

In 2006 I published *The Cosmos: A Historical Perspective*, a book based on my courses' lecture notes and aimed at a broad academic and student audience [5]. One subject discussed in this book that has engaged me more recently is the emergence of modern cosmology in the period from about 1912 to 1935 [6]. The first decades of the 20th century witnessed one of the most remarkable coincidences in the entire history of science. On the one hand, there was the development of a new mathematical theory of gravity by Albert Einstein, followed by his publication in 1917 of a geometric model of the universe based on this theory [3]. In developing this model Einstein was not at all engaged with astronomy. Rather, he was concerned with certain questions in the foundations of physics related to something known as Mach's principle (see [19]). At the same time that Einstein was working on his new theory, the astronomer Vesto Slipher had established in Flagstaff, Arizona, a program of spectroscopic observation of spiral nebulae. Slipher's research program had no connection whatsoever with any of the developments underway in theoretical physics. To the astonishment and sometimes disbelief of astronomers, Slipher found that the nebulae possess large redshifts, orders of magnitude larger than those of any celestial objects in our galaxy. It was also found in these observations and subsequent ones that there is at least a rough correlation between the faintness of the nebulae and the size of their spectral shifts [16; 17; 18]. Slipher's observations were perhaps the most unprecedented and significant discovery in the whole history of astronomy.

Figure 2. Vesto Slipher (1875–1969) and the Clark 24” Refractor at the Lowell Observatory in Flagsta f, Arizona. Wikimedia Commons and *Clio: Your Guide to History*.

As the 1920s unfolded there were exciting developments in both observational astronomy and relativistic cosmology [11, 12, 13, 20]. Astronomers recognized that the spiral nebulae are very distant objects external to the Milky Way galaxy. Further spectroscopic work by Edwin Hubble and Milton Humason at Mount Wilson established that there is a linear relationship between distance and redshift, a result presented in 1929 in a famous paper [8]. Meanwhile researchers in relativistic cosmology devised various geometric models, “invented universes” in the words of one historian [10]. In 1922 the Russian mathematical physicist Alexander Friedmann published a geometric model of an expanding universe. Although Friedmann did not refer to contemporary astronomical work, it is important to note that he cited writings by the Dutch mathematical astronomer Willem de Sitter and English astrophysicist Arthur Eddington in which Slipher’s observations were discussed. Later in the decade Georges Lemaître developed mathematical models similar to Friedmann’s, but he presented them as actual physical descriptions of the universe.

Eddington was an admirer of the general theory of relativity and its formulation in terms of differential geometry. In his 1920 book, *Space, Time and Gravitation: An Outline of the General Theory of Relativity*, he wrote, “a geometer like Riemann might almost have foreseen the more important features of the actual world” [1, p. 167]. It is then not surprising that Eddington believed that the distance-redshift law, although discovered by astronomers, was foreshadowed in the work of theorists.

In his 1933 book *The Expanding Universe* he wrote, "These observational results are in some ways so disturbing that there is a natural hesitation in accepting them at their face value. But they have not come upon us like a bolt from the blue, since theorists for the last fifteen years have been half expecting that a study of the most remote objects of the universe might yield a rather sensational development" [2, p. 2].

One of the leading pioneers of relativistic cosmology was the American mathematical physicist Howard P. Robertson. In 1949 the volume of essays *Albert Einstein: Philosopher-Scientist* was published to honor Einstein on the occasion of his 70th birthday. Robertson contributed the essay "Geometry as a branch of physics." Although Robertson was primarily concerned here with questions in the foundations of geometry, he did refer to the general theory of relativity. He called attention to the empirical success of this theory, presumably referring to eclipse observations and predictions of the motion of the perihelion of Mercury. However, he put forward the following remarkable assessment: "Einstein's achievements would be substantially as great even though it were not for these minute observational tests" [15, p. 329]. It is difficult to imagine a more complete statement of the theoretical viewpoint than this one.

Historian of science Paul Forman maintains that an historic shift occurred at the end of the 20th century, from a conception of science and theory as primary relative to technology to a conception of technology as primary relative to science [4]. Forman sees the shift from the primacy of science before about 1980 (a primacy which he repeatedly describes as "preposterous") to the primacy of technology after 1980 as the defining feature of a wider shift in the cultural zeitgeist from modernity to postmodernity. In the writings of scientists such as Eddington and Robertson one finds an unequivocal commitment to the primacy of theory relative to technology, of science relative to practice. Both the emphasis on theory among cosmologists from the 1920s through to the 1970s, and the exponential growth of technology and observational work since then are consistent with the cultural and historical schema identified by Forman. By the early years of the new millennium the noted philosopher Don Ihde could assert that "Science is embodied in its technologies, and technologies determine what is science" [9, p. 431].

General relativity and theoretical physics have continued to play a major role in cosmology, a fact that is evident in current inflationary theories of the early universe. The discovery of cosmic acceleration in the 1990s—the most significant event in cosmology since the discovery of the cosmic background radiation in 1964—required theoretical mathematical calculations of the rate of universal expansion. Nonetheless, technological innovation and an embrace of practice is a pervasive characteristic of modern cosmology. Nobel prizes are awarded to observational cosmologists such as Arnold Penzias, James Peebles or George Smoot and not to theoreticians such as Robert Dicke, Alan Guth or Edward Witten. The place of theory has been usurped by technology. Indeed, the eminent astronomer Martin Harwit sees the technological character of modern astronomy as its defining and redemptive feature [7].

Figure 3. Artist concept of the James Webb Space Telescope (2022), an infrared orbiting telescope. NASA.

