The TRIUMPHS Society: TRansforming Instruction: Understanding Mathematics via Primary Historical Sources

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CSHPM Notes brings scholarly work on the history and philosophy of mathematics to the broader mathematics community. Authors are members of the Canadian Society for History and Philosophy of Mathematics (CSHPM). Comments and suggestions are welcome; they may be directed to the column's editors:
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Learning a discipline by reading the original writings of the field's most important and innovative thinkers is a long and honored tradition in the humanities and social sciences. What better way to grapple with postmodern historiography than by reading Michel Foucault? What would one read other than the *Dialogues* to become familiar with the ideas of Plato? Why would someone even try to learn about Shakespeare's plays and sonnets without reading them oneself?

But what about learning mathematics? Can you imagine asking your students to read Cantor in an introductory topology course? What about reading Cauchy in a course on linear algebra? And how about reading Gauss in a second semester calculus course? Oh my! And yet this is precisely what the members of a small but growing community of mathematics instructors in North America have not only been proposing *should* be done for several decades now, but are actually doing themselves, using classroom modules known as Primary Source Projects (PSPs) [1].

Based on a guided reading approach to primary sources, PSPs are intended to replace the standard textbook treatment of core topics in the undergraduate mathematics curriculum, and to do so in about the same amount of time as a typical textbook-lecture approach to teaching a given topic [2]. Each project melds excerpts from primary sources with carefully-crafted tasks that interrupt students at specific points in their reading of the source material. This design lends itself especially well to student-centered instructional strategies (e.g., small-group discussions) as an alternative to lectures. PSPs also include commentary on the historical context and mathematical significance of the questions that the historical author set out to explore.

It is known that the quantity $a^n + 1$ always has divisors whenever n is an odd number or is divisible by an odd number aside from unity. Namely $a^{2m+1} + 1$ can be divided by $a + 1$ and $a^{p(2m+1)} + 1$ by $a^p + 1$, for whatever number is substituted in place of a. But on the other hand, if n is a number which is divisible by no odd number aside from unity, which happens when n is a power of two, no divisor of the number $a^n + 1$ can be assigned. So if there are prime numbers of this form $a^n + 1$, they must all necessarily be included in the form $a^{2^m} + 1$. But it cannot however be concluded from this that $a^{2^m} + 1$ always exhibits a prime number for any a; for it is clear first that if a is an odd number, this form will have the divisor 2.

Figure 1. Primary source excerpt from Euler's 1738 article "Observationes de theoremate quodam Fermatiano aliisque ad numeros primos spectantibus" ("Observations on a theorem of Fermat and others concerned with prime numbers"), featured in the 2017 TRIUMPHS Primary Source Project "Primes, Divisibility, and Factoring," by Dominic Klyve. Digital Commons at Ursinus.

Since 2005, the development and classroom testing of PSPs has been supported by three grants from the US National Science Foundation, the latest of which was *TRansforming Instruction in Undergraduate Mathematics via Primary Historical Sources* (*TRIUMPHS*). As a result of TRIUMPHS' efforts, approximately 100 PSPs were developed for use in university mathematics courses ranging from beginning algebra to topology, all of which are freely available for download and classroom use at https://blogs.ursinus.edu/triumphs/. Through these projects, students are given the opportunity, for example, to study the derivatives of the sine and cosine functions directly from Euler, uniform convergence of function series from the writings of Abel, and ideals directly from the words of Dedekind. Classroom testing of these and other PSPs by over 120 university mathematics instructors in 200+ classrooms at a wide variety of institutions across the US and Canada also took place under the auspices of the TRIUMPHS grant [3].

Of course, teaching with primary sources is not without its challenges . . . but neither is it without its benefits, for students and instructors alike. For instance, here are just a few of the

benefits of using PSPs that instructors who served as site-testers under the TRIUMPHS grant reported:

- Certainly students were more likely to discuss their own difficulties in interpreting problems from the PSP than they are in discussing difficulties on the other problems in our class. (Since it was acknowledged that these were "different" problems.) So hearing the students' concerns and difficulties was beneficial.
- I think the students also learned some virtues of persistence, since there was more "groping around in the dark," and they didn't believe that I had the absolute right answer ready to deploy. (As opposed to the course notes, which I wrote myself.)
- Many students remarked that they really liked reading from a primary source (as opposed to the textbook, I presume) because it made the math seem more humane. I interpreted this as evidence that reading the primary source helped students begin to break down their static views of mathematics.
- As an instructor, I experienced that students can do more than what we think. It's hard for Calculus I students to read and understand math articles, but they did it!
- Both the students and I gained a deeper understanding and appreciation of the historical development of basic ring theory. The students also gained a deeper understanding of the concepts.
- I didn't anticipate it, but implementation of the PSP led to several conversations about how mathematics is developed, what research mathematicians do, and going on to do graduate studies in mathematics.
- It gave both me and my students a chance to pause amidst the early onslaught of analysis definitions and theorems to discover what motivated them historically. This course can be relentless in its pace, and this time spent, however brief, on background was enlightening to my students.

