CMS NOTES

Mathematical Constitution and Theory Uniqueness in Physics

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Galileo famously suggested that "mathematics is the language in which God has written the universe" [15, p. 5].[1] For Galileo, that language was geometry, and he claimed that the physical universe would be incomprehensible without an understanding of it [6, p. 4]. The crux of Galileo's sentiment holds weight in even the farthest reaches of physical theory. The field of physics, perhaps more than any other scientific discipline, is furnished with laws and theories written in the language of mathematics. From the complex Hilbert space of quantum mechanics to the non-linear partial differential equations that constitute the field equations of Einstein's theory of general relativity, the smallest and largest things in the universe reveal themselves to us in the language of mathematics.

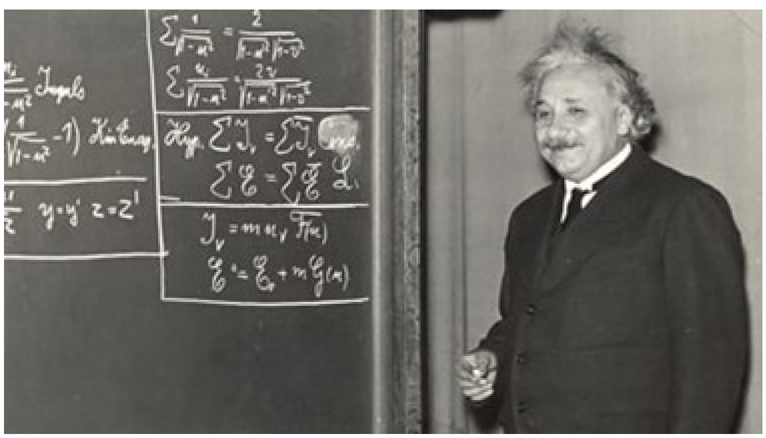

Figure 1. Albert Einstein in Pittsburgh, 1934. Carnegie Mellon University; see also [14].

Physics Nobel Laureate Eugene Wigner remarked in "The Unreasonable Effectiveness of Mathematics in the Natural Sciences" that this intimate relationship between physics and mathematics is both nontrivial and completely baffling. He posed the philosophically rich question of whether the appropriateness of mathematics for describing and predicting physical phenomena is merely coincidental or indicative of a more fundamental relationship between the two [15]. Wigner gestured at two possible explanations for the unreasonable effectiveness of mathematics in natural science, which can be summarized thus:

(i) The mathematical formulation of physical theories is not unique. Their effectiveness is purely coincidental. Alternative mathematical (or non-mathematical) frameworks could support the same, similar, or better results.
(ii)The mathematical formulation of our physical theories is unique. Their effectiveness is explained by the fact that they are the *correct* formulations.

Figure 2. Eugene Wigner, 1963. Nobelprize.org.

Let us call (i) the representability view and (ii) the constitution view. I call (i) the representability view because it implies that mathematics operates in physics by representing the way the world is and describing it formally. Furthermore, the term "representability" has two forceful implications. The first is that mathematics appears to be an astoundingly *suitable* mode for capturing physical facts. (Alternative modes of representation, such as culinary expression or dance, appear less apt for this task.) The second implication is that there exists a degree of interpretation when we represent physical facts mathematically. Our chosen mathematical frameworks are merely one of the many possible interpretations. Analogously, we can think of a garden scene and its many possible representations, such as a sketch, a photograph, or a detailed list of everything present. Each representation is a distinct interpretation of the same object—vastly different in its attributes and yet similar in its functional properties. The representation view asserts that mathematics serves as one of many possible representational vehicles for physics.

I call (ii) the constitution view because it entails a fundamental relationship of constitution between mathematics and physics.[2] According to the constitution view, the same properties, relations, and identities that furnish mathematics are those found in nature. That is to say, when physicists refer to mathematical constructions, such as Hermitian operators or Fock space, they are picking on mathematical constructions that define nature's fundamental structure. To invoke an analogy popular in philosophical literature, Plato famously wrote in *Phaedrus* that "like an animal, the world comes to us predivided. Ideally, our best theories will be those that 'carve nature at its joints'" [12, p. 1]. On some interpretations of the mysterious applicability of mathematics, the constitution view can be construed as echoing Galileo's sentiment that 'the universe is written in the language of mathematics' [6, p. 4].[3] The tapestry of physical reality is woven from a mathematical fabric.

Among philosophers of science, the representation view is the popular choice [e.g., 9; 3; 11]; however, there remain a handful who champion the constitution view. A topical debate among philosophers of physics and mathematics concerning this distinction is that of mathematical explanations. The debate is characterized by the following question:

> How do mathematical frameworks, models, and theories improve our understanding of the physical world if they are not physical themselves?