Cosmology advances today through a myriad of technological tools—radio, optical, infrared, x-ray and gamma-ray telescopes; satellite probes and orbiting observatories; computer modeling and simulation; adaptive optics, LED devices, neutrino detectors and gravitational wave interferometers. The Webb Space Telescope is providing an array of dazzling images and is also probing the structure of the early universe. But for all the prominence of technology, we should in the final analysis remember the role theory also plays in interpreting these findings and reveal- ing the nature of the cosmos.

References

[1] Eddington, Arthur E. (1920) *Space, Time and Gravitation: An Outline of the General Theory of Relativity*. Cambridge: Cambridge University Press.

[2] Eddington, Arthur E. (1933) *The Expanding Universe*. Cambridge: Cambridge University Press.

[3] Einstein, Albert. (1917) Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie. *Sitzungberichte der Königlich Preußischen Akademie der Wissenscha ten zu Berlin*, 142–152.

[4] Forman, Paul. (2007) The primacy of science in modernity, of technology in postmodernity, and of ideology in the history of technology. *History and Technology* 23, 1–152.

[5] Fraser, Craig. (2006) *The Cosmos: A Historical Perspective*. Greenwood Publishers.

[6] Fraser, Craig. (2022) Vesto Slipher, nebular spectroscopy and the birth of modern cosmology 1912–1922. *HOPOS: The Journal of the International Society for the History of Philosophy of Science* 12, 146–169.

[7] Harwit, Martin. (2013) *In Search of the True Universe: The Tools, Shaping and Cost of Cosmological Thought*. Cambridge: Cambridge University Press.

[8] Hubble, Edwin, and Milton Humason. (1929) A relation between distance and radial velocity among extra-galactic nebulae. *Proceedings of the National Academy of Sciences* 15(3), 168–173.

[9] Kaplan, David M., ed. (2004) *Readings in the Philosophy of Technology*. Lanham, MD: Rowman & Littlefield.

[10] Kerszberg, Pierre. (1989) *The Invented Universe: The Einstein-de Sitter Controversy (1916–17) and the Rise of Relativistic Cosmology*. Oxford: Oxford University Press.

[11] Kragh, Helge. (1996) *Cosmology and Controversy: The Historical Development of Two Theories of the Universe*. Princeton, NJ: Princeton University Press.

[12] Kragh, Helge, and Robert W. Smith. (2003) Who discovered the expanding universe? *History of Science* 41, 141–162.

[13] Kragh, Helge, and Malcolm S. Longair, eds. (2019) *The Oxford Handbook of the History of Modern Cosmology*. Oxford: Oxford University Press.

[14] North, John D. (1965) *The Measure of the Universe: A History of Modern Cosmology*. Oxford: Clarendon Press.

[15] Robertson, Howard P. (1949) Geometry as a branch of physics. In *Albert Einstein: Philosopher-Scientist*, edited by Paul Schlipp and Albert Einstein, 314–332. Evanston, IL: Library of Living Philosophers.

[16] Slipher, Vesto M. (1914) Spectrographic observations of nebulae. *Popular Astronomy* 22, 146.

[17] Slipher, Vesto M. (1915) Spectrographic observations of nebulae. *Popular Astronomy* 23, 21–23.

[18] Slipher, Vesto M. (1917) Nebulae. *Proceedings of the American Philosophical Society* 56(5), 403–409.

[19] Smeenk, Christopher. (2014) Einstein's role in the creation of relativistic cosmology. In *The Cambridge Companion to Einstein*, edited by Michel Janssen and Christoph Lehner, 228–269. Cambridge: Cambridge University Press.

[20] Smith, Robert. (1982) *The Expanding Universe: Astronomy's 'Great Debate' 1900–1931*. Cambridge: Cambridge University Press.

Craig Fraser is a professor emeritus at the University of Toronto. His primary area of research is the history of mathematical analysis and dynamics from the 18th to the early 20th centuries. A secondary focus of interest is the history of astronomy and cosmology in the 20th century. Fraser is on the editorial board of the journal Historia Mathematica. *He is the immediate past president of the Canadian Society for History and Philosophy of Mathematics and is former chair of the International Commission on the History of Mathematics. His most recent publication (with Michiyo Nakane) is "Canonical Transformations from Jacobi to Whittaker,"* Archive for History of Exact Sciences *(2023).*

Notes on Transitions from Arithmetic to Mathematics

Education Notes

John Mason (Open University & University of Oxford)

Education Notes bring mathematical and educational ideas forth to the CMS readership in a manner that promotes discussion of relevant topics including research, activities, issues, and noteworthy news items. Comments, suggestions, and submissions are welcome.

Kseniya Garaschuk, *University of Fraser Valley (kseniya.garaschuk@ufv.ca)*
John Grant McLoughlin, *University of New Brunswick (johngm@unb.ca)*

The content here augments the Powerpoint (PPT) slides of a presentation to The Fields Institute Mathematics Education Research Forum in January 2023. The slides can be viewed via the link here: http://www.pmtheta.com/jhm-presentations.html

The session title was

School Arithmetic is to Mathematics,
as Making Sounds is to Music:
some pedagogically supported transitions
from arithmetic to mathematics

My starting point is that arithmetic (calculations with numbers) is NOT in itself mathematics. Mathematics is the study of relationships, so arithmetic becomes mathematical when the objects of study are relationships between numbers and beyond.

Making sounds is making sounds; when sounds follow in some sort of sequence, they may become or may be experienced as a tune, and when they are made together, they may be experienced as harmony. Both tunes and harmony are relationships between sounds.

So too, school arithmetic is concerned with naming numbers and performing calculations, accurately and swiftly. But that is analogous to making sounds. What really matters, what created mathematics, is recognition and study of relationships between numbers.

Given this opening task, what is your immediate action?