Figure 2. Founding members of the TRIUMPHS Society. Back row (L to R): Adam E. Parker, Michael P. Saclolo, Kathleen M. Clark, Mark Watford, Kenneth M Monks. Front row (L to R): Daniel E. Otero, Dominic Klyve, Janet Heine Barnett. TRIUMPHS Society website.

Building on the success of TRIUMPHS and its predecessor grants, the recently-formed TRIUMPHS Society aims to:

- bring together practitioners and others interested in the use of primary historical sources in the teaching and learning of mathematics;
- encourage and support the development and use of classroom resources based on primary historical sources;
- share teaching experiences and publicize research based on the implementation of such resources; and
- promote the proliferation of primary source-based pedagogy in mathematics through conversation and professional development.

Society activities to date have included workshops and other presentations at annual MAA MathFest meetings (see, for instance, the materials from our 2024 workshop "On the Shoulders of Giants"), as well as the International 2024 History and Pedagogy of Mathematics conference in Sydney, Australia (sponsored in part by the Society). Through its peer-reviewed journal, *The Annals of the TRIUMPHS Society*—edited by Michael Saclolo (St. Edwards University) and Kenneth M Monks (College of Southern Nevada)—the Society further intends to publish PSPs and similar classroom-ready materials designed to teach specific mathematical topics by engaging students with excerpts from primary sources, artifacts related to the development of such projects, and articles on scholarship related to the use of such materials. If a topic is related to teaching and learning mathematics with primary sources, then it is potentially of interest to the journal!

So, how can you become part of this exciting movement? Find out more about teaching and learning with PSPs by joining us at our next workshop, "Engaging and inspiring students in the mathematics classroom by teaching with Primary Source Projects," to take place during MathFest 2025 in Sacramento, CA (August 6–9). And for more information about the new TRIUMPHS Society and journal, including how you can become a member for just US$12 annually, visit https://triumphssociety.org.

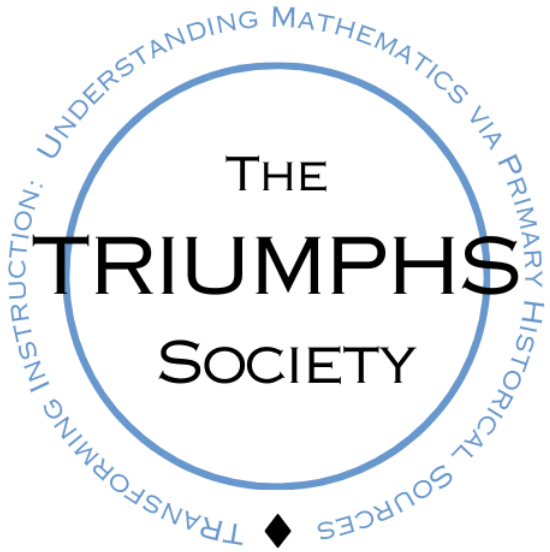

Figure 3. TRIUMPHS Society logo. TRIUMPHS Society website.

Notes

[1] Calls to incorporate original (primary) source readings into mathematics education have occurred on the international level for some time as well; see, for instance, Jahnke, H. N., A. Arcavi, E. Barbin, O. Bekken, F. Furinghetti, A. El Idrissi, C. M. S. da Silva, and C. Weeks. (2000) The use of original sources in the mathematics classroom. In *History in mathematics education: The ICMI study*, edited by J. Fauvel & J. van Maanen, 292–328. New York: Kluwer Academic.

[2] The genesis of this approach is due largely to Toronto-born David Pengelley (New Mexico State University, emeritus) and described in Barnett, J. H., J. Lodder, and D. Pengelley. (2014) The pedagogy of primary historical sources in mathematics: Classroom practice meets theoretical frameworks. *Science & Education* 23, 7–27. See also Jankvist, U. T. (2014) On the use of primary sources in the teaching and learning of mathematics. In *International handbook of research in history, philosophy and science teaching*, edited by M. R. Matthews, 873–908, on pp. 887–888. Dordrecht: Springer. Additional justification and background is available in Barnett, J. H. (2018, December) Why Use Primary Sources in a Mathematics Classroom? *Notes of the Canadian Mathematical Society* 50(6), 16–17.

[3] Additionally, the seven-institution TRIUMPHS grant (2015–2023) researched the impact of PSPs on students and instructors. For a report of some of the key results related to TRIUMPHS research goals, see Clark, K. M., C. Can, J. H. Barnett, M. Watford, and O. M. Rubis. (2022). Tales of Research Initiatives on University-level Mathematics and Primary Historical Sources. Special Issue on Exploring the Significance of the History of Mathematics in Mathematics Education: Recent Developments in the Field. *ZDM—Mathematics Education* 4, 1507–1520.

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