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Is AI after our jobs as research mathematicians?

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Abstract. Can ChatGPT, available to ordinary users, contribute to mathematical research? I describe an experiment in which I asked it to find a shorter proof of a theorem on finite ordered sets.I had just found such a proof myself.

Given the relevant papers and a brief prompt, it produced a short proof in about five minutes, using an approach entirely different from mine. The experience made the question in the title rather personal.

First they came for the socialists, and I did not speak out—because I was not a socialist.
Then they came for the trade unionists, and I did not speak out—because I was not a trade unionist.
Then they came for the Jews, and I did not speak out—because I was not a Jew.
Then they came for me, and there was no one left to speak for me.

Martin Niemöller

Luddites of the world, unite. After Karl Marx

1. Introduction

In two recent editorials, Robert Dawson [1, 2] gives an interesting narrative of what happened when OpenAI decided to spend millions of dollars solving a whole bunch of old mathematical problems.It hit pretty close to home.

Let me explain. In 1963, the following result was published [12]: Let L be an algebraic lattice. Then there exists an algebra A such that the congruence lattice of A is isomorphic to L. Soon thereafter, this result became known as the Grätzer-Schmidt Theorem. At about the same time, the term "finite Grätzer-Schmidt Theorem" was introduced. This was not a theorem, but an open problem: If L is finite, can A be chosen to be finite?

2020 Mathematics Subject Classification. Primary 06A07; Secondary 06B05. Key words and phrases. Artificial intelligence, ChatGPT, mathematical research, ordered sets, order-preserving maps. And the expenditure of millions of dollars resulted in a negative answer to this question. But this article is not about OpenAI's expenditure of millions of dollars solving old mathematical problems. It is about the ChatGPT that is available to you and me.

2. Customer Support

Most of us first experienced AI replacing people in customer support. Most of it was not very good, and we had to shout, "I want a live agent!" many times before we were connected to somebody who could answer our questions. But this is changing. A few weeks ago, I was very anxious to try out some new features of macOS 27, so I installed a beta version. Unfortunately, it was so buggy that I had to go back to macOS 26. This is not easy.

You can easily upgrade an operating system on a Mac, but downgrading it is very complicated. I called up Apple and got the AI customer support, which held my hand through the difficult and, complicated procedure with complete success. First they came for the customer support specialists, and I did not speak out—because I was not a customer support specialist.

3. Programmers

A few months ago, there were several newspaper articles explaining why new graduates with computer science degrees cannot get jobs as program-mers: AI programs better. Then they came for the programmers,and I did not speak out because I am not a programmer.

4. Research mathematicians

After I uploaded an article to arXiv, it warned me that I was allowed only two articles per month; there were too many submissions. We have all heard from mathematics journals how they are inundated with AI-written articles.
But how do we measure the effectiveness of the AI available to you and me in writing mathematical articles? I was given a unique opportunity to test this.Let me set the stage. For the ordered sets A and B, let AB denote the or-dered set of all order-preserving maps from B to A, ordered componentwise.

In 1978, Dwight Duffus proposed the following problem in his Ph.D. the-sis [3]: Let A and B be finite nonempty ordered sets. If AA is isomorphic to BB , does it follow that A is isomorphic to B? Equivalently, can we recover A from AA?
The References include publications on this topic from the late 1970s and the 1980s. J. D. Farley reignited interest in the problem in the early 2020s. This problem has recently been solved by J. D. Farley and G. Grätzer [9]. The proof is fairly long and it utilizes deep results of J. Hashimoto [13], L. Lovász [14], and R. McKenzie [15]. It was a logical next step to look for a

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build, so height(XY) = |Y | height(X).

Call an interval [a, b] of X of maximum height if its height is height(X), and let MaxInt(X) be the set of these intervals. Intervals are bounded, and bounded ordered sets are exactly the ones whose powers are tame.Let Y1, . . . , Yr be the components of Y . If [f, g] is a maximum-height interval of XY , the count above forces each [f (y), g(y)] to have maximum height in X, so f (y) is minimal and g(y) is maximal in X; comparable minimal elements coincide, so f and g are constant on each component, and [f, g] ∼= IY1 1 × · · · × IYrr , Ii ∈ MaxInt(X). Conversely every choice of the Ii arises, and the correspondence is a bijection. For connected A this says that the maximum-height intervals of E = AA are the powers LA with L ∈ MaxInt(A): the exponent A reappears, attached to a bounded base. Two difficulties remain, and the proofs diverge exactly here. (i) There are many maximum-height intervals; for disconnected A they are mixed products. What well-defined object does E yield? (ii) One needs a logarithm with L(LA) ∼= A × L(L) for bounded L, and then a way to divide by the unknown L(L).