$48 + 69 – 49 – 68 = ?$

People o ten start calculating: 48 + 69 then subtract 49 etc. They enact the first action that becomes available to them. An alternative is to become aware at some point, perhaps as they sub-vocally read the numbers, that there is a +40 and a –40, a +60 and a –60 etc. so that the answer is 0, without explicit calculation. Pedagogically and mathematically this is the use of pausing to consider alternatives to the first available action, in case there is something more efficient just below the surface.

A second task followed on:

748 + 369 – 769 – 348 = ?

Alerted to looking out for relationships, you might immediately see that the answer is again 0. The reason is that each digit appears in the same tens-place in both a + and a – number. But to do this requires either a sensing of a relationship while sub-vocally reading, or intentional movement of attention between details of the numbers, seeking out relationships.

A third task followed

748 + 369 + 251 – 761 – 358 – 249 = ?

> Something similar is available. I then asked people to construct their own task 'like this'; then another; then another. Well aware that this sort of task was unlikely to challenge the people present, I proposed that their aim in the session could be to try to catch how their attention shi ts, and how the way in which they attend, shifts. For it seems to me that learning mathematics is essentially about learning what to attend to, and in what ways.

Interlude

Methodological Stance

I actually began the presentation with some remarks about my fundamentally phenomenological stance, emphasising that what was available from the session was what people noticed about how their attention shifts, in relation to the 'attention conjecture':

> *when teacher and learners are attending to di ferent things, and even when attending to the same things but di ferently, communication is likely to be impoverished.*

Put another way, breakdowns in classrooms may o ten be due to di ferent people attending to di ferent things or attending to them differently. For example, while the teacher is attending to an example as an instance of a general property, learners may be attending to specific details in the example, or to relationships between those details. What the teacher says may not connect with or make sense to the learners. Consequently as a teacher it is vital not only to be aware of what I am attending to, and how I am attending to it, but also what pedagogic actions could be invoked in order to direct learner attention appropriately.

The session involved a sequence of tasks which can be found in the PPT. These notes act as re lections on the experience of undertaking them.

Attention

I have found the following distinctions useful in attending to how I am attending to things mathematically:

> *Holding Wholes (gazing)*
>
> *Discerning Details (which can then become wholes for further gazing)*
>
> *Recognising Relationships (amongst discerned details; amongst relationships; …)*
>
> *Perceiving Properties as being instantiated*

> *Reasoning on the basis of agreed properties*

The shift from recognising a relationship to perceiving it as an instance of (a more general) property may be the single most important experience to make mathematics engaging and learnable. Captured in the slogan 'seeing the general through the particular', and its converse, 'seeing the particular in the general' (Mason & Pimm 1984), these shi ts can reveal mathematics as a constructive, creative human endeavor, rather than a collection of procedures to internalise, and in particular, can turn the tedium of arithmetic into the wonder of mathematics.

The overall structure of the session is to remind people of three pedagogic actions which can help ease the transition from arithmetic to algebra, bearing in mind the observation of my friend and colleague Dave Hewitt, that in order to do arithmetic, you have to think algebraically.

Tracking Arithmetic

In the PPT I used two different contexts to illustrate the principle of tracking arithmetic, which was inspired by the writing of Mary Boole (Tahta 1972). The idea is to choose one or more parameters in a task and to isolate them from calculations, so that their presence is constantly visible. Once the calculations are finished, each parameter is exchanged for a symbol: at first, a single parameter is replaced with a little cloud, representing 'the number that someone (I usually refer to my wife at this point) is thinking about'. The notion, the experience, of generality is immediate. Doing this a few times is rarely problematic in classrooms, and using the cloud has helped me show algebra refusers that there is nothing frightening or abstract about algebra.

A third context in the PPT analyses an ancient Egyptian task, showing how I think it was meant to be used with learners as an instance to be generalised. I use tracking arithmetic to achieve the intended generalisation.

Expressing Generality

For me, algebra is about expressing and manipulating generality, despite most textbooks since the 15th century describing algebra simply as 'arithmetic with letters'. Questions such as

> *Why would you manipulate letters? When will that be of value to me?*

lie at the heart of algebra-refusing. Algebra has, over the span of my career, been the principal watershed in mathematics for learners, with fractions a close second. Experiencing the expressing of generality, not just a few times, but on every possible occasion, helps to internalise expressing generality as one of the things that mathematicians do, because of the power it unleashes. Individual exercises turn into classes of problems with a common approach (the generality, of which each is an instance). I have been known to claim that

> *a lesson without the opportunity for learners to express mathematical generality is not a mathematics lesson.*

Multiple Expressions of the Same Generality

Algebraic manipulation arises for me because when di ferent people express the same generality, they o ten express it quite differently. Different people discern different details and hence different relationships. There ought to be a way to go between expressions, without having to resort to the original situation. In other words, the rules for manipulating algebra turn out to be the same as the rules for manipulating arithmetic and so provide a taste of perceiving properties which are instantiated in calculation, whether arithmetic or algebraic.

In the PPT I use a slightly unfamiliar context, namely hexagons, and because I was inviting people to catch shifts in their attention, I chose to display several ways of expressing the same generality rather than inviting people to spend time expressing that generality for themselves. Both pedagogies call upon shifting attention of course, but I wanted to remind participants that there are different pedagogical actions that can be initiated.

The task is to express how many hexagons would be required to surround a display of r rows and c columns of hexagons, as illustrated in these two examples.

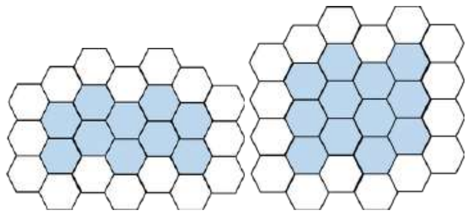

Figure 1: Hexagonal arrays with 2 rows and 5 columns, and with 3 rows and 4 columns

I anticipated that time would need to be spent negotiating and coming to terms with the notion of an 'array' of hexagons with r rows and c columns. Recognising the 'presence' of arrangements of shaded hexagons conforming to 2 rows and 5 columns in the first diagram, and 3 rows and 4 columns in the second is likely to lead to considering what is the same and what is different about the two diagrams, about the specified number of rows and columns in each and hence about the relationships which determine what an array is.

