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The history of mathematics is plural; thus, mathematicians have expressed differing views about what mathematics is and whether it has changed over time. For instance, contrast Henri Poincaré's 1908 statement:

If we wish to foresee the future of mathematics, our proper course is to study the history and present condition of the science [7, p. 19].

with the 1938 reflection of Jean Cavaillés, a French philosopher of mathematics:

The mathematician does not need to know the past, because his vocation is to refuse it . . . in the measure where he rejects the authority of the tradition, does not recognize an intellectual climate, in this measure alone, he is a mathematician [quoted in 3, p. 5, translated by the author].

or with Gaston Bachelard's observation:

A truly new mathematical idea is also an immediate reorganization of all the ancient ideas [quoted in 3, p. 5, translated by the author].

Although these quotations sum up to only a few sentences, they reveal the complexity and variety of opinions on the history of mathematics held by scholars. In the remainder of this column, we offer several additional examples for readers to ponder. We close by suggesting several resources for delving more deeply into the nature of mathematics in the past, present, and future, as well as for contemplating relationships between history of mathematics, mathematical research, and the teaching and learning of mathematics.

The Bourbaki group was created in France in 1934 by Henri Cartan, Claude Chevalley, Jean Delsarte, Jean Dieudonné, André Weil, Jean Coulomb, René de Possel, Charles Ehresmann, and Szolem Mandelbrojt. Over the next decades, they published collectively *Éléments de mathématique*, a series of modern textbooks in mathematics. They collected their notes about the history of mathematics in their 1960 *Éléments d'histoire des mathématiques*, issuing the following warning:

> Finally, the reader will not find in these notes practically any biographical or anecdotal information on the mathematicians we are talking of; we have mainly looked for and emphasized each theory as clearly as possible [1, p. iii, translated by the author].

Figure 2. Cartan, de Possel, Dieudonné, Weil (standing), Mirlès, Chevalley, and Mandelbrojt (seated) at the first official meeting of the Bourbaki group in 1935. MacTutor.

Dieudonné went on to edit a two-volume "abbreviated" history of mathematics (*Abrégé d'histoire des mathématiques*) in 1977. Although his project was also of most interest to pure mathematicians who were not historians, he adopted an approach that was less rigid than that stated by Bourbaki. Specifically, he no longer equated mathematics with only abstract concepts and rather saw it as unfolding within a human context:

> No more than the other sciences (and despite its reputation of abstraction), mathematics is not a disembodied science, and it would be absurd to separate completely a history of ideas from that of the men who introduced them. An annex at the end of the volume gives some biographical indications about most of the mathematicians quoted during the path of the text [4, translated by the author].

Indeed, in 2024 most mathematicians would deem it essential to associate Leonhard Euler or Jean D'Alembert with the period of the Enlightenment, for example, or to link Augustin-Louis Cauchy's royalist ideas to his career and research.

Let us see how the American Morris Kline presented his notion of the history of mathematics in his very popular three-volume history, first published in 1972:

> This book treats the major mathematical creations and developments from ancient times through the first few decades of the twentieth century. It aims to present the central ideas, with particular emphasis on those currents of activity that have loomed largest in the main periods of the life of mathematics and have been influential in

promoting and shaping subsequent mathematical activity. The very concept of mathematics, the changes in that concept in different periods, and the mathematicians' own understanding of what they were achieving have also been vital concerns [5, preface].

For Dirk J. Struik, mathematics was a vast adventure of ideas, with its history reflecting some of the noblest thoughts of countless generations. Yet, in his 1967 *A concise history of mathematics*, he confessed his difficulty with fulfilling the role of historian:

> The selection of the material was, of course, not based exclusively on objective factors, but was influenced by the author's likes and dislikes, his knowledge and his ignorance. As to his ignorance, it was not always possible to consult all sources first-hand; too often, second- or even third-hand sources had to be used [8, p. 1].
>
> Our story ends around 1945, for we feel that the mathematics of the last decades of the twentieth century has so many aspects that it is impossible—to this author at any rate—to do justice even to the main trends [8, p. 1].