5.2. The long proof. Logarithm. Farley and Grätzer build it from structure theory. For bounded P every map in P Y is joined to a constant map, soP Y equals the suborder Con(P, Y)

of components containing maps constant. on each component of Y . McKenzie's normal form writes every connected directly indecomposable D uniquely as D ∼= Con(ED, FD) with ED absolutely indecomposable and FD connected; a power by connected Y keeps ED and replaces FD by FD × Y . With Hashimoto's factorization P ∼= D1 × · · · × Dk, put Tow(P) = FD1 + · · · + FDk . Then Tow(P Q) ∼= Tow P + Tow Q, Tow(P Y) ∼= Y × Tow P (P bounded), and height(Tow P) < height P for nontrivial P . This is a genuine logarithm on all connected ordered sets, resting on two deep theorems. One interval. For (i) the long proof selects a single interval. It constructs, in three lexicographic steps (exponent vectors of Hashimoto factorizations, then normal-form pairs, then multiplicity functions), a linear ordering ≤ of the isomorphism types of connected ordered sets with P < Q ⇒ P × R < Q × R and P Y < QY . Then Max(X), the ≤-greatest element of MaxInt(X), is well defined and satisfies Max(XY) ∼= Max X × Max Y and Max(XY) ∼= Max(X)Y . Reconstruction. With L = Max A and R = Tow L, the rules give, from E alone, U = Tow(Max E) ∼= R × A, V = Max U ∼= Max R × L, W = Max(Tow V) ∼= Max R; the last step uses the height drop to see which summand of Tow V ∼= Tow(Max R) + R carries the top intervals. Cancelling W in V gives L, hence R; cancelling R in U gives A. Cancellation is again Hashimoto: encode an or- dered set as a polynomial in indeterminates indexed by indecomposables and cancel in the polynomial ring. Iterating the same operations handles towers such as (AA)A and A(AA), and every bracketing of at most four copies of A.

5.3. The elementary replacements. Both short proofs keep the interval lemma and discard the rest. Logarithm from lower covers. Let J(B) be the elements with exactly one lower cover.

For bounded B, the elements of BY with a unique lower cover are the maps equal to some j ∈ J(B) on a principal up-set ↑ y and to oB elsewhere: lowering a map at a minimal point of its support yields a lower cover, so uniqueness pins down one point, one value, and no variation above. Hence J(BY) ∼= J(B) × Y ∂ ; and since a cover in a product moves one coordinate along a cover, J(BC) ∼= JB + JC. With L(B) = J(B)∂ this is exactly the three-line behaviour of Tow, now for bounded B only, including the height drop, and it takes a page to prove. All intervals instead of one. The interval lemma is a bijection MaxInt(XY) ↔ MaxInt(X)r, and the top intervals of XY are the products I × J. Any symmetric function of all the intervals therefore inherits product and power rules, with no ordering of isomorphism types. Cancellation by counting. For connected Z let hom(Z, X) be the number of order-preserving maps Z → X; it is additive on sums, multiplicative. on products, and positive. Lovász (1967): if hom(Z, X) = hom(Z, Y) for all connected Z, then X ∼= Y ; the ordered-set case has a short inclusion– exclusion proof. So XT ∼= Y T implies X ∼= Y : apply hom(Z, –) and divide by hom(Z, T) > o.

5.4. First short proof: averaging. Let R+ be the nonnegative rational combinations of connected ordered sets, multiplied via direct product, an ordered set being the sum of its components. For X of positive height put Av(X) = 1 | MaxInt X| X I∈MaxInt X L(I), and extend to R+ by averaging over the top-height terms. The interval lemma and the L rules give Av(RS) = Av R+Av S and Av(XY) = Y ·Av X, and the height drop gives height(Av R) < height R. With R = Av A and S = Av B, A · R = Av(AA) = Av(BB) = B · S, hence R + Av R = S + Av S. Since Av R lies strictly below the top height of R, the top part of R+Av R is that of R; averaging it returns Av R, and subtracting returns R. So R = S, and cancelling R in AR = BR by counting gives A ∼= B. The unknown logarithm is never computed; it identifies itself through its height drop and is divided out.

5.5. Second short proof: three invariants. To stay with integers, let M(X) be the disjoint sum of a copy of each maximum-height interval of X; then M(XY) ∼= MX MY and M(XY) ∼= M(X)Y . Put P = MA, with m components, and W = M(JP), with k components. From E = AA alone form U = J(ME), V = MU , Q = M(J ∗ V), with J ∗ counting upper covers. Following J through a power with disconnected base gives U ∼= µ(JP × A∂), V ∼= µ(W × P ∂), Q ∼= µk W ∂ , µ = mc–1, c the number of components of A; the height drop again isolates the sum- mand of J ∗V carrying the top intervals, and height A = height U – height Q. Now count with a fixed connected Z, once for A and once for B. The counts for Q and V divide to give kB hom(Z∂ , PA) = kA hom(Z∂ , PB), hence kB PA ∼= kAPB and kB JPA ∼= kAJPB . Inserting this in the count f or U , cancelling positive numbers, and using |A| = |B| (equal heights and |A| height A = |B| height B) leaves hom(Z, A∂) = hom(Z, B∂) for every con- nected Z. Lovász's theorem finishes.

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