Notice that in the first diagram the columns rise and fall alternately, while in the second, they fall and rise alternately. This a dimension of possible variation which leaves the overall notion of an 'array' invariant. Once a sense of array is established, these two diagrams can be seen as instances of the property of 'being an array' of hexagons. Drawing attention to the action of considering what is the same and what is di ferent, or what is allowed to change and what not, are key pedagogic actions which, once internalised by learners, become mathematical actions which they can enact for themselves in the future.

Interestingly some people wondered whether the fact that 2 + 5 = 3 + 4 had any relevance (I had not noticed it in preparation), illustrating how different people attend to different things, and how, if the teacher is present, there are ongoing issues of when and how to intervene in order that learner attention is directed in fruitful directions.

I then initiated the pedagogical action of inviting participants to make sense of various expressions of generality. An alternative would have been to invite participants to express their own generalities and to illustrate these with shadings, but I knew this would take longer, and my concern here was in providing opportunities to catch shifts of attention within limited time. Furthermore I wanted to emphasise the need to check expressions on other examples, and to develop a narrative which justifes the conjectured expression.

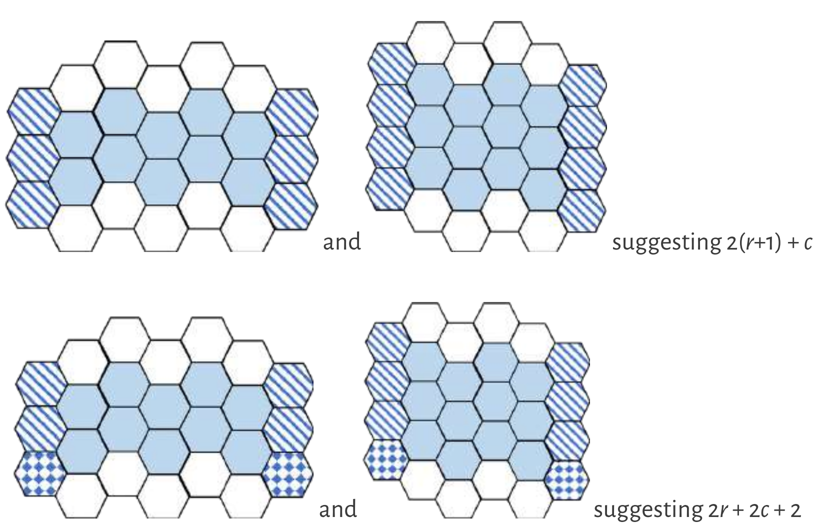

Figure 2: two pairs of 'seeings' to be interpreted as expressions of generality.

For example, in Figure 2, the crosshatched hexagons on each side of the rst pair of arrays are one more than the number of rows, because of the way hexagons pack. The white hexagons correspond to the columns of the array. This action constitutes 'reading an expression in the context of the situation', and so justi es the expression in general by seeing the general through the particular. This is the essence of tracking arithmetic in shi ting from recognising relationships between discerned details, and perceiving a property as being instantiated in the particular examples.

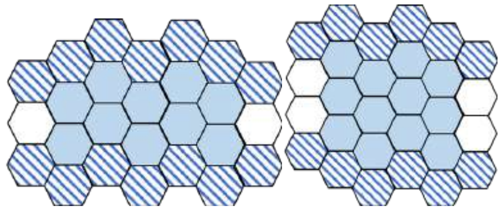

Figure 3: a further pair of shadings

I intended the first diagram in Figure 3 to illustrate how easy it is to be misled by what is fixed and what is variable when expressing generality. Looking only at the 2 by 5 array it might be tempting to conjecture $2(c+2)+2$ for the number of bordering hexagons, seeing the white hexagons as fixed rather than depending on the size of the columns of the array. Looking at the second diagram as well, however, reveals that the final 2 ought instead to be $2(r-1)$ in general. It is never sufficient simply to 'express a generality'. It is essential to treat it as a conjecture which has to be justified with some sort of a narrative linking the situation with the expression. Hence the importance of personal narratives for establishing and for beginning the internalisation of a way of thinking, and the value in checking against a further example can both be seen.

The PPT slides have two further opportunities for expressing generality and for equating different expressions as motivation for algebraic manipulation, all involving hexagons.

Mathematical Version of Tunes and Harmony

Sundaram's Conjecture

Sundaram's grid is one of my favourite contexts for inviting recognition of relationships, expression of generality, and the use of manipulation to verify a conjecture. The situation has low threshold (I have used it with primary teachers) and high ceiling (different directions for exploration and generalisation).

28	47	66	85	104	123	142	161	180	199
25	42	59	76	93	110	127	144	161	178
22	37	52	67	82	97	112	127	142	157
19	32	45	58	71	84	97	110	123	136
16	27	38	49	60	71	82	93	104	115
13	22	31	40	49	58	67	76	85	94
10	17	24	31	38	45	52	59	66	73
7	12	17	22	27	32	37	42	47	52
4	7	10	13	16	19	22	25	28	31

Figure 4: Sundaram's original grid

Figure 4 shows a grid of numbers in which each row and each column form arithmetic progressions. This means that the invitation is to see the grid as extending effectively infinitely both to the right and up. Sundaram's claim (Honsberger, 1970; Ramaswami Aiyar, 1934) is that if you take the entry in any cell, double it and add 1, the result will be composite (not prime).