In addition to being a representational vehicle of the physical, mathematics also seems to explain it. That is to say, physicists rely on mathematical apparatus to facilitate an understanding of what nature is like and why it behaves in the ways it does. For example, in thermal physics, renormalization group methods are a mathematical technique used by physicists to explain the unexpected identical behaviour of fluids and magnets near their critical temperatures.[4] In cases of mathematical explanation, it appears that the math does more than merely represent, it also acts as an epistemic tool for unearthing physical properties and mechanisms that would otherwise be inaccessible. The central question is: In virtue of what mechanism does mathematics provide physical insight?

Proponents of the representation view are prone to defer to a sort of similarity relation between mathematical and physical systems**,** but they tend to be reluctant to admit of any deeper relationship. In other words, mathematics is a "good enough" tool for procuring physical insights. On the other hand, proponents of the constitution view typically assert that what enables the mathematical explanation of physical facts is the realization of mathematical properties in physical systems. To the constitutionalist, the goal of physics is just to reduce natural systems to a form in which salient features and behaviors are highlighted [4]. These features and behaviours are mathematical by constitution. Explanation obtains when this underlying structure is unearthed and made sense of by physicists and applied mathematicians [7, p. 487].

Let us further characterize the mathematical explanation debate by way of another example: Noether's Theorem. Mathematician Emmy Noether discovered an important result used in physics and mathematics about the relationship between symmetries and the conservation of quantities in physical systems. It is relevant to our discussion because it demonstrates how the mathematical representation of a physical system can be leveraged to yield genuinely physical insights. At base, Noether's Theorem says that when the action of a physical system has a differentiable symmetry, there is a conserved quantity in the system. That is, if I have a system that I can represent with a Lagrangian and its Lagrangian contains a symmetry, then the physical system contains a conserved quantity. For instance, suppose a physical system's behaviour is invariant under changes to its orientation in space. Its Lagrangian is then rotationally symmetric. By Noether's Theorem, the physical system's angular momentum is always conserved.

In Noether's Theorem, the Lagrangian is the central mathematical entity that defines physical systems. It should be noted that the theorem is only valid for systems that can be modeled by a Lagrangian alone. What is explanatory about the theorem is that if I want to know what types of systems conserve a quantity Q, I can easily compute the types of Lagrangians for which Q is conserved by looking at the relevant symmetry. Systems that are representable by these Lagrangians conserve Q. Here we see that a property (symmetry) of the mathematical entity (Lagrangian), which defines the physical system, sets criteria for having a specific physical property (conservation of Q).

Figure 3. Emmy Noether, circa 1900. Wikipedia.

At this junction, advocates of the constitution view will want to say that—without committing to the idea that the Lagrangian *defines* the physical system—this relation remains mysterious. Why would characteristics of a mathematical function dictate the conservation laws of nature? Moreover, Noether's Theorem is only valid within the scope of systems that are representable by a Lagrangian alone. In other words, the constitutional mathematical entity dictates a defining characteristic of the physical. This also is reminiscent of Galileo's idea that the universe is written in the language of mathematics.

The aim of this short article has been to provoke thought about the relationship between mathematics and physics. Too often, we take for granted the mysterious and wonderful way in which these two disjoint fields intertwine to deliver knowledge about our elegant universe. Ultimately, it is as important to grapple with these great questions as it is to accept that their answerability is dubious. Nonetheless, they remain worthy of our consideration and central to the way we understand the world. Eugene Wigner captured this sentiment forcefully:

> The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve. We should be grateful for it and hope that it will remain valid in future research and that it will extend, for better or for worse, to our pleasure, even though perhaps also to our bafflement, to wide branches of learning [15, p. 9].

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[1] See [8] for a discussion of the veracity of Galileo's quotation.

[2] The language of this name is inspired by the central argument made in [2].

[3] It is worth noting that a host of alternative interpretations of Galileo's famous words exist. The mathematical constitution of physical facts is one way to explain the uncanny effectiveness of mathematics; however, there are also epistemic, metaphysical, and explanatory means to do so (to name a few!). See, for example, [5]; [9]; and [13].

[4] This example is inspired by Robert Batterman's 2002 monograph *The Devil in the Details* [1], as well as Collin Rice's *Leveraging Distortions: Explanation, Idealization, and Universality in Science* [10]. See both for excellent expositions.

Tessa Ng is a masters student at the University of Toronto studying philosophy of science. Her main research interests are in the philosophy of physics, as well as mathematics and the history of both subjects.