To See the World in a Grain of Sand

Editorial

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Recently I, and some European colleagues, have been spending a lot of time on tetrahedra, and when they fall over.

You might think that there wouldn't be much to say about a tetrahedron. Four vertices, shrinkwrapped. Twelve real numbers specify the whole thing, including its position. If you're not concerned with position or size, five parameters suffice. But... it's amazing the number of rabbit-holes that that five-dimensional moduli space contains, and we're exploring just one of them. I won't say it's the most earth-shattering material any of us has worked on: it isn't. But we're still surprising ourselves with what we find.

Mathematics is notoriously full of questions that are easy to ask and hard to answer. Why that should be is in itself a deep question. Partly, it's because a proper answer to a well-defined mathematical question is just as hard-edged as the question was, so that we can keep following arbitrarily long chains of arguments. Other subjects – based on reality – don't always work that way. At some point not only does the biologist have to deal with the presence of the platypus, but also the absence of many perfectly plausible species, such as the unicorn and the Ogopogo, that would allow interesting conjectures to be tested if they only had the decency to exist. This means that the discoveries of biology are, in general, more important to the taxpayer, – no NSERC grant used up on the case of the spherical massless horse – but perhaps means that the articles in Nature resemble each other more strongly than the articles in the American Mathematical Monthly. We can build weird baroque temples to the imagination without worrying whether they will bear their own weight, precisely because they weigh nothing. Provided the individual joints are rigid, the structure stands.

Is this playing the game on the easy setting? Possibly, but it's a different game. Other sciences can treat a statistical argument based on a sample size of a thousand as effectively conclusive; we treat a conjecture that holds in the first trillion cases as "plausible," knowing that the rigor that lets us keep chaining deductions forever hasn't been satisfied. (And any graduate student should be able to cite a few examples of nontrivial conjectures that do hold for the first trillion, or ten-to-the-trillion, cases and then fail.)

And, as a result, we can find interesting problems in the stability of a tetrahedron, the sum of a pair of prime numbers, or any of a million other sources.

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