In order to justify his conjecture, it is necessary to find an expression for the entry in the *r*th row and the *c*th column, and then to show that doubling and adding 1 leads to an expression which factors non-trivially. Indeed, treating the grid as effectively infinite in all directions, Sundaram's conjecture can be shown to hold everywhere (extending to the left and down as well) except in one or two specific rows and columns.

Posing your own problem is usually much more interesting than responding to someone else's challenge. Stop for a moment and see what further questions come to mind.

SPOILER ALERT! I asked myself how many entries and in what positions can be specified in a grid so that it can be completed to a unique Sundaram- like grid in which each row and each column is an arithmetic progression. (Notice the shi t from recognising relationships to perceiving a property, and then considering other instances.) How would the Sundaram conjecture have to be modified for other Sundaram-like grids? Also, select any four cells on the vertices of a parallelogram, and consider the difference between the sums of diagonally opposite cells of the parallelogram. How is this related to the size of the parallelogram?

An applet which makes it possible to construct different Sundaram Grids, formulate and check Sundaram-Conjectures and check the parallelogram property is available with the PPT at the website given above.

Series and Parallel Arithmetics

I wanted to provide something that would challenge sophisticated mathematicians in the audience, well aware that I might not have time to get to them in the session.

Challenge

The analogy between sound is to music as arithmetic is to mathematics brought to mind the notions of series (as in tunes) and parallel (as in harmony). It turns out that there are actually two completely parallel arithmetics of fractions, well worth exploring, and deserving of much more care than there is space for here. (See Ellerman web references for elaboration.) They arise in traditional Medieval and Victorian word problems based on a multiplicative relationship such as

> *distance = speed x time*
>
> *(number of objects) = (objects per person) x (number of persons)*
>
> *Voltage = current x resistance*
>
> *Volume = low x time*

Suppose then that $p = r \times a$, read as "it takes an amount *a* at rate *r* to produce *p*" or as "*p* is produced from *a* due to a resistance of *r*".

Consider the following situations:

> *If r1 and r2 are happening together for the same amount a, then p = (r1 + r2)a is the combined effect achieved, so the combined rate is r = r1 +r2 and this is the familiar addition of rates (fractions) known as series (ordinary) addition.*

If r1 and r2 are working jointly to achieve a fixed p, $\frac{p}{r_1}$ *and* $\frac{p}{r_2}$ *are the corresponding amounts then required to produce p individually.*

Working together, $p = r\left(\frac{p}{r_1} + \frac{p}{r_2}\right)$ *which makes the joint (parallel) rate* $r = \frac{1}{\frac{1}{r_1} + \frac{1}{r_2}}$ or $\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2}$.

If a1 and a2 are each required to achieve p separately, then they operate at rates of $\frac{p}{a_1}$ *and* $\frac{p}{a_2}$ *respectively.*

Working together to achieve p, $p = \left(\frac{p}{a_1} + \frac{p}{a_2}\right)a$ *so*

together they need an amount $a = \frac{1}{\frac{1}{a_1} + \frac{1}{a_2}}$ *to achieve p together. These are known as parallel addition, by analogy to electrical resistance.*

Pedagogically, considerable time would of course be required working with specific multiplicative relationships and the associated discourse in order to internalise these actions. It is the heart of many ever-popular word problems.

A mathematical move is to perceive parallel addition as a property, and to explore the arithmetic arising from using it as the 'addition' in a ring (two binary operations with standard arithmetic properties).

Denoting 'parallel addition' by $r_1 : r_2$ then

$r_1 : r_2 = \frac{1}{\frac{1}{r_1} + \frac{1}{r_2}}$ *expresses parallel addition in terms of series addition*

$r_1 + r_2 = \frac{1}{\frac{1}{r_1} : \frac{1}{r_2}}$ *expresses series addition in terms of parallel addition.*

What is perhaps somewhat surprising is that parallel addition of fractions (rates; ratios) satisfies all the properties of series (ordinary) addition, such as commutativity, associativity, and distributivity of multiplication over the addition. So there are opportunities to get a glimpse of mathematical structures as sets of objects satisfying certain properties.

Closing Remarks on Research in Mathematics Education

My last two slides (there was only time to show one of these) raised some concerns and observations about research in mathematics education.

First, what is research in mathematics education for? Whom does it serve?

Obvious responses include

> *Academic careers?*
>
> *Publications are the primary basis for appointment and promotion*
>
> *Path of personal development?*
>
> *As a practitioner, trying to make more coherent sense of my practice and its implications for learners*
>
> *Improving the experience of learners?*
>
> *Surely this is the originating force to engage in research, though the others may come to dominate*
>
> *Classifying learners and classifying situations?*
>
> *Each theoretical frame consists of a collection of distinctions (eg levels, or stages, or competencies, or achievements, or what is noticed, or …), leading to assessment and evaluation of both learners and teachers*

The result of an extensive body of observations and studies is the growth of theories. What is the role of theories?

> *Making predictions?*
>
> *Theories are generally expected to make predictions: if such and such conditions are present, then such and such is likely (will?) be the outcome.*
>
> *Informing choices?*
>
> *Through recognising specific details, suitable pedagogical and mathematical actions may become available to enact*

I see frameworks as sets of labels for distinctions which can be made by an observer. Frameworks very often acquire the label 'theory', meaning that authors and researchers seem to be saying that the distinctions ARE what is going on, rather than simply possible distinctions to be made by an observer. I am mindful of Humberto Maturana's famous adage: "everything said is said by an observer "(Maturana 1988).

The question is whether making those distinctions actually makes a difference (Bateson 1973) in how the teacher acts, and hence how the learners' experience is enriched. That is why I adopt a fundamentally phenomenological stance, concerned with the lived experience of teaching and doing mathematics. Given the complexity of human beings, aspiring to make predictions seems to me to overlook the essential humanity of teachers and learners. Frameworks of distinctions are what I find useful.