In 1986 the French authors Amy Dahan-Dalmédico and Jeanne Peiffer did not hesitate to title their book **One** *history of mathematics: roads and mazes*. They commented:

> "History," this term takes on two senses at least. Historiography on one side: a narrative according to a chronological order of what happened in such or such domain of human activity. . . . Genesis on the other side: development, persistence, and transformation of the thing itself that the activity concerns [2, p. 7, translated by the author, who also added emphasis to the title].

Figure 3. Dahan-Dalmédico and Peiffer (fifth and sixth from left) among other historians of mathematics at the Oberwolfach Research Center in 1988. Photo by Enid Grattan-Guinness, Oberwolfach Photo Collection.

Finally, we consider the thoughts voiced in 2000 by George Phillips, who is a professor at the University of St Andrews in Scotland. It appears that he selected topics in the areas of mathematics that particularly interest him, but he challenged the still-common student misperception that mathematics has not changed over time:

> This book is intended for those who love mathematics, including undergraduate students of mathematics, more experienced students, and the vast number of *amateurs*, in the literal sense of those who do something for the love of it. . . . It is fascinating, for example, to follow how both Napier and Briggs constructed their logarithms before

many of the most relevant mathematical ideas had been discovered [6, p. v, emphasis in source]

> I have often been asked, "How can one do research in mathematics? Surely it is all known already!" If this is your opinion of mathematics, this book may influence you towards a different view that mathematics was not brought down from Mount Sinai on stone tables by some mathematical Moses, all ready-made and complete. It is the result of the work of a very large number of persons over thousands of years, and with no end in sight [6, p. vii].

If any or all of these quotations have intrigued you, there are many ways to enter further into the communities of history (and philosophy) of mathematics! The Canadian Society for History and Philosophy of Mathematics (CSHPM) was founded 50 years ago, in 1974, in order to promote research and teaching in the history and philosophy of mathematics. We are a sister society to the Canadian Society for the History and Philosophy of Science (CSHPS), the British Society for the History of Mathematics (BSHM), the Canadian Philosophical Association (CPA), and of course the Canadian Mathematical Society (CMS). A meeting is organized annually, and we publish annals of contributed papers as well as a semiannual newsletter. More than a decade's worth of keynote lectures appeared in 2005. Many members have published articles, monographs, and textbooks, including founder Kenneth O. May (bibliography), Duncan Melville (Mesopotamian mathematics), Len Berggren (medieval Islamic mathematics), Glen Van Brummelen (trigonometry), Robert Bradley (Leonhard Euler), Craig Fraser (Hamilton-Jacobi theory), and Israel Kleiner (abstract algebra), to only scratch the surface of the breadth of interests and productivity of our members.

Figure 4. Participants in a joint meeting of CSHPM and BSHM in Dublin, July 2011. CSHPM/SCHPM *Bulletin*.

In France, the Séminaire d'Histoire des Mathématiques de l'Institut Henri Poincaré was founded in 1948. Its objectives are to maintain and develop the links between mathematicians and historians, and to be a place of exchange for historians of mathematics. Regular lectures and symposia are now under the direction of François Lê and Maarten Bullynck. GDR 3398, History of Mathematics, was established in 2011 by the Institut national des sciences mathématiques et de leurs interactions (INSMI) of the Centre national de la recherche scientifique (CNRS) to support research in history of mathematics in a variety of ways, such as the annual European conference and workshop for graduate students, Novembertagung. GDR 3398 also helps support France's journal, *Revue d'Histoire des Mathématiques*, and digital library, Numdam.

Outside of North America, the Instituts de recherche sur l'enseignement des mathématiques (IREM) have been active at producing textbooks and supporting the use of history in teaching mathematics at the lycée level since 1969. Work on interweaving the history of mathematics with the teaching and learning of mathematics is also pursued by the International Study Group on the Relations between History and Pedagogy of Mathematics, or HPM for short, which meets every 4 years as a satellite to the quadrennial International Congress on Mathematical Education (ICME) gatherings. Those eager to join the international community of scholars and educators who explore the effectiveness of using the histories of mathematics to support student learning will be pleased to know that HPM's next conference is this summer, July 1–5, in Sydney, Australia. To discuss research into the histories of mathematics with mathematicians, historians, and philosophers, please join CSHPM June 15–17 at McGill University for the 2024 Congress of the Canadian Federation for the Humanities and Social Sciences.

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