I have long maintained that an architectural image of mathematics education is not appropriate: research does not contribute to building a structure of 'knowledge'. This is evident from looking at the topics of research papers over the last 50 years. The same topics come up, o ten but not always cast in fresh discourse. Certainly each generation has to recast insights of the past in its own vernacular. But mathematics education for me is a context for personal development, with the underlying assumption that developments in learners' experience will follow as a consequence. It is not a matter of replacing old insights with fresh and more precise ones, but rather that each teacher has to develop their teacherly-self with their own sensitivities to notice, with associated mathematical and pedagogical actions to initiate. It is a matter of developing a positive relationship between the teacher, the content (mathematics and mathematical thinking), and the learner.

The PPT ends with a long list of my own publications that kept coming to mind as I prepared the session. These are but a drop in the ocean of useful and insightful observations of many different authors. But what matters to me is not the 'body of knowledge', but rather the development of the mathematical being of each teacher, each learner, and each researcher.

References

Bateson, G. (1973). *Steps to an Ecology of Mind*. London: Granada.

Ellerman. (accessed Jan 2023). www.ellerman.org/series-parallel-duality-part-i-combating-series-chauvinism/

Ellerman. (accessed Jan 2023). www.ellerman.org/sp-duality/

Honsberger, R. (1970). *Ingenuity in Mathematics*. New Mathematical Library #23. Mathematical Association of America. p 75.

Mason, J. (accessed Jan 2023). www.pmtheta.com/jhm-presentations.html

Mason, J., & Pimm, D. (1984). Generic examples: seeing the general in the particular. *Educational Studies in Mathematics*. 15. 227-289.

Maturana, H. (1988). Reality: the search for objectivity or the quest for a compelling argument. *Irish Journal of Psychology*. 9 (1) p25-82.

Ramaswami Aiyar, V. (1934). "Sundaram's Sieve for Prime Numbers". *The Mathematics Student* **2** (2) p73.

Tahta, D. (1972). *A Boolean Anthology: selected writings of Mary Boole on mathematics education*. Derby: Association of Teachers of Mathematics.

Author's email: john.mason@open.ac.uk

Keywords: Shifts of Attention; Noticing; Tracking Arithmetic; Expressing Generality; origins of algebraic
manipulation

How to Support Students and Colleagues Experiencing Abusive Relationships

MOSAIC

Karen Meagher (University of Regina) *Chair, Women in Mathematics*

Preamble

This MOSAICS article is a hard, but important topic. Last September, for the NSERC Science Literacy Day, I was part of a group that organized an event called "Careers in Mathematics: They're Everywhere!" We invited several women with interesting careers in math to talk about their experience and give younger folks career advice. These women spoke about their different paths and the different challenges they faced, this included some very personal and very hard conversations.

One of our speakers shared her experience surviving a violent abusive relationship with her supervisor many years ago when she was a student. She has long since left the relationship and has gone on to have a tremendously successful career. But it was important that she share this story because people who are in abusive relationships need to know that they can get out and thrive afterwards. Our speaker noted that one thing that made the whole situation even worse was that her colleagues didn't realize what was going on.

Professors, and other people in leadership roles, need know how common these situations are, and they need to know how best to support a student or colleague in an abusive relationship. So, for this MOSAICS article, I did an interview with Lynn Thera, Master of Social Work, Registered Social Worker. Lynn is the Coordinator, Sexual Violence Prevention and Response, at the University of Regina Sexual Violence Prevention and Response Office. We talked about sexual violence and abusive relationships with a focus on what happens on campuses across Canada, with a focus on what professors need to know and how to support students going through this. There are also comments from a student about what approaches helped her when she was leaving an abusive situation. This is a difficult topic, but with a little bit of knowledge we might be able to help a student in need.

Interview

Me: How prevalent is intimate partner violence?

Lynn: When we look at sexual violence, we're looking at about one in four.

Me: How prevalent are sexual assault and harassment on campuses?

Lynn: The latest statistics, I think it's four years old now, over 70%, 72%, I believe, of individuals that go to university identify that they've either experienced or witnessed sexualized behaviours or unwanted sexualized behaviour.

Me: That's an astoundingly high number.

Lynn: Yeah, when people hear that number, they think that's not a possibility, like there's no way. But the truth of the matter is that those are the numbers, and if you come from within the LGBTQ2s, BIPOC communities and people with disabilities will experience more violence.

Men also experienced intimate partner violence and we don't really have a good understanding of the numbers. What we do know is that more women will end up hospitalized or dead as a result of violence, so the kind of violence is different. [Women] are more likely to fear for our lives in intimate partner violence.

Me: That's interesting. I had a friend who had an issue where there was sexual harassment, and some male colleagues didn't take it seriously. I wonder if they didn't understand the type of violence and were interpreting it as what would happen to them.

Lynn: One hundred percent. When we look at the way women walk the world, it's different than the way men do, just simply because we're raised to know that the world isn't safe for us. We know people that have been sexually assaulted or have experienced intimate partner violence.
When we look at sexual harassment we all know situations where we feel powerless because there's a power differential. We have a society that's based understanding of violence on a bunch of myths that aren't really true. [Those myths] create victim blaming or the silencing of individuals that experience violence. We often blame the survivor and say, "What did she do?" or, "She's just being silly. He didn't mean it that way." Those are all comments we use to silence people.

Me: That sounds terrible, what services are available on most campuses? And how are you a part of that?

Lynn: We have the Sexual Violence Prevention and Response Office, and so does almost every other university. Part of my job is education; the other half of my job is supporting individuals that have experienced sexual assault or intimate partner violence.

Me: Are university policies effective? We know that power structures are very important, and understanding the context is important. It's hard to encode that into rules and regulations.

Lynn: When I came in, one of the first jobs I had to do is create a new sexual violence policy. The good thing about this new sexual abuse policy Sexual Violence/ Misconduct is that it mentions prohibited relationships and that means certain relationships are considered prohibited because of the power differential between students and professors, supervisors, and coaches. These relationships can be had but there needs to be approved by supervisors- the policy aids in clearly exploring the power differentials in certain relationships. Policy is that people with power — that means professors — have to understand their privilege and their power. And you can't have a [romantic] relationship with your graduate student because you have to understand your power and privilege.

Me: Is it standard for universities across Canada to have a policy that explicitly explores power differentials?

Lynn: It's fairly new, but it's becoming more of a thing that's being included.

We look at what happened with the Me Too Movement and that sort of the voicing of the violence women are experiencing what is actually being seen. Things that used to be silenced are no longer being silenced, so no, it's no longer acceptable.

Me: Are there signs or indicators that a professor could use to spot if a student or colleague needs help?

Lynn: It's really important to recognize that part of the violence is keeping secrets. That's part of the reason it keeps going. So, you might see signs, but it's not going to be like bruising. What you might find is the person will say that they have to be home at a certain time, or their partner is looking for them; so you might see controlling behaviours.

Most people in abusive situations feel that they're not being seen. So what I say is, don't go to them and say, "Hey, I think you're in an abusive situation." Your best bet is to say things like, "Do you need anything? If you ever need anything, let me know." A simple comment like, "Are you okay, is there anything I can do for you?" is enough for that person to see there's an opening for when they need it. People won't necessarily come to you when you think they will. They might come a year later.

Me: So what resources are available on this campus or most campuses across Canada for people who are in abusive relationships or experiencing gender based violence?

Lynn: There is this office [the University of Regina Sexual Violence Prevention and Response Office] and counseling services, I can connect them to residences or shelters. Shelters can help with social systems and the legal issues that come with domestic violence. The Student Union is also a good bet, but there's not a lot of extra funding.

Me: What advice would you have for a student or staff member if they have a problem?

Lynn: I'm here for students with a problem and most of our offices are the same way. We understand the complexities of intimate partner violence. We understand the complexities of a sexual assault. I get a lot of professors that will do a soft referral so they'll tell the student to "go talk to the Sexual Violence Prevention and Response."

Me: How effective is the campus reporting process, and what can be done to make it more effective?

Lynn: Well, I think that the system works really well. The problem is our society. 98% of people who have experienced violence will not go through legal systems. And I'm not saying people should go through the system. I'm just saying we're silenced because we're often blamed.

Me: What sort of damage is done with abusive behaviour and sexual harassment and gender based violence?

Lynn : There's a lot of damage that can be done if you feel like you're not being taken seriously because you're a woman. Or if you feel uncomfortable with somebody because they're making comments. You tend to become quieter; you don't feel like you belong.

When you feel unsafe in an environment, you feel less likely to communicate and you are less likely to talk about what's going on. If you experience sexual violence when you're in university you're likely to nish your degree or you will take an easier degree, or your marks will su fer. This has a long-term e fect on your ability to make money later in life.

Me: Any final comments?

Lynn: The world is changing. We need to start giving voice to these things that are happening. I think more people are aware of the problems and are trying to support those that have experienced violence. Things that were acceptable aren't going to be acceptable or aren't acceptable and are being called out.

Side bar

Below are some comments from a student Nita* (name changed to protect her privacy). Nita recently completed her PhD in math, but in the first year of her program, she left an abusive marriage. Below are her comments about what helped, what is still needed, and advice to people in a similar situation.

Things that helped

> *When I told my department head, the first thing he asked me was if I needed to take a break and gather myself. It is a big support when your advisors or course instructors address that you are going through a rough patch and are willing to give you some time to catch up. Unlike some who tell you to downgrade because you are not fit for a PhD.*
>
> *When I left my home, I was homeless but immediately received emergency funding from Graduate Studies. This helped a lot with my rent for the first few months and kept me from sleeping in my office. I appreciate the emergency funds and/or scholarships for anyone who is getting out of domestic abuse.*
>
> *I liked the security at university, campus security made me feel safe. I did not want to leave campus for the rst few months. In my case, my ex and his family tried showing up during my lectures and I had to call campus security twice to escort me to my room.*
>
> *The counselling services at university, which are free for students, were really helpful. The women's centre on campus is a safe space where you are surrounded by women going through similar things, and you get emotional support without any judgement.*

What is missing

> *I called all women's shelters in my city and they did not have an opening for another six months. I think an emergency shelter, or rooms in residence at the university for students getting out of domestic abuse would be a great initiative, especially if it could be available at a lower price.*

Advice to Others

> *Talk to a counsellor if you are confused about leaving an abusive partner.*
>
> *It is important to let your advisors in on what is going on. If the abuse is coming from your advisor, then opening up to another professor, head or woman in the department whom you trust and who can guide you.*
>
> *I have now learned that there are several resources and services on campus such as the women's centre, counselling services, wellness centre, and emergency bursaries from student's a fairs which are a godsend to students in need.*
>
> *Make friends. My biggest support was my group of friends who stood by me for four years. I don't know if I would have been able to get through this without them.*

Call for Submissions: CMS Notes Mathematics, Outreach, Society, Accessibility and Inclusiveness Column (MOSAIC)

MOSAIC

The Canadian Mathematical Society (CMS) invites you to submit articles to be featured in the MOSAIC column of the CMS Notes.

MOSAIC (Mathematics, Outreach, Society, Accessibility, and Inclusiveness Column) is directed by the CMS Equity, Diversity, and Inclusion (EDI) committee.

The column offers a space of expression for you to ask, listen, learn, share experience, and propose solutions to build a more diverse, just, and stronger mathematical community. For instance, you are welcome to submit an article sharing challenges and successes in enacting EDI initiatives within your university, with competitions, outreach activities, or other events.

Your email submission should include your article in both Word and PDF formats. Please submit your article to the EDI Committee at mosaic@cms.math.ca

Canadian Mathematical Society
Société mathématique du Canada

2023 CMS Winter Meeting | Montréal, Quebec

CALL FOR Sessions

The Canadian Mathematical Society (CMS) welcomes and invites session proposals and mini-course proposals for the 2023 CMS Winter Meeting in Montréal from December 1-4, 2023.
In accordance with the CMS mandate to propose conferences that are accessible and welcoming to all groups, diversity amongst organizers and speakers is strongly encouraged. Diversity includes topics of interest, career stages, geographic location, and demographics.

CALL FOR SESSIONS:

Proposals should include:
(1) Names, affiliations, and contact information for all session co-organizers. Early career researchers are encouraged to propose sessions.
(2) A title and brief description of the topic and purpose of the session. This can include an overview of the subject.
(3) The total number of expected talks, with a list of possible speakers and/or papers in the theme. Sessions should strive to respect the above CMS policy of accessibility and diversity. These groups include, but are not limited to, women, Indigenous Peoples, persons with disabilities, members of visible minority/racialized groups, and members of LGBTQ2+ communities. Intention to include new PhD's and to make the session accessible to graduate students is also encouraged.

Open Call for Abstracts: The CMS will continue the open abstract submission process that was recently introduced to support session organizers in their important work and in their efforts towards inclusivity and diversity.

The CMS kindly asks session organizers to consider all eligible abstract submissions for their session, as up to 30 speakers per session can be accommodated.

The scientific sessions will take place from December 2-4, 2023.

Deadline: Proposals should be submitted by **Monday, July 31, 2023** to the Scientific Directors and the CMS Office should be cc'ed. There will be a second deadline of **September 1, 2023**, but earlier submissions will be considered first. Their contact information is as follows:

François Bergeron : bergeron.francois@uqam.ca
Simone Brugiapaglia: simone.brugiapaglia@concordia.ca
Alina Stancu: alina.stancu@concordia.ca

Sarah Watson: meetings@cms.math.ca

Canadian Mathematical Society
Société mathématique du Canada

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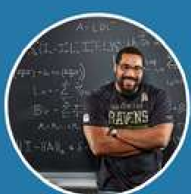

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Call for University Hosts: Winter '25 / Summer '27

The Canadian Mathematical Society (CMS) welcomes and invites host proposals from Canadian Universities for the 2025 CMS Winter Meeting, and the CMS Summer Meetings for 2027.

CMS will provide all logistical support and contract negotiation with local venues. CMS is looking for Canadian Universities that are willing and able to showcase their department and University to students and faculty from across Canada. It is asked that proposals include the following information:

1. Location

- How would people get from the airport to the venue?
- What are the reasons your city may be of interest to Canadian Mathematicians?

2. Site

(For summer meetings) Describe the University where the meeting would be held.

- Which building would the meeting be in and how many rooms are available for meeting sessions and plenaries? What technological support is available in session rooms?
- Will these rooms be available during the proposed dates?

(For winter meetings) Do you have a venue in mind for the meeting, is your University available to host the meeting onsite? If not, CMS will find a property outside the university.

3. Lodging

Is your university able to offer any residence lodging during the conference dates? CMS will take care of contracting and negotiating with hotels.

4. Host University

Please describe your institution and department briefly.

- What funding support will the Host University have for the CMS Meeting?
- Is the University available for regular calls and updates on the meeting's progress?
- Can the Host University commit and provide at least one scientific director for the meeting? What level of participation do you think there might be from academics at your institution?

The CMS Meetings typically run from Friday to Monday on the first weekend in June and December but we are open to other possibilities. Summer meetings typically have 250-350 registrants and winter meetings typically 400-600 in larger cities. Please admit your submissions to Sarah Watson (meetings@cms.math.ca).

Call for Speakers

2023 CMS Summer Meeting

The Canadian Mathematical Society (CMS) invites you to submit an abstract to participate in one of the **planned sessions at the 2023 CMS Summer Meeting** which will take place from June 2-5, 2023.

The Canadian Mathematical Society has created an open abstract submission process to support session organizers in their important work and in their e forts towards inclusivity and diversity. We encourage applications from members who identify as part of traditionally under-represented groups, including, but not limited to: Women, Indigenous Peoples, Persons with Disabilities, Members of Visible Minorities and/or Racialized Groups, and members of the LGBTQ+ Community. The CMS also welcomes applications from Graduate Students.

Deadlines

Applicants must submit their abstracts for approval to the session organizers using our abstract submission form no later than **Friday, March 31, 2023**. Please submit your abstract to only one session.

The CMS encourages organizers to review submitted abstracts on an ongoing basis and to accept all eligible speakers. Successful applicants must register for the meeting and submit their abstract to the CMS website by **Monday, May 1, 2023.**

Call for Nominations for the 2023 Fellows of the CMS

The Fellowship recognises CMS members who have made excellent contributions to mathematical research, teaching, or exposition; as well as having distinguished themselves in service to Canada's mathematical community. In exceptional cases, outstanding contributions to one of these areas may be recognized by fellowship.

The CMS aims to promote and celebrate diversity in the broadest sense. We strongly encourage department chairs and nominating committees to put forward nominations for outstanding colleagues regardless of race, gender, ethnicity and sexual orientation.

Nominations should include a reasonably detailed rationale and be submitted by **March 31, 2023**.

All documentation should be submitted electronically, preferably in PDF format, by the appropriate deadline, to **awards-prizes@cms.math.ca**

For more information on this award, please visit: https://cms.math.ca/awards/fellows-of-the-